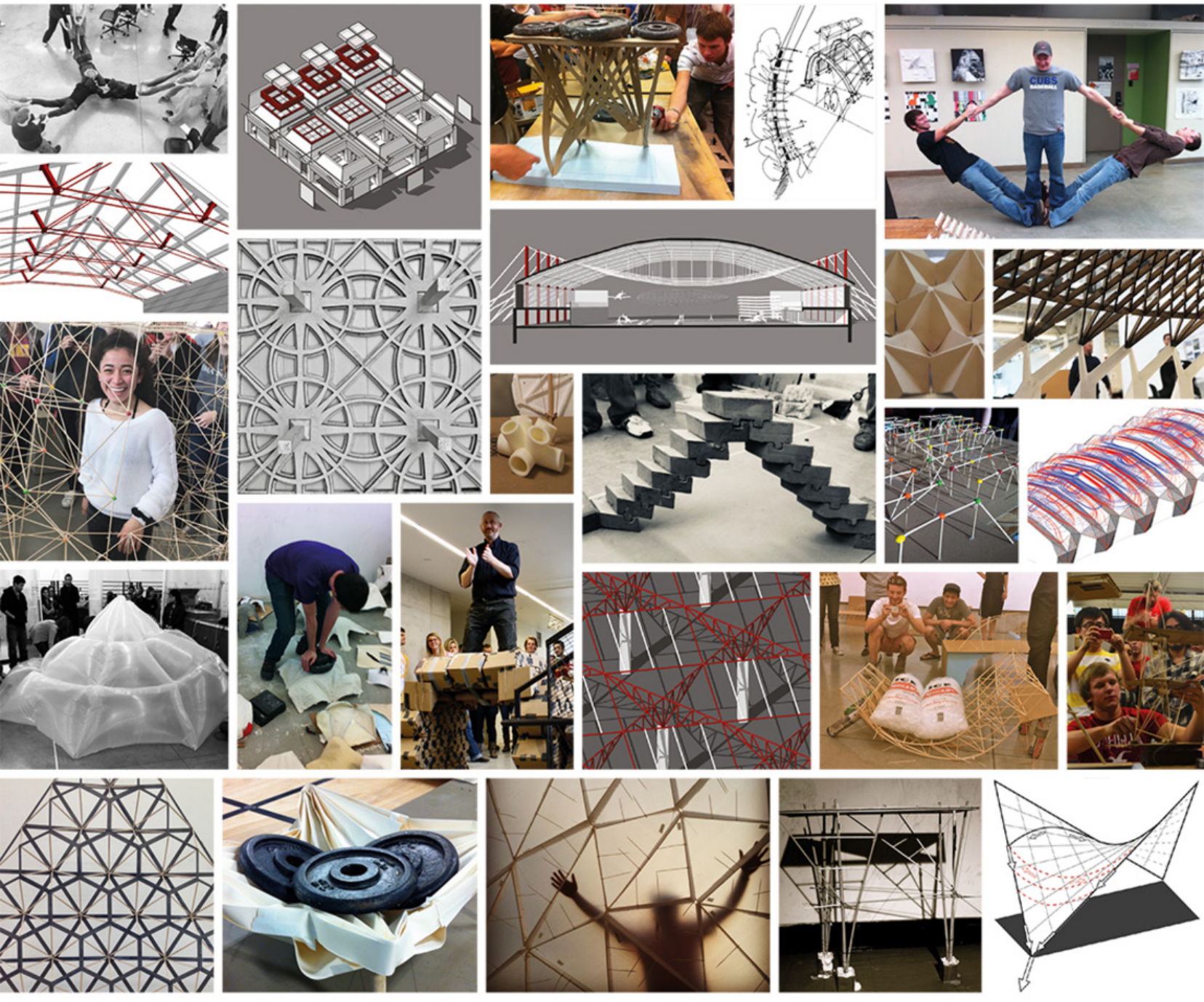
STRUCTURES BY DESIGN

THINKING, MAKING, BREAKING



ROB WHITEHEAD

ROUTLEDGE

Structures by Design

Structures by Design: Thinking, Making, Breaking is a new type of structures textbook for architects who prefer to learn using the hands-on, creative problem-solving techniques typically found in a design studio. Instead of presenting structures as abstract concepts defined by formulas and diagrams, this book uses a project-based approach to demonstrate how a range of efficient, effective, and expressive architectural solutions can be generated, tested, and revised.

Each section of the book is focused on a particular manner by which structural resistance is provided: Form (Arches and Cables), Sections (Beams, Slabs, and Columns), Vectors (Trusses and Space Frames), Surfaces (Shells and Plates), and Frames (Connections and High-Rises).

The design exercises featured in each chapter use the Think, Make, Break method of reiterative design to develop and evaluate different structural options. A variety of structural design tools will be used, including the human body, physical models, historical precedents, static diagrams, traditional formulae, and advanced digital analysis.

The book can be incorporated into various course curricula and studio exercises because of the flexibility of the format and range of expertise required for these explorations. More than 500 original illustrations and photos provide example solutions and inspiration for further design exploration.

Rob Whitehead, AIA, LEED AP is an architect and Associate Professor of Architecture at Iowa State University where he teaches structural design, design-build, and integrated design studios. He was named one of 30 Most Admired Educators by Design Intelligence (2014). He received the Iowa Educator Award (2014) by the Iowa Chapter of the American Institute of Architects (AIA), and the Emerging Faculty Award (2011) by the Building Technology Educators' Society (BTES). His structural design course was awarded the Association of Collegiate Schools of Architecture (ACSA) Creative Achievement Award. He is the co-author of *Design-Tech: Building Technology for Architects* (2014) and currently serves as the Wells Concrete Construction Research Fellow.

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Structures by Design

Thinking, Making, Breaking

Rob Whitehead

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Resist the forces that would undo the beautiful and meaningful things in this world.

Be diligent. Be creative. Make something that matters.

This book is dedicated my beloved ones: Theo, Sophia, and Kelly

And to Jayme: You would have liked this book.

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Preface

Starting with Structures

This book proudly begins with a confession of bias. I believe that:

- Part of the richness of great architectural design comes from a creative, and scrupulous attention to structural principles.
- Learning to appreciate and understand structural principles will enrich a designer's ability to generate thoughtful, creative, and technical solutions to vexing problems.
- Structural rigor should become an integral part of any creative design process, and that structures should be taught as a design discipline in conjunction with comprehensive architectural education.

When in doubt, start with structures.

Like many young architectural students, the first time I recognized the potential impact of integrating structural principles into building designs was not in a structures classroom, but in a history book. Across the centuries, as buildings grew larger and taller, they developed a creative defiance of gravity. Building materials and forms changed. They became lighter, more structurally expressive, and elemental. When these architectural forms evolved in conjunction with building sciences and practices the technical, functional, and creative aspects of the design became inextricably linked.

This revelation became a lamentation when I realized that the richness found in these convergent solutions seemed largely absent from the traditional structural education of architects. If structural design principles can substantially influence architectural design *qualitatively*, then why would structural design courses for architects be so focused on *quantitative* analysis? The answer, perhaps, lies in misconceptions about the process of learning.

Effectively teaching structural design to architecture students is an important but complicated endeavor. In practice, structural engineering is very design-oriented with a focus on problem solving. This ability to solve problems relies on detailed knowledge of math, physics, material science, and construction methodologies. The engineer Christian Menn described structural design, "falling between two opposite poles: natural science and art. Natural science is static, exact, and indisputable . . . in contrast, art is dynamic, open, creative, and disputable."

Historically, in education, technological knowledge has been taught as a prelude to design experimentation. Frequently the engineering-based information comes first—and never leaves. As a result, traditional structures classes focus on sizing and selecting discrete structural elements. Competency is based on finding the "right" answer with little effort aimed at creating and improving design and performance. At its core, it erroneously treats design and technologically-based problem-solving as separate endeavors.

When structural design is presented primarily as a search for answers that are either right or wrong, students may mistakenly develop an adversity to experiment or innovate. This habit leaves them unprepared to deal with the interactive and synergetic nature of design practice, in which their ability to critically integrate structures into their designs is an important responsibility. Separating structural design from the creative act of design, or assuming that technically-based assessments of a structural design proposals aren't creative, are wrongful assumptions that I reject. I hope you will as well.

Think, Make, and Break + Repeat

This book was written to offer an alternative approach to the structural design education of architects—one that presents learning opportunities not as a choice between design and technology, but a confluence of them. It's based on a simple idea that structural design is a creative act and therefore, the process by which structures is taught and learned should be more akin to a design studio. The central narrative of the book is

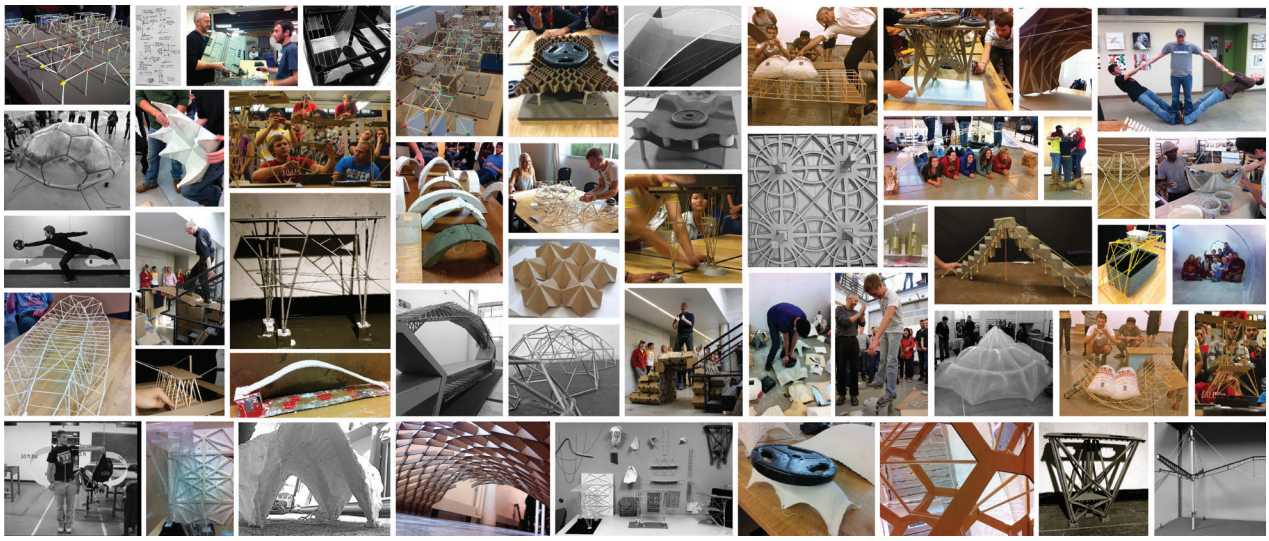


Figure P.1 Student experiments from Structures by Design (SxD) at Iowa State University's Department of Architecture

based on the courses taught in Iowa State University's Structures by Design (SxD) modules. This "think, make, and break + repeat" format encourages students to take a creative and evaluative approach to design—one that puts experimentation and active learning strategies at the forefront of the design process. (Figure P.1)

> *Thinking requires purpose. Making reveals developmental intent. Breaking is a search for answers and improvements*. Design isn't linear, so thinking, making, and breaking is often repeated cyclically.

Using the Book

This book's goal is to teach structural design to architects in a manner that develops intuitive understanding about how forces, materials, and structural behavior correspond with a range of architectural forms. Effective, efficient, and expressive structures optimize these relationships into a responsive proposal. This relies on three intra-related domains of knowledge and the requisite skills to creatively apply them:

> *The ability to see and feel the oft-hidden behaviors of structures under loading.*
>
> *The ability to apply these structural lessons towards proposals in creative and appropriate ways.*
>
> *The ability to critically evaluate the behaviors of these proposals with a desire to "hack" or improve its qualities.*

Ultimately, the goal is to develop fast-thinking and fluid problem-solving design procedures through which designers can generate and assess by qualitative and quantitative consequences of their design decisions—an effective feedback loop balancing the creative and technical.

Although a textbook is inherently a static format, its use need not be—these lessons are best applied in active learning environments like labs. Readers are encouraged to generate their own designs for the problems introduced in each chapter (ideally with different options than the ones presented). This textbook is intended for a flipped (or otherwise participatory) course format. To facilitate in-class discussions, each chapter provides questions or additional exercises to complete. These are deeper dives into each topic and are classified as Thinking, Making, or Breaking types of activities. The book's narrative assumes a fundamental ability to apply certain architectural and mathematical knowledge as part of design development—skills that will evolve over time as additional sources of information and tools are recruited into the learning effort.

Architects don't need to follow the same process of design as engineers to share their desire for effective and efficient solutions. If we understand the relationship between architectural form and structural behavior, our work can develop collaboratively in accordance with structural principles, not against, or in spite of them.

But as I said, I'm biased.

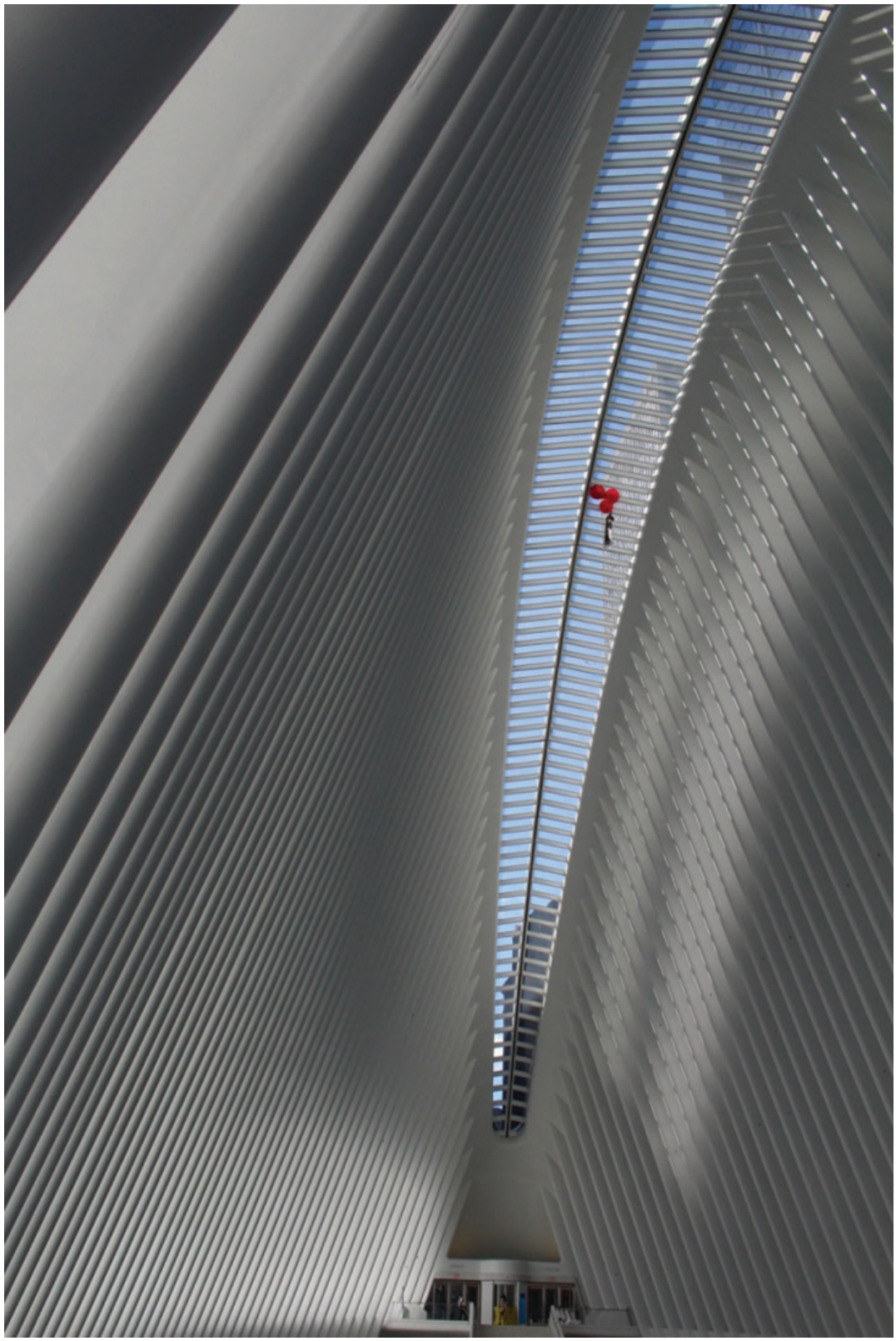


Figure A.0 The Oculus PATH Station, New York City

Acknowledgments

My first thank you is to the students from the Structures by Design (SxD) courses at Iowa State University's Department of Architecture. I'm indebted to the industrious and creative efforts of those who've engaged in this work with such enthusiasm and patience. I've learned a great deal from you all and I have high hopes for the future you are helping to create.

This course sequence evolved from the progressive "Sci-Tech" graduate building technology curriculum at ISU developed by Profs. Tom Leslie and Jason Alread more than a decade ago. Co-teaching with these extraordinary educators and eventually being asked to write most of the Structural Design section for the 2nd edition of *Design-Tech: Building Science for Architects* was a formative moment in my development.

The idea for writing this book was hatched during a Building Technology Educator's Society (BTES) conference as fellow structural educators lamented the lack of resources for design-based learning exercises in structures. Prof. Jeana Ripple suggested that perhaps *I* was the one who should write such a book. She's persuasive. Many other colleagues and friends have willingly offered creative and critical insight into structural design, architecture, and instruction as well, including: Profs. Ed Allen, Marci Uilhein, Erin Carraher, Tyler Sprague, Shelby Doyle, and Kevin Dong. A special thanks to Nathan McKewon, M.J. Johnson, and Mohamed Ismail for their help in creating some of the illustrations and for giving me honest insights into how students prefer to learn. I appreciate you all.

The idea of writing a book is considerably easier than the challenges of actually doing so. The process can be fraught with self-doubt, dead-ends, hilariously awful sentences, and flourishing moments of elation. During these ups and downs, Prof. Tom Leslie has been my touchstone, mentor, and friend. He taught me to stop speculating and start writing. He reminded me that writing isn't what's important—re-writing is. He always encourages me to be concise, so I'll simply say: Thanks, Tom.

A special thanks to the person who changed my life several times: Cal Lewis, FAIA. He was a great man who demonstrated that designing with kindness and integrity produced great buildings and close personal bonds. You'll be missed, but I promise your legacy will live on through those who were fortunate enough to know you.

Finally, none of this could have happened without the tireless love and support of my dear wife, Kelly Roberson, and my incredible children, Theo and Sophia. Few people are lucky enough to be supported as I was—despite the long hours, distracted conversations, the miscellaneous ramblings scrawled on our chalkboard, and the scraps of paper strewn about the house. The hardest part of writing the book was the emotional toll of missing you. I love you with all my heart.

Figure 0.0.0 Kevin Roche inside the TWA Terminal model (Eero Saarinen, 1958)

CHAPTER 0.0

Introductory Endeavors

The Case for Making and Breaking

The process of designing structures should be aspirational and creative. Well-designed structures give integrity to architectural forms, improve a building's performance, strengthen experiences, and amplify the beauty of order. Architectural qualities improve because of structural encumbrances, not in spite of them.

The Challenges of Structural Design

Structure is the primary form-defining, volumetric, and expressive element in architecture—as such it is inextricable from design.

Structural design consists of simple and essential physical challenges: *Stacking and spanning structural components in an efficient and stable manner.* There are many potential solutions that require more than just math and science—they need designers' critical perspectives. Ove Arup described engineering as "a creative activity, involving imagination, intuition, and deliberate choice." Although often maligned as an impartial search for right and wrong answers to architectural problems, structural design is more subjective and open-ended. And that's good thing.

Structural design is the artful resolution of technical constraints into effective, efficient, and expressive buildings. Like other design disciplines, it relies upon insight and acumen to find creative solutions to architectural problems. Unfortunately, at the beginning of projects—when we have the greatest freedom to apply structural influence to architectural design—we are often equipped with the least amount of technical knowledge. (Figure 0.0.1)

Responsive designers learn to synthesize quantitative and qualitative aspects of architecture, structure, and construction together using critical thinking and integrated problem-solving early into the project. Creativity defies the linearity of analytical thinking; it demands an iterative and exploratory process of developing, testing, and redesigning. It is guided by a creative vision, an enthusiasm for the efforts, *and lots of practice and patience.* (Figure 0.0.2)

Earnest, Industrious, and Creative Efforts

> *All truths are easy to understand once they are discovered; the point is to discover them.*
>
> (Galileo Galilei)

The pre-requisite condition of structural design, specifically the need to transmit and resist forces through a physical form, creates complications if this "ideal" structural form isn't also aligned with architectural

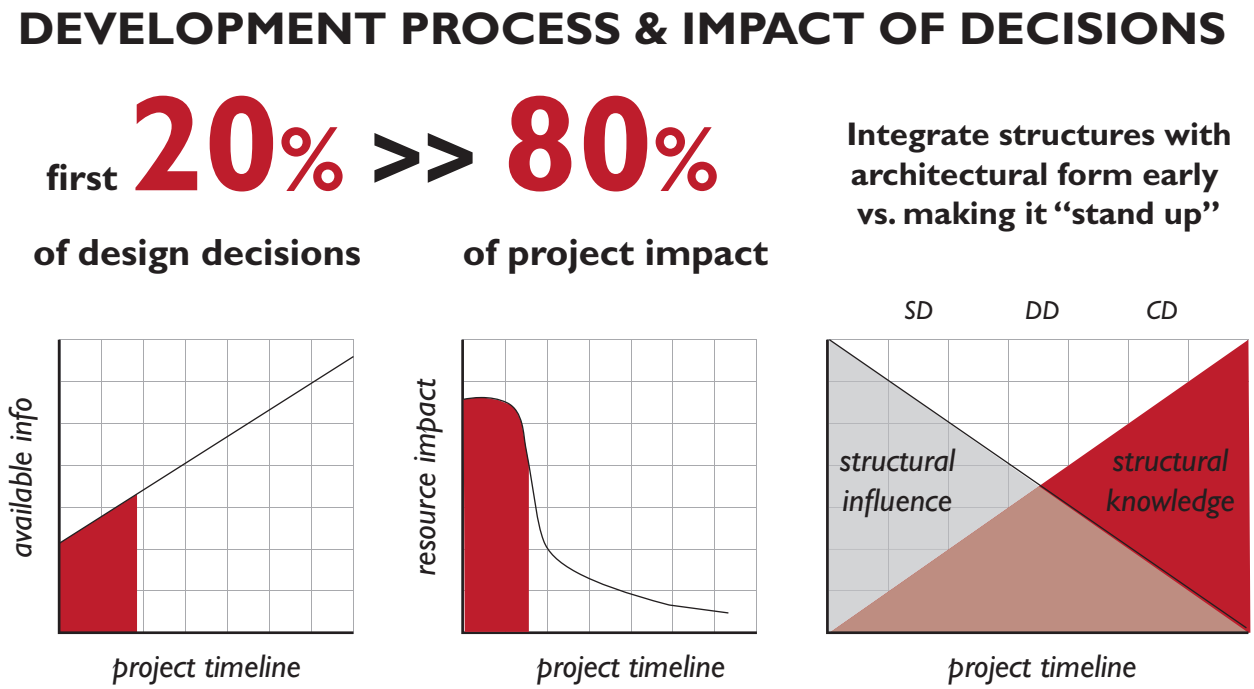


Figure 0.0.1 The importance and impact of early integration of structural knowledge

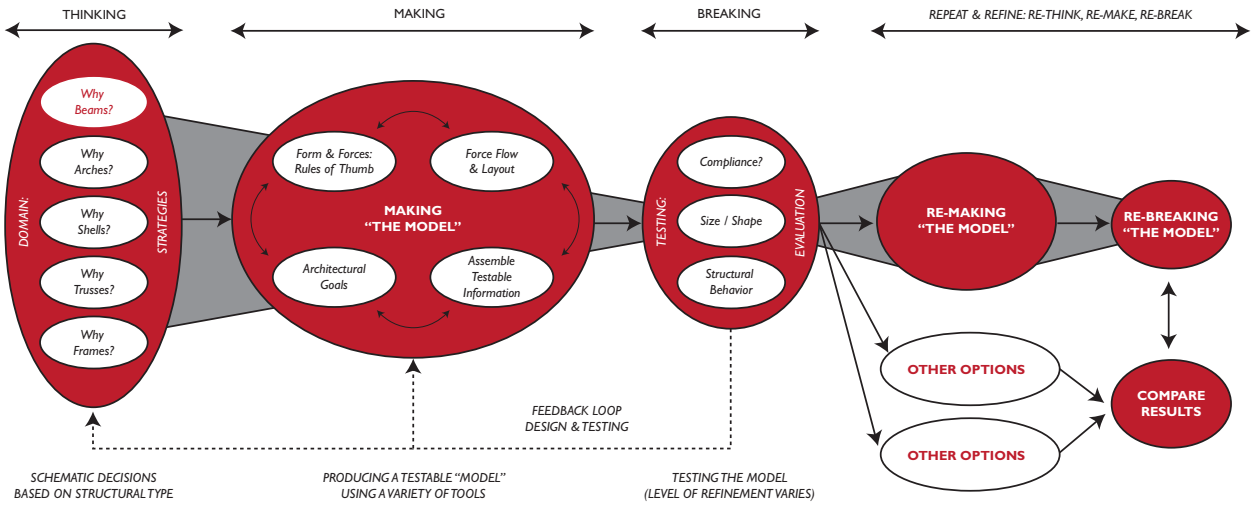


Figure 0.0.2 Think, Make, and Break, + Repeat flow chart

intentions. Or vice-versa. Getting the form right makes everything else easier.

Avoiding these conflicts is easier said than done, and sometimes even experts struggle to harmonize creative and technical aspects. For example, the widely-heralded Sydney Opera House (1957–1973) was in fact cumbersome to design, develop, and build because of the discrepancies between its competition-winning architectural idea and the feasibility of structuring and constructing its sail-like forms.

The Opera House's designer, Jørn Utzon, hadn't conferred with an engineer before submitting (and winning) the proposal and the project engineers, Arup & Associates determined that the project couldn't be built as designed. The basic problem was that the roof's form didn't transfer loads properly. Although the roofs looked like shells, they acted like curving beams and wouldn't work as designed. For six years, while the project's foundations were already under construction, Arup's office explored *eleven different structural schemes* for the roofs using traditional and geometrically-descriptive drawings, scale testing models to determine structural behavior and force distribution, and computer-aided calculations using programs written by the project engineers.

Figure 0.0.3a Exterior view of Sydney Opera House.

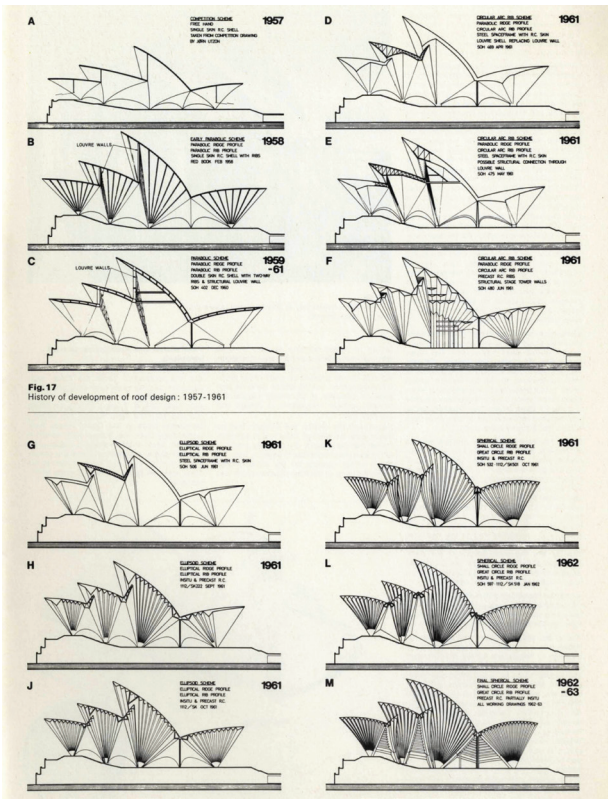


Figure 0.0.3b Various roof schemes explored by Arup's office (1957–1961)

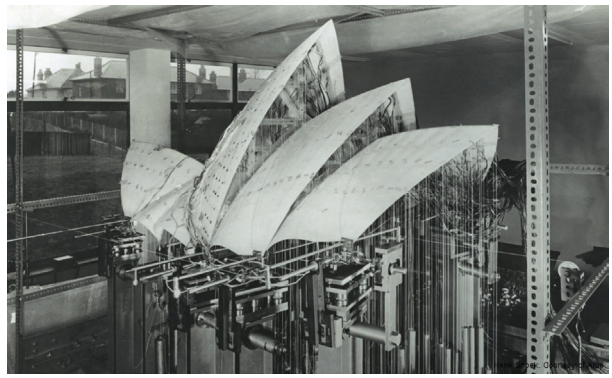


Figure 0.0.3c Testing of a large-scale model to confirm structural calculations

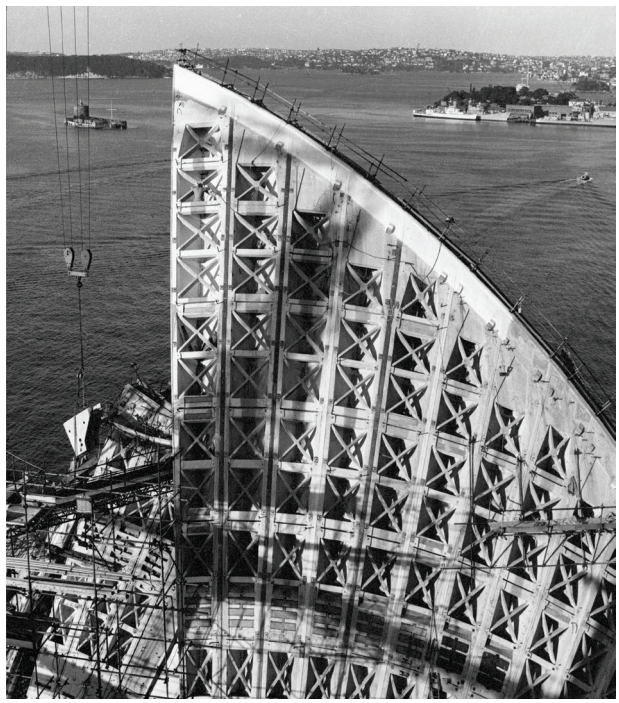


Figure 0.0.3d Arrayed precast structural ribs under construction

Eventually, these explorations revealed a system by which each curve could be assigned to the same "great circle" and the roofs could be built from a repeated fan-like array of 2,400 precast concrete arches and ribs. The innovative design work continued through construction (after Utzon resigned in 1966) with small-scale models and full-scale construction assemblies to test the sequencing of staging and pre-stressing the precast concrete elements. This "adventure in building" (as Arup called it) was completed sixteen years after it started. Although the process was difficult, interesting innovations in structural design, documentation, and analysis emerged from the process. If the structural form had matched the forces, many of these problems would have been easier to solve. (Figure 0.0.3)

This process may seem anomalous, but it wasn't. Over five hundred years earlier in 1436, a watchmaker named Filippo Brunelleschi was selected by a design competition to complete the design and construction of a uniquely-shaped dome over the Cattedrale di Santa Maria del Fiore in Florence, Italy. It was to be higher and wider than any dome that had ever been built. The octagonal base had fixed the design proportions of the dome one hundred and fifty years earlier—leaving a puzzle no one had been able to solve.

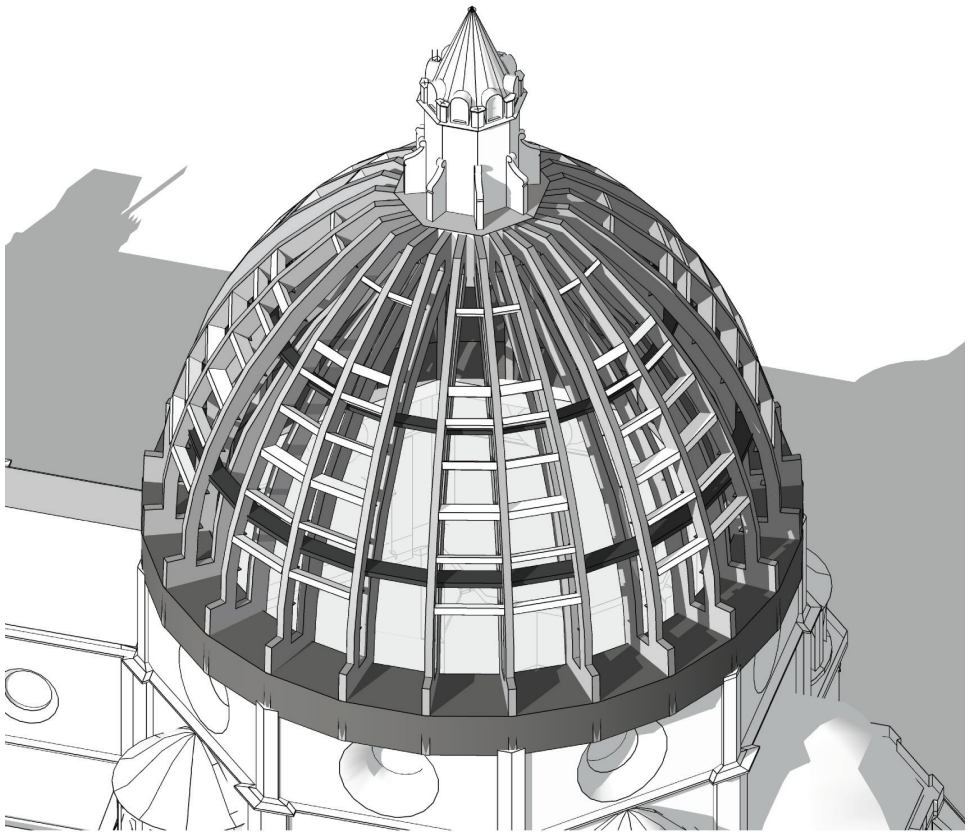
Modern, scientific-based understanding of structural and material behavior was still centuries away and the project had no precedent to draw upon. Brunelleschi proposed an ingenious double-shell dome that allowed the outer shell to meet the geometric design standards, and an inner shell that could be formed to better align with a more apt structural form, using large chains embedded within the dome's perimeter to resist outward thrust.

No one had ever constructed a dome like this without a supporting timber frame below; but this dome's height made such scaffolding impossible. Brunelleschi had to invent new ways to construct the dome as it rose; he even designed the hoisting machines that lifted the four million bricks to the workers. To test his ideas, and to convey his design intent to the craftsmen, he had a large wood and brick model constructed (one that he left intentionally incomplete to ensure his continued involvement and control of the project). Brunelleschi studied, tested, and documented construction problems using drawings, physical models, and prototypes—just as Arup did a half a millennium later. The solution worked admirably. It is still the largest masonry dome in the world. (Figure 0.0.4)

We can learn from Arup and Brunelleschi by looking at *how* they developed their work. Their problems

Figures 0.0.4a Exterior view of the cathedral dome (Duomo)

Figures 0.0.4b Wooden model of the dome



Figures 0.0.4c Section showing double shell of dome with multiple layers structural elements

were so vexing that they used every means available to them, from analog to algorithms. They created multiple representations of their projects in "models" to test their decisions' validity, and they learned from testing to improve, innovate, and refine their work. How they solved problems, and what they revealed through their efforts is so exemplary that it forms the basis of this book's core lessons:

Lesson #1: *Matching architectural forms to materials based on structural principles makes everything easier. Concepts come before calculations, so* ***Think****.*

Lesson #2: *Design requires earnest, industrious, and creative efforts (and a diverse set of tools) to manifest an idea. Learn by* ***Making****.*

Lesson #3: *Making mistakes is part of a healthy design process. Ideas are improved through testing. Embrace the productive role of failures when* ***Breaking*** *your work.*

Lesson #4: *Repeat as necessary: Breaking helps you rethink and rethinking helps you re-make . . .*

The vast number of potential solutions to which these lessons apply is daunting. We need to first understand how structures work before we attempt to make them work better.

Why Think*? Structural ideas must be purposeful. Development of forms without intent is aimless. A well-informed, fluid problem-solving mindset allows creative and responsive solutions to be quickly considered.*

Structural engineering is primarily concerned with predicting the location and magnitude of forces that flow through a system. Its goal is to determine whether the structure's elements can resist these forces without breaking or displacing too much. This process allows designers to understand how much stress certain elements will have to resist and how the structure and its elements will behave under loading.

By determining how much force is acting at certain points within a structural system, we can design specific elements to resist these anticipated forces. Learning how to design and develop structures under these conditions can be complicated, because it relies upon knowing many factors such as form, material, size/shape of elements, and connections—all unknown when design begins. Unfortunately, these analytical obligations for how *engineers* design discrete structural elements don't form a generative basis for architectural design—the engineering process is often designed to be more confirmative. Architects need to use different design methods than engineers to help connect structural performance to architectural considerations. (Figure 0.0.5)

A more effective way to learn structural design is to consider the primary mechanisms by which structures provide resistance, and to explore the patterns by which this resistance is provided. Fortunately, there are discernable patterns in how structures resist forces that are related to their forms and materials. These patterns have clear architectural implications. (Figure 0.0.6)

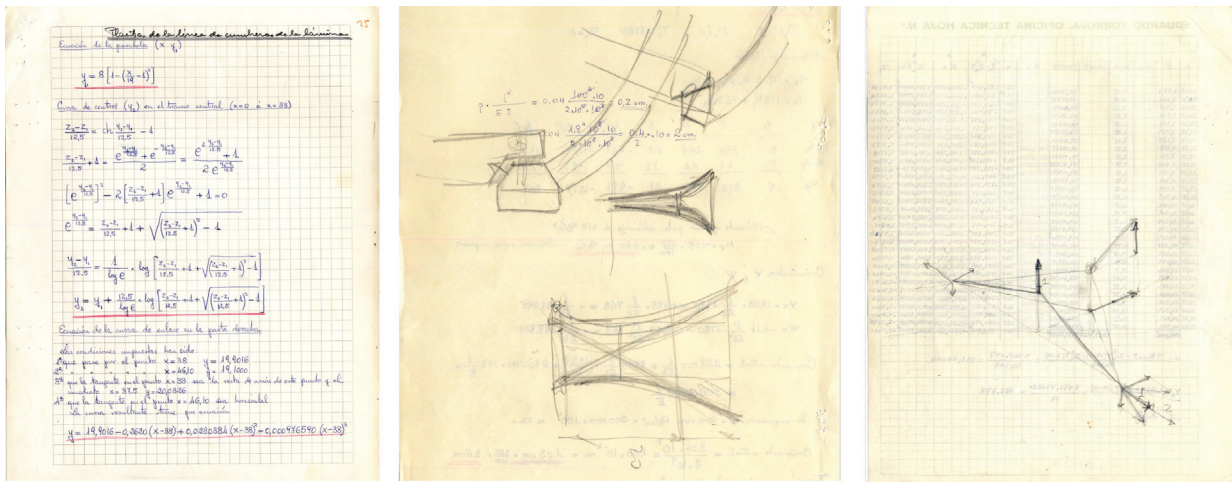


Figure 0.0.5 Eduardo Torroja's calculations and sketches of force vectors and details for Club Tachira (Caracas, 1957)

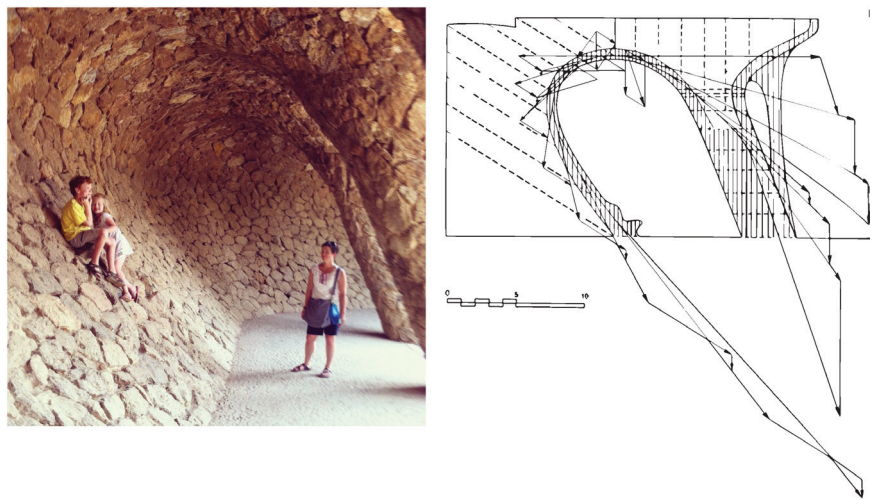


Figure 0.0.6 Certain analytical methods can also provide generative geometry. Graphic statics vector analysis showing force flow and resulting final design at Park Güell (Gaudi, Barcelona, 1914)

Architects and engineers endeavor to solve problems by arranging building forms and materials in certain ways. Because these decisions impact structural performance, they should be the targets of our initial efforts. For example, even though a tent or an arched vault might be used to enclose the same amount of space, they transmit loads differently (compression versus tension), requiring different materials. Understanding these differences leads to purposeful design.

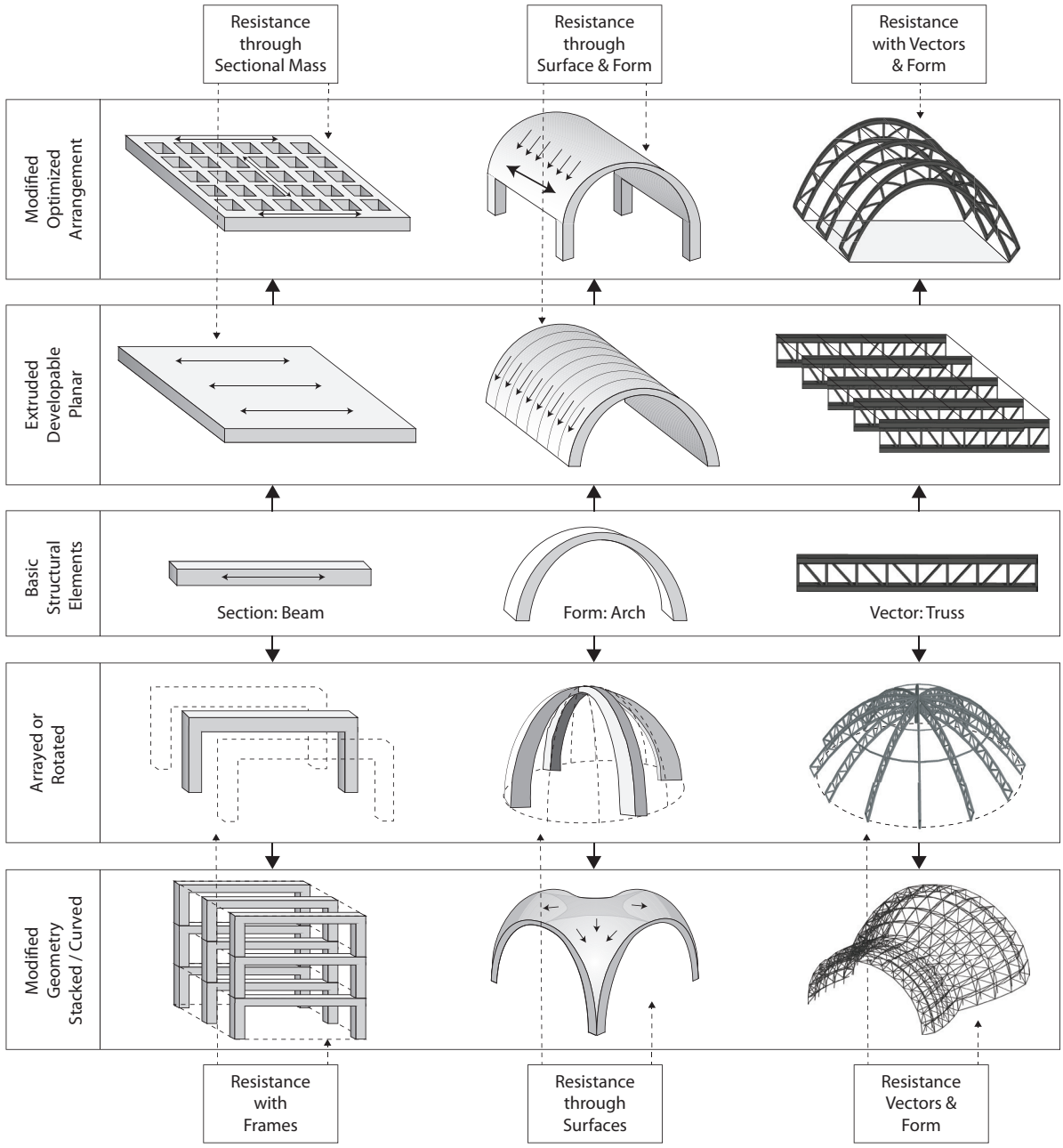


Figure 0.0.7 Formal relationship of selected structural elements when extruded, arrayed, stacked, rotated, or curved to create enclosed volumes

We can classify and differentiate structures from each other in many ways: By rigidity, by geometry, by optional assemblies, etc. But understanding that an arch behaves differently than a beam or a truss can be an ally in design. This book is based on how structural resistance is provided: **Form, Section, Vector, Surface,** and **Frame**. By grouping structural options based on how forces are resisted, we can understand fundamental behaviors and better explore and evolve options based on that information. (Figure 0.0.7)

Why Make? *Thinking and making are necessarily connected. Whenever ideas are translated from conceptual abstractions into testable simulations, impactful lessons become possible.*

Architectural design is about creating prototypes or "models" to test our proposals' responsiveness. Because architects don't make buildings—we make abstracted and/or simplified *representations* of buildings—we need to be vigilant about our tools, what simulations they produce, and what we are trying to learn. Once we understand that different structural systems and elements resist and transfer loads in particular ways, and that these suggest certain formal and material qualities that correspond with architectural, spatial, or

Figure 0.0.8 Students at Black Mountain College testing a prototype for a geodesic dome mock-up they'd designed and built with Buckminster Fuller (1948–1949)

even experiential considerations, we can develop our proposals. Christian Menn suggested that, "The most important basis for achieving balance and harmony in works of structural engineering is a clearly organized three-dimensional visible form that originates directly from the structural concept." We'll find that the classifications of structural form and behavior in this book will suggest tools and procedures that are more helpful in certain chapters than others (an origami-like paper model isn't as helpful in designing a truss as it would be in designing a folded plate structure). (Figures 0.0.8 and 0.0.9)

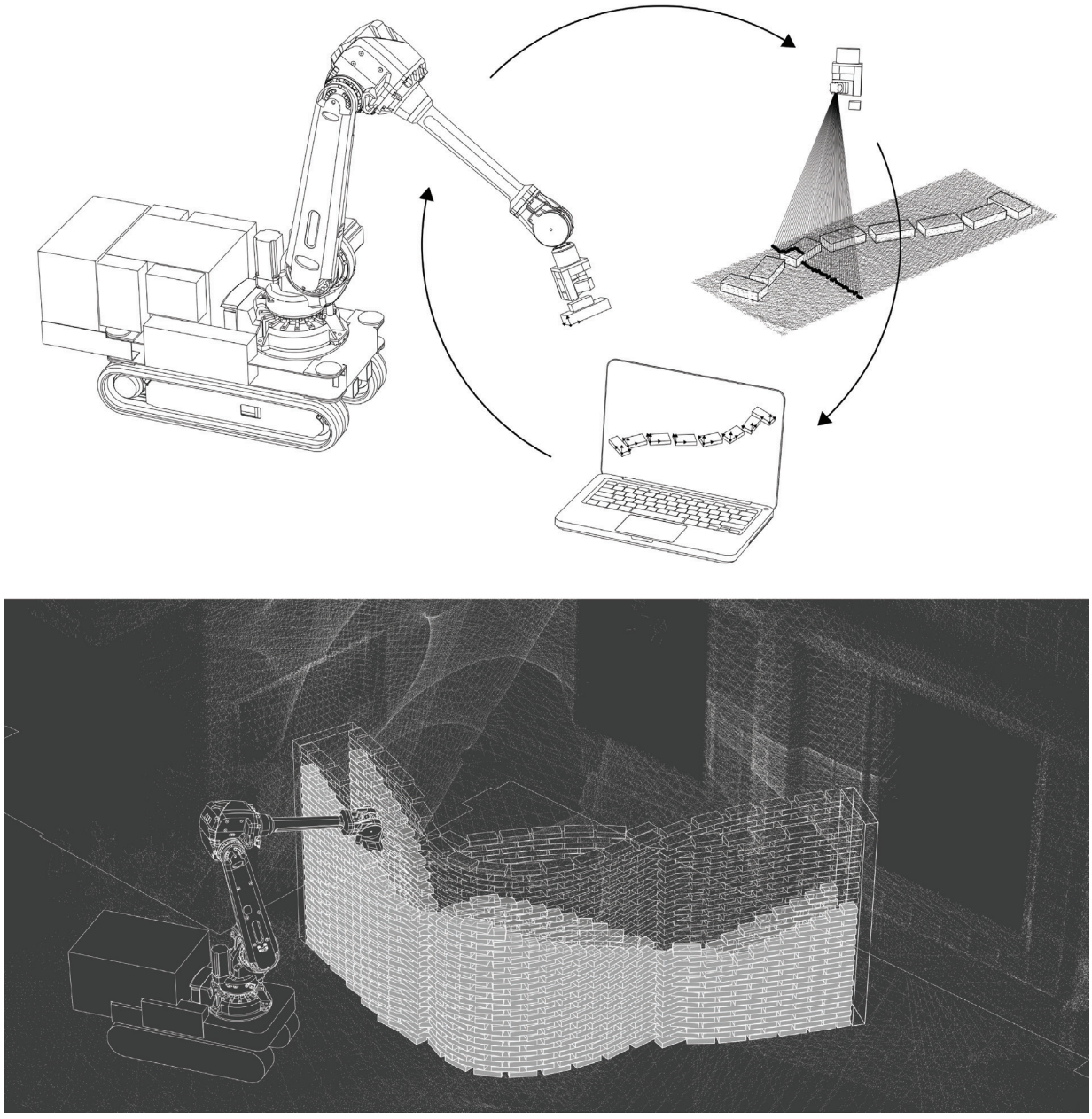


Figure 0.0.9 Gramazio Kohler Research (ETH Zurich) experiments with brick wall design and fabrication using computational design strategies and robotics

Tools and Models for Design

Our design tools should teach us through their use. Architecture's history can be seen through the evolution of tools we've used to document, develop, and represent our work—especially when these tools are shared with other professions and industries. Tools have changed, and their usefulness can be marginalized by technological changes (e.g., no slide rules will be used here) or appropriateness (e.g. rules of thumb are easier than calculating stresses in a thin shell membrane).

Tools can become obsolete, particularly if their feedback loops are prolonged, or if their methods of use are cumbersome (or fraught with potential inaccuracy). But some tools that aren't technologically advanced, like sketching, may still be most effective, especially if they are linked to the ability to think. Specific tools will vary by project type and by designer, and will often change throughout a project as our expected level of refinement advances (e.g., sketches to CAD). Selecting a tool should be associated with the simulation (or "model") being created and the feedback anticipated from its production and testing. The adage "use the right tool for the job" is an apt reminder. (Figure 0.0.10)

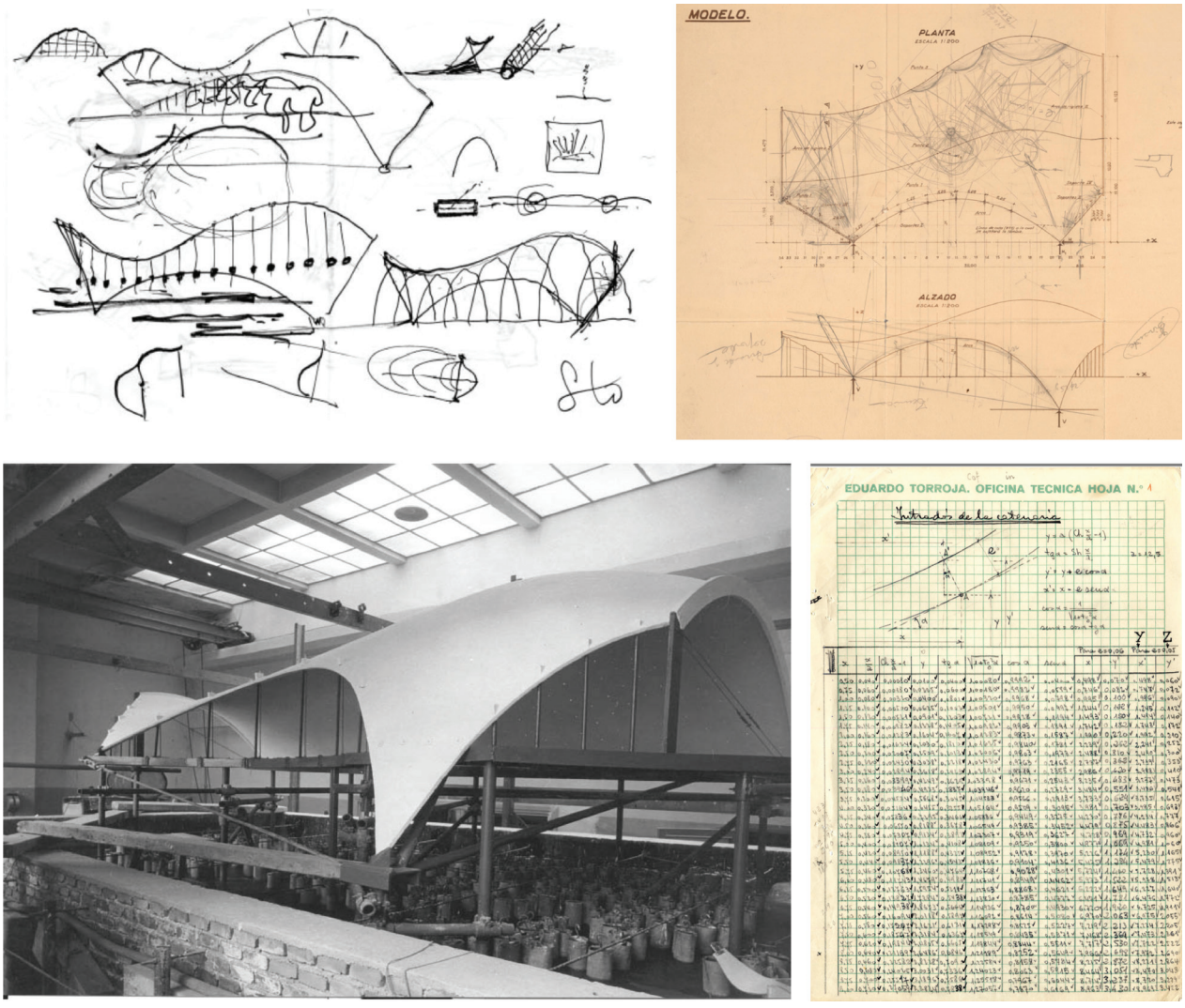


Figure 0.0.10 Torroja's design and testing process for Club Tachira used a variety of tools to produce various models for testing, revisions, and confirmations (Caracas, 1955)

Throughout the book, we will use tools that convey architectural and structural information. Our bodies will help us feel forces, sketches and drawings will convey ideas graphically, diagrams will illustrate structural behavior, rules of thumb for geometry and sizing will kick-start our efforts, charts will help us size elements, equations will help us assess quantitative information, physical models and prototypes will help us to see and test our ideas, and computational simulations (including digital models and/or algorithmic parameters) will process more information and options than traditional representations.

Types of Models

There is an important distinction between a tool and a model. A tool is a method—a model is a product. We blur this distinction in design with words like "drawing, model, and calculation." Models need not be physical manifestations. They may just be a simulation of a building (or part of a building) that includes enough information to be assessed. This could describe a spreadsheet or physical prototype. Throughout this book, the word *model* will be used to describe what has been produced by our tools.

All models are produced with a spirit of inquiry. But not all models produced by these tools are made for the same reason. There are five purposes for "models" we produce:

Representational: These models illustrate design ideas that aren't intended to be "tested" technically. These models frequently contain technical information that helps us assess them visually and experientially. (Figure 0.0.11)

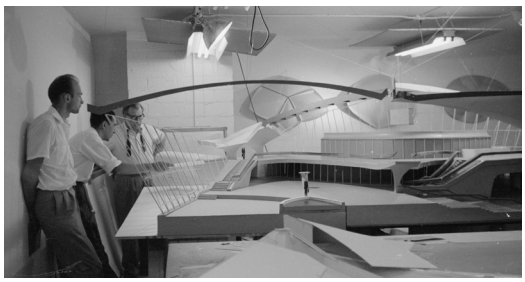


Figure 0.0.11 Large-scale representational model of TWA Terminal created by Eero Saarinen and associates (Korab)

Educational: Educational models are built as learning-aids or for demonstration purposes. They are simplified representations of structural elements that are illustrative of structural principles that may move in exaggerated ways or may remove unnecessary detail to be more diagrammatic. Graphic representations of structural forces as vector arrows that convey information about structural behavior are also educational models. There is a long tradition of using these diagrams in useful ways. (Figure 0.0.12)

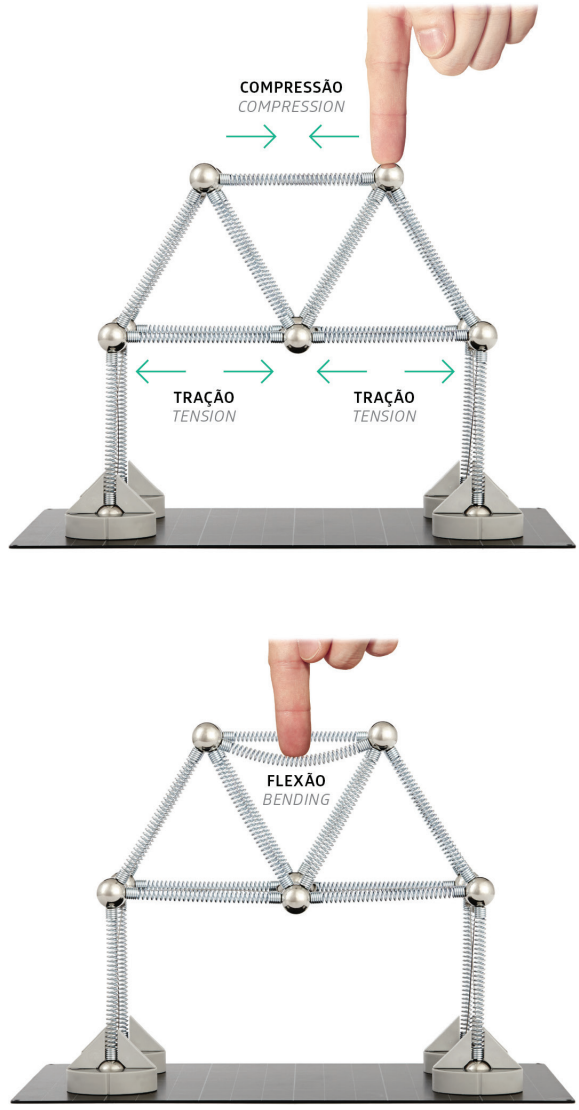


Figure 0.0.12 The Mola structural model kit demonstrates force flow and stability by using a series of magnets and springs to recreate structural behavior

Generative: Extrinsic forces that are applied to these "form-finding" models may suggest an ideal geometry. These models are useful for structures where the form is a primary source of load transfer and resistance (such as arches) and where traditional methods for form-finding are ineffective or unclear. Contemporary computational simulations, for instance, create a broader range of generative models for complex three-dimensional structures than physical models. (Figure 0.0.13)

Confirmative: These prototypical models are created to generate and collect measurable data. They need not be complete representations of a building (e.g., testing a beam or prototyping a roof connection) or be physical models at all (a set of calculations is a confirmative model). They ultimately become the basis for assurances that compliances have been met. (Figure 0.0.14)

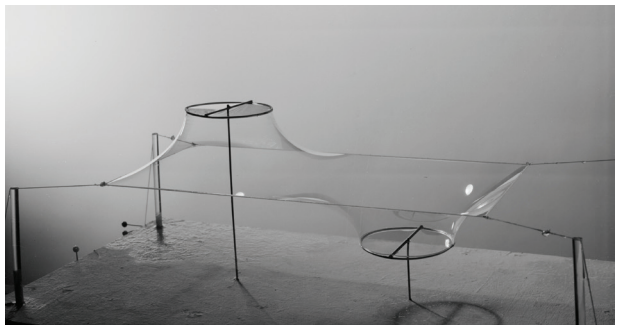


Figure 0.0.13 Frei Otto used soap bubbles to find the idealized geometry of a fully tensioned surface structure (1961)

Figure 0.0.14 Testing a cantilevered folded plate system for Hipodromo de la Zarzuela stadium (Torroja, Madrid, 1935)

Optimizing: This type of model is a hybrid of generative and confirmative goals. Simulations target measurable criteria and run tests to find an idealized (or acceptable) compromise between factors (e.g., weight versus stiffness, longer members versus more connection, and even optimized custom forms based on specific loading conditions). Many different procedures are possible to optimize a design, but computational simulations have made generating and evaluating possibilities faster and more visually apparent. Algorithmic-based parametric software programs generate options that can optimize building performance. (Figure 0.0.15)

Certain tools are designed to produce models that blur these classifications, delivering more impactful information. Many generative models are representative and confirmative, which makes them ideal for educational purposes too. *The specific intent is to find tools that create models that allow for a series of responsive feedback loops between architectural and structural principles so these problems are generated and tested in conjunction with each other.*

One of this book's goals is to help designers explore ways to design with the rigor of quantitative data, physical form, and materiality *without* diminishing or ignoring qualitative factors. The ultimate goal is to be able to think critically about these topics using a diversity of tools that suggest responsive solutions in a feedback loop. As design evolves from schematic decisions about the type of structure (thinking), into configuration of the elements (making), and evaluation (breaking), designers will need different types of tools to create and test the models. (Figure 0.0.16)

Why Break? *Failures are only a problem if they are unexpected. Evaluations are intended to reveal more than just compliance—they generate new information that can be used to improve future proposals.*

"All models are wrong. Some are more useful." Structural Axiom.

For centuries, designers endured unpredictability and potential failures in the structures they built. New design tools, materials, manufacturing methods, construction processes, and educational/practice models developed in an effort to improve these conditions. Contemporary engineering emerged through its ability to apply the scientific method to building through

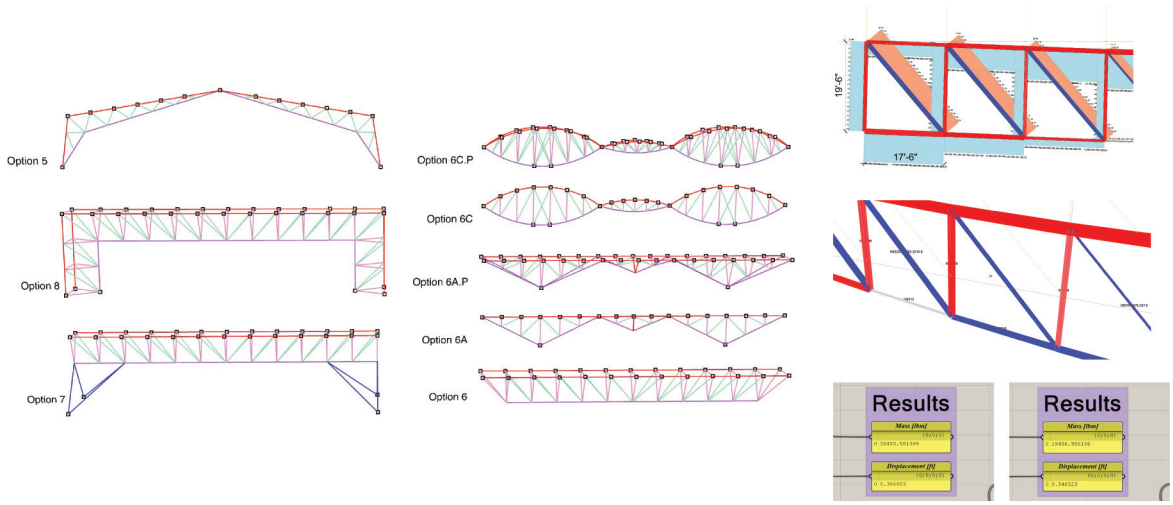


Figure 0.0.15 Digital simulations used to find optimized member sizes for various options for truss designs, (Mohamed Ismail)

two separate, but complimentary forms of inquiry: Theory (reliance on mathematical evaluation) and Experimentation (use of physical testing and prototypes). Theorists and Experimenters have both relied upon their abilities to think and observe with purpose, to hypothesize about why things work, and to subject their work to rigorous evaluation and examination. Contemporary work has emerged from this tradition. The important part isn't the manner by which testing and evaluation occurs—but that it occurs and that we learn from these efforts.

Structural design is concerned with determining if the structure is: *Strong enough, stiff enough, stable, and/or suitable*. Because we are concerned about these performance standards, we need understand the consequences of certain design choices—testing gives us this. Formal testing for technical performance typically occurs on confirmation models, particularly if we want to determine how strong or stiff a proposed element is—calculations and diagrams are helpful testing tools. (Figure 0.0.17)

Many of the same tools we use to create models can also be used to test them. For example, drawings can represent a truss's design *and* analyze the internal forces on the members by applying analytical information to the drawing (e.g., graphic statics or force diagrams). The goal is a fast and productive testing system synced to other phases of the design process using theory and experimentation. Sometimes we can "break" our proposal by applying logic and

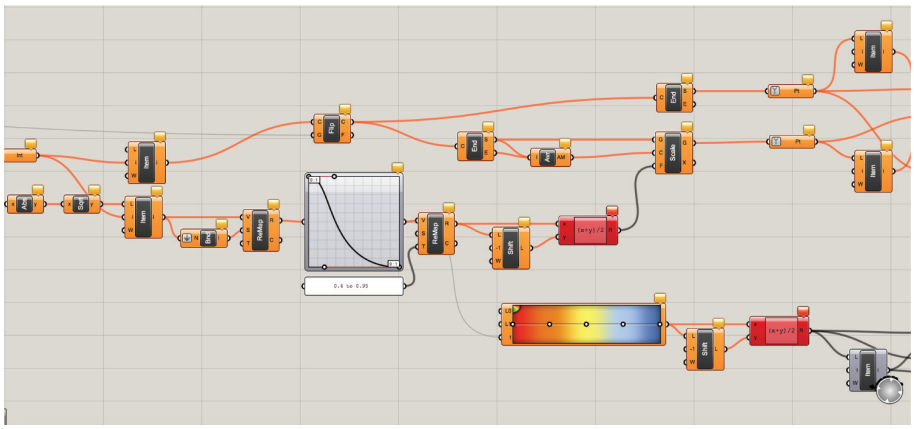


Figure 0.0.16 An algorithm showing the escalating levels of considerations and influences on decision-making—it also serves as a model for a digital analysis program

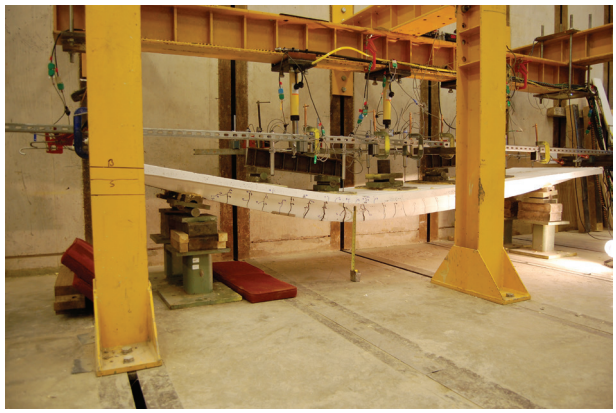


Figure 0.0.17 Testing efficiently shaped fabric-formed beam, University of Bath (UK)

structural principles to find a better solution (e.g., a ruler is harder to bend when its long side is vertical). Sometimes a visual assessment will suffice (particularly if a physical model cracks or if a force diagram doesn't show equilibrium). At other times, we may need a calculation-based assessment or a digital visualization of the force concentration. (Figure 0.0.18)

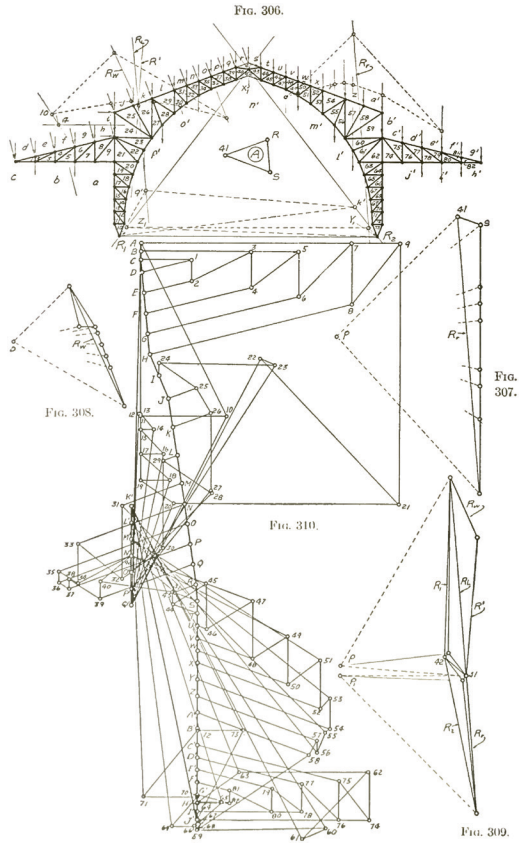


Figure 0.0.18 Graphic analysis method that links the internal stresses of the truss with its geometry which gives immediate feedback between form and forces (W.S. Wolfe)

One of the paradoxes of learning about structures is that beginning designers are often hesitant to engage in the work for fear of getting it wrong. But excluding oneself from the active effort of design and evaluation means learning nothing. *Failure means feedback and feedback is helpful*. The testing process is not a way of assessing right from wrong. It is a way of offering insight into how a structure works, or doesn't, and of suggesting alternatives.

Physical models are less exact ways of measuring strength and stiffness than calculations, but they can help assess structural stability and architectural experience in a manner that gives a broader range of feedback. We will use physical models for form-finding exercises and as ways to "see and feel" how structures work. (Figure 0.0.19)

Digital models should be produced with software that allows us to assess them structurally. This is important, not just because some digital modeling tools can create forms that can ignore basic structural principles, but also because we'll want the digital model to be more than just representative. Certain software tools are powerful allies in shortening a feedback loop, managing multiple parameters, and providing analytical data (visual and computational). Optimal structural solutions, especially digitally-generated ones, may conflict with architectural criteria, which is why evaluation is one step in a reiterative process of designing and redesigning. (Figure 0.0.20)

As we will see, there is a strategy of matching tools, models, and testing methods with different structural systems. Throughout history, as designing and analyzing buildings evolves in accuracy and refinement, it creates corresponding improvements in building performance—and even new means of expression.

Think, Make, and Break: *Repeat as Necessary*

When thinking, making, and breaking are applied towards design exercises, we face the question: Is our work intended to be compliant, conventional or exploratory? Designing a building that simply strives to stand up isn't very complicated or fulfilling, especially when we can look for ways to "hack" these conventions in creative ways. The term hacking implies either that something is hidden or inaccessible, and these lessons should, of course, be

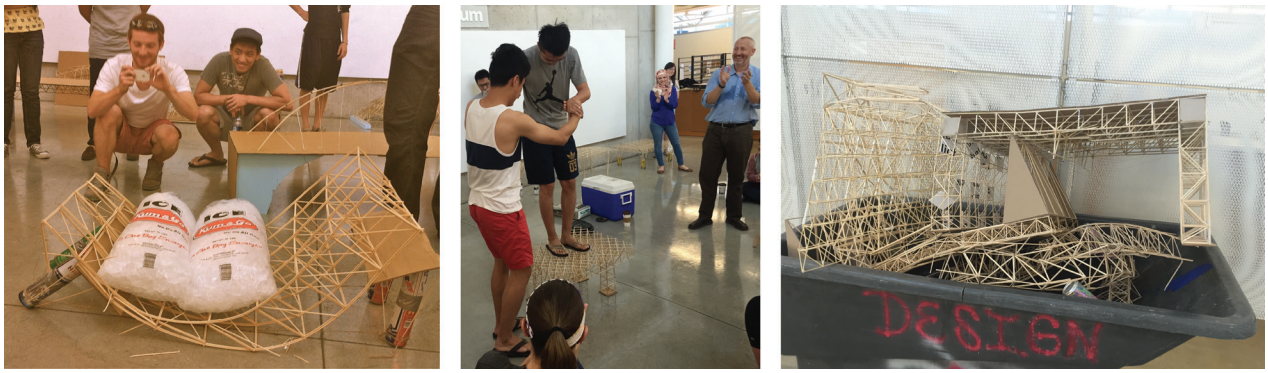


Figure 0.0.19 Examples of student modeling and testing of truss framing (Structures by Design (SxD), Iowa State University)

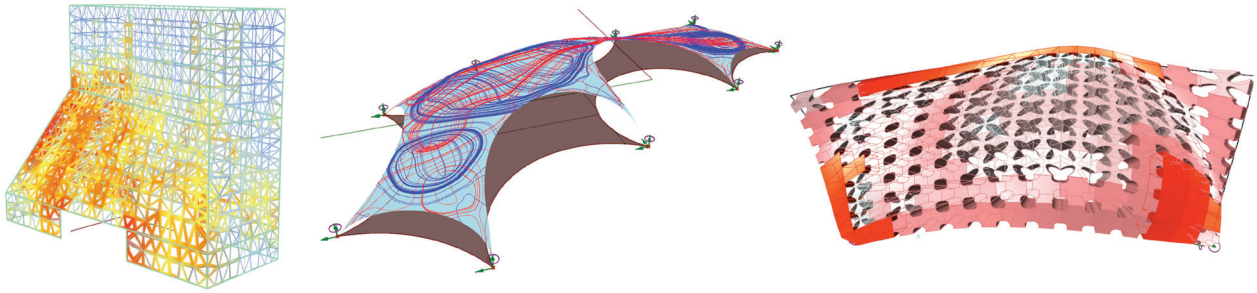


Figure 0.0.20 Digital simulations used to find (from left to right): Stress and material distribution, idealized forms that are responsive to forces, and complex forms generated from parametric modules (Jeana Ripple, Karamba 3d)

neither. But the spirit behind the term implies that there's often an easier, or better, way to maximize the potential of certain scenarios, especially if you know key information or work harder to apply it. Anthony D'Angelo suggests that the process of inquiry be rigorous: "When solving problems, dig at the roots instead of just hacking at the leaves." We will proceed with this spirit in mind.

Finally, the Think, Make, and Break process is intended to be open for interpretation. Like any iterative process of design, certain lessons may emerge more quickly or saliently than others depending on the tools you use and the questions you are trying to answer. As you proceed, look for ways to deconstruct the core information and skills required to design structures with creative and critical acumen.

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PART 1

Learning to Think, Make, and Break

Figure 1.0.0 Body-like column forms at Dulles Airport Terminal (Saarinen, 1964)

CHAPTER 1.0

Body Structures

The Feel of Things

Are you sure this is architecture?

(Beginning design student, during lab, at Iowa State University)

You probably know more about structural design than you'd expect. The trials and errors of our physical experiences have prepared us to connect what we "feel" to what we'll learn. By enacting particular poses in a mindful way and recording the results using principles of structural design, we'll build a foundation of knowledge and develop a reliable learning tool to assist our design efforts.

Begin with Basics

Learning structures may seem difficult and complicated *but it doesn't have to be*. Profound lessons about structural principles can be felt and understood by being mindful of what our bodies can teach us (or have already taught us)? Our bodies anticipate and adjust to the different structural conditions we face daily through trial and error. Throughout our lives we've subconsciously learned to "design" our body into responsive and effective structural forms. As Eduardo Torroja explained, "The process of visualizing or conceiving a structure is an art. Basically it is motivated by an inner experience, by an intuition." We are all experienced structural designers already, we simply need to be mindful of the lessons we've learned.

The Feel of Things: Basic Support

Standing up straight, holding a child on your hip, or leaning over to pick up grocery bags without tipping over may not seem like profound accomplishments or structural lessons, but these simple acts pull, push, and twist different parts of the body—all fundamental structural actions. Because we'd rather not drop the child or the groceries or fall over, we've learned to adjust our "structural frame" to best resist these forces acting upon us—we adjust our body's form because we can't change our body's "material" matter. Throughout our lives, responding to the basic challenges of structures: Stacking, spanning, and stabilizing, becomes routine. (Figure 1.0.1)

While these actions *can* be described, calculated, and diagrammed using mathematical and scientific principles, we didn't wait to perform them until we had learned the structural principles behind them. This would be absurd, of course, but this has been the most common approach to structural education. As the architect, engineer, and shell builder, Felix Candela, observed, "what could be the progress of mankind if nobody were allowed to perform any

Figure 1.0.1 Body forms respond to common structural challenges

jump or movement without a previous mathematical determination of the force that must be asked from a certain muscle?"

Compared to our bodies, conventional tools used in structural education (charts, diagrams, formula) aren't easy to learn, can be laborious to use, and in their most basic representations they portray structures as flat, not spatial. Our bodies, though, can make simultaneous adjustments of different variables and evaluate the three-dimensional consequences of these changes quickly. In his essay, "The Feel of Things", renowned structural engineer Fred Severud argued that body structures were more effective than calculations initially because, "Without having to go through all these mystifications, you can feel what happens in a structure."

Consider how easily we can learn about the differences between arches and beams by simply acting out the differences. *Try it*. When you put your body in a traditional plank pose, your straight-backed torso acts like a beam spanning between your feet and hands. It's hard to maintain this pose for long. Your hips sag due to the bending stress you feel. You may allow your hips to sag like a tensioned cable or to

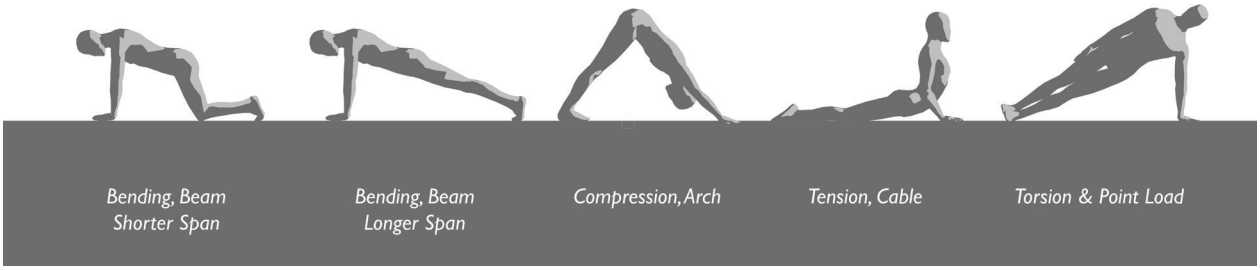

Figure 1.0.2 Different poses create different stresses

bend your waist and lift your hips into a compressed arch shape. Note that although you are spanning the same distance in all three forms (as a beam, cable, and arch) each pose feels different as a result of your body's changing form. This may not seem remarkable but these simple acts reveal a profound lesson: *There is an inescapable relationship between the form of a structure and how it behaves.* (Figure 1.0.2)

THIS is the fundamental lesson that will be repeated throughout the book. When you understand relationships between forms and forces, you can design structures to be more responsive. Learning how to be a proactive structural designer will require different experiments and lessons; we will start with where we all started our structural education—our bodies.

Design Challenges: Spanning and Stacking

For the remainder of the chapter you will be challenged to create different types of structural solutions as individuals and in groups using primarily your bodies. These poses will reveal basic structural principles and behaviors. The structural challenges are simple and intentionally open-ended:

Challenge #1: ***How high can you reach***? *Measure from the ground to the highest point.*

Challenge #2: ***How far can you span***? *Clear span is the distance between supports (your feet), but you should also try to maximize the overall length your bodies can reach from end to end.*

THINK, MAKE, AND BREAK: ASSESSING ACTIVITIES

One of the critical lessons to learn about the Think, Make, and Break method is knowing what information you are hoping to find at each stage of the process. Before you begin, think strategically about what you are planning to do and why it might, or might not work. All useful experiments start with a considerate hypothesis ("think"). Next, go through the entire process of "making" all these structures; you may learn certain strategies from one exercise that helps you in another. As you act these out, be mindful of your posture, placement of "supports," and locations inside your body where you feel stresses. Gather objective data by documenting these poses with photos, notes, and measurements taken from different perspectives so you can see your body as a three-dimensional construct—compare how the structure behaves or "breaks" back to your hypothesis. To make sure the act of "breaking" is productive, you'll need to be specific about what sort of information you are hoping to learn as a result. Record your activities in a lab report including answers to specific question about "what I learned" based on the assigned objectives. (Figure 1.0.3)

(continued)

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Figure 1.0.3 Body structure experiments (SxD, Iowa State University)

Breaking: Confirming Experiences

Consider how you can translate your physical activities to useful lessons. Some of these poses may seem routine, difficult, or fun, but look beyond the experience itself. If a pose is difficult or doesn't work, try to determine why. The goal is to visualize and document the hidden structural behaviors that you are experiencing into the language of structural design principles. Enacting these will produce results and reactions, but unless you record them, the lessons will be fleeting. The digital representations of body structures shown in this chapter will also include examples of how to describe, diagram and analyze structural principles: *Forces, loads, equilibrium, supports and states of stress*. The goal is to align the math and science of structures with your experiences. Document your work accordingly.

With each pose, answer the following questions:

Question #1: *What is the primary type of stress you feel? Where do you feel it*?

Question #2: *How did the form change as the group size increases? Under what conditions was it beneficial to have more people*?

Question #3: *What complications arise the higher you go or the farther you span (falling down, tipping over, increased stress, "construction" complications, etc.)*?

Question #4: *Is it "efficient?" Consider the relative efficiency of the number of people compared to the height or span achieved. Can three people reach three times higher than one person? Or span three times as far? Why, or why not*?

Question #5: ***Details, Part I***: *How does the angle of your supporting legs, their relative orientation, and/or the position of your feet change in successive poses? How did these "columns" have to adjust in order to maintain equilibrium as the structure grew taller or wider*?

Question #6: ***Details, Part II***: *Look at how you make "connections" to other bodies—is it a rigid and stiff connection or is rotation allowed? What is the most common "weak point" for the structure you are enacting* (*and why*)?

Before you begin, consider that your bodies are useful but imperfect testing devices. There are only a few structural situations that we can accurately mimic with our bodies—we can't become a truss or a tent—and we are limited by our sizes and abilities. Remember that our bodies are designed for dynamic movement. We have joints with many degrees of potential rotation that actual structures don't have, and these joints make static positions difficult to maintain. Of course, no structural exercise is worth physical injury. Proceed mindfully and stay within your limits—some of the following diagrams aren't realistic or advisable.

Forces, Loads, and States of Equilibrium

These exercises give us a chance to answer one of the most elementary structural questions: "*How do structures stand up?*" The purpose of a structure is to provide a stable framework that channels loads to the ground. Therefore, in order to stand up, your body structure relies upon a supporting skeletal frame to be strong and stiff enough to resist forces acting upon it. (Figure 1.0.4)

When your body is standing still, it is in a state of *static equilibrium*. All the forces acting on your body are being supported and channeled in a way that keep you from falling down (i.e., in *translational equilibrium*) or tipping over (i.e., in *rotational equilibrium)*. Equilibrium means that the sum of all the forces acting upon the structure equal zero—in this case, the weight of your body pushing down is equal and opposite to the amount of resistance provided by the floor (translational) and there are no lateral/sideways forces trying to overturn the body (rotational). (Figure 1.0.5)

As Mario Salvadori suggests, it's helpful to visualize forces as a flow of water through a network of pipes. We can diagram the direction that this "water" flows with vector arrows to make sure that our "pipes" have enough capacity. Inside a structure, there is an internal struggle of forces that try to push, pull, bend, and/or twist it while supports try to resist these stresses. A static structure masks a constant internal battle for equilibrium. We can diagram the state of equilibrium when you're standing by drawing two equal and opposite arrows: One that represents the force flow of body weight down to the ground and the other representing the load-bearing capacity of the floor—or

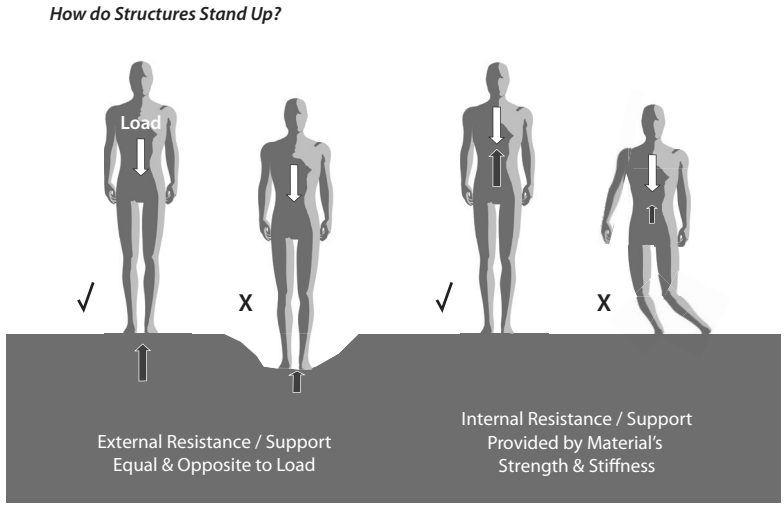
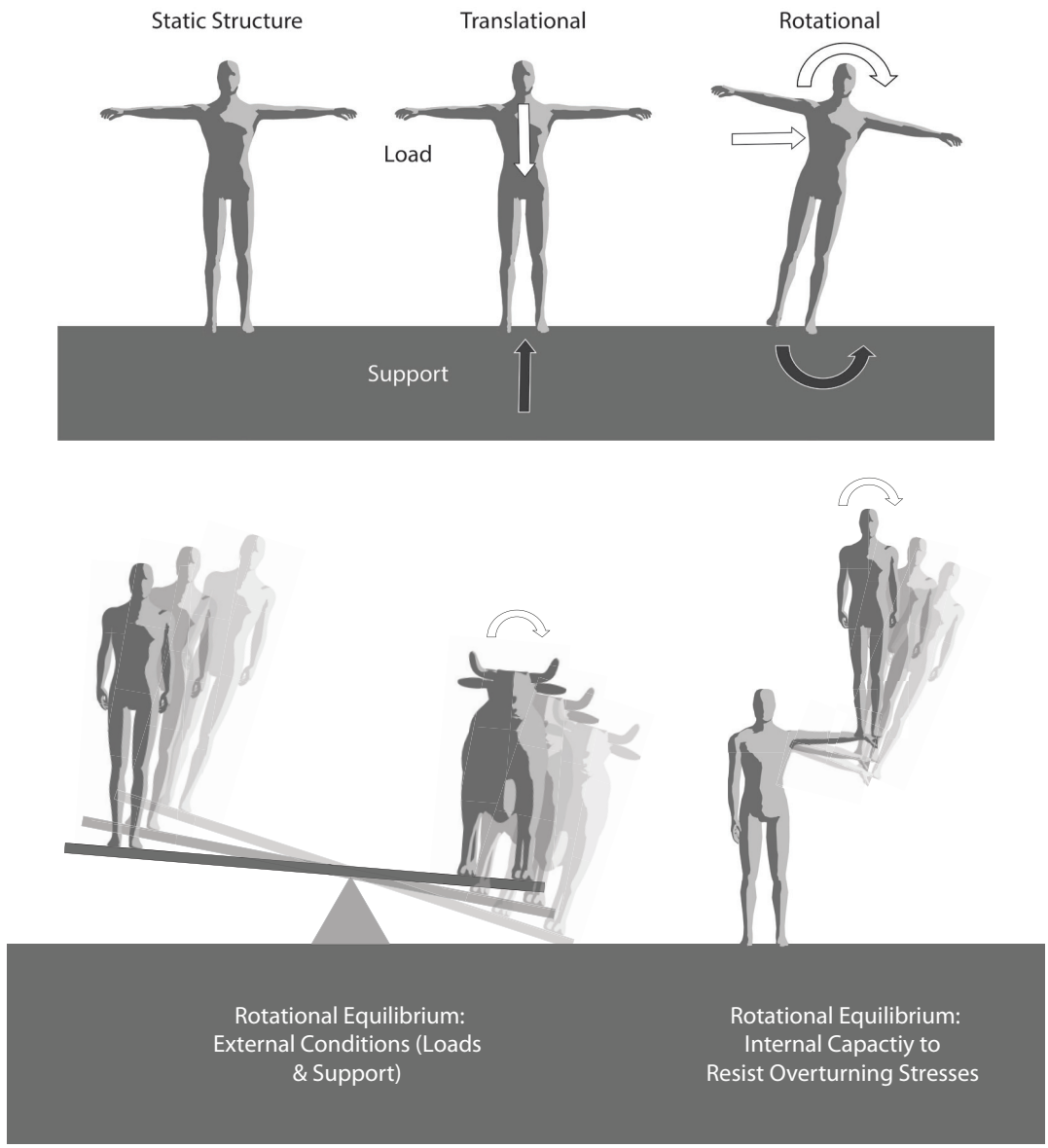


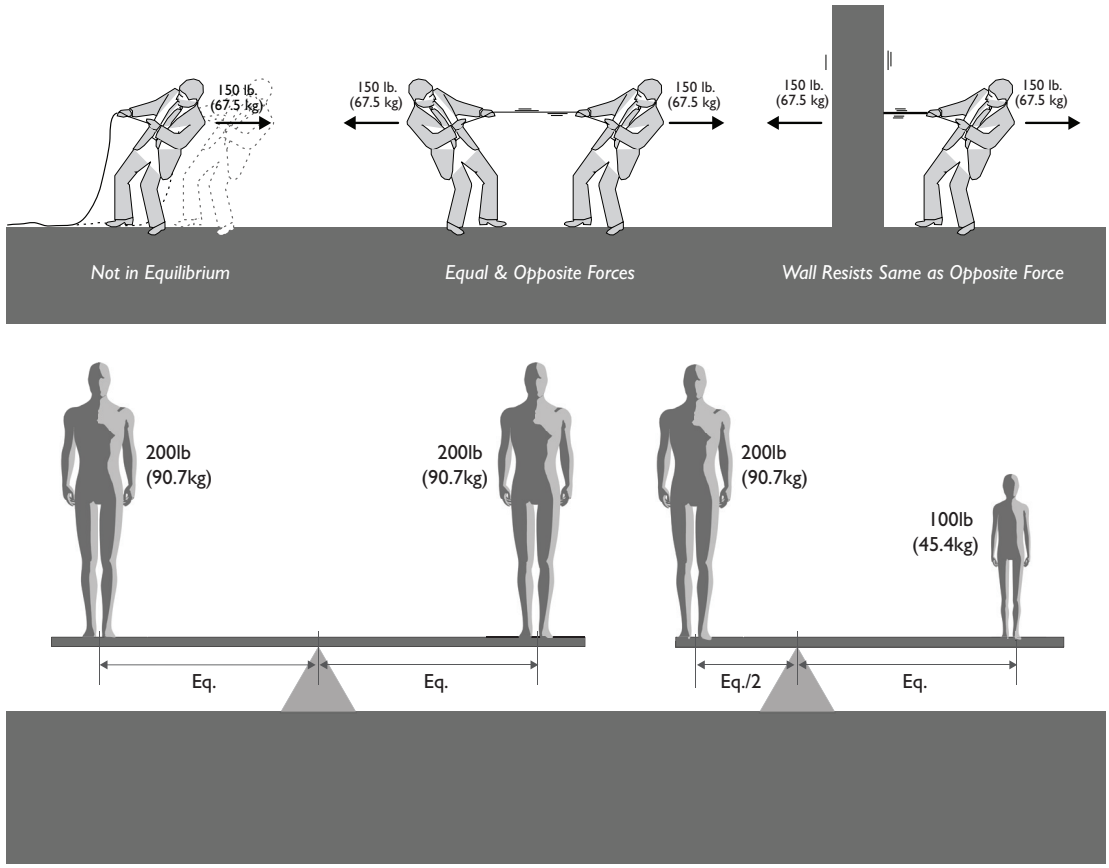
Figure 1.0.4 The external conditions of support and internal structural capacities are both required for a structure to stand up



Figures 1.0.5a and 1.0.5b Static structures maintain translational and rotational equilibrium

opposing arrows when pulling a rope attached to a wall. Rotational equilibrium need not be the result of identical forces acting on the body as long as the load (P) and moment arm length (d) between two forces are equal and opposite. (Figure 1.0.6)

We use force arrows, or vectors, to graphically demonstrate these forces' magnitude and direction. The units of these forces are in weight (pounds (lbs), kilograms (kg), etc.) and the arrow head indicates direction. The length of the vector indicates the amount, or magnitude, of the force. Vectors are useful tools for evaluating loads visually. For instance, if a parent, a child, and a hefty cat are standing side-by-side on a floor, we can represent their weights as vector arrows with three different lengths. The forces are acting in a vertical downward position starting from the center of gravity of each body and they are supported by equal and opposite forces from the floor below. Although this floor surface may have the capacity to resist with a greater force than we've shown, it only needs to exert a resistance opposite of the forces acting upon it so the resisting arrows are drawn to accurately reflect equilibrium. (Figure 1.0.7)



Figures 1.0.6a and 1.0.6b Equilibrium requires equal and opposing forces in both direction and magnitude

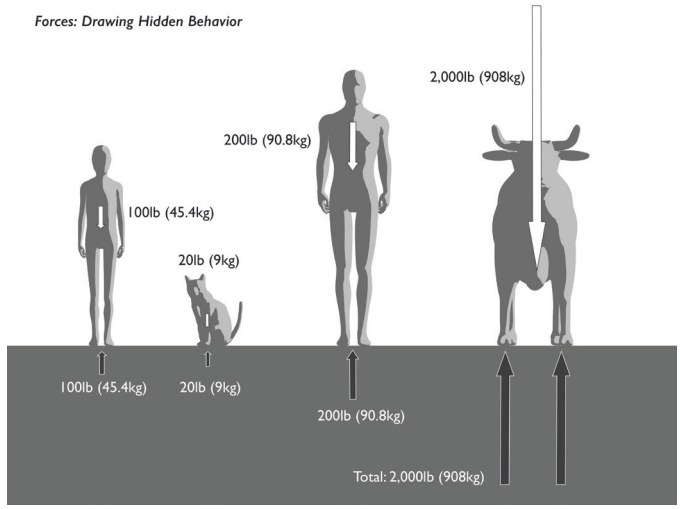


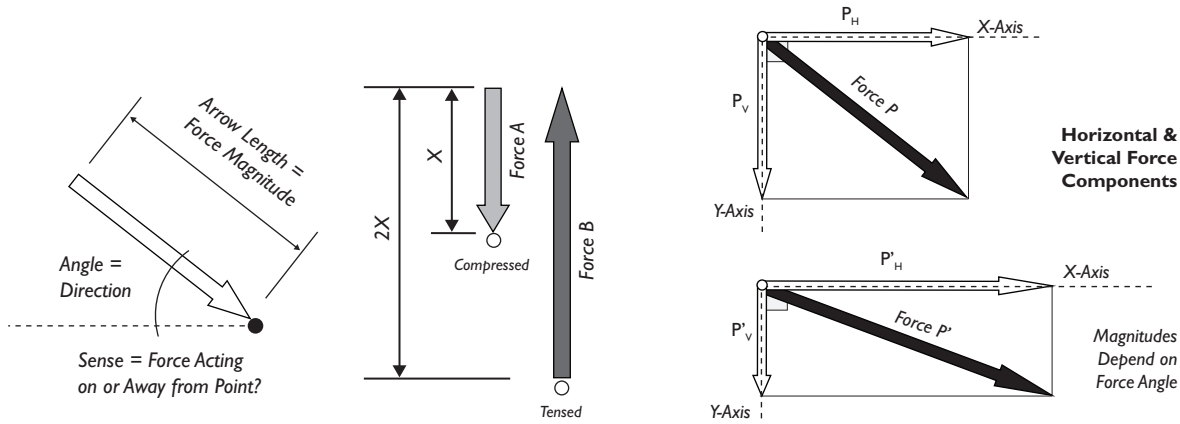
Figure 1.0.7 Different masses are represented with different arrow length, ideally to scale for the sake of visual and mathematical comparison

MAKING: TOOLS OF THE TRADE

Statics and vector mechanics: Statics is a branch of mechanics we use to study physical objects at rest, like buildings. And although buildings may move because of wind or seismic forces, we assume they aren't accelerating downward or sideways. We will concern ourselves with the forces and loads acting on these bodies, and the efforts it takes to establish equilibrium. Compared to other mathematical explorations, we've developed the ability to visually document these loads, forces, and internal stresses with arrows. This provides a helpful tool when documenting, assessing, or understanding structural behavior.

To Discuss: How (and when) were force vectors understood? At what point was it documented with arrows? Before this science, how were the structural designs of buildings "confirmed" before they were built?

To Understand: What are concurrent forces? What is a resultant force? (Figure 1.0.8)



Figures 1.0.8a and 1.0.8b Basic principles of force vectors

Types of Loads: Dead, Live, and Environmental

The essential role of a structure is to receive an applied load, resist it, and transfer it to where it can be discharged without compromising the desired form. This is the flow of forces.

There are different types of loads that need to be resisted: *Dead loads, live loads, and environmental loads*. Dead loads are permanent loads, such as the weight of your body. Live loads are non-permanent loads that come and go, for example if the cat jumped on your back during a yoga pose—the cat may not always be there, but the supporting structure needs to be designed to accommodate this possibility. It's easy to remember the difference between these loads if you picture a building turned upside-down and shaken; the live loads would fall away and the dead loads would remain. (Figure 1.0.9)

Building codes dictate design standards for loads based on structural systems, materials used, building occupancy and location. Environmental loads are live loads that are caused by natural phenomena: Thermal conditions, wind, snow/rain, and/or seismic conditions. These forces are often unpredictable and they vary in direction, magnitude, and duration—anyone that has carried a portfolio across campus in a wind storm can attest to the impact of these forces. These loads complicate the process of diagramming equilibrium in our basic body structures, so, for now, we will assume everyone is safely inside and return to these loading conditions later. (See Appendix, Tables A.1–A.3, for specific weights by type, area, and volume.)

Point Loads and Distributed Loads

Loads applied at one specific location are known as point loads and we diagram them with single arrows. Forces that act across an area, known as *distributed loads*, are diagrammed as regularly spaced force arrows, and are listed with units that reflect the amount of weight per area (e.g., lbs per linear foot, per square foot, etc.). You can recognize the difference in how these loads "feel" and the effect it would have on how a structure behaves (e.g., a person lying on your back versus standing on your back). Distributed loads can be dead loads (e.g., the weight of a floor), live loads (e.g., boxes of drafting supplies and books set on the floor), and/or environmental loads (e.g., wind blowing against a building). If the load is the same across the entire area, it is called a *uniformly distributed load*; but certain loads, like wind loading on a building, vary in magnitude across the area. These *uniformly varying loads* are diagrammed with different length arrows (usually triangular). (Figure 1.0.10)

Once we know how forces flow through different structural elements, we can design our structures to receive, transfer, and disperse the load in the manner that supports our architectural strategies of form, space, and function.

Challenge #1: How High? Transmissivity and Load Transfer

Part of the challenge of this first exercise is to see how high you can reach. This means that one person may

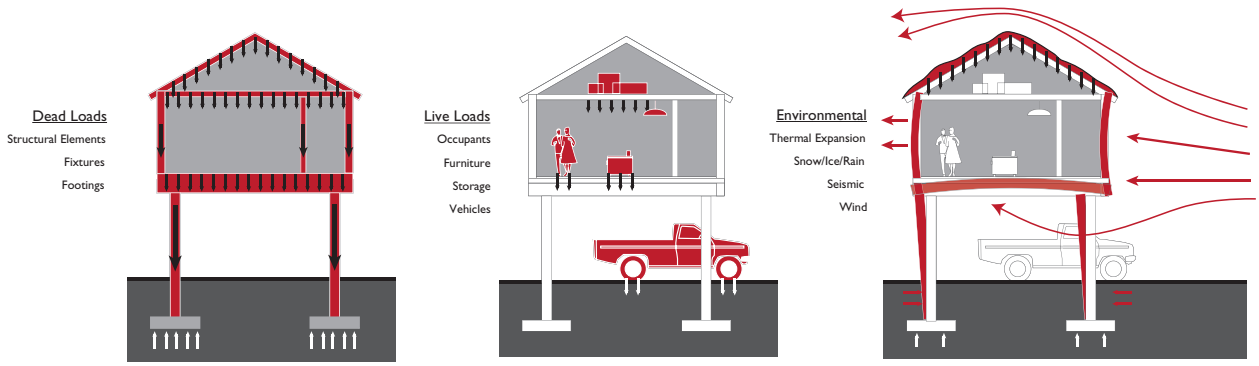


Figure 1.0.9 Types of load classifications: Dead, Live, and Environmental

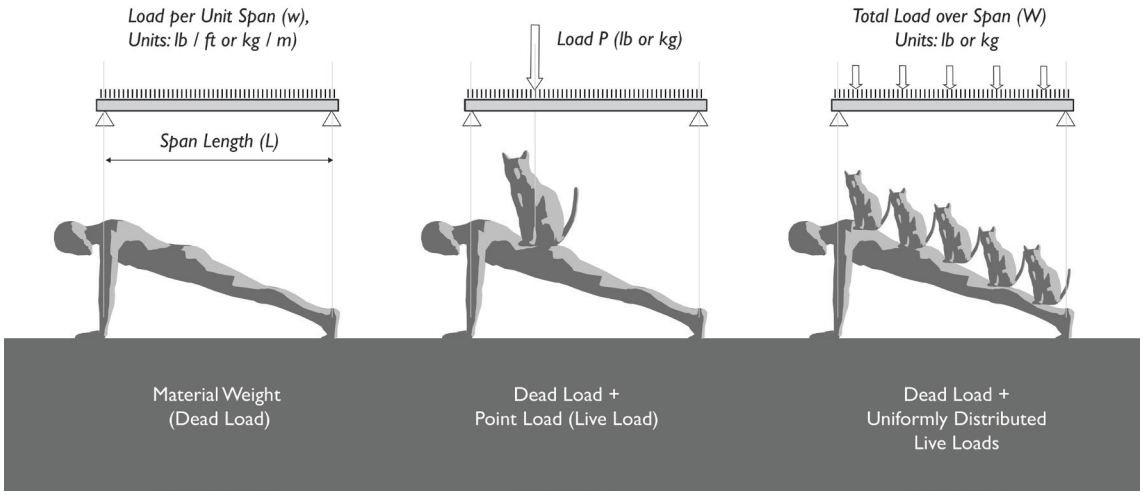


Figure 1.0.10 Diagramming uniform and point load conditions

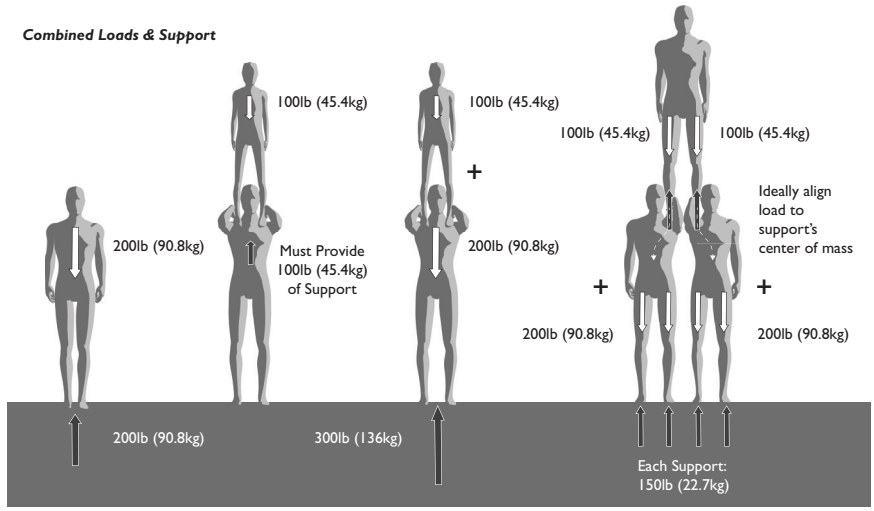


Figure 1.0.11 Diagramming vertical load accumulations

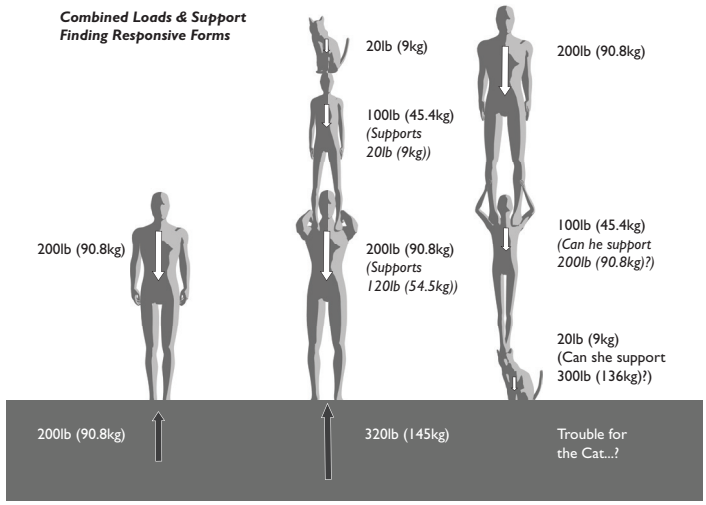


Figure 1.0.12 Lowest support elements must provide support for loads above

attempt to climb on the shoulders of another. What changes when people are stacked atop each other? The load from the top person transfers down—theoretically a very tall tower of people could be built this way. The ability to have one force pass through another in this way, it's *transmissivity*, is key to understanding structural behavior. When multiple people (loads) are stacked atop each other, the forces are added together as the loads transfer down. If the cat rests on the head of the child standing on the shoulders of the adult, we know that all three arrows representing their weights will combine together as a single force to be resisted by the ground. (Figure 1.0.11)

But stacking elements isn't without consequence. Even though the forces pass through from the top element to the lower ones, each supporting element must exert enough resistance to support the loads above it. If we reversed the order to have the parent stand on the child who then stands on the cat, we would still be transferring the same load to the ground, but this would be a bad idea since the cat and child won't be able to support the larger weights above. (Figure 1.0.12)

States of Stress: What do You Feel?

When structures channel their loads through a framework, the structural elements are pushed, pulled, bent, or twisted and they change shape accordingly (e.g., compressing, lengthening, etc.). We say that these loads *stress* the structure, and that the structure *strains* under this stress. Stresses include: *Compression, tension, bending, shear, and torsion*. All of these stresses can be understood by returning to the basic yoga poses. (Figure 1.0.13)

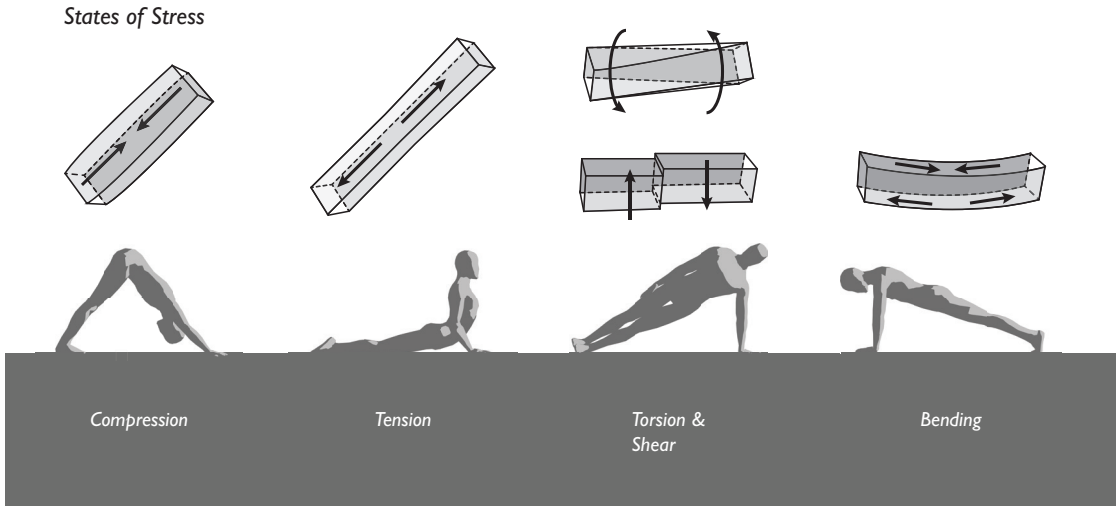


Figure 1.0.13 Pulling, pushing, twisting, and bending and their corresponding types of stress: Tension, Compression, Torsion, Shear, and Bending

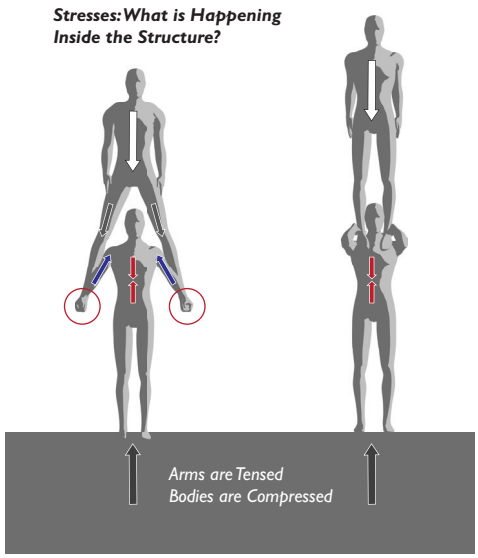


Figure 1.0.14 Both bodies are supported but internal stresses are different (pushing versus pulling). Because the stresses differ, the strains will as well (compress versus elongate)

Some types of stress will be present in certain arrangements but not others. *Strain* is a measurement of how much a structure grows, shrinks, or deflects under loading. When one load is stacked atop another, structural elements are pushed down, or compressed. If these elements were, instead, hanging off the side of bridge like a ladder, each body would be pulled apart and elongated in a state of tension. In the plank pose, your body is in bending and you feel the shear stress on the shoulders. If the heavy cat on your back shifts off to the side on your hip, your body may twist—this is torsion. Understanding these states of stress and how they feel is a good way of confirming what force vectors acting in a structure are doing. For example, consider supporting someone on your shoulders under compression or having them hang onto your ankles as you dangle, pulling you in tension—the amount of force in each scenario is the same, but the states of stress are different and your body will behave accordingly. (Figure 1.0.14)

THINKING (AND BREAKING): STRUCTURES AROUND YOU

A real-life example of these principles can be seen in the human tower structures, called Castells, built during Catalan festivals in Spain. The tallest towers can reach up to ten tiers tall. There is a structural logic and tact to their arrangements. The lightest and smallest children are on top and the strongest are at the bottom. Everyone stands as straight as possible and positions themselves on the shoulders of those below to ensure pure compression. More people are added at the lower levels to support this extra weight. Because the combined compressive stress at the bottom of the pile is so high, there is a concern that the "columns" at the bottom of the pile may buckle, so a group surrounds the base, pushes in and props up these people.

To Do: What other structural principles do you observe about the structures and/or how people are arranged?

To Discuss: What affect does "building" the Castell have on the final form? (Figure 1.0.15)

Figure 1.0.15 Castellers de Villafranca build a human tower, October 2016 (Nur Photo, Getty)

Body Tower: Adjusting Form for Support and Stability

Most groups will find the challenge of stacking three people vertically too cumbersome, unstable, or simply not worth the effort for the marginal gains in height. The amount of extra height gained by adding an additional person is tempered by increased instability—either collapsing or falling over. These towers usually fail because they reach the limits of strength and stability; at a certain point the person

on the bottom won't be strong enough to keep their joints (hips, knees, etc.) from buckling (coupled with the feeling of impending failure if the column of bodies tips over). Common alternative poses broaden the base and spread out the load between more supporters—as either a body pyramid or a modified tower. Pyramidal shapes make sense as a way to share loading across more "supports" and to create a wider base for added resistance against instability. (Figures 1.0.16 and 1.0.17)

Figure 1.0.16 Potential failure of tall tower with a potential solution

Look closely at the arrangement and details of the supporting elements to see how intuitive solutions are structurally useful. The two supporting teammates have their feet spread apart to distribute the load and to resist tipping over (the direction of the toes usually points diagonally outward to further assist). The person being supported may be concerned about falling forward or backward, so the supporting teammates will intuitively secure the points of rotation—ankles and knees—by holding them in place—a lesson we'll see when we learn about frames in Chapter 6.0. (Figure 1.0.18)

Free-Body Diagrams and Resultant Vectors

The difference in how we draw force vectors for supporting elements, particularly those at angles, can help us understand how the magnitude of forces can change based on geometry even if the weight of the load stays the same). For instance, when we spread our

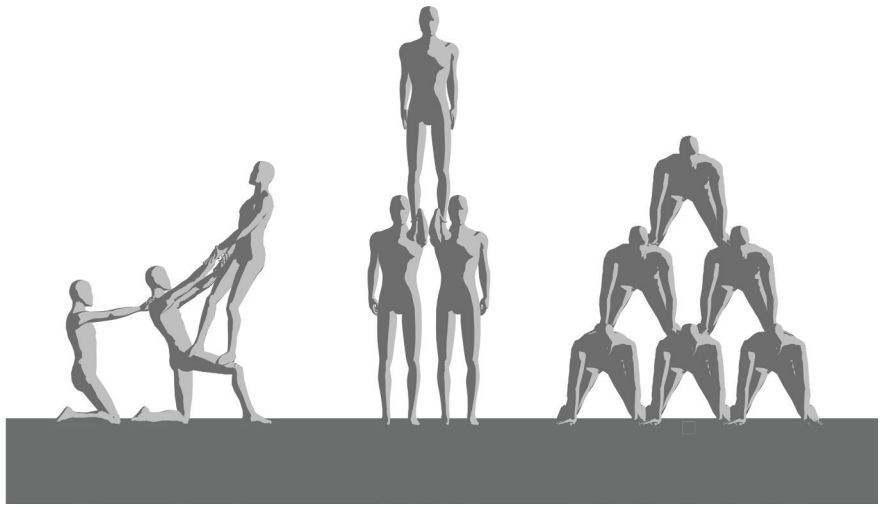


Figure 1.0.17 Building taller and wider helps to share support and stability, but staging the construction of the tower becomes difficult

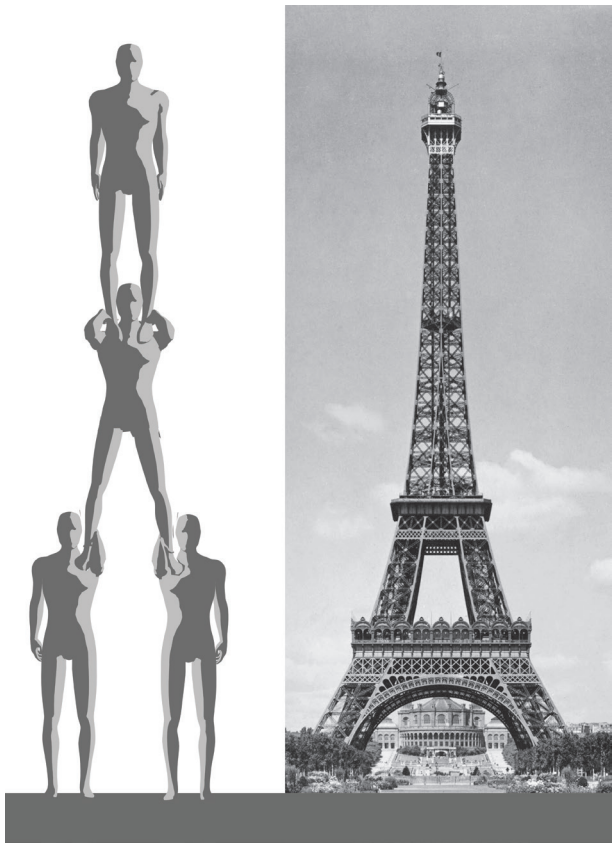


Figure 1.0.18 When secured together, multiple bodies act as a single, effective, structural element

legs apart into a wider stance, we can feel a difference in the forces in our legs, but why? Our weight (i.e., the downward force) hasn't changed, and we have the same number of support points. The answer is in the geometry of the force vectors; a diagonal force has horizontal and vertical forces so it is larger than just a vertical force. We can use a graphic representation of forces vectors acting on the body, called a *free-body diagram*, to describe this phenomenon and to calculate the different forces. (Figure 1.0.19)

Free-body diagrams can evaluate or calculate forces. We will return to this tool often because it presents useful mathematical and geometric information. In this case, we can draw a free-body diagram to find the unknown forces acting within the supporting angled leg. From basic statics (and vector mechanics) we know that angled force vectors can be drawn as the hypotenuse of a force triangle that has both horizontal (H) and vertical (V) force components. In our scenario, we know the vertical force component (V) is one-half of the overall downward force (weight), assuming each leg provided equal support. If we also know the angle of the leg then we can draw the remaining portions of the triangle and solve for the hypotenuse (i.e., the resultant force) using trigonometry or a graphic method.

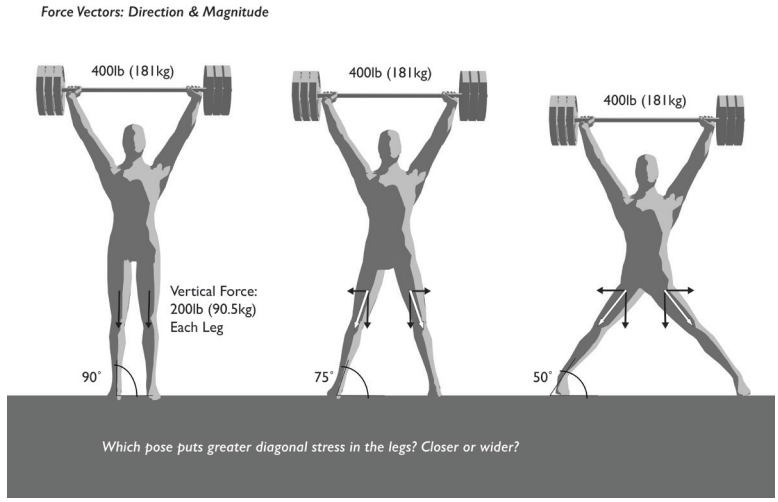


Figure 1.0.19 What are the consequences of a wider or narrower stance when supporting the same vertical load?

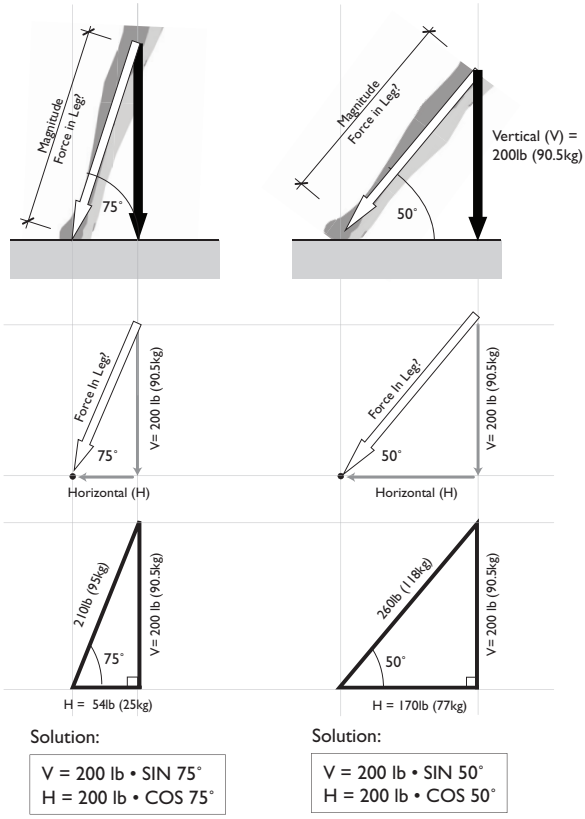


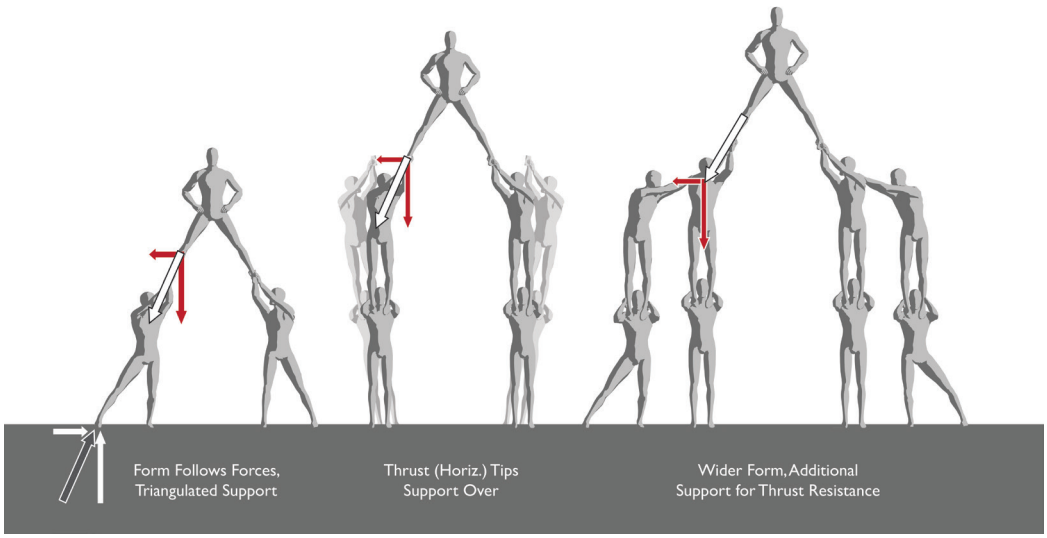
Figure 1.0.20 Mathematical and graphic analysis helps us evaluate the results

Because the length of our vectors represents the magnitude of the forces, even without solving the exact forces, we can determine that the angled resultant force within the leg will be larger than the vertical force. A wider stance has more diagonal force and more horizontal thrust. As a result of our calculations, we can see that the resultant force in the leg is nearly 30% higher. (Figure 1.0.20)

Horizontal and Vertical Forces: Moment Force and Thrust

So far, we've shown downward acting forces as single point loads and vectors, but not all structures or bodies are supported this way. Multiple points of support and/or angled support (legs), may be necessary to span farther, which changes the vector directions. An A-frame structure with a person in the middle doesn't add any more weight or enclose more space below, but it comes with a consequence (please don't try it)— an outward thrust.

Drawing the forces, we notice that the vector arrows are no longer acting directly downward, but at an angle. Vector mechanics can help us see the consequences of these angled forces. The legs are diagonal so the forces being transferred down are no longer



Figures 1.0.21 Taller and wider structures create the potential for overturning due to horizontal "thrust" forces

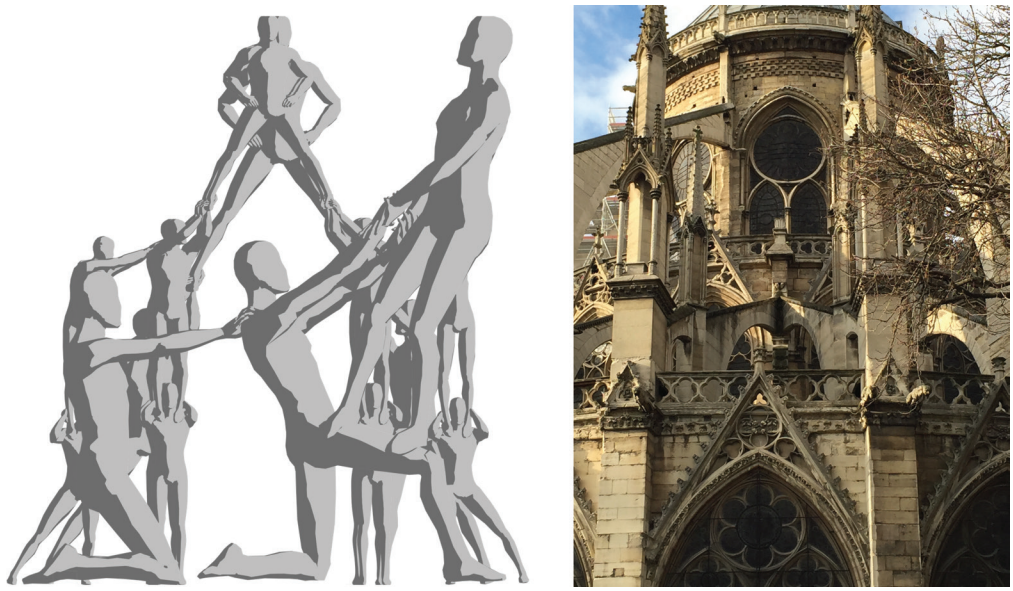


Figure 1.0.22 Body poses, like gothic structures, need various sizes and orientations of bracing for stability

just vertical—they also push out horizontally. A person would be aware of this added horizontal force, or *thrust*, if the ground were slippery. (Figure 1.0.21)

This horizontal force would also have consequences if we decided to add more people to this theoretical arrangement to make it taller. If this horizontal force pushes outward at a certain distance above the ground, it threatens to overturn the supports. The governing threat is no longer just the vertical forces, but the horizontal forces and the overturning "moment force" they are creating. A *moment force* is defined as a force multiplied by its distance from the point of support (or lever arm); the taller the structure or greater the force, the higher the moment force. In this scenario, the wider the legs of the middle person, the higher the horizontal force, and therefore the higher the overturning moment force.

The challenge "reaching high" while spanning requires adjustments in the structure's physical form—namely spreading out the load across a wider base and bracing from overturning. Additional buttresses (people) could be added to the outside and linked horizontally. This basic body structure, one crafted from trial and error of personal experience supplemented by structural properties gives us a human version of a gothic cathedral. (Figure 1.0.22)

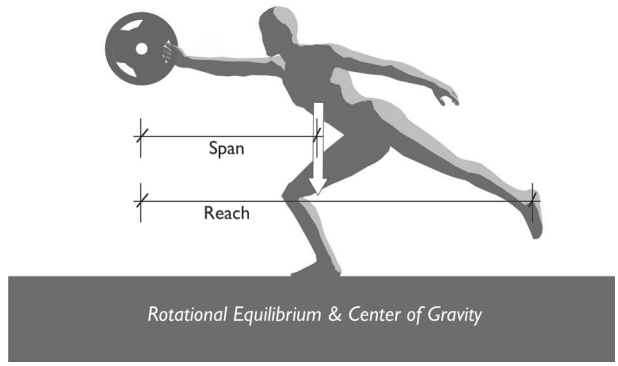
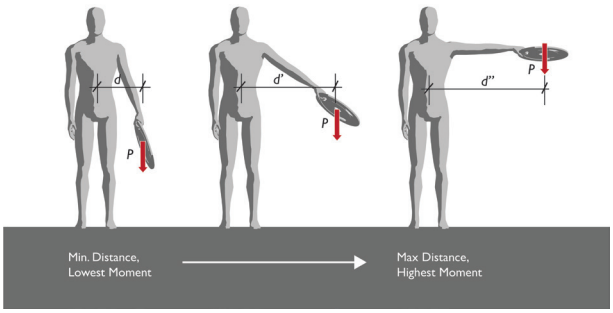
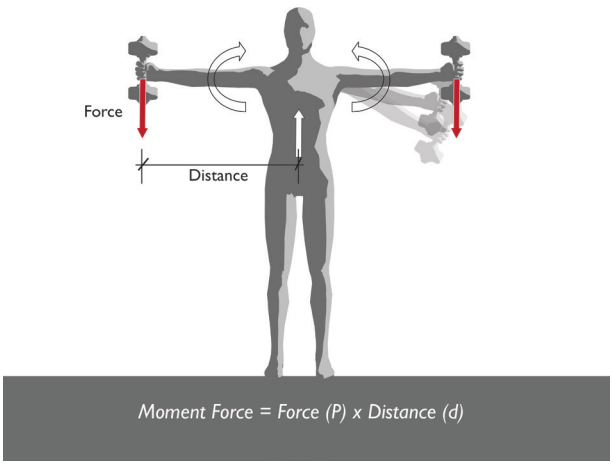


Figure 1.0.23 Balancing loading and support with a one-person pose

Design Assignment: How Far Can You Span or Reach?

One-person pose: What is the farthest you were able to reach from a point of support? Standing in one spot and reaching and leaning will move your *center of gravity* out past the point of support and tip you over. To maintain *rotational equilibrium*, balancing





Figures 1.0.24a and 1.0.24b The load (P) is the same but resisting moment force (M) differs because of the varying distance from the support

the forces on the either side of the support, you could balance on one foot, reaching out with your arm while kicking your leg backwards, balancing into a T-shape (a common approach for long-necked and tailed dinosaurs). As long as you stabilize the pinned joints in your ankle and hip, this single leg pose should give you the longest overall reach from one point of support. Or, if you are able to find a wall or a column to hold and secure your feet against, you can lean out and reach from one point of support by using your body as a cantilevered triangulated bracket. (Figure 1.0.23)

When a structure spans between supports it bends in a specific manner (compressing on top and tensing on the bottom) as it transfers loads. But it feels different if you just reach out, especially with a weight in your hand. Your arm isn't acting like a typical beam that spans between two supports because it is cantilevered from your shoulder. What you are feeling is another expression of a moment force. Moment force is the product of a force (P) times a distance (*d*, known as a *moment arm*), from a fixed reference point. As you move the weight and your arm away from your hips it gets harder because you are creating a longer lever arm between the weight and the support. Your shoulder muscles have to resist this overturning force. If you could add a diagonal element from the end of your arm to your hip, you'd resolve the forces without an overturning moment—you'd also have mimicked the solution for a typical cantilever shelf support. (Figure 1.0.24)

BREAKING: CONFIRMATIONS AND CALCULATIONS

When a force turns/rotates an object, a moment force has been applied. Moment forces are found in common tools like levers or wrenches in which the effort needed to move an object is reduced by the force multiplier of the distance between the force and the object.

To Do: Raise a modest weight in your hand (5lbs (2.27kg)) with a straight arm, using your shoulder to move it upward from your hip towards a horizontal position. Calculate (and diagram) the moment forces for at least three stages of this activity. Where do you feel the stress the most? What portion of your body is providing the resistance? *Diagram It*: What would the effect be if you doubled the weight?

To Discuss: Find examples of structural forms that are like these double-cantilever "reaching out" poses and describe how they are stabilized against these stresses. (Figure 1.0.25)

(continued)

(continued)

Figure 1.0.25 Expressions of structures reaching out: Madrid-Barajas Airport Terminal 4 (2006) and abandoned service station, near Canyonlands Utah

MATERIAL MATTERS

Instead of the static or monolithic structures we find in buildings, our bodies have a series of connection points that allow us to move. Our ability to remain rigid under loading is a function of our muscles. But the form of skeletons, and even the bones themselves can reveal information about structural design.

To Do: What sorts of differences can you notice about the skeletons of different animals when you compare a small four-legged creature to a large bi-pedal one?

To Discuss: What types of structural systems are certain bones analogous to? Consider the how the shape and form of the femur, collar bone, and skull respond to the forces they are resisting. When skeletons are assembled together, the heaviest/biggest bones need to be positioned where they would resist the most weight; how does this logic affect the skeletons of different species? (Figure 1.0.26)

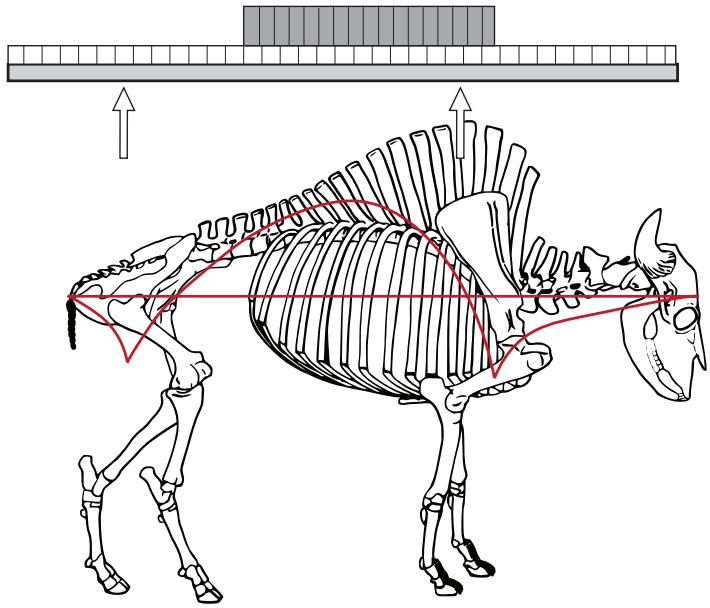
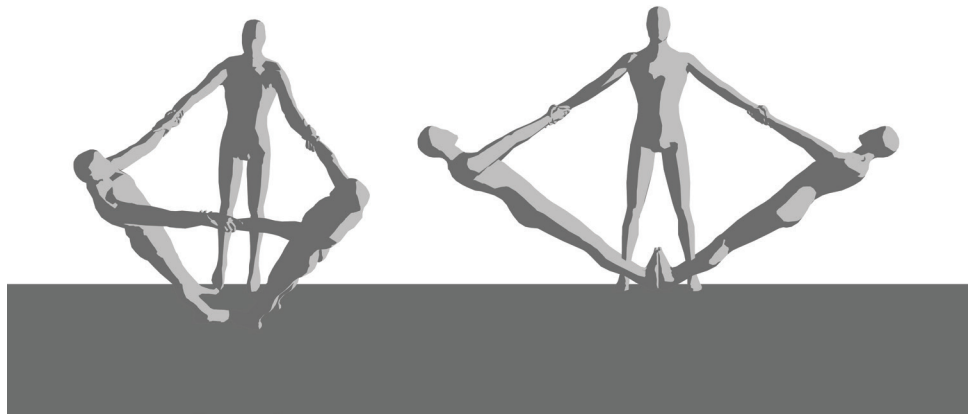


Figure 1.0.26 When the dead load of the buffalo is graphed like a beam, the shape and depth of the bones correspond with a diagram that shows bending intensity. Internal stresses are resisted by biological form (from Thompson's *On Growth and Form*)

Rotational Equilibrium, Brackets and Arches

Two-person pose: If two teammates reach out in opposite directions, joining hands horizontally and placing their feet together (to make a triangular support) they create the longest possible reach from a single point of support. In practice, adding a second person to this exercise complicates the ability to maintain rotational equilibrium if the two teammates aren't equal heights or weights, since the leaning out pulls the body structure out of equilibrium (i.e., the force arrows are no longer equal and opposite). Teams usually alter their form to account for these differences (typically without being conscious of these choices) by not leaning out as far or by lowering their stance. (Figure 1.0.27)

A third person can be added in the middle of this arrangement to increase the overall reach even further. This type of double-cantilever arrangement is common in bridge designs where a central "tower" holds up equally weighted elements to the side. The most famous "prototype" of a double-cantilevered scheme came from the designers of the Forth Bridge in Scotland in 1887, by Sir John Fowler and Sir Benjamin Baker. They re-enacted that structural form using their bodies, a Japanese engineer in the middle, and a handful of cricket bats. Compressive



Figures 1.0.27a and 1.0.27b Differences in height and weight need micro-corrections in the geometry of the form (arms, hands, and hips) to maintain rotational equilibrium

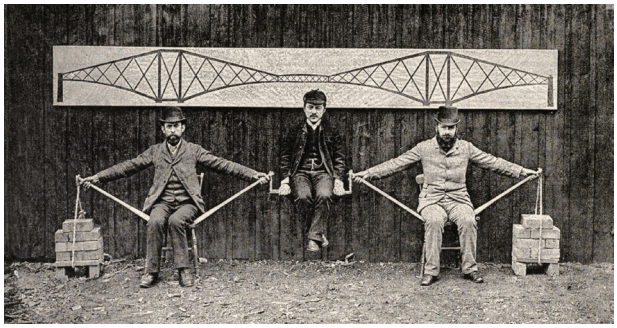


Figure 1.0.28 Postcard depicting the double-cantilever structural strategy of the Forth Bridge, Scotland

elements and tension elements combined with counterweights kept the human structure in equilibrium. (Figure 1.0.28)

Two people can also lean in and push against each other to form an arched shape. The force vectors in the double-cantilever and arch pose are similar. In the double-cantilever, the horizontal thrust force at the feet are counteracted at the same point, but when the feet are separated to make an arch, the horizontal thrust has to be resisted from outside. This is difficult to stage unless your heels are against another surface because of

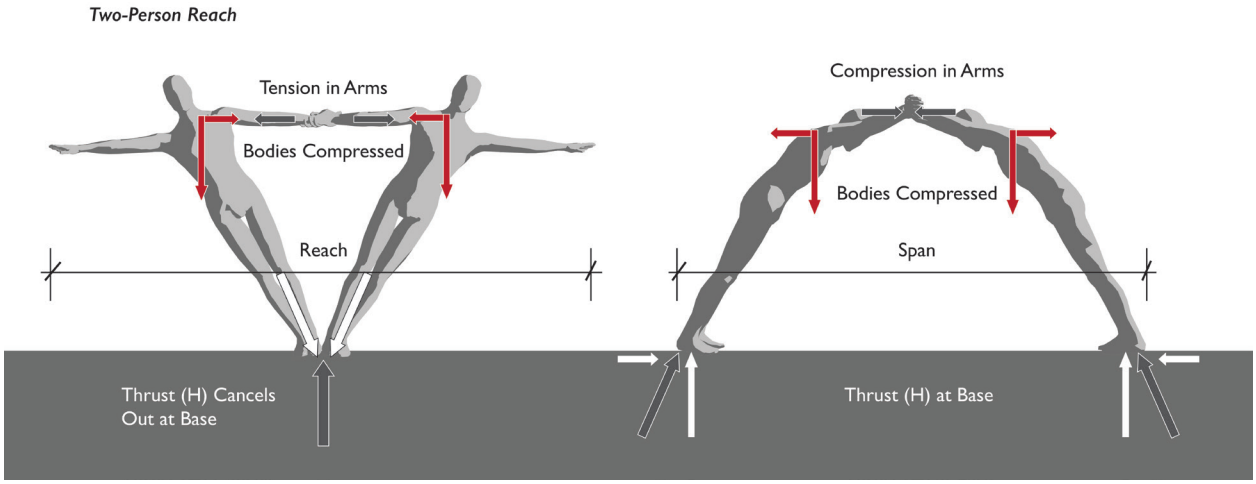


Figure 1.0.29 Two-person reaching poses; different forms create different stresses

the limitations of our hinged ankle. As with any arch, outward thrust at the feet is a concern, particularly if the two teammates aren't the same mass and height. (Figure 1.0.29)

Longer Spans: Transitioning from Beams or Cables

Three-person span: Besides forming an arched shape, there aren't any other viable ways to make a two-person body structure that spans between two supports. Because our bodies aren't rigid, we can't splice ourselves together to make one long plank pose without being supported in the center. A third person will have to volunteer their body to act as spanning member. But again, the limits of our bodies as non-rigid elements restrict our ability to act like beams. There is no difference in clear span between doing a plank pose on our own and doing the same pose while being held by two others—as we extend our arms, resisting bending stress in the shoulders and abs becomes incredibly difficult. (Figure 1.0.30)

We know that joints and hinges in our bodies will bend easily when loaded perpendicularly (in bending). But if we are pulled apart parallel to our body, our joints become taught, like a chain, resisting tension. If the middle person in the three-person plank extends their arms and legs, the two supporters can lift them off the ground if they pull with enough upward and outward force. It helps to also lean back. The success of structures that use pure tension or compression in their spanning members depends on finding the right form. This strategy, one that many teams will arrive at intuitively, was used by Eero Saarinen in one of his most celebrated designs, the Dulles Terminal building (1958–1963). (Figure 1.0.31)

Because the stress in the spanning body is tension and not bending, more bodies can be theoretically added to the middle as long as they can be securely linked together. If you try this, you'll notice two things. First, the tension in the bodies becomes so great that it pulls apart at the weakest link in the "chain" of bodies—the hands. Second, the additional weight in the middle requires even more people to pull outward just to lift the bodies off the ground. In long span bridges with geometries that rely on hanging cables, this outward force is generated from other cables that are pulled to each side and secured into the ground. The vertical supporting elements, or towers, in these cases only resist the vertical forces in compression. (Figure 1.0.32)

Intrepid students may wish to try out one final pose. In it, three people lie on the ground in a triangular form with their heads/arms towards the middle, and they link their arms together (hands alone won't be strong enough to resist tension). Each person is pulled backwards by their feet and the entire structure lifts off the ground. Theoretically more and more bodies can be added to the pose to complete a full circle. Ideally everyone would hold onto a "tension

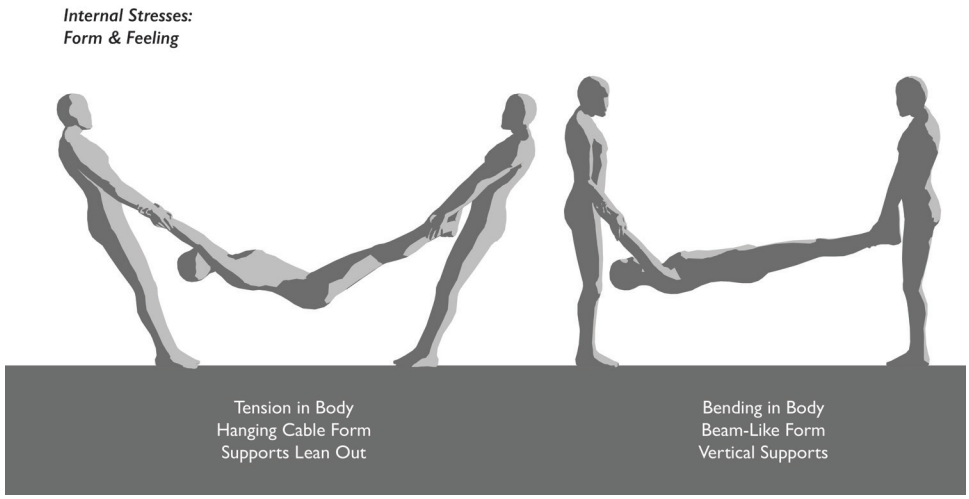


Figure 1.0.30 The same span and supports create different internal stresses, depending on the geometry of the spanning body. The straight body has bending while sagging creates tension

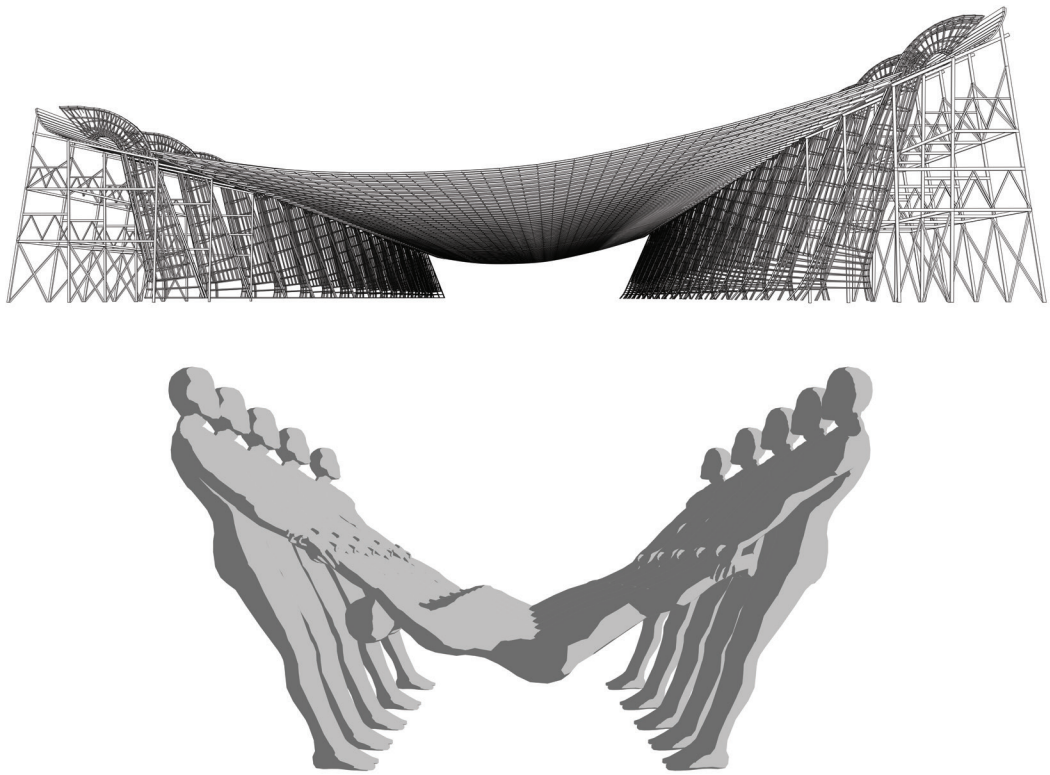


Figure 1.0.31 The Dulles Airport Terminal formwork and corresponding body pose

ring" in the center to prevent it from pulling apart AND everyone on the perimeter would be linked with a rigid compression ring to keep them from falling inward. This cable-based structural system has been used in long span arena projects like Madison Square Garden. And yet, even students can feel and understand the first principles of structural behavior that this design is based upon. (Figure 1.0.33)

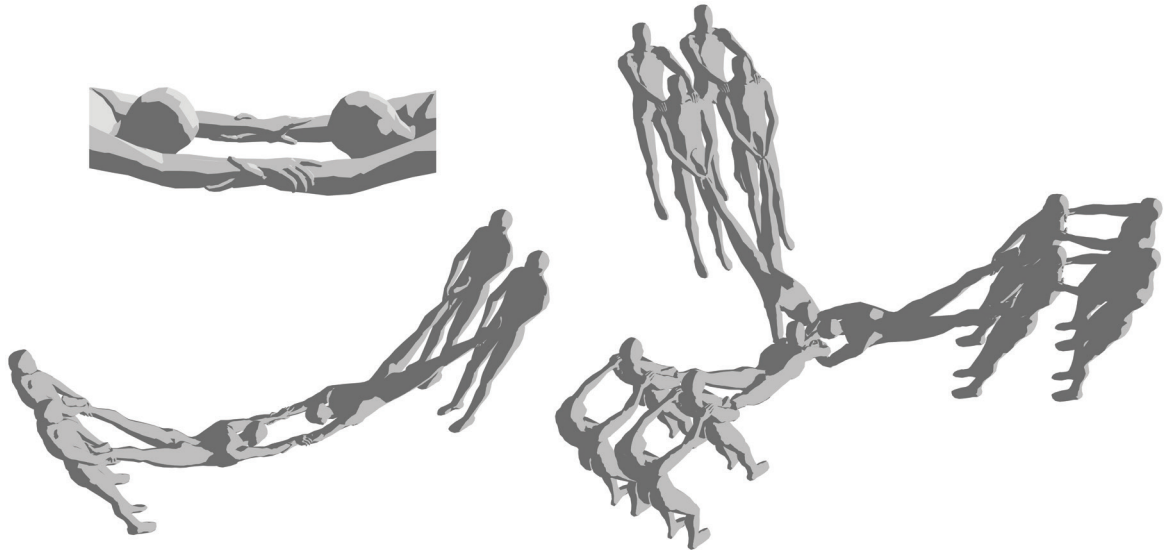


Figure 1.0.32 Multi-person poses reveal the intensity of stresses in connections

Figure 1.0.33 When an outer compression ring, hanging cables, and inner tension ring are balanced, long spans can be achieved—in bodies and buildings. Shown: New York State Pavilion, 1964 World's Fair

BODY POSES AND STABILITY

Our bodies are useful tools in helping us to understand the relationship between forces, stresses, and form because we intuitively find ways to adjust our bodies to minimize stress. Our bodies also change form to keep from tipping over. As we learn to design structures that are stable, we'll find that stability is created by adjustments to physical form. We can use our bodies to recognize strategies for stability.

To Do: Stand up and have a teammate gently push you forwards and then side-to-side at the shoulder level. Next, make adjustments in your physical form to resist these forces (if possible). What changes did you make?

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To Discuss: If you were pushed elsewhere (waist, knee-level, etc.), what effect would this have on your ability to resist overturning (would it be easier or harder)?

Hint: Think about moment forces. (Figure 1.0.34)

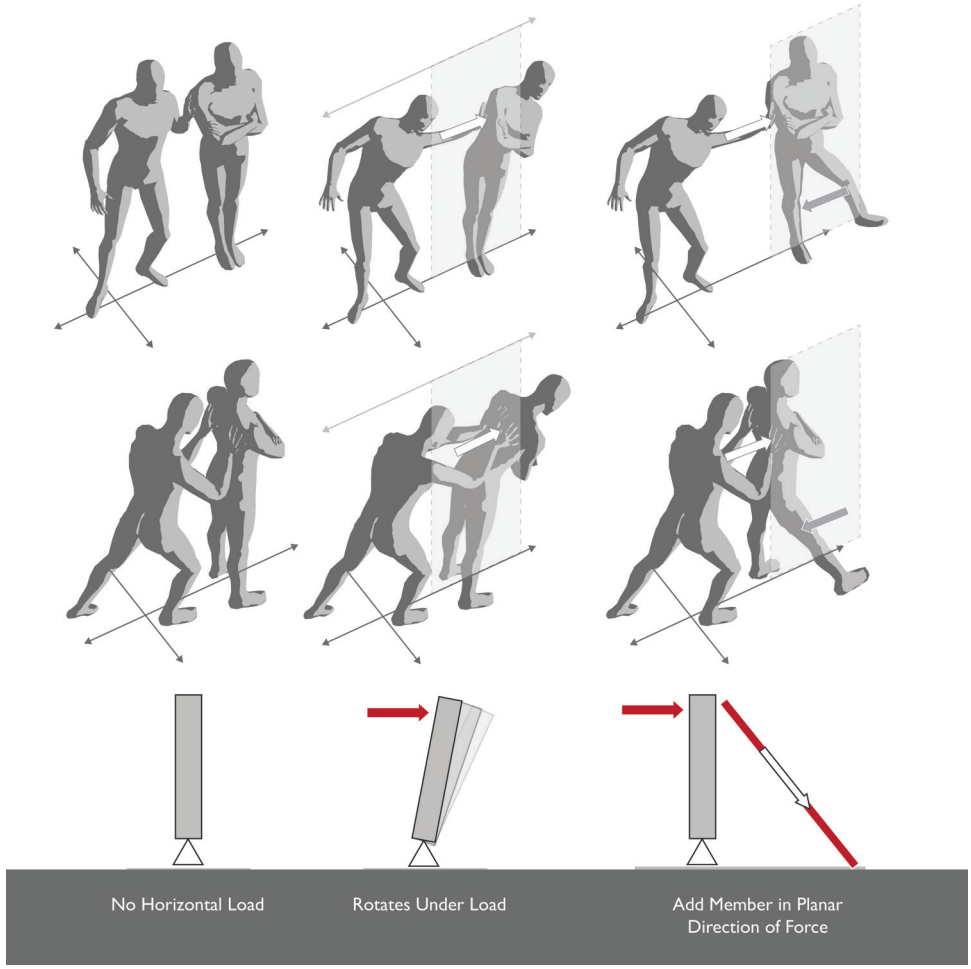

Figure 1.0.34 Because we have hinges throughout our body, we can't stand firm against lateral forces. Resistance needs to be provided within the same plane as it is applied

A Tool for Testing Structure and Form

There are many types of structural conditions that you'll understand by visualizing them or acting them out physically. In one class lab you've learned about forces, been able to feel and understand the difference between types of stresses, understood how equilibrium is necessary to support other elements, and how basic acts of stacking and spanning require cunning to match body position with forces. Ultimately, the single largest determinant for the types of stress you felt, and how easy or hard each pose was to hold was related the structural form because your structural materials (muscles and bones) stayed constant.

These "body structures" and the lessons they illuminate are tools to use throughout your studies. For conditions that defy the bounds of your body, you can still use these experiences to understand stresses and structural behaviors you've learned

first-hand. Learning to translate these experiences into free-body diagrams and vectors will be a way of documenting structural conditions, testing (or calculating each condition), and improving your structural proposals.

Frequently Asked Questions

What is the benefit of learning methods of elemental analysis techniques like free-body diagrams? Don't structural engineers simply calculate everything using advanced computer modeling systems? Digital models are often used to confirm building calculations, but the majority of an engineer's design work is still based on essential concepts of structural behavior. Diagramming force vectors is an essential building block tool because it shows the relationship between forces, stresses, and responsive forms. Digital models and calculations rely on this basic knowledge.

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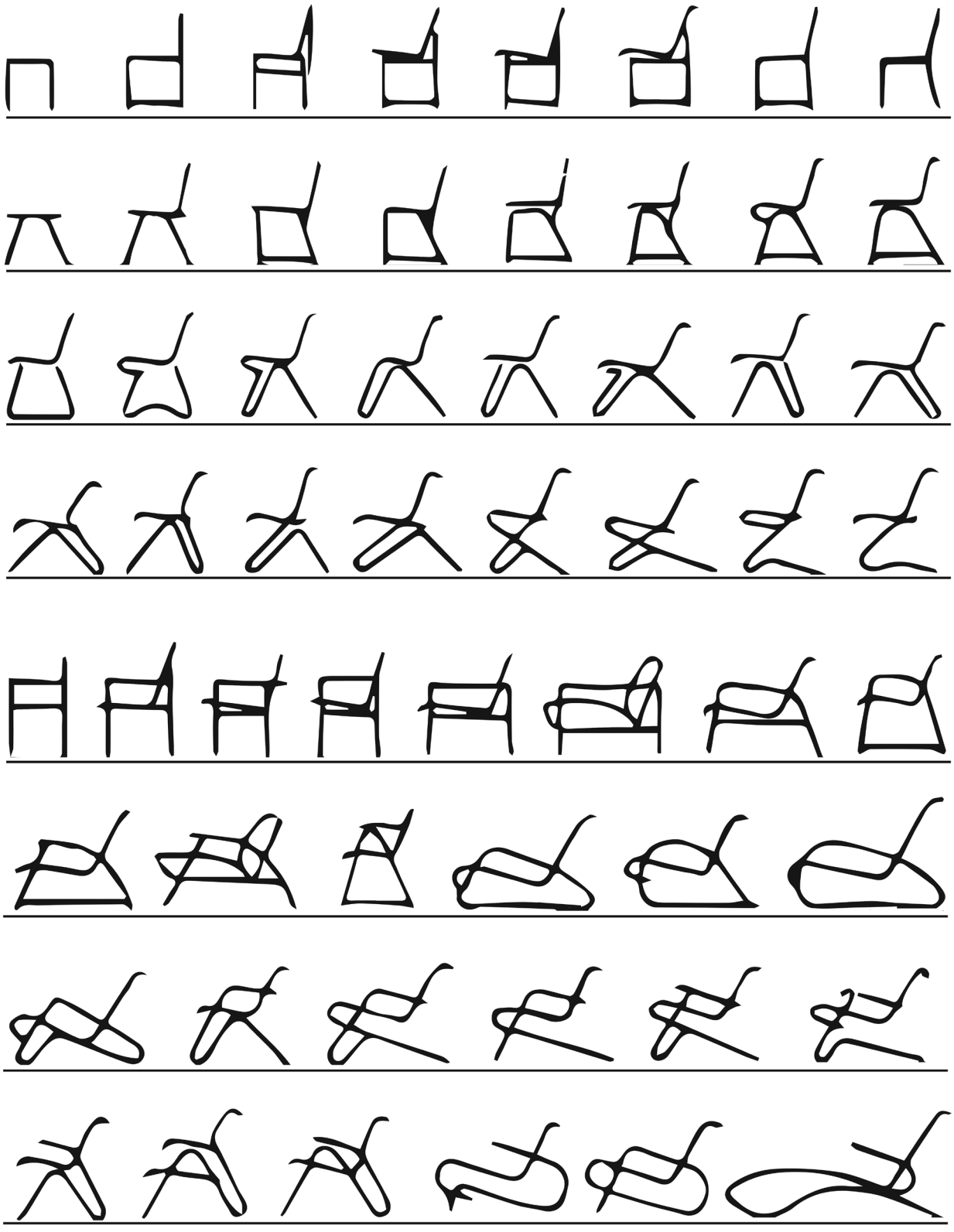


Figure 1.1.0 Illustration based on Erich Dieckmann's "Design of a Metal Tube Chair," Bauhaus, 1931

CHAPTER 1.1

Seating Structures

Elemental Design Options

Good design emerges from a creative conjunction of structural integrity and functional considerations. There are many ways to create a structure that merely stands firm—particularly for a modest challenge like a chair—so in these cases, designers will need to know how to discern what makes a "good" design. One possibility is to use the formative qualities found in structural and material principles to guide design decisions.

Useful Constraints: Structures as a Design Generator

The transition between learning structural principles and applying them is fraught with indecision and uncertainty. Structures may be seen as right and wrong answers, whereas design is intentional exploratory. It's unnecessary to distinguish between two activities—*creation and evaluation*—that are essentially linked. Structural design *is* design. It has a more defined range of constraints and considerations, but all useful design exercises do, too. Structural design is simply an exercise in problem-solving that considers how the form and materials can be combined to provide support.

In this chapter, we'll solve a modest structural problem—seating. We've used our bodies as a structure and now we'll make structures to support them. We'll see how basic structural principles can be used to develop and assess our work. It isn't as easy as it sounds. Mies Van Der Rohe describes the chair as, "a very difficult object. A skyscraper is almost easier." By the end of the chapter we'll see how structural form, materials, and the means by which forces are transmitted can be classified into groups that will be useful for our ongoing work. (Figure 1.1.1)

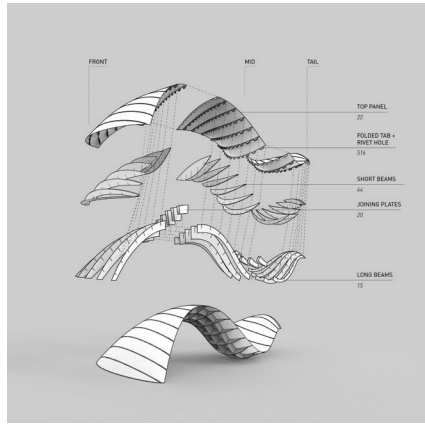


Figure 1.1.1 Aluminum Monocoque shell seating (Nate Peters and Lara Tomholt, Hybrid Formations Studio, GSD, 2017)

Design Assignment: Seating Structures

A project team has been commissioned to design an open-air observation pavilion at the end of the Keys View trail at Joshua Tree National Park in southern California that will provide shelter and seating options for visitors. The team has proposed an open-air concrete pavilion set atop (and spanning over) a ridge with views on all sides. The Park Service would like to offer choices for comfortable seating throughout the observation areas, either for groups or for individuals. The total number of seats may surge with the changing seasonal traffic of visitors (ranging from a minimum of ten seats to thirty).

The initial sketches are intentionally unrefined and open-ended. They show one type of seating fixed atop the slab and another hanging below the roof—roughly classified as "Stones" and "Strings". Other options for seating are included in the sketches with conventional and unconventional forms. Ultimately, the form and materials are required to be responsive to user comfort and structural performance, and the design should be informed by the long-standing tradition of experientially rich, environmentally considerate, and technically proficient structures found throughout U.S. National Park system. *Simply put, the seating should be effective, efficient, and expressive.* (Figure 1.1.2)

Beyond Form and Function

How should we begin? George Nelson suggested that, "Every truly original idea—every innovation in design, every new application of materials, every technical invention for furniture seems to find its most important expression in a chair." We can't simply create a form that matches the required function shape because there are so many different ways to accommodate the act of seating. But we can gather information about the dimensions of the body at rest to determine how best to support it.

Anthropometric and ergonomic sources provide important information about the acceptable ranges of heights, depth, and angles of repose for seating. But this data is broad and interpretive. Body sizes vary and what constitutes "comfortable" seating is subjective. Ergonomic professionals argue over optimal seating

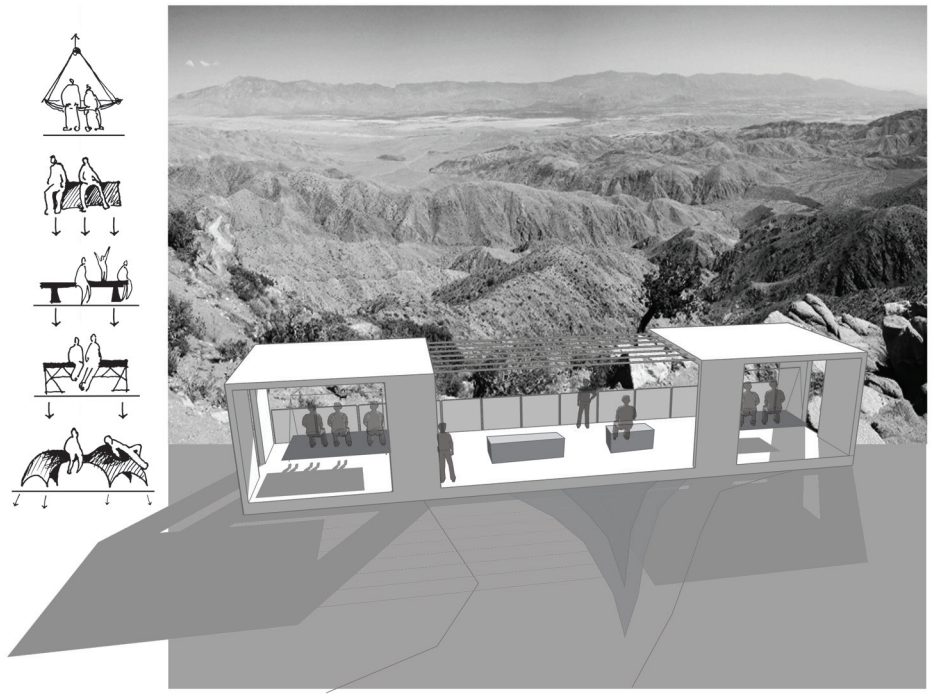
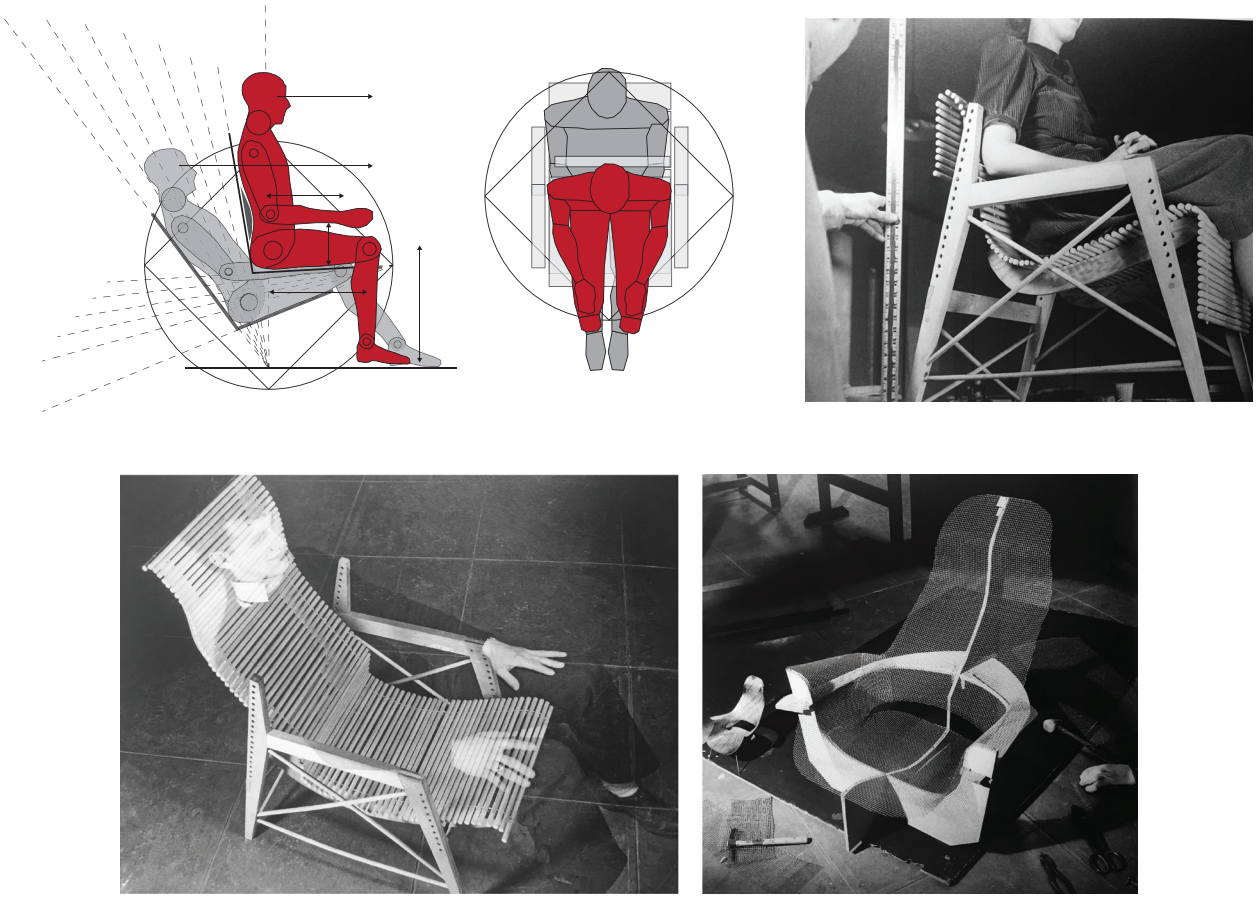


Figure 1.1.2 Proposed massing of pavilion and initial sketches for types of seating



Figures 1.1.3a and 1.1.3b General ergonomic diagrams of seating angles, heights, and geometries. Seating experiments for the Organic Design in Home Furnishing competition by Charles Eames and Eero Saarinen at Cranbrook Academy, 1940

postures (or if there even is such a thing), especially for casual seating. There are few objectively "right" answers to be found in this data, but most research concludes that comfortable seating provides users with *options* for how and where to sit. Perhaps this explains the breadth of seating design options that have developed across time. (Figure 1.1.3)

Ergonomic information will only take us so far. Seats are essentially just small elemental supports, so perhaps we should start with structures.

THINK: STRUCTURES AROUND YOU

Chairs are perhaps the most pervasive of all structures around us. By supporting our weight, they solve a simple task, and yet they accomplish this in many different ways. Consider the number of historically important precedents for chair designs, particularly those that evolved simple, utilitarian seating structures into innovative objects of design by embracing the relationships between forms, materials, and construction methods—an enduring set of considerations that are still relevant today. (Figure 1.1.4)

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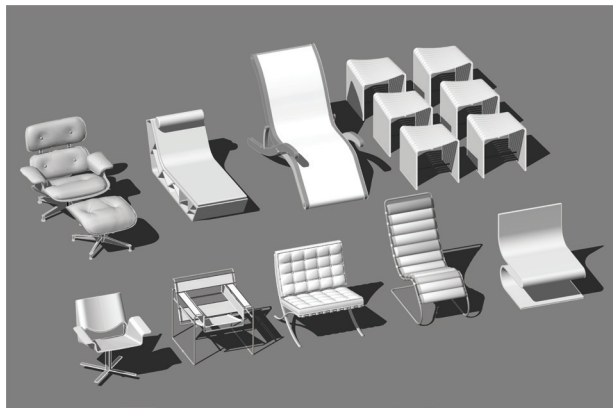


Figure 1.1.4 Various styles, sizes, and structural strategies for seating including works by Eames, Aalto, Breuer, Van der Rohe, etc.

To Do: Before we begin designing, take the time to identify at least two different (and interesting) options for seating designs. First, collect the basic information: What are its dimensions? What materials are used? How does it provide support in the simplest terms?

To Discuss: What tools would have been needed to create and eventually test this proposal? What was the design strategy for combining materials and support?

We'll continue studying these options throughout the chapter in more detail.

Structural Basis for Evaluation: "S-Words"

Seating would be an easy problem to solve if your only concern was structural viability—the loads are light and the spans are small. Seating structures need to keep people from falling down and/or tipping over, but nearly any common object can provide this level of support. We'll focus on the challenge of creating *better* structural options. The word "better" may imply a subjective standard, but it need not be.

A structural basis for design allows us to develop information about the forms and materials of the seating, and to determine if these proposals are more *effective or efficient* than other potential options. This means that we'll need a structural basis for evaluation—common criteria that we can use to assess the different qualities of structural options.

We'll use four "S-words": *Strength, Stiffness, Stability, and Suitability*. These words are useful because they focus our assessment on the two central considerations of structural design: *Materials and Form*. Strength and stiffness are measures of material qualities while stability and suitability result from choices about form and configuration. (Figure 1.1.5)

Each of these S-words can be assessed separately as a structural element undergoes loading, but initially they should be evaluated together to get a broader view of overall behavior. As an example, a pyramid isn't *strong* because of its form; it is strong because of the stone that makes it up; it is *stable* because of the form. Compliance with one factor doesn't guarantee compliance with others. Consider a two-legged chair; even if the legs were strong enough and stiff enough, it wouldn't be geometrically stable and therefore it wouldn't be *suitable*. (Figure 1.1.6)

Materiality: Strength (Stress) and Stiffness (Strain)

Determining if a material is strong enough seems like a straightforward question: "Can it support the

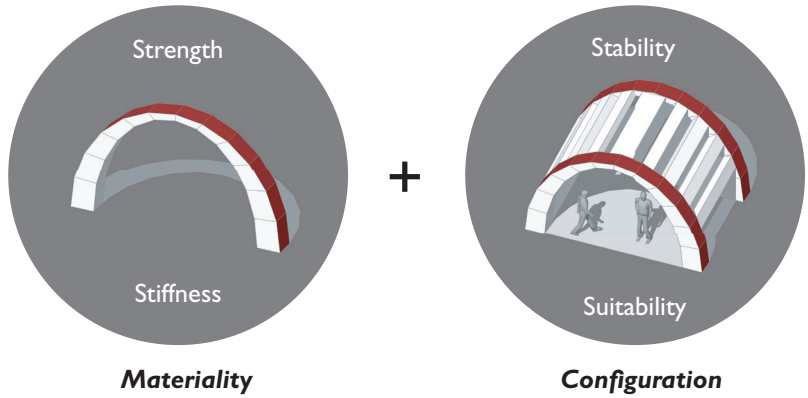


Figure 1.1.5 Material qualities and configuration tactics are assessed differently

loads placed on it or not?" To determine strength, we determine a material's inherent capacity to resist loads applied to it. Different materials will have different capacities to resist stresses (e.g., steel is stronger than wood). But strength is not a universal measurement for all conditions the structure may endure.

As you'll recall from the body structures, there are five types of stress: *Compression, tension, shear, bending, and torsion*. Not all materials have the same "strength" under all types of stress. Stone, for example, is stronger than wood under compression, but it fails easily under tension and therefore has no reliable tensile strength. We define strength as the *allowable stress* of a material, a property that is listed in tables according to the type of stress—usually compression, tension, and shear. (Figure 1.1.7)

Stiffness is a measurement of how much a material deforms (or strains) under loading. When stressed, *all* materials change shape—perhaps imperceptibly. It helps to imagine the molecular qualities of each material as a three-dimensional grid of springs that are pulled apart or pushed together. All matter is springy, and when materials are subjected to stresses (loads) their "springs" become shorter or longer. Because this relationship between a material's strength and stiffness is inextricably tied together,

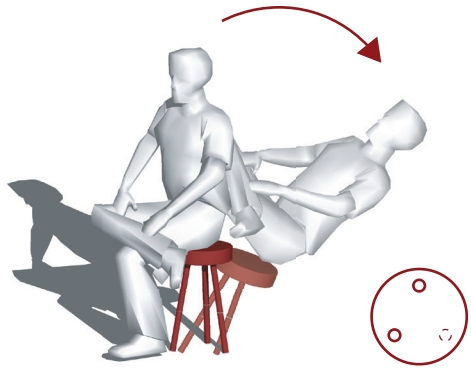


Figure 1.1.6 Be mindful of the critical threats for failure that render all other structural criteria immediately obsolete

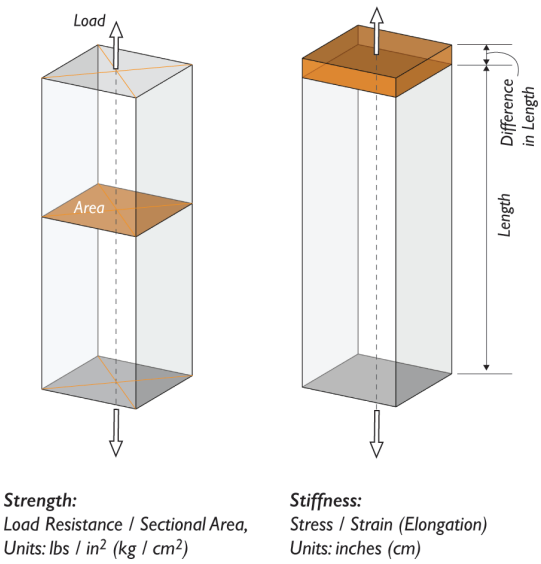


Figure 1.1.7 Stress is what is felt internally when a force is applied to an object. Strain is what happens *as a result* of the stress. Compression (stress) shortens (strains) an object. Stress and strain values vary by material

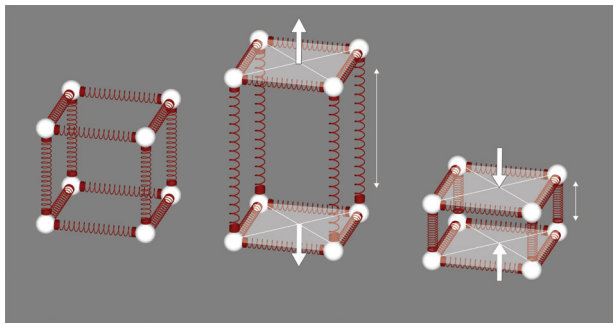


Figure 1.1.8 All materials are "springy" which allows us to visualize relative stiffness in tension and compression (measured as the Modulus of Elasticity)

we can plot this relationship on a stress/strain graph to understand the relative behavior of materials under loading. There is a directly proportional relationship between the applied loads and the amount of elongation (or strain)—doubling the load will double the elongation. (Figure 1.1.8)

Like an actual spring, when the load is removed the spring may return back to its original length—unless the load exceeds a certain capacity, in which case the spring will be permanently deformed. All materials have a capacity for loading up to which they'll return to their normal/original length when the load is removed. This is known as *elastic behavior*. Up to the *yield point*, the material will behave elastically, but stressing a material beyond this point will permanently deform the material—this is known as *plastic* behavior. If the material is subjected to additional stress past its yield point it will behave erratically and will fail at the material's *ultimate strength* value. (Figure 1.1.9)

We rely upon empirical testing to know how much stress a material can safely endure and still behave in a predictable way without undergoing permanent deformation—this value is a material's *allowable stress capacity* (f_a). Because we don't want structural elements to permanently deform under loading, we

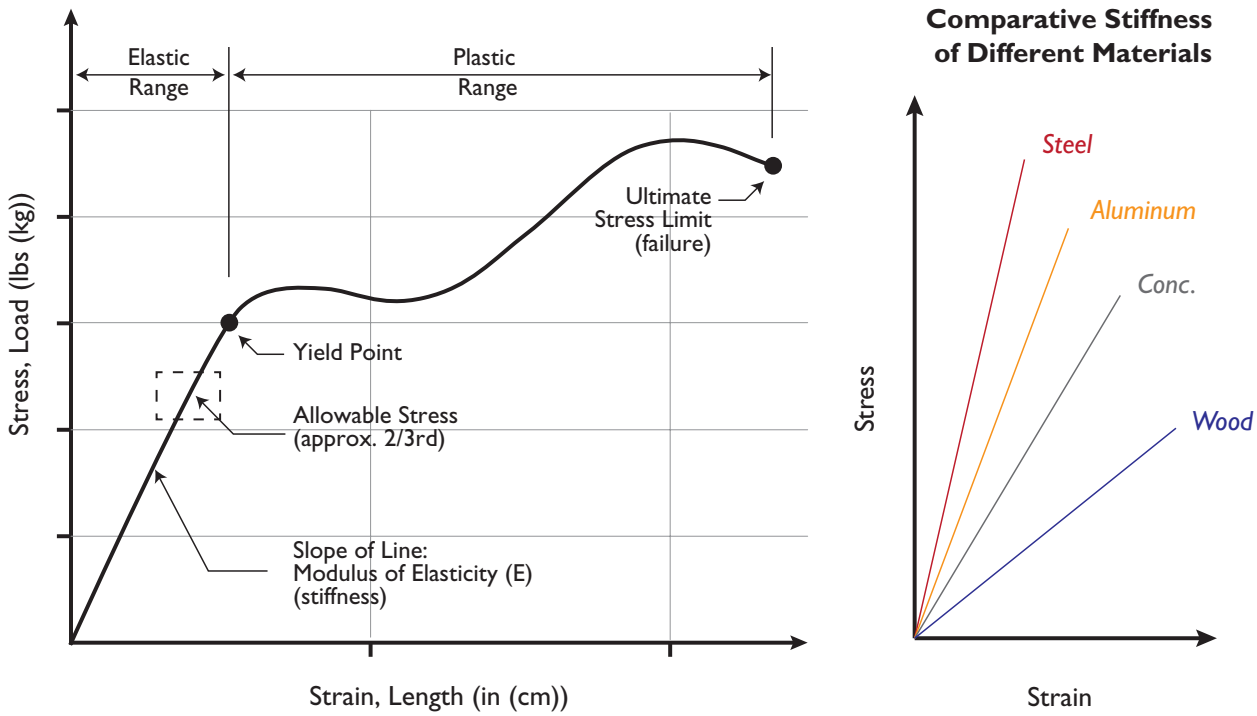


Figure 1.1.9 Material behavior graphed as a stress/strain relationship. Stiffer materials have steeper line slopes. Behavior changes past the yield point to become unpredictable and permanently deformed

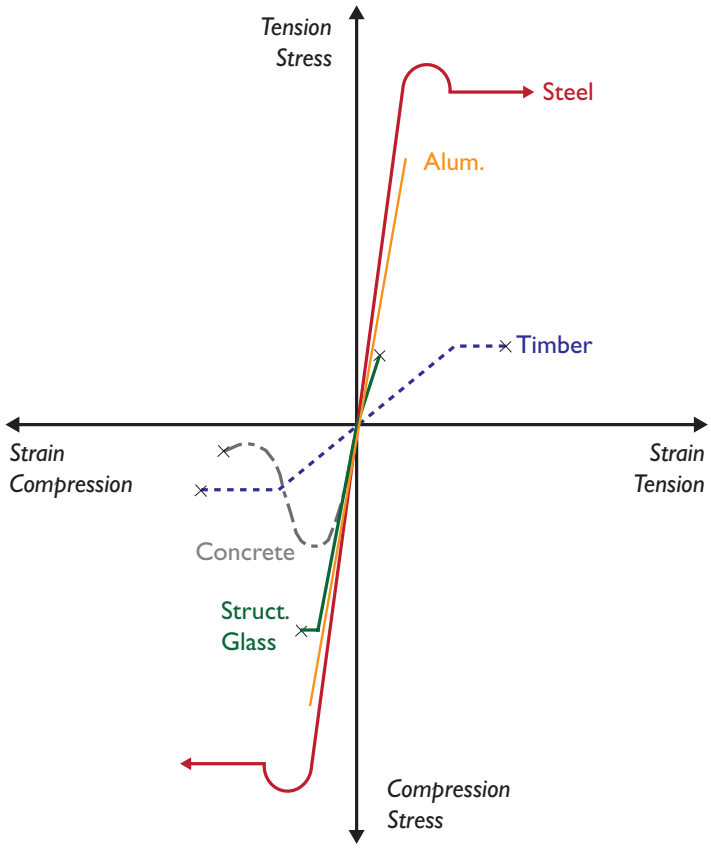


Figure 1.1.10 Stress/strain graphs vary by material and type of stress

use an *allowable stress value* (f_a) that is intentionally restricted to around ⅔rd of the total yield capacity of each material. All of these material behavior factors are combined in a graph showing stress/strain and listed in material charts. (Figure 1.1.10)

Table A.4 in the Appendix lists various material properties: Density, allowable stresses in bending, tension, compression, and shear, and thermal conductivity.

Configurations: Stability, Suitability, and Shape

Resistance against gravity isn't our only concern—if it was, a two-legged chair would be possible. We also need to worry about *stability*. Stability is easy to visualize. We've all sat on loose-framed chairs that shift from side-to-side or that easily tip over. How the legs connect to the seat, and to each other, is an important factor in creating stability in a chair. These wooden chairs are stabilized by joints that are connected (keeping them rigid and not allowing the chair to become a parallelogram instead of a rectangle) and with the cross-bar footrests at the base of the legs.

Stability is achieved by fixing points in space in equilibrium to each other. Equilibrium is dependent on finding ways to resist potential movement in the x, y, and z axes. Stability is achieved by fixing these points in space either through rigidity between horizontal and vertical connections or by introducing an element that fixes them through triangulation—ideally within the same x, y, or z axes plane as the applied force. (Figure 1.1.11)

Without adequate strength, stiffness or stability, the chair won't be *suitable*. The craft of determining whether or not a seat is suitable can be parsed with distinctions of comfort, but for our purposes, as long as we aren't proposing something obviously absurd

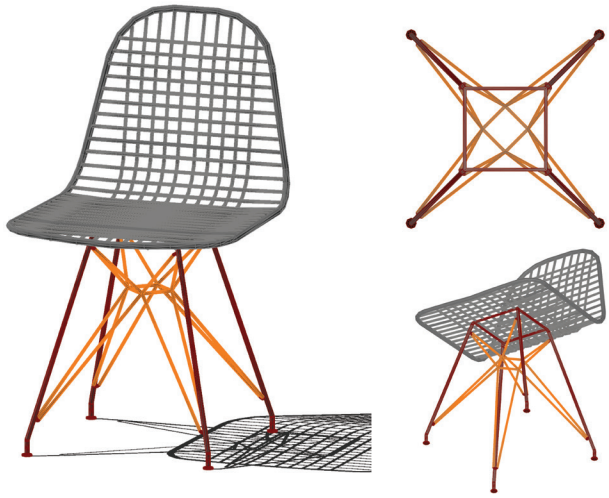


Figure 1.1.11 The stability of an Eames chair is central to the design. Support legs are stabilized bi-axially and symmetrically with triangulated tubes

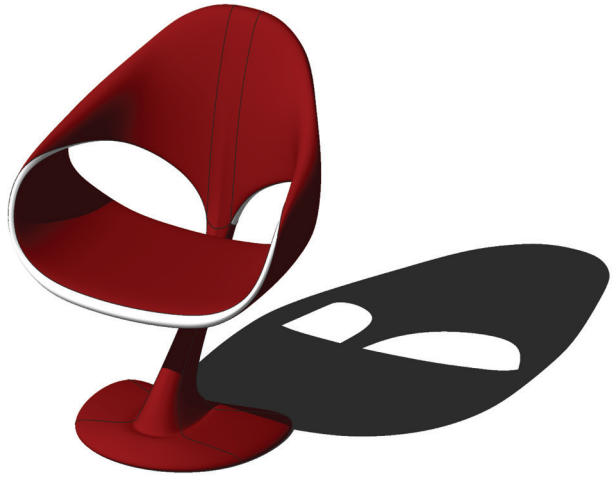


Figure 1.1.12 The Whale Chair uses one continuous surface as the base, support stem, chair back, and seating surface

(like a pointed rock), we'll factor this consideration into our overall strategy. Suitability may involve issues of durability and serviceability as well. Assessing the suitability of structures becomes more complicated than that, but this is a simple place to start.

There is a fifth, more inclusive "S-word" related to structural performance: *Shape*. Including this factor is a reminder that the structure's shape can be intentionally manipulated to direct how forces are resisted while stiffening and stabilizing the structure *and* creating suitable spatial enclosures. Consider how important the role of shape is in transforming an arch into a vault or a dome, or a planar surface into chair. (Figure 1.1.12)

MAKING: SOME ASSEMBLY REQUIRED, DRAWING THREE-DIMENSIONALLY

Structural assemblies are three-dimensional forms that transmit forces through a variety of mechanisms across their volumetric shape. Look closely at the form, connections, and arrangement of elements in the chair you've selected to study and you'll see that the structural roles of certain elements are combined and aesthetically obscured into a single composition (e.g., angled legs that support *and* stabilize the chair).

To enable this spatial flow of forces, structures are frequently assembled from composites of different elements. If the form, behavior, and assembly process all involve three-dimensional considerations, then that's how we'll diagram and analyze the structures we study. All contemporary practice models in architecture and engineering use three-dimensional models, so whenever possible, we'll do the same.

To Do: Make a three-dimensional representation of the chair examples you identified, accurately representing the separate pieces and connections. Diagram how the forces are collected, grounded, and stabilized across and down the structure using arrows.

To Discuss: What are the critical connection points for the chairs? Do these connections transfer loads? Create stability? Both? (Figure 1.1.13)

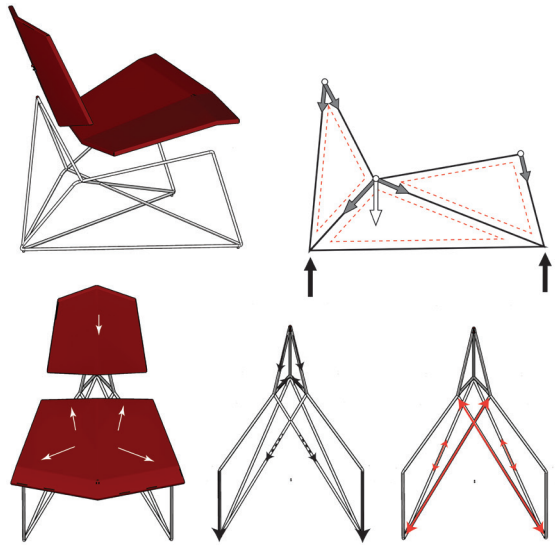


Figure 1.1.13 The Air Chair's seating surface collects the loads and distributes them to a series of triangulated tubes that act as both load grounders and load stabilizers

Resistance through Form and Materials: "Strings and Stones"

Applying this structural basis for design and analysis back to our design problem will depend on understanding how forces move through a structure and how these affect our evaluation of their material performance.

There are two categories of seating proposed—those that rest atop the slabs and others that hang below. Because the goal is to successfully pair a structurally responsive form with materials, it makes sense to start with one solution that is compressed and one that is tensed. No matter how complicated *any* structure becomes, at a micro-level, it is essentially just resisting being pushed and pulled.

The most influential determinant of a structure's behavior within a form is the material. If two identically-formed arches are placed side-by-side, one made of stone and the other with sponges, they will clearly perform differently. One traditional axiom of structural education is: *You can't push on a rope* (qualifier: you *can*, but the rope isn't helpful in providing resistance). Within this statement lies a larger idea that materials should be matched with the stresses they'll endure. Because we know our proposal will either be pushed or pulled, we can select materials that are able to resist these stresses. (Figure 1.1.14)

There is an advantage in limiting the type of stress within a body to either tension or compression—it is easier to resist being pulled or pushed than being bent or twisted. If we wanted to limit stress in the seating to only tension or compression, we'd have to develop forms that resist forces running parallel to the primary

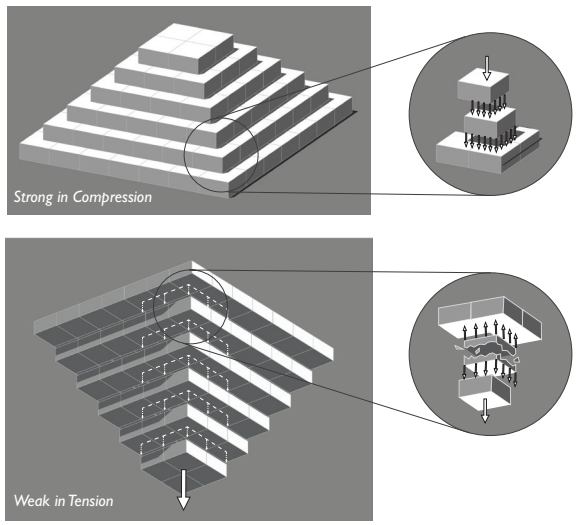
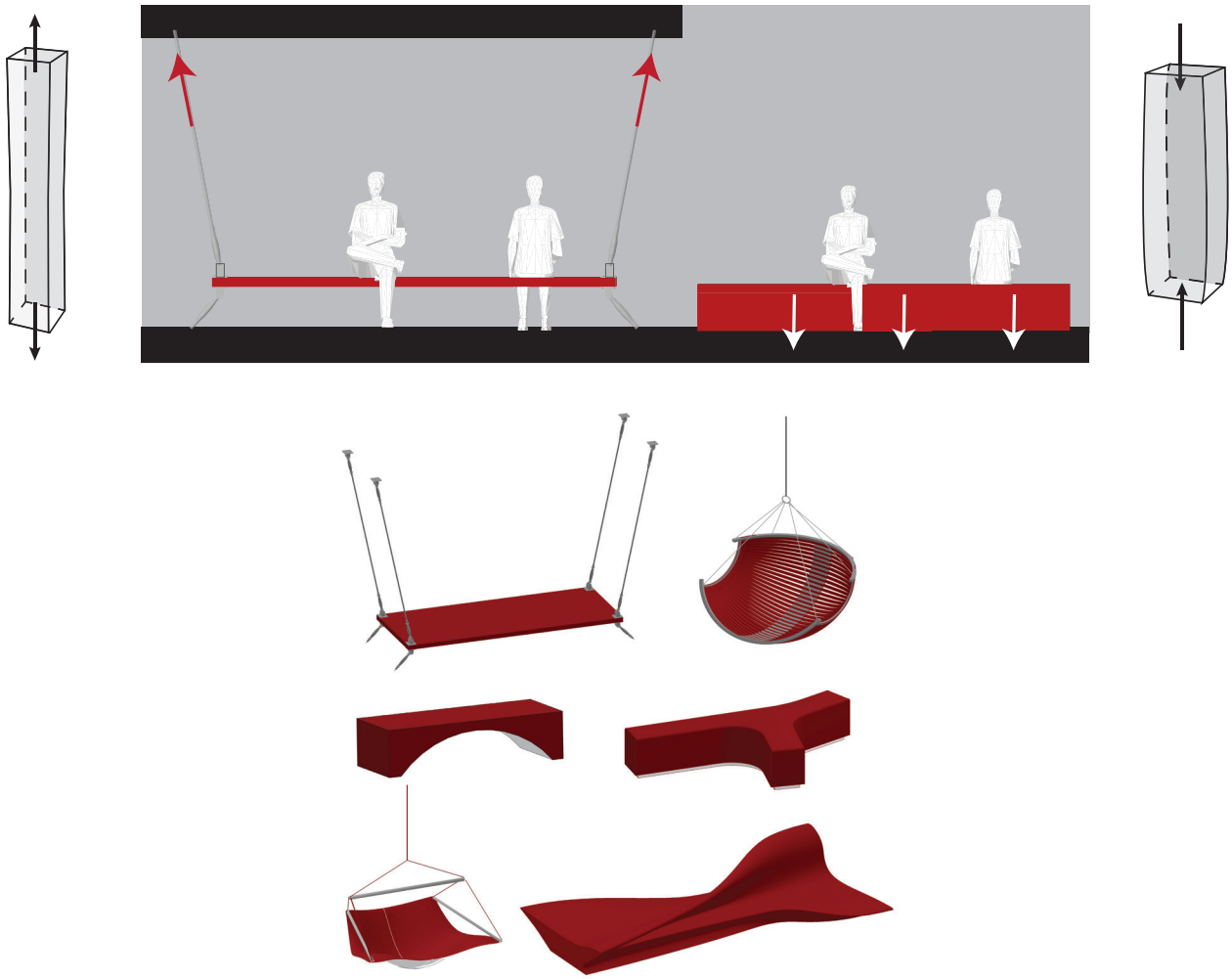


Figure 1.1.14 A stone pyramid loses its "strength" when inverted and tensed because of the material weakness of stone



Figures 1.1.15a and 1.1.15b String (tension) and Stone (compression) schematic proposals for seating—the forms follow the forces

material axis (or axially). We'll call these options *form-resistant structures*, or "Strings and Stones," as a reminder of the essential pairing of form and materiality to the loading conditions. By understanding these basic behaviors first, we'll eventually understand the differences in how section-resistant structures that we'll call "Sticks," and surface-resistant structures that we'll call "Shells" provide support and resistance. (Figure 1.1.15)

MAKING: MATERIAL MATTERS

The load-bearing capacity of a material isn't our only structural consideration. Certain forms and assembly options will be better suited to one material than others. Wood, steel, concrete, and plastics are traditionally our primarily structural materials. Within each category of materials lie critical distinctions that influence their relative strength and stiffness. Depending on the carbon content, for instance, some steel alloys can be one hundred times stronger than others. The same applies to different admixtures in concrete. Throughout the book, we'll look at the material qualities of our proposals from the perspective of structural behavior (strength and stiffness), options for assembly (standard sizing, types of connections, fabrication issues), economics (affordability, durability), and their environmental profile. Readers are encouraged to

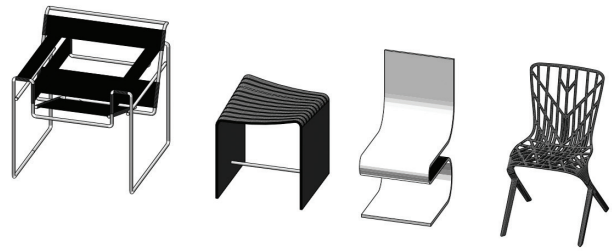


Figure 1.1.16 Consider how these designs use leather, steel tubes, wood, plastic sheeting, and 3D printing to reinforce their design ideas and ergonomic goals

supplement material information provided here with additional readings, research, and activities that enrich their knowledge of material science and assemblies. (Figure 1.1.16)

To Do: Returning to the chairs you've studied, identify the materials that are used for each component—be specific (what type of "wood" or "metal"?). Describe the material qualities: Stiff, brittle, flexible, cold, strong, etc. Hypothesize about why these specific materials were chosen for each particular "role." Are the connections made of different materials than the other elements (why or why not)?

To Discuss: The material qualities of a chair are inseparable from the experiential qualities of the chair (comfort, texture, aesthetics)—for the chairs you've studied, have complementary materials been selected to achieve both ends?

Design Option #1: "Strings"/ Tension Structures

Let's begin with rudimentary proposals for hanging chairs: One with a flexible fabric/string support system and another with a rigid flat seat. Initial sketches reveal the formal limitations of the fabric "hammock-like" options, namely the loss of suitability as the geometry of the seating experience changes based on whether one, two, or three people sit in it. These types of seats can only be used in very specific ways by a limited number of people. Alternatively, if we use a rigid seating surface, we can still hang the structure from above but we'll have more flexibility in how the seats are used, supported, and stabilized.

Although we can envision how a hanging seat works in broad terms, we'd need a more detailed assessment to evaluate the four measures of its effectiveness: *Strength, stiffness, stability, and suitability*. What are the threats to this type of design? If the supporting ropes aren't strong enough, the seat will experience a catastrophic failure and fall. If the ropes aren't stiff enough the seat will drop under loading (perhaps becoming uncomfortable). And unless we modify the configuration, the hanging seat will be an unstable "swing set" that could tilt front/back, side/side and spin. Before we deal with stability and suitability of form, we will need to understand the consequences of material selection. (Figure 1.1.17)

Form-active tension structures, particularly those with non-rigid surfaces, move with the changing loads of different people and their placement. Consider how surfaces and supports can respond to these different conditions. The support "strings" should include materials that are good in tension such as metal wires, cotton and nylon ropes, or even thin metal rods. These "ropes" will be connected to the ceiling through a clevis-like clip that is embedded in the concrete—the concrete structure will eventually resolve the load back down to the ground. (Figure 1.1.18)

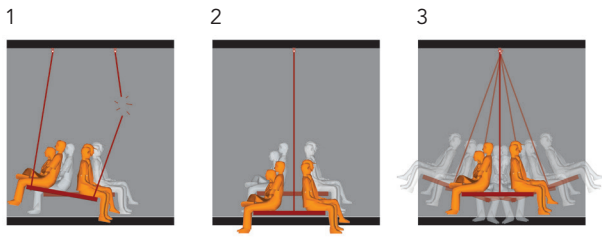


Figure 1.1.17 Hanging structures have three distinct threats: 1. Failure is catastrophic and complete, 2. Too much sag affects functionality, and 3. Inherently instable from shifting loads

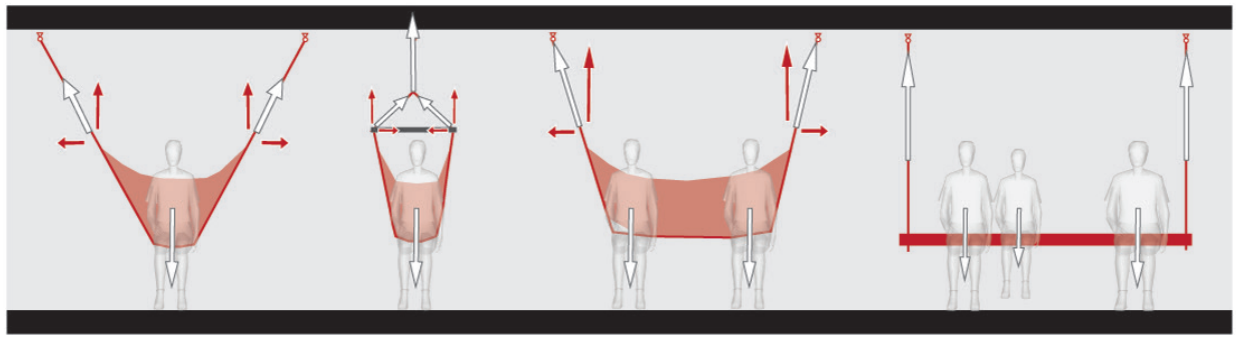


Figure 1.1.18 Diagramming the changing geometries of support forces with rigid and non-rigid surfaces

Determining Strength and Stiffness: Form-Resistant Structures

How much load needs to be resisted by the supporting ropes? We'll assume the rigid seat is sized appropriately to transmit the loads (P) to the supports and that the load will be evenly distributed to the supports at all four corners (i.e., load of P/4 for each cable). The forces in the supports will vary depending on whether they are straight or angled. We can use a free-body diagram and the *graphic statics* method of vector mechanics to determine how the arrangement and angle of the ropes will affect the forces that need to be resisted. Assuming the seat equally distributes weight, vertical supporting ropes on each side will each take half of the total load. When these ropes angle inward, they will still need to resist this same amount of vertical force, but because they are angled, they'll have to resist the resultant force of both horizontal and vertical forces. (Figure 1.1.19)

But will it be strong enough? In our hanging chair, the load is running parallel to the support material, which causes internal tension. This makes the relationship between the materiality, area, and load easy to understand. Once we know the load it has to resist, we can select a rope and can check that its allowable stress and its cross-sectional area will provide enough capacity to resist these stresses. Mathematically, this is a simple relationship:

$$f = P / A$$

Stress (f) is a measure of how much load (P, (lbs or kgs)) is applied across a certain cross-sectional area (A, (in^2 or cm^2)). We have to select a material with an allowable stress (lbs/in^2 or kgs/cm^2) higher than the ratio of load to area found in the actual conditions. Once we find the allowable stress of a material, we can compare this with maximum loads and bearing area for rope supports. (See Appendix, Table A.4.)

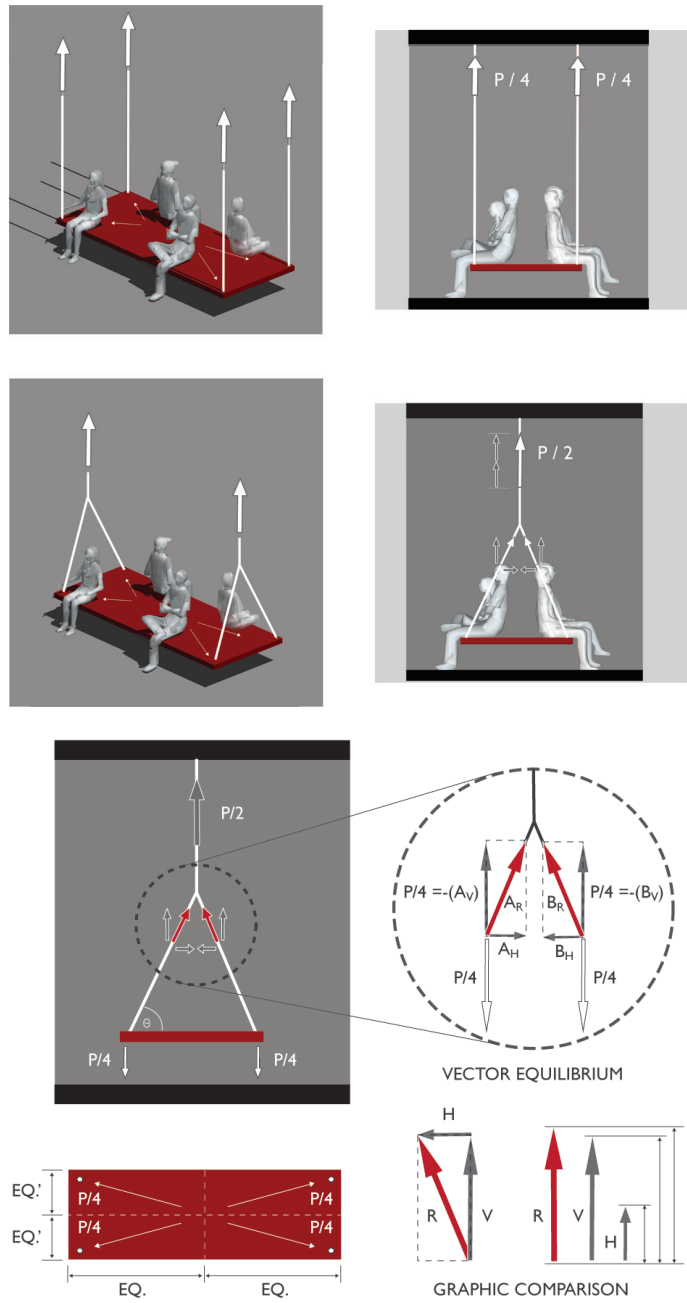
We can adjust any of these factors if there is a problem—make a larger cable, add more ropes to increase overall area, or simply change to a material with a higher allowable stress. If we start with allowable stress (based on material) and area, we can solve for the maximum allowable load. Or we could choose a material, set a maximum limit for allowable load and solve for overall area. Any, or all, of these adjustments are possible. (Figure 1.1.20)

We need to make sure the cable or rope doesn't elongate too far under this stress. Because our proposal may include materials that range from stretchy ropes, like nylon, to less pliant metal strands, the relative *stiffness* of the material varies considerably and will be an item of concern.

Elongation (e) (in inches (in) or centimetres (cm)), is a mathematical relationship between the product of the applied load (P, (lbs or kg)) and the original length of the rope (L, (in or cm)), divided by product of the overall area (A, (in^2 or cm^2)), times the material's stiffness factor, known as the modulus of elasticity (E, (lbs/in^2 or kg/cm^2)):

$$e = P * L / A * E$$

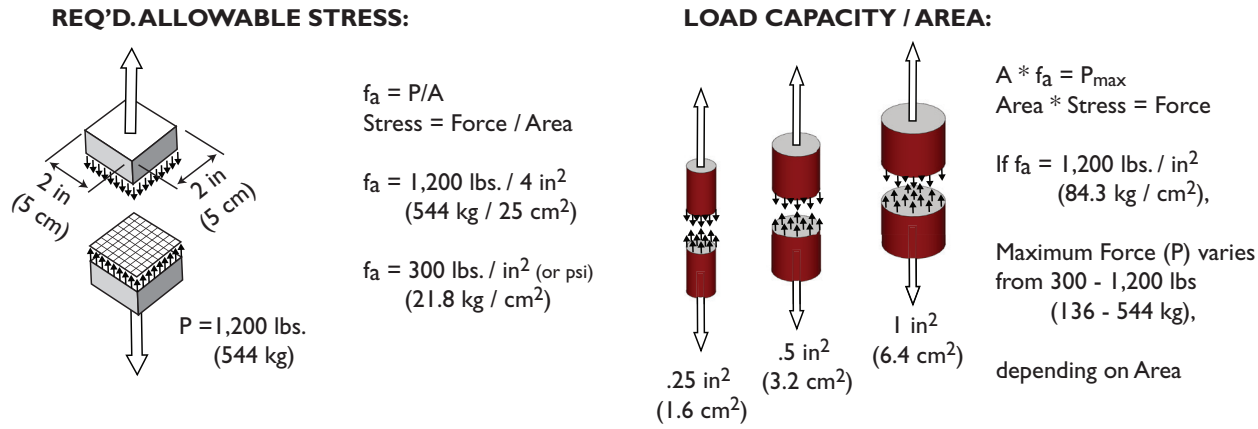
Intuitively this formula makes sense: A greater load increases the elongation if all other factors remain the same, while a stiffer material under the same load elongates less. (See Appendix, Table A.4.) (Figure 1.1.21)



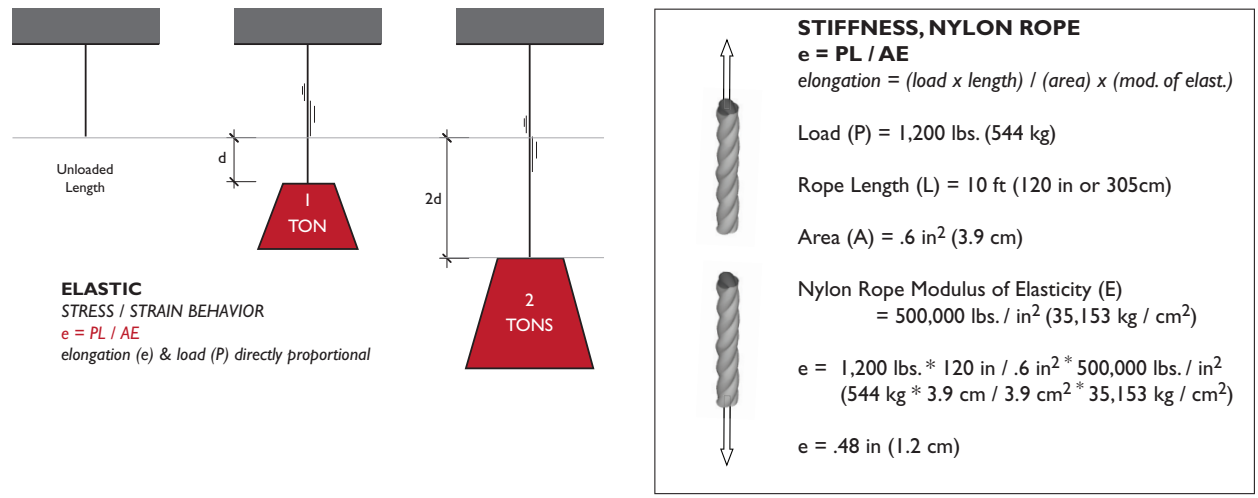
Figures 1.1.19a and 1.1.19b Support diagrams of four cable support option versus two cables. Graphic analysis allows us to compare stresses within cables based on geometry of configuration

Design Option #1: "Strings" Evaluation

It's important to identify the most likely problem inherent in each scheme. For tension structures, the inherent lack of stability is its greatest threat to suitability. A string or a raised platform holding up a weight can both be designed to be "strong enough," but the slightest touch could make the hanging seat move—making it unstable. Unlike the previous factors that we were able to discern with calculations, stability can be studied with the use of a physical



Figures 1.1.20a and 1.1.20b The simple calculations show us the direct/inversely proportional relationships to evaluate the size, load, and material factors in tension systems. In comparing materials, we find that for the same load a steel cable will require less area than a nylon rope



Figures 1.1.21a and 1.1.21b Strain is directly proportional to stress. Finding the results of testing nylon ropes for stiffness to avoid excessive deflection/elongation

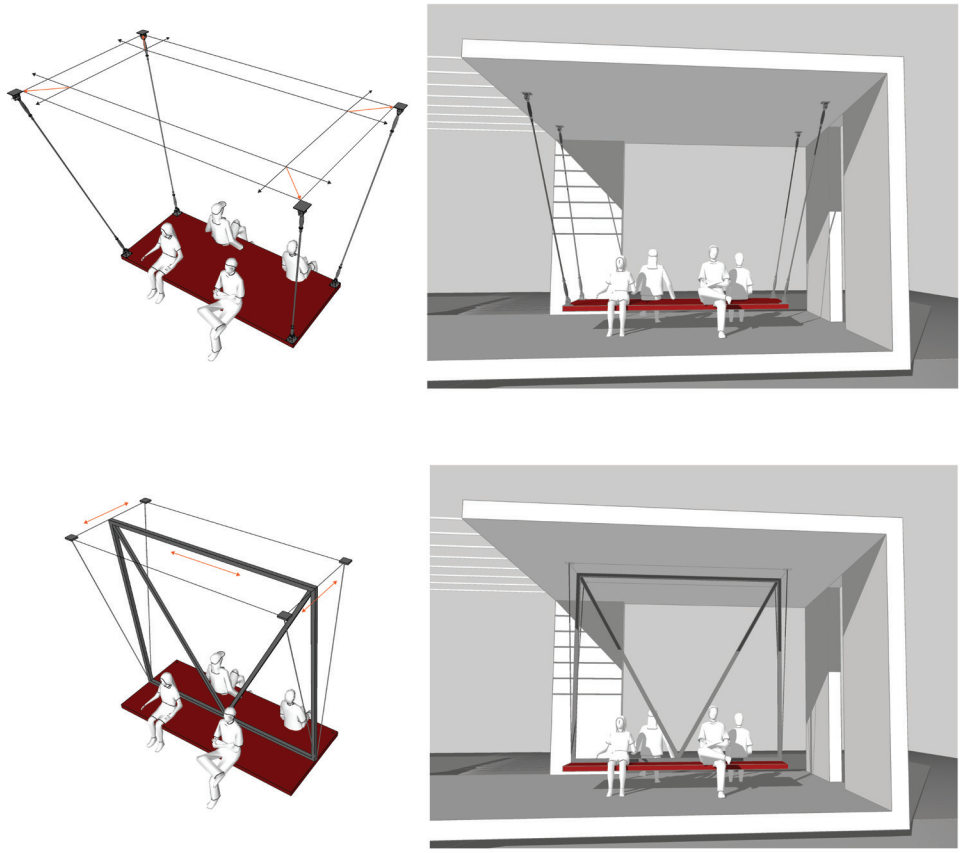


Figure 1.1.22 Lightweight and heavy stabilization options: Pulling cables to four corners and a rigid frame. Both solutions are stable yet quite different experientially—that's the point of including structures into design deliberations

prototype. We can also confirm its stability graphically and geometrically by looking at the x, y, and z axes to determine if we have fixed certain points in space to each other.

Some movement may be acceptable, but the swing shouldn't move too freely. We can increase the number of support points for the seating surface or simply angle our "supporting cables" to make them "stabilizing cables." Ideally, we'd pull four cables outward in opposite directions to prevent pitching and rolling (picture this as vector arrows that pull in opposite directions to restrict movement). By having stabilizing cables double as supports, the strength and stiffness of nylon supports is less concerning, because we have more of them. As a second option, we could create a rigid triangulated frame of supports and stabilizing members. Triangulation fixes these points in space side-to-side and front-to-back. The frame would have to rely on the rigidity of the frame to stabilize it against movement caused by forces perpendicular to its plane. (Figure 1.1.22)

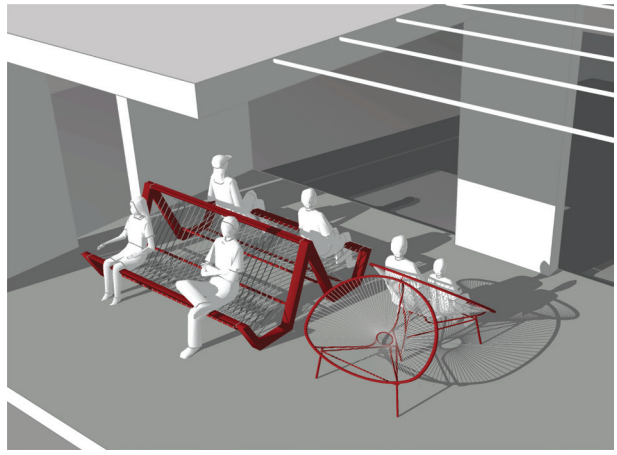


Figure 1.1.23 Hybrid seating options: Strings hanging from stick frames support the bodies

We could get the ergonomic benefit of comfort provided by strings without the hassles of instability by hanging a flexible tension-resistant material (such as string, cables, fabric, leather, etc.) within a rigid support frame provided by "Sticks." There are many options for a hybrid system in which a metal or wooden frame is infilled with a mesh-like net that collects and transfers the load to the frame. (Figure 1.1.23)

Design Option #2: "Stones"/ Compressive Structures

"Stone," or compressive seats, are perhaps our most primitive and enduring seating option. A natural granite stone would match the setting, would be warm in the sun, and would provide ample support with little effort—it is also a bit unimaginative and perhaps uncomfortable. Labeling an entire category of materials as "Stone" is misleading if we intend to include other materials in this category that behave similarly under compression such as masonry units and concrete. Stones are effective materials *unless* they are subjected to tension and bending—their brittleness can cause them to fail catastrophically in these cases, so they have high *factors of safety* applied to their use in tension. The allowable stress table shows no acceptable level of allowable tension stress for stone and concrete.

Our proposal is a solid piece of stone fully resting atop the slab. It is over-structured—the form is based on functional and not structural dimensions. We could intuit that the stone will be strong, stiff, stable, and suitable enough. After all, the stone isn't even spanning between the supports—a seated person's weight would transfer directly down through the bench into the ground. If needed, we could calculate the stone's stress and strain, but it doesn't seem necessary—stone has an allowable stress value (f_a) of over 15,000 lbs/ in^2, which means we could support all the anticipated weight on a miniscule amount of stone area. The stone is sufficiently stiff to be suitable. (Figure 1.1.24)

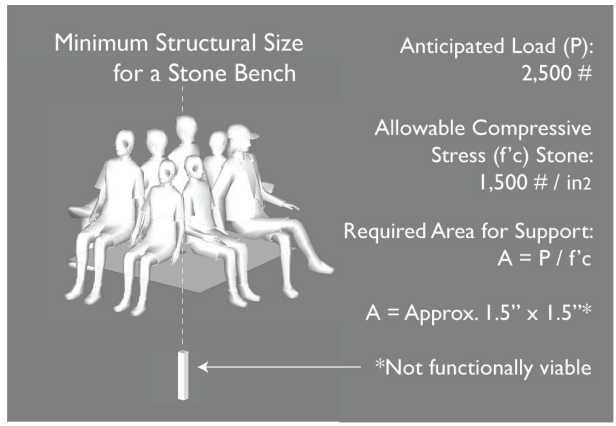


Figure 1.1.24 A large group of people could be supported with only a minimally sized compressive element—but it isn't functionally viable to do so

Shaping Compressive Structures for Efficiency

Could we reduce material? We want to maintain a large seating surface but we *could* carve the bench's supports into "legs." If the legs were placed under the four corners, each would support just one-fourth of the total load and each one would only need to be one-fourth of the total required area. We'd only need a small area to support the load on the legs (fractions of an inch (or cm)—but any sideways movement would probably cause the legs to buckle, or to shear off.

If we carve away material, the bench will no longer be a compressive structure—it will become a beam that spans between supports. While stone is good in compression, it cannot carry significant bending stresses, because bending inflects tension. The smaller the seat becomes, the taller the legs get and the more risk we have for failure. Stone is brittle, so when it's used in a beam, it risks splitting apart under tension.

There are, perhaps, more elegant options for carving away excess material, such as making it into a shallow arch, but arches are difficult to construct, and they create outward thrusts at their bases. Perhaps, stones are more effective the *less* they are modified. Materially-conservative historical stone structures make more sense from this perspective. (Figure 1.1.25)

Solid materials that are cast, like concrete, give us more options. Because concrete begins in a liquid state that is poured into a formwork, it can take any shape—making materially efficient profiles or ergonomic forms easier. Lighter materials, like

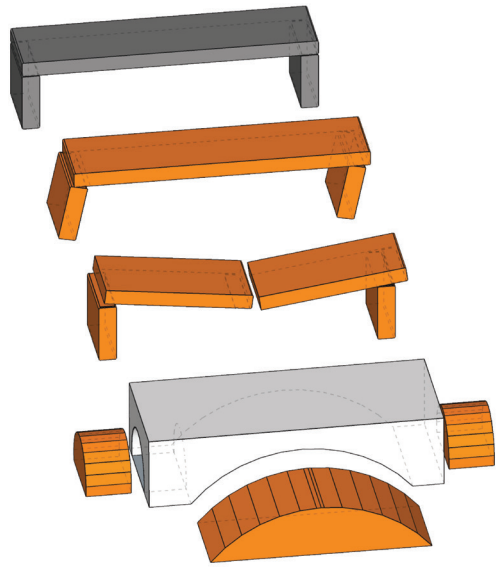


Figure 1.1.25 Potential failures in compressive structures: Bending/breaking, instability at supports, and potentially intensive modification

plastic, could also be cast (or molded) into similar forms. We could increase possible forms by adding reinforcing to concrete to resist the tension stresses found in bending—these could be considerably thinner, but they would no longer be purely compressive structures—they'd be beams or shells. (Figure 1.1.26)

Design Option #2: "Stones" Evaluation (and Options for Wood)

Even the most lightweight stone and concrete benches would be too heavy to easily move or adjust. Perhaps the most viable option would be to make the seats out of wood. Wood is a lighter material that could be easily modified (e.g., cut, carved, and connected) *and* it performs equally well in compression and tension. But wood is an *anisotropic* material—it has a directional primacy to its structurally capacity and it is vulnerable to loads that run parallel to its grain (steel is *isotropic*).

We could maintain the purity of compression-resisting forms for the seats, such as a carved-out solid log or a wicker bench, but because wood can be easily modified, easily connected *and* can resist bending stresses, we could explore many other possible forms. When wood spans between supports and becomes its own framework, it behaves likes "Sticks." If we use plywood sheets, wood can be folded or curved to make "Surfaces." These will change the way the forces move through the structure, and how we'll evaluate their behavior. (Figure 1.1.27)

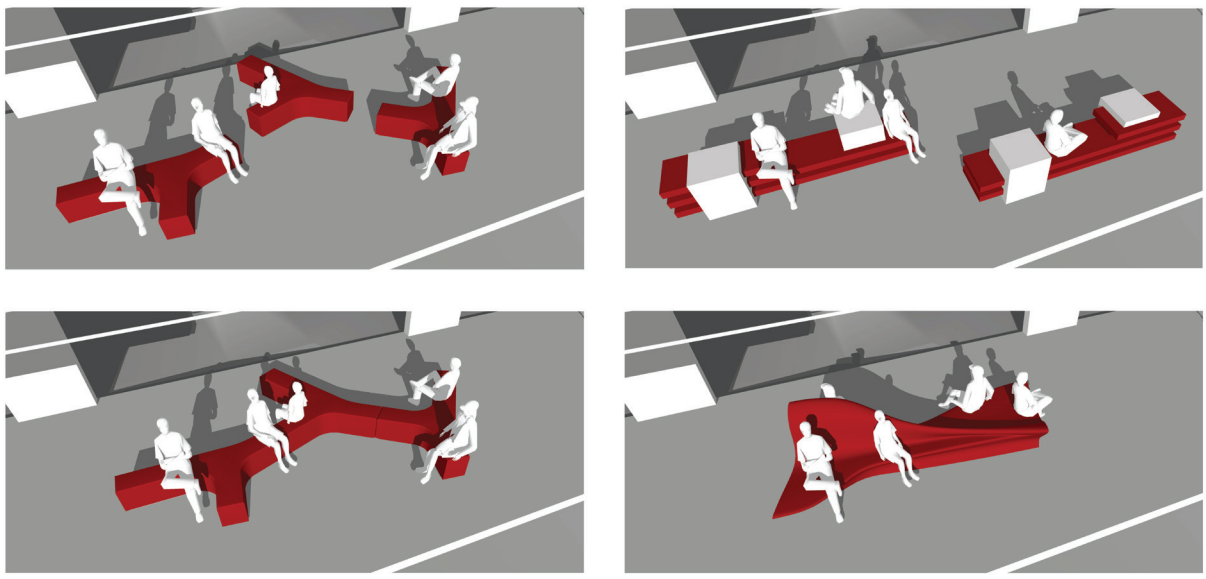


Figure 1.1.26 Design options for compression-based seating solutions are based on more aesthetic and ergonomic criteria than structural performance

Figure 1.1.27 Wood is a lightweight, relatively strong (strength-to-weight ratio) material that can be easily modified and assembled in many ways

Design Option #3: "Sticks"/ Bending Structures

For our third type of seating, we'll explore a framework of "Sticks" to make portable free-standing wooden seats. Chairs that use a framework of "beams, slabs, and columns" are quite common—and yet remarkedly varied in materials and expression. Before we develop these options, we need to understand how this support and transfer of forces works. (Figure 1.1.28)

Analyzing a simple wooden chair can reveal the support functions of the different framing elements. This chair, like all structures, has components that fill three major resisting and transmitting roles: *Load collectors, load grounders, and load stabilizers*. Forces flow through these elements from smallest to largest as the chair collects and transmits the sitter's load. In other chairs, this seat may be more or less rigid, but we count on it maintaining enough integrity to transfer the loads to a location where they are consolidated and "grounded" downward. Alvar Aalto described the design importance of connecting the horizontal and vertical elements together, "I believe this is absolutely decisive in giving the style its character. And when joining with the horizontal level, the chair leg is the little sister of the architectonic column." (Figure 1.1.29)

This convenience in form has consequences in structural behavior. These structures are no longer loaded parallel to their supporting elements, now they are being loaded perpendicularly to them, as a beam. Because they are loaded non-axially, beams have different ways of resisting loads—relying on the shape of their cross-sectional area to do so. Unlike the cables or

Figure 1.1.28 Frames in furniture, like buildings, can be straight or curved, repeated or arrayed, and designed to act like beams and columns or structural slabs

Figure 1.1.29 The framing of a chair reveals the method by which loads are transferred (red), grounded (black) and stabilized (orange)

stones that resisted stresses equally across their cross-section, beams have some areas along the span (and in the cross-section) that resist more stress than others. The relationship between the span of the seat and the location (and number) of legs will determine the amount of stress each element has to resist—a longer span between legs means more stress in the load collector. (Figure 1.1.30)

Because these elements are subjected to bending loads they need to be analyzed differently than form-resistant structures. To determine strength and stiffness, we need to know how much load is applied, the location and number of supports, where the stresses are highest along the span, the material's capacity to resist stress, the cross-sectional area, *and* the cross-sectional shape to determine whether or not the design is strong or stiff enough. This requires a different set of design strategies and analytical techniques, which we'll learn about later in Part 3 of the book.

Wood and steel are viable options for these structures. They are good at resisting bending and they

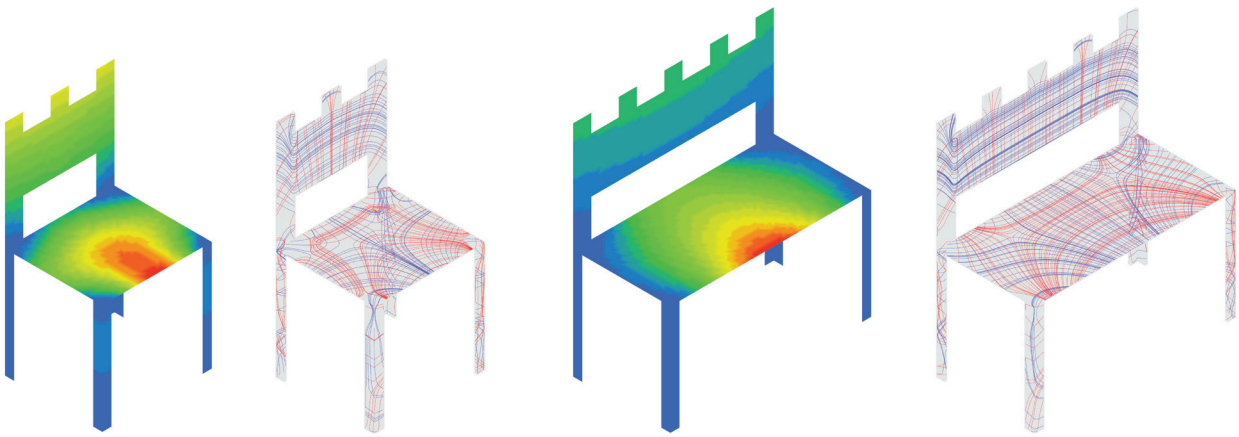


Figure 1.1.30 The consequences of stress and deflection when changing the length of span between supports of a chair versus a bench. Displacement in grades of color, and intensity of stress distribution (red is compression, blue is tension) based on frequency of lines

can be configured into a supporting framework. Once we switch from a monolithic material into a framework of smaller components and multiple connections, we need a strategy for shipping, assembling, and stabilizing the entire system. Stability, after all, is a quality of geometric configuration, not materiality, and instability, once again, is a liability for seating. Stick framing or planar assemblies of wooden sheet elements are two primary options for wooden seating options. (Figure 1.1.31)

Typically, chairs are assemblies of different parts with multiple connections. These parts need to be stabilized together. Sometimes their connections can be "fixed" to keep the chairs stable (e.g., welded connections on a steel frame) or fastened as "pins" with bolts or nails. Reliable connections for rigidity and stability allow chairs to be mass-produced as a series of interchangeable parts. The now ubiquitous flat-packed furniture trend of the 21st century which is the logical extension of a structural idea has become a staple of modern design. (Figure 1.1.32)

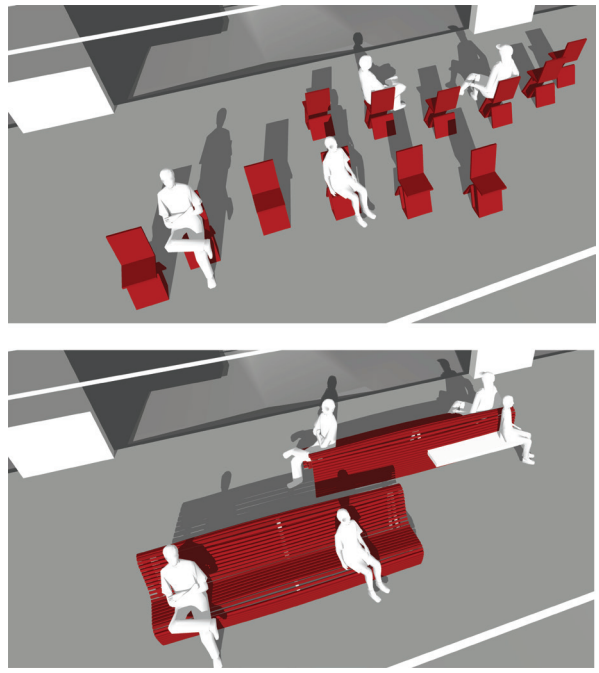


Figure 1.1.31 Design options with wood framing can be analogous to slabs or columns and beams. Because wood elements are easily assembled, various iterations and sizes are possible

Figures 1.1.32a and 1.1.32b Manufacturing techniques change how seating is packaged and assembled. Shown: Thonet 3 (Michael Thonet), and Rib Chair (whitehead design workshop)

BREAK: FOLLOW THE FAILURES

Think back on your experiences with seating to a time when the seat "failed" or was deficient in a noticeable way. What was the problem? Was it an issue with design negligence, material failure, or fabrication?

To Do: For the chairs you've studied, speculate on where they are most likely to fail structurally.

To Discuss: How does this point of potential failure you've identified correspond with your previous work? Does this happen where forces are concentrated? Or where materials change? Does this observation/hypothesis suggest ways the chair designs could be improved?

Design Option #4: Triangles and Trussed Framing

Framing doesn't have to be strictly horizontal or vertical. One of the simplest ways to provide for stability is to include diagonal bracing members. When these elements are triangulated, they offer new formal and structural opportunities (e.g., from high-rise buildings to chairs and bicycles!).

A triangle is the most stable form because it locks three points in space together so that one point can't move without being resisted by the other two points. Diagonal elements also provide support for vertical forces—as both load stabilizers and grounders. An interlocking group of triangular frames could create a simple option for a bench. (Figure 1.1.33)

Although triangulated structures span like beams, they don't behave the same because they are not resisting bending the same way. These are vector-based structures, also known as trusses. As we'll see in later chapters, their support elements are only pushed and pulled axially (like Stones and Strings) depending on their configuration. We can apply the same type of assessment of strength and stiffness as we did with Strings and Stones, and combine this with the way forces are transferred across their length as we did with Sticks. Part 4 of the book will cover more detailed ways of designing and analyzing trusses. (Figure 1.1.34)

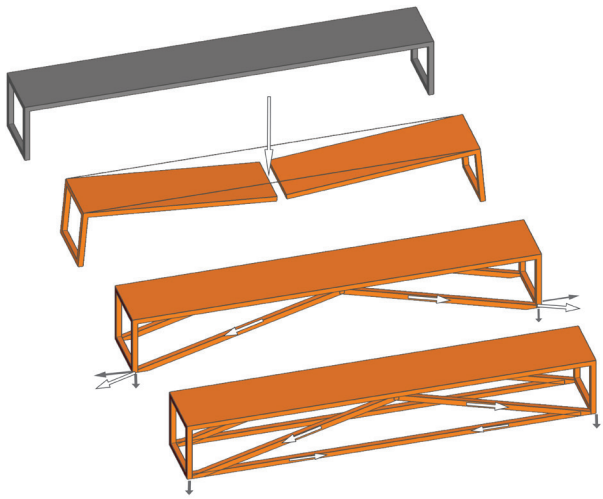
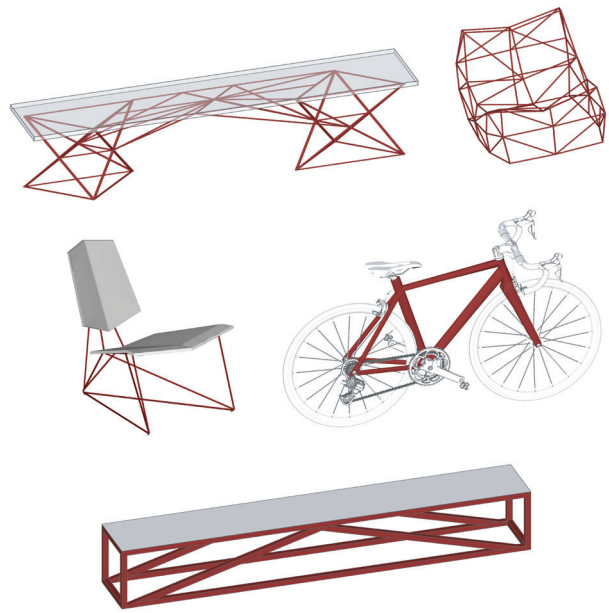


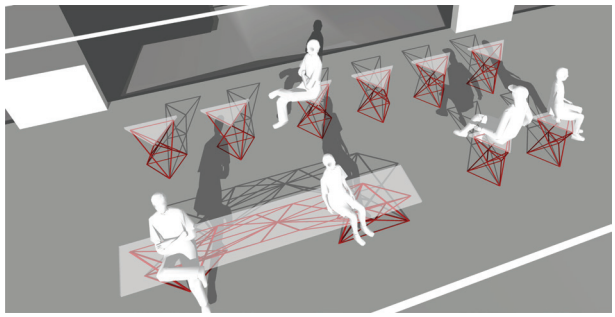
Figure 1.1.33 Triangulation helps resolve difficult structural issues of support and stability without excessive supports

Design Option #5: "Surfaces"/ Supporting Shells

Instead of a solid bench or a composite frame, we could create the seat and back rests as parts of a continuous shell that we'll call a "Surface." These lightweight and thin shells are good options for chairs as they provide continuous support for bodies. They rely on the form, and on the surface material's rigidity to transmit loads and resist deformation. In shells, curved surfaces act as load collectors, load grounders *and* the load stabilizers—all while creating an expressive form. (Figure 1.1.35)

Because shells resist loads monolithically they are hard to analyze. If the shell's surface (known as its "membrane") is stiff enough and shaped appropriately, stresses will be resisted within the thin sectional





Figures 1.1.34a and Figure 1.1.34b Triangular components make lightweight frames that can be combined to create planar or volumetric elements as the span and function requires

Figure 1.1.35 Seating options using surfaces. Curved and rigid shells develop load-bearing capacity far beyond the material's capacity as a planar element

area of the membrane's surface. This shell transfers loads efficiently while creating a structural surface that supports the seat. Shells are naturally more supportive, stiffer, stable and expressive as a result of their curvature and the rigidity of their materials. Plastic, concrete, and plywood can all be formed into double-curved shapes to provide the material resistance needed.

Shell chairs weren't always so easily produced. After their "Organic Home Furnishing Competition" winning designs for curved wooden chairs faltered 1941, Charles Eames and Eero Saarinen (with help from Harry Bertoia) experimented with double-curved shell surfaces for seating. Eames perfected the system with plywood, Saarinen achieved it with an injected plastic molding process, and Bertoia made his out of a thin mesh of curved metal bars. These weren't simply innovative structural and ergonomics solutions; they experimented with relationships between form, materiality, and production. In these chairs, selecting a structural strategy was the beginning, and not the end point of their designs. (Figure 1.1.36)

Design Option #5: "Surfaces" Evaluation

This example shows how one idea for a supporting surface can be manifest in three viable (and enduring) solutions. In our proposal, we could cast the shell forms from a lighter weight and durable material like plastic, fiberglass, or carbon fiber to make a double-curved surface. These surface structures could be lightweight, materially efficient, durable, and movable. They could be ergonomic as well, because their geometric variance offers numerous ways to sit or lie on them. (Figure 1.1.37)

Figure 1.1.36 (Left to right), Eames Molded Plywood Chair (Herman Miller, 1947), Bertoia Diamond Lounge Chair (Knoll, 1952), and Saarinen Tulip Chair (Knoll, 1955)

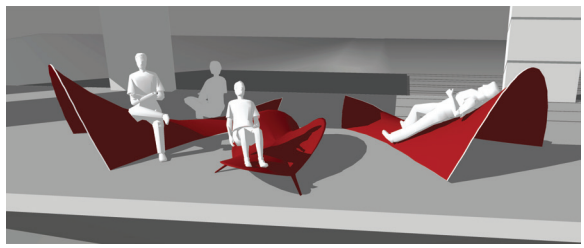


Figure 1.1.37 Because their surfaces can be curved and folded, shells can accommodate various types of seating options and forms with relatively thin materials

Perhaps a curved or folded paper structure wasn't what the Park Service had in mind . . . or maybe this logical reasoning based on structural principles is exactly what was intended. Overall this range of options is attractive because it is so different from the original "Stone" proposal. Structural design is a discipline that can reveal a multitude of different (and hopefully "better") solutions.

Classifications of Form and Materials: Overall Evaluations

The potential iterations we've generated show how searching for "better" solutions is more challenging and fulfilling than creating an object whose only objective is to stand firm. Although a straightforward problem—a seating structure to support the body—the more we searched for options that were responsive to the problem using structural principles, the more expansive and wide-ranging the proposals became. Responsiveness of a structure is a crafted outcome of these efforts to optimize relationships between architectural ideas, material, form, and forces. (Figure 1.1.38)

Grouping structures together, however broadly, based on the mechanism by which they provided support narrowed our choices. We found four overall categories:

FORM: ***Strings*** *were the most lightweight, easily transportable, materially efficient. Because their form follows tension, we could match structural expression with structural performance. But stability was a problem and strings were not, ultimately, suitable.* ***Stone*** *seats were strong, stiff, stable, and suitable, but they weren't efficient. Any attempt to improve their efficiency compromised practicality.*

SECTION: ***Sticks*** *implied a structural frame assembled of smaller pieces that could resist bending. Different frames, or framing materials, can give a designer great latitude, but the separate parts need to be connected and coordinated to collect and ground*

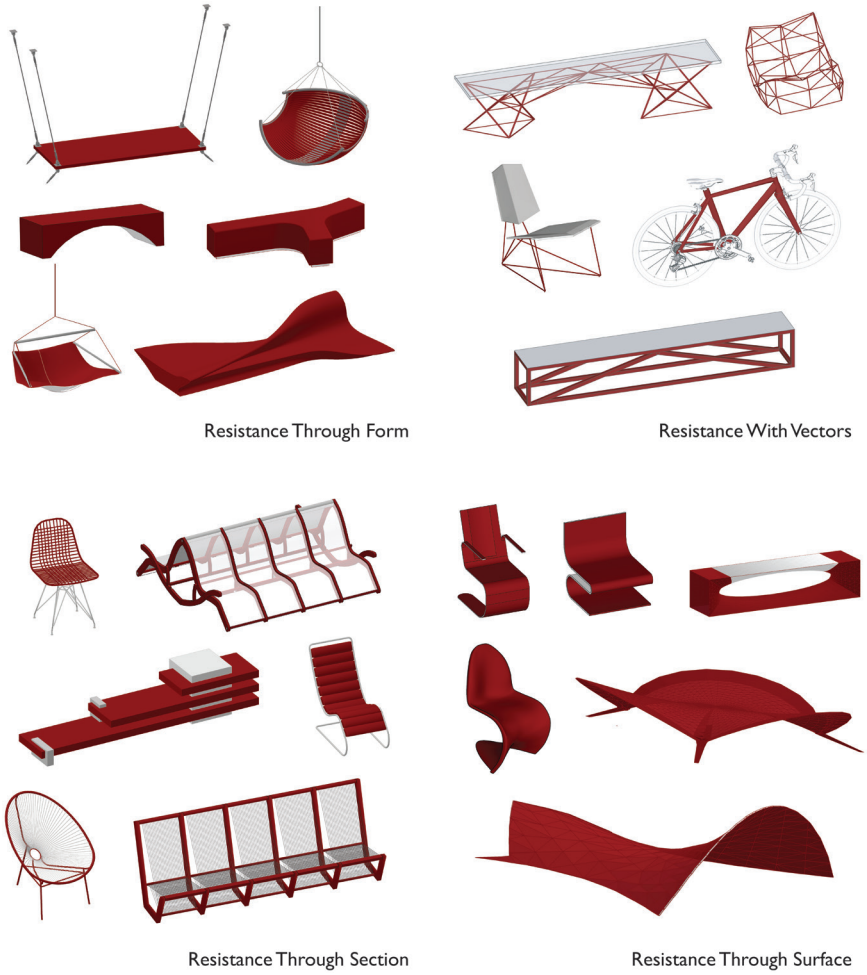


Figure 1.1.38 A selection of our various options can be grouped by their method of providing structural resistance: By Form, Section, Vectors, or Surfaces

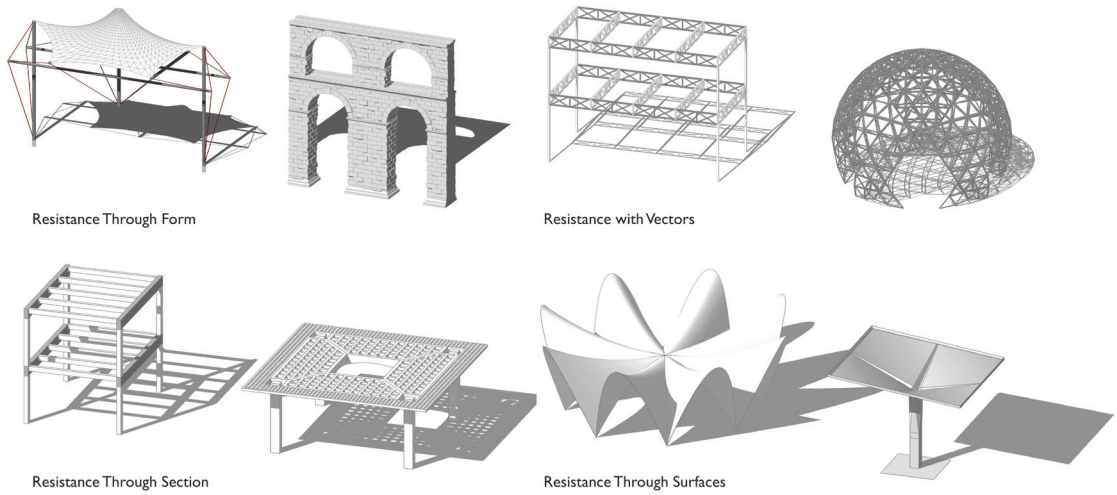


Figure 1.1.39 These categories for how resistance is provided (Form, Section, Vectors, and/or Surfaces) are useful in understanding building structure classifications

the forces, and to stabilize the frame. Because Sticks are made of multiple components connected together to form a frame, we'd be concerned about that frame remaining stable.

VECTOR: ***Triangles*** *have an inherent efficiency because they are lightweight, strong, and naturally stabilizing. But there is a formal determinism that emerges from using these components. The triangle becomes the primary feature of the geometry and expression.*

SURFACE: ***Shells*** *or other membrane structures are thin and efficient forms that resist loads through their stiffened surfaces. We'd be concerned about the stiffness of the shell material, the ability to find the right combination of an ergonomic and structural form, and the fabrication of the double-curved surface.*

To complete the proposal, the design team would need to assess the pros and cons of these categories as functional *and* structural items. (Figure 1.1.39)

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CHAPTER 1.2

Building Structures

Effective, Efficient, and Expressive

Before we can learn to make structures better through design, we'll need to understand not only how they work technically, but the means by which their responsiveness and overall performance can be assessed. Structures can be classified based on how they resist and transmit loads (Form, Section, Vectors, Surfaces and Frames). Doing so allows designers to better understand the means by which resistance is provided and responsive options for materials and form.

The Task of Structural Design

What are structures? Technically, structures are the portions of a building designed to resist the loads imparted upon them by supplying the strength, stability and relative rigidity required to prevent buildings from collapsing or failing to meet required operations. The primary purpose of structures is to provide protection and support—a challenge not to be taken lightly—but structures need to do so much more.

Designing an object that simply stands firm isn't enough when integrating structures into the discipline of architecture. When done correctly, structures give form to spatial volumes that are inextricably connected to the function, aesthetics, materiality, assembly, economics, and experience associated with architectural design. Structural design's challenge is to develop and apply technical *and* creative aptitude towards this inclusive and expanded range of performance standards. (Figure 1.2.1)

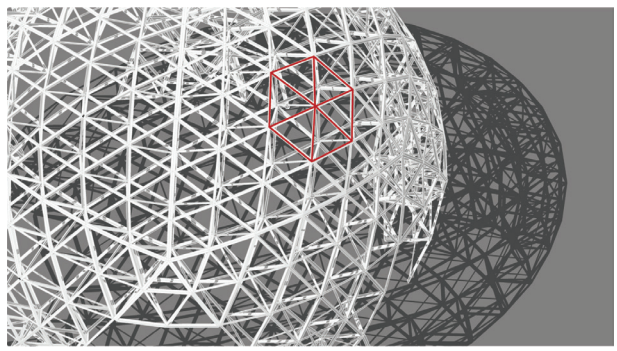


Figure 1.2.1 The pattern, light, volume, and mathematical principles of a geodesic dome are inextricable

THINK: BEFORE WE BEGIN

What Is a Structure? Quickly sketch a few images of what the word "structure" brings to mind.

To Discuss: In a few words, describe the three most important tasks that a structure must perform. What information would you need to know to determine if the structures met these criteria or not?

Structural Form and History

For most of human history, architecture, structure, and construction have been nearly indistinguishable from each other—so much so that, as Eduard Seckler observed, the terms "building, structure, and construction" are used interchangeably (varying as verbs and/or nouns)—and incorrectly. This reveals the challenges of understanding and assessing the design of "buildings." There are shared concerns between a building's design, it's structural framework, and the process by which it is constructed. These distinctions have differences which are useful when creating architecture. (Figure 1.2.2)

A structural system is a central—and indispensable—component of a building. By focusing on the structure, we can make a distinction between these terms without disassociating one from another. We are interested in the final product and the process by which it has been designed, supported, and constructed. Tectonically-based histories of architecture discuss the changing nature of this relationship between architecture and structures in two primary ways:

Intended integration: This narrative describes the degree to which designers sought to integrate (or disassociate) the structure from the formal and/or spatial intent of architecture. This can be: exposed, concealed, or celebrated, with either corresponding or conflicting volumetric relationships to the architectural spaces. Many ancient structures (like pyramids or nomadic houses) and industrial machines have clearly associative relationships between their structure and expression. These terms are more descriptive than generative—they aren't enough to refine potential solutions as we begin the design process. (Figure 1.2.3)

Evolution of structural elements: This approach considers the physical manifestations of structures and how, through the passage of time, forms and materials have been arranged to resist stress and

Figure 1.2.2 Design + Structure + Construction. Model of Gothic cathedral bay at Architecture and Heritage City museum, Paris

Figure 1.2.3 The blimp and mosque both have "spatially integrated" and "celebrated" structural strategies. These classifications alone aren't sufficient to distinguish between such different solutions (Graf Zeppelin in Cairo, 1931)

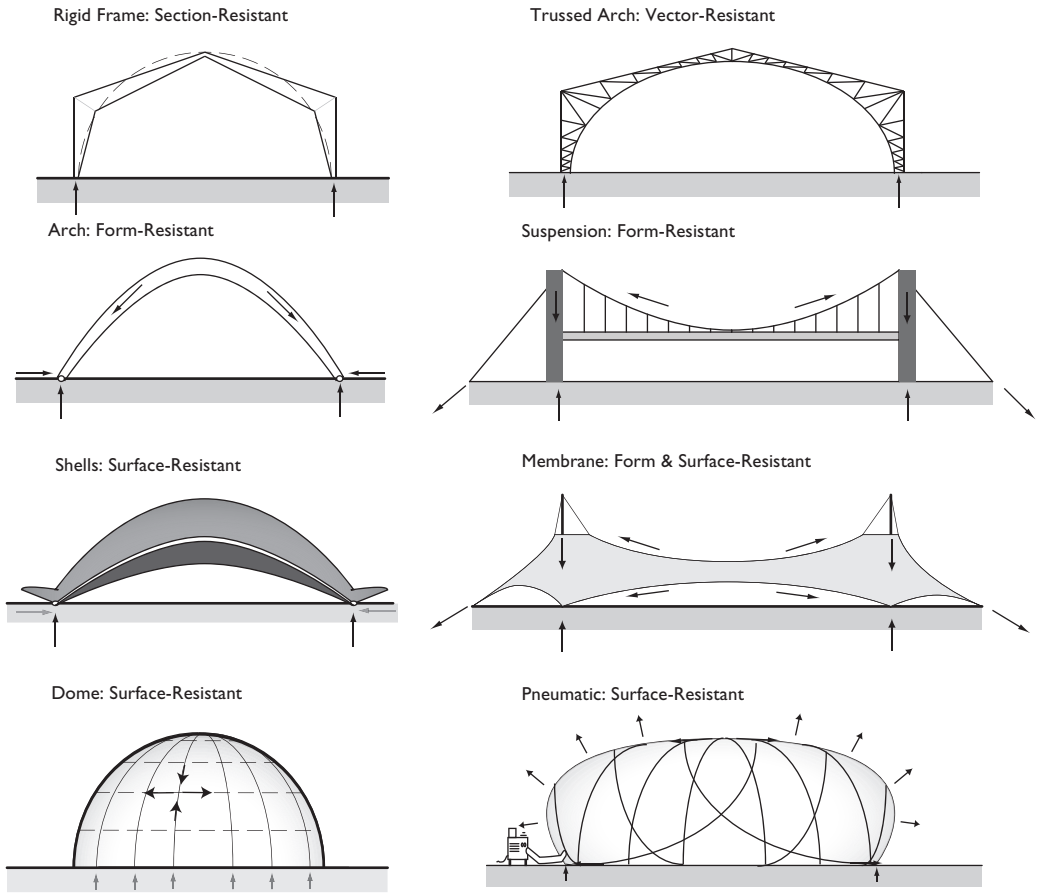


Figure 1.2.4 There are formal relationships between the forms of different structures. They share common morphologies with variances based on materials and internal stresses

transmit forces into discernible patterns. As scientific understandings and material advancements occurred, these behaviors became recognized and elements evolved in their use and implementation (e.g., arches became vaults and eventually shells)—an evolution which produced the groupings of structural morphologies upon which this book's format is based. This approach should inspire continual experimentation in potential structural form, perhaps based on a broad set of performance standards. (Figure 1.2.4)

All buildings will have to reconcile structural integrity, functional purpose, and intention of aesthetic expression with their manner of the building's construction—therefore, we won't be choosing between how the structure looks and how it works in our evaluations.

Performance Standards of Structures

The first codes for building performance date back nearly 4,000 years ago to the Code of the Hammurabi of ancient Babylonia. This code held home builders liable for the integrity of the structure they'd built. A broader, and more considerate evaluation for how structures should be assessed was codified 2,000 years ago, by the Roman architect and military engineer, Vitruvius. In *De Architecura*, he proposed that "buildings" needed three interdependent qualities: *firmitas*, *utilitas,* and *venustas*—*firmness*, *commodity*, and *delight*. Firmness ensures structural integrity, Commodity

strives for functional accommodation for occupants and building systems, and Delight is the expression and experience of beauty through design. These suggest that buildings can be evaluated by comparing the purposes and performances of finished buildings.

What we expect from our buildings, and our building professionals, has evolved. Designing for "viability" isn't enough. Rigorous building codes, design standards, professional training, and material testing have contributed to a focus on health, safety, and welfare. We can span farther, reach higher, and build more effectively than at any other time in history, all while maintaining astonishingly low rates of structural collapses due to design error.

This requires creative insight, a willful adherence to a design perspective of structural principles and forms, and guiding principles to assess what makes one solution better than another. Structural designs look at qualitative and quantitative standards for their evaluation—instead of the vague Vitruvian canons, we'll use a parallel (but more descriptive) set of standards to evaluate structures. We'll replace "Firmness, Commodity, and Delight" with *Effective, Efficient, and Expressive*. (Figure 1.2.5)

THINKING: FORM FOLLOWS FORCES

To Do: Look back at the form you sketched at the beginning of the chapter. Was it conventional, utilitarian, easy to conceive? Or more amorphous and harder to represent? What factors did you cite (e.g., not falling down, not tipping over, helping the building function, etc.)? Did those match the Vitruvian canons?

Effective Structures: "S-Words"

> *You ought to make such models . . . and examine them over and over so often, that there shall not be a single Part in your whole Structure, but what you are thoroughly acquainted with.*
>
> (Leone Alberti, 1404–1472)

"Firmness" isn't a descriptive measure of structural integrity. Because structures resist and transmit loads through their form and materials, their assessment should logically measure these factors by using the "S-word" structural criteria: *strength, stiffness, stability, and suitability*.

Strength and stiffness measure a material's structural qualities. Stability is the result of configuration and connections. The fourth "S-word," *suitability* reminds us to evaluate the appropriateness and serviceability of the design. Compliance with one factor doesn't guarantee compliance with others. A column made of rubber may be strong enough under loading, but its lack of rigidity would be a big problem for stiffness and stability. (Figure 1.2.6)

Figure 1.2.5 Updating Vitruvian terminology to describe the performance goals of contemporary buildings

Learning how to design building structures, we aren't simply looking for compliance (although compliance is a requisite), we are looking for ways to also improve and measure how effective they are at achieving certain performance goals.

Effectiveness: Strength, Stiffness, and Science

Even the earliest structural designers and builders knew that material qualities were the most important indicator of a structure's performance. They evaluated materials through *Theory* (based in mathematical models) and *Experiments* (based in physical models). Efforts to define strength and stiffness emerged from the earliest experiments in physical sciences and forces (e.g., a stick loaded with weights across its length) and corresponding ways of describing these tested results with mathematical models. These methods are still used today.

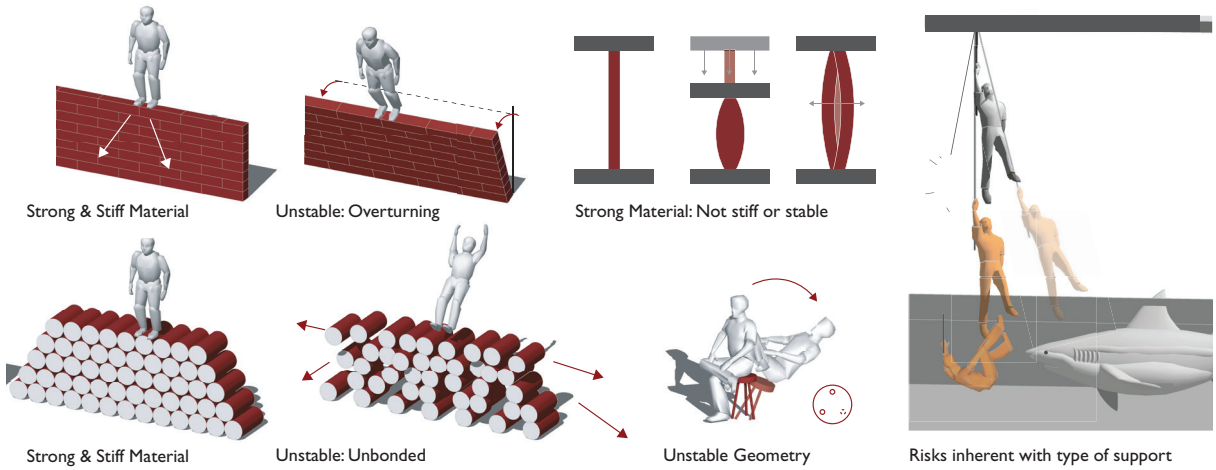


Figure 1.2.6 Consider how to assess and determine compliance for strength, stiffness, stability, and suitability in different scenarios

Sometimes materials would break under loading. Other times structures wouldn't break, but they would deform excessively. Imagine trying to understand the structural behavior of materials when two different objects of the same size and material would perform differently under the same loading (e.g., an iron stick versus an iron spring)! Or how confusing it would be to discover that the "strength" of an iron bar under tension was drastically different from the same iron bar under perpendicular loading as a beam. (Figure 1.2.7)

Over the next two centuries, scientists and builders developed specific methods by comparing Theory and Experiments. Two different means of structural assessment emerged (and remain): *Ultimate strength* and *elasticity*. Ultimate strength establishes the loading limits for a material based on its behavior under loading. Elasticity describes the stiffness of a structure based on the ratio of stress and strain and the consolidation of stresses. This analysis predicts how much a structure will deform under loading, infers the stresses within the system from this data, and compares this to allowable material capacities.

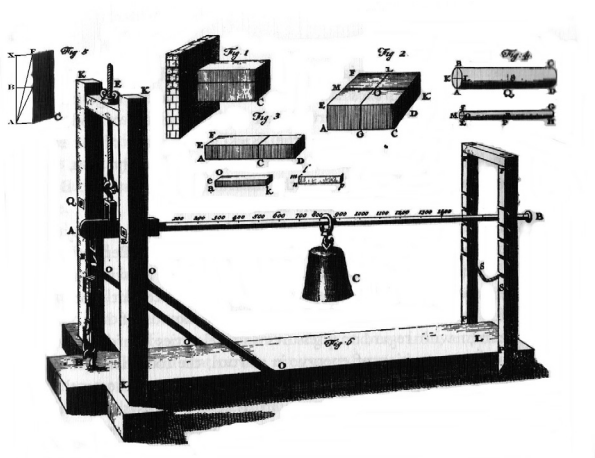


Figure 1.2.7 One of several testing machines created by Petrus van Musschenbroek to measure the physical behavior of materials under stress (1729)

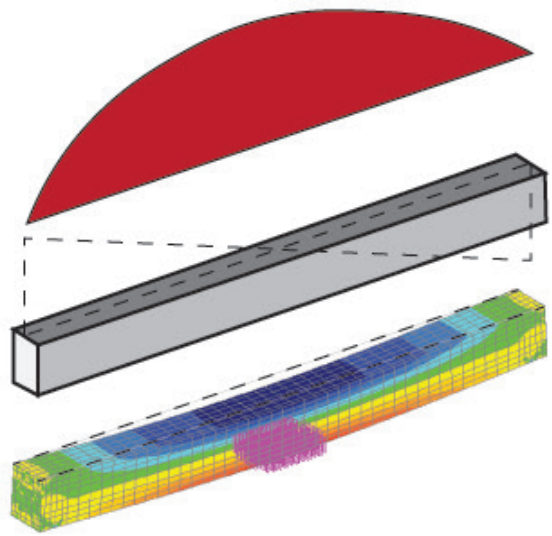


Figure 1.2.8 Determining locations of maximum stress and elongation under loading of a concrete beam using a moment diagram and a digital simulation

These methods are still the basis for evaluating the material behavior of structures, although new tools allow this analysis to happen more quickly, on more complex forms, with a color-based representation of stresses and deformations represented. (Figure 1.2.8)

Effectiveness: Shaping the Design for Stability and Suitability

Stability and suitability are based on the physical configuration of a structure. Stability is a three-dimensional puzzle that anticipates disruptions to translational and rotational equilibrium. Picture yourself on a ladder in which the sides are secured to the rungs with a single screw. We hope the ladder is strong enough to not fall down (z-axis) but it may tip over sideways—or if the ladder racks into a parallelogram shape (y-axis), or isn't angled correctly it could fall backwards (x-axis). Each of these threats to stability could be fixed if additional structural members were added in each direction. (Figure 1.2.9)

Suitability is analogous to the Vitruvian definition of commodity. We'll include it as part of "effectiveness" as a reminder that architectural and structural designers need to share the same overall goals. Assessing suitability isn't subjective if constructability and building performance are considered. Some structures could be technically effective but functionally unsuitable—a strong, stiff, and stable structure for a theater that places a column in the middle of the seating isn't suitable. A proposal that can't be constructed isn't suitable either. (Figure 1.2.10)

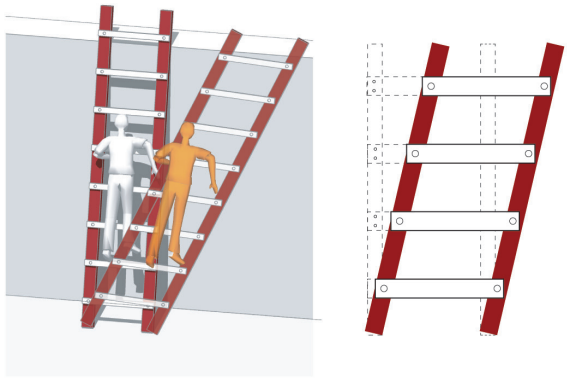


Figure 1.2.9 Selecting proper fasteners and their placement can stabilize assemblies that are otherwise unstable. Example: One pin allows rotation while two pins stabilizes the connection

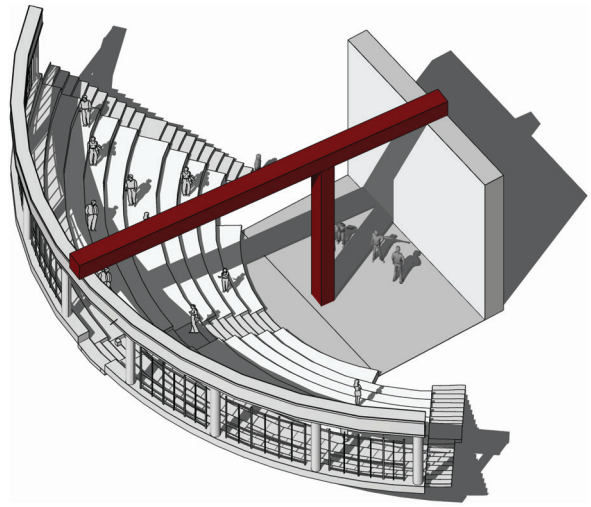


Figure 1.2.10 Despite its other structural qualities, it ultimately isn't suitable as a theater with a column in the middle

The fifth "S-Word," "Shape," is a term that affects more than just stability and suitability. By changing the shape—either to make it more stable or suitable—we may be changing the way forces are resisted and, therefore, the means by which it is evaluated. An arch can be stabilized by shaping it into a vault or a dome. All three have similar structural considerations, but they need to be evaluated differently. Structural design tries to optimize the formal relationship between architectural intentions and structural behavior. (Figure 1.2.11)

Efficient Structures

In the *History of Technology*, Arnold Pacey explains the importance of evaluation as part of the design process:

> *Although ideas may arise in all sorts of ways that may be described as intuitive or participatory, there is always an obligation to translate them into more rigorous, often mathematical formations, so that others may understand and check them, and explore their precise implications.*
>
> (Arnold Pacey)

These evaluations can be used to understand the relative efficiency of a solution.

Efficiency is a more apt and contemporary measure of structural performance than the pre-engineering standards of Commodity. Historically, when projects aspired to build higher or span farther, the most

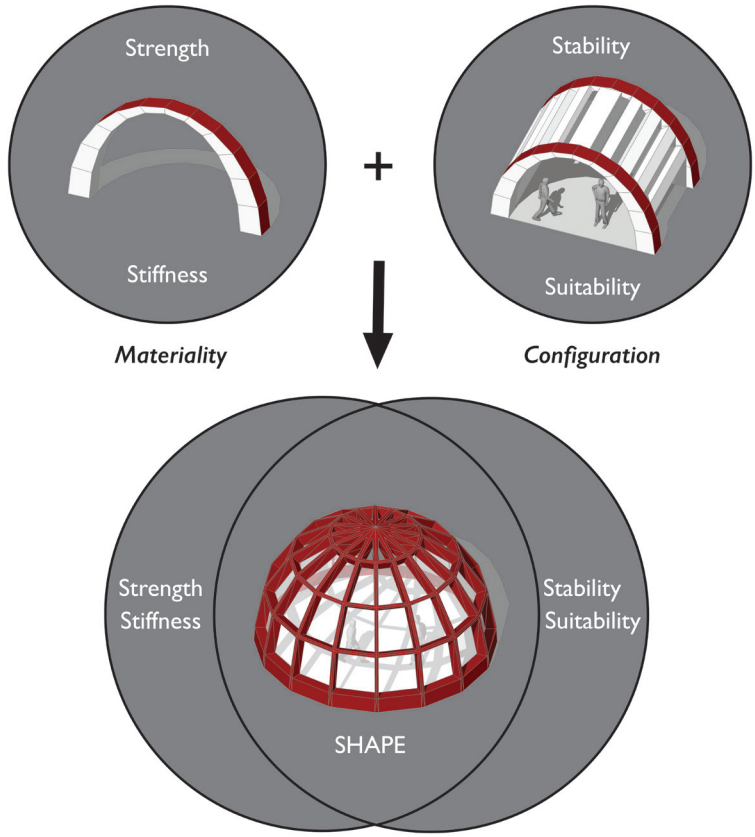


Figure 1.2.11 When all "S-Words" are integrated, the structure becomes more than an assembly of parts

common approach was to add more material, hoping that a more massive structure would suffice.

Of course, more materials don't guarantee structural integrity; the only things they guarantee are more resources, time, cost, and structural weight. Because these structures weighed more, they had to support more—a vicious cycle of wastefulness, called the *scale effect*, that caused the collapse of many projects. Viollet-le-Duc's 19th century theoretical spirit of rationality, expressed in *Designing for Efficiency,* was a repudiation of these older practices and *is* now a cornerstone value modern engineering. Thus emerged the common contemporary axiom of: "Any engineer can design a building that can stand up. A really great engineer can design a building that just barely does." (Figure 1.2.12)

Most commonly, efficiency measures material utilization. It evaluates how much a building weighs, how hard the materials work, and the relative economy of construction. If a structure transmits loads effectively, then less material is needed to provide resistance. The introduction of new materials (e.g., aluminum, carbon fiber, etc.) and/or improvements in existing material performances (e.g., high-strength steel) are also important factors in making structures stronger, stiffer, and lighter—and thus more efficient.

Shape is also a central consideration of efficiency. The performance of a structural element is determined by its shape and its material properties, so by adjusting the structural geometry of elements we can increase their relative strength and stiffness without adding additional materials—in other words, make it more efficient. Folding a piece of paper, for example, allows it to support weight. Structural improvements can be made in cross-section (such as I-beams, corrugated slabs, and trusses), in longitudinal profile (e.g., suspended cable bridges), and/or spatially (like arched vaults or domed trusses structures), by manipulating shape assemblage. Modifications that simultaneously consider strength, stiffness, stability, suitability, and shape should be the goal. (Figure 1.2.13)

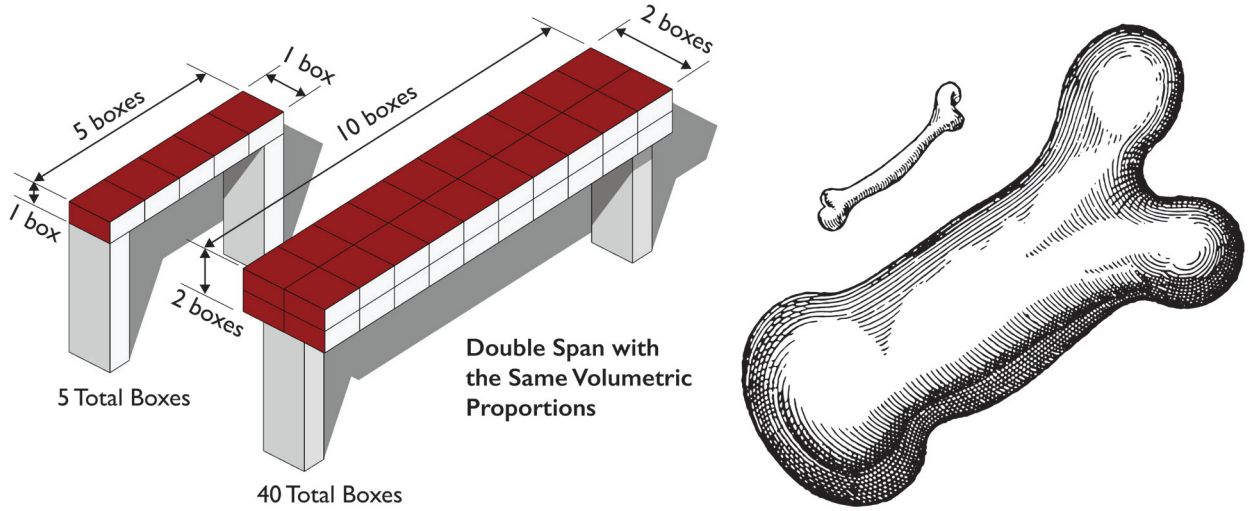


Figure 1.2.12 Galileo's description of the bones and the square-cubed law of volume demonstrates that increasing size has consequences. (Illustration from Galileo, Discourses and Mathematical Demonstrations Relating to Two New Sciences, 1638.)

Interpretative Standards of Building Efficiency

In addition to measuring quantity, efficiency should consider the sourcing of materials, their durability, their embedded energy, ecological profile, availability, and qualities of constructability. A lighter material isn't necessarily more efficient. It depends on how it is used.

Consider two divergent structural expressions of "efficiency:" the 1,000-year-old Clochan "Bee-hive" huts of Dingle Peninsula, Ireland and Buckminster Fuller's experiments with pyramidal frames from the 1950s. The dry-stacked Irish structures are heavy, but the stacked stones are local, easily sourced, placed in a shape that uses their strength against compression, assembled using a simple construction method, with a material that protects well against environmental conditions, and are still standing centuries later. They're clearly efficient. Fuller had an alternative definition; he saw efficiency as a function of optimizing materials through geometry. In one of his sketches, several smaller planar triangles, when combined, form a volume of space from planes with fewer structural elements than their original parts. By linking these pyramidal triangulated elements together, these lightweight structures can create large-scale enclosures (such as domes and spaces frames) from small, discrete parts. Again, it is clearly efficient, in this case because it does more with less. (Figure 1.2.14)

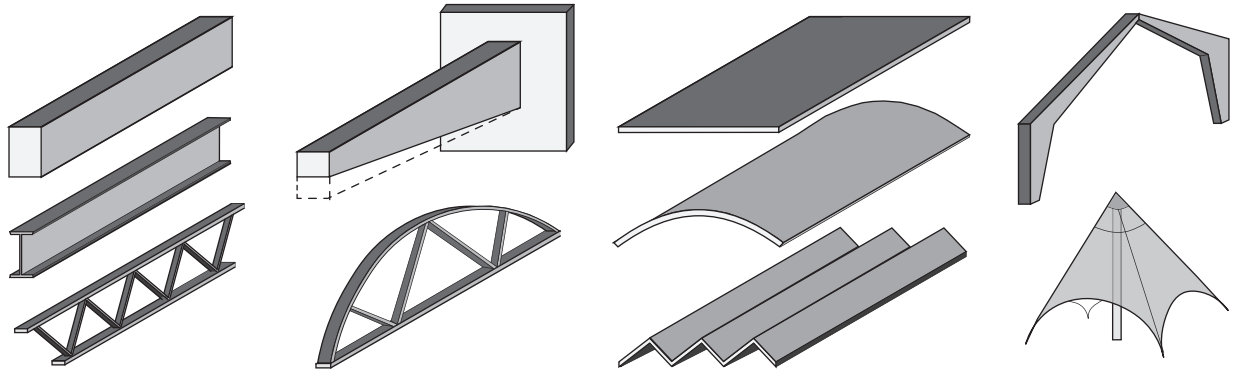
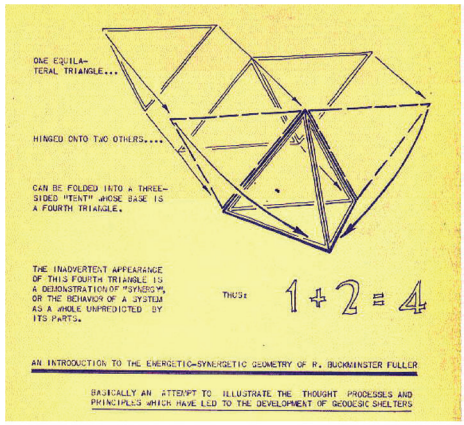
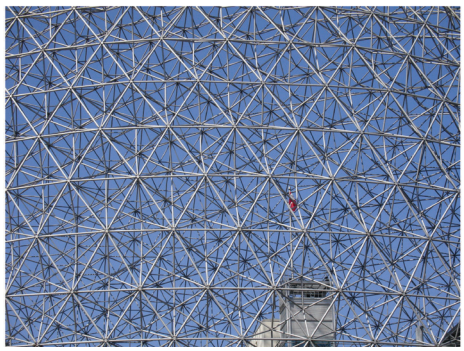


Figure 1.2.13 Strategies for modifying shapes to reduce materials and increase efficiency of resistance





Figures 1.2.14a and 1.2.14b Despite formal and material differences, the Clochan hut and geodesic domes both utilize their resources in a mindful and efficient manner

When strategies for structural form and materials are combined, they make both effective and efficient structures—and when these intentions are manifest into a corresponding physical form, the structure also becomes expressive.

Expressive Structures

Structures shouldn't be evaluated solely on their mathematical performance. Pier Luigi Nervi implored designers to consider that, "Technical correctness constitutes a sort of grammar of architectural speech, and just as in spoken or written language, it is impossible without it to advance to a higher form of literary expression." While structural integrity and functional compliance can be evaluated objectively, the source and measure of aesthetic "Delight" is more elusive.

The role of aesthetics and expression has evolved across time and practice, intersecting with larger questions about architectural culture and construction. In the 1917 book, *On Growth and Form*, D'Arcy Thompson argued that part of nature's beauty was the way it moved towards optimized structural forms (e.g., urchins, shells, and skeletons). This helped inspire the contemporary practice of biomimicry in which the mathematical and formal qualities of natural forms are used to suggest structural solutions. (Figure 1.2.15)

Some early 20th century Modern schools of cross-disciplinary design, such as the Bauhaus and Cranbrook Academy of Art, saw "Delight" as a consequence of creatively designing for Firmness and Commodity (including the means of production)—not as a separate consideration. By mid-century, progressive-thinking technophiles, designers, and builders saw technical and aesthetic considerations as inextricable. To them, structural effectiveness and efficiency *were* the source of their expression. In their thinking, beauty could come from the rigors of structural form, in how materials performed, and even in how structures were fabricated and assembled. (Figure 1.2.16)

Currently, digital design and analysis tools that couple high-performing structural materials and advanced digital fabrication techniques (3D printing, robotics, etc.) can create complex and expressive forms—sometimes with an advanced relationship to structural form and other times without. This

book won't spend time discussing explorations of form devoid of structural and construction considerations—learning to provide responsive solutions is a significant enough challenge. However, these powerful digital design, analysis, and fabrication tools are great assets and when used correctly, will be important allies in this pursuit. (Figure 1.2.17)

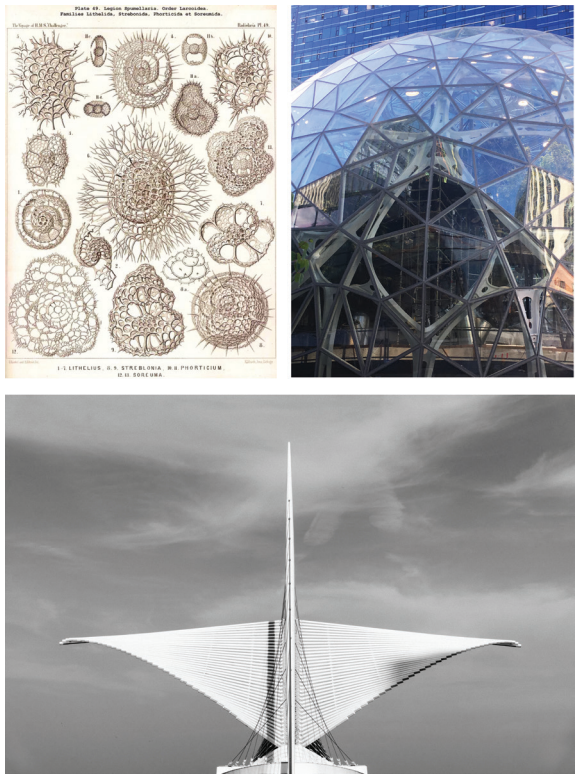
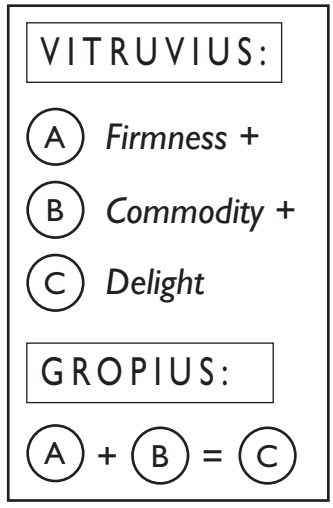


Figure 1.2.15 Thompson's Spumellaria Legion forms, the Amazon's Seattle Spheres (2017), and the Milwaukee Art Museum Quadracci Pavilion by Santiago Calatrava (2001)



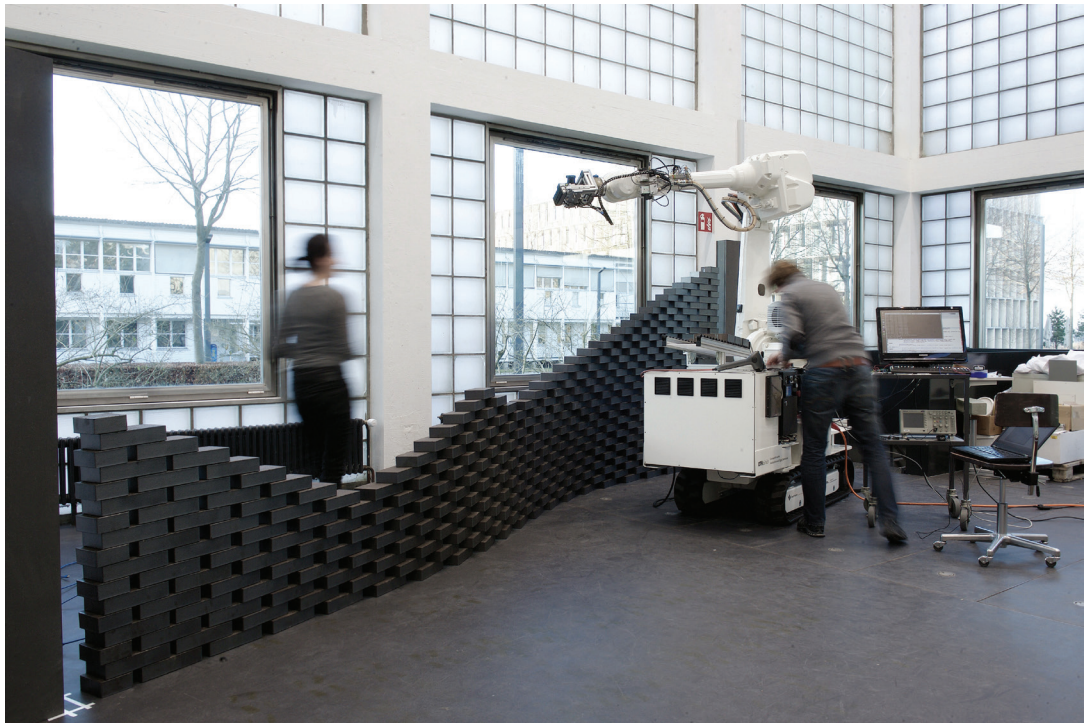
Figures 1.2.16a and Figure 1.2.16b Good Design by Eliot Noyes (MoMA) and Robert Venturi's updated interpretation of Firmness, Commodity, and Delight in a post-Bauhaus world of design

LEARNING BY LOOKING

Find information about a building that you think is interesting structurally and answer the following questions:

To Do: Describe the building (e.g., architect/designer, name, location, year built, etc.) and why you selected it. What strategies would you use to try to understand how the structure works?

To Discuss: How would you describe the relationship between the structure you see and the "architectural design"—are they distinguishable from each other in any particular way?



Figures 1.2.17a and 1.2.17b Efficiency and expressiveness in construction can be crafted using rules of geometry and hand-built construction to simplify complex forms (e.g., shell by Felix Candela), or automated to the point where complexity in form isn't consequential to traditional rules of labor (Gramazio Kohler Research, ETH Zurich)

Structural Classifications: Influential Examples

Good design emerges from a creative conjunction of structural integrity, functional consideration, and beauty—whether this is called *firmness, commodity, and delight* or *effectiveness, efficiency, and expressiveness*, the importance of these priorities endures. Because responsive designs optimize relationships between structural form and materials, the book format follows a structural classification system based on this relationship: Form, section, vector, surface, and frame. Each has different reasons why it would (or wouldn't) be selected, which we can codify by assessing their effectiveness, efficiency, and/or expressiveness in architectural and structural terms. We'll understand how to evaluate these factors in each category. As you read the following descriptions of exemplary projects, ask yourself:

How was the structure designed to be effective?

What are the major load-bearing members, how do they transfer loads, and how are these related to the structural form (Intended Integration)?

What materials were used, and how do these correspond with the structural system?

Was one factor (strength, stiffness, or stability) more challenging to resolve than others?

How w**as the structure designed to be efficient?**

What were the major performance goals for efficiency?

By what standards would efficiency be evaluated?

How was the structure designed to be expressive?

Does the form follow the forces?

What was the designer's source for inspiration in the expression (technical responsiveness, aesthetics, proportion, style, etc.)?

FORM: Dymaxion House (1930–1950), Buckminster Fuller

Fuller believed that by harnessing our available technology, humankind could evolve by transforming our patterns of "making" to be more efficient. Fuller applied his ideals to numerous objects and structures, including one of his first experiments, the Dymaxion (or 4D) House. Fuller described the open-ended nature of his search for design: "I didn't set out to design a house that hung from a pole . . . I started with the Universe as an organization of regenerative principles . . . I could have ended up with flying slippers."

Fuller believed that reducing the weight of a conventional house and simplifying its construction would make housing more affordable. This caused him to deviate from conventional norms for housing design towards form-resistant systems and materials because they were lighter and more efficient. His "house hanging from a pole," was elemental, efficient and radically different. Its materials and details ensured that the lightweight structure would be more affordable and much easier to assemble than traditional home designs. (Figure 1.2.18)

The house's components were to be shipped inside a hollow central mast. Once erected, this mast would serve as the entrance to the house (via elevator within the mast), a connection point for utilities, and of course, the main support for the house. The 1,700ft^2 (158m^2) hexagonal-shaped living space that encircled the mast had compression ring "hoops" at the roof and floor, held in place by six high-carbon steel tension cables. These cables extended up above the living space, becoming visible in the open-air roof-top platform space and were secured to the apex of the central mast. To ensure stability—a concern in any form-active tension-based structure— the cables continued down to the ground, crossing in triangular patterns

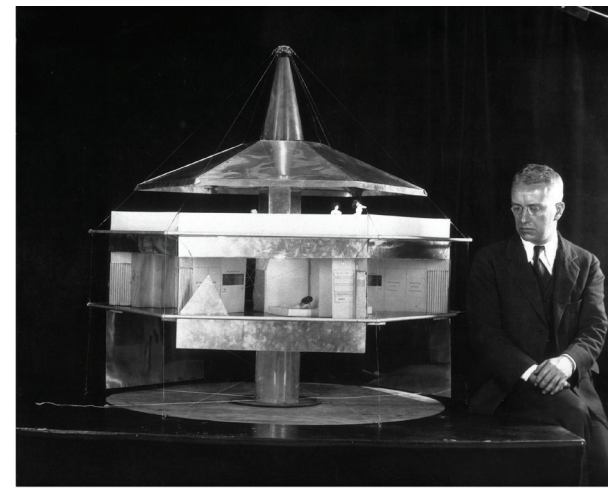


Figure 1.2.18 Fuller's demonstration model for Dymaxion House (1932)

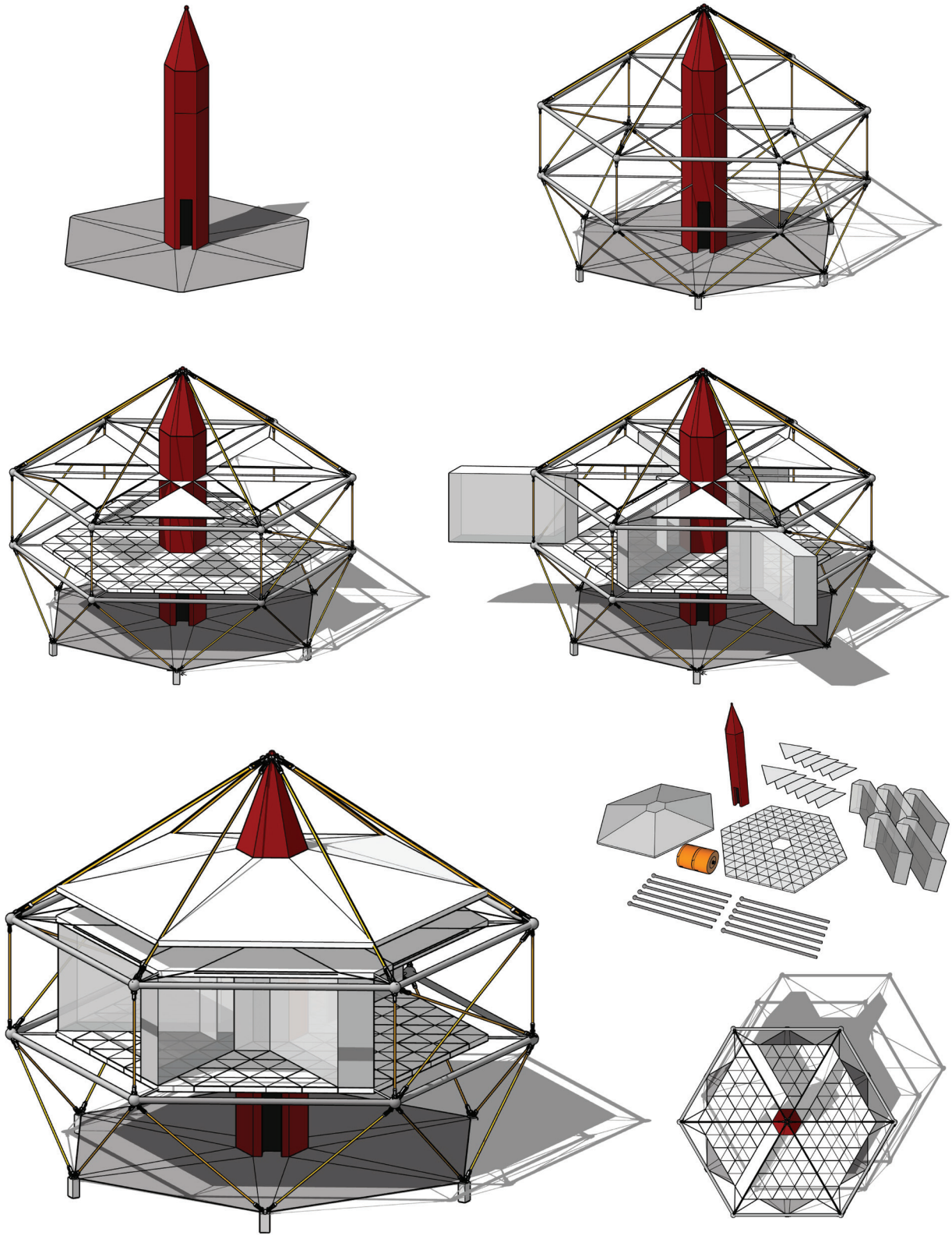


Figure 1.2.19 Sequential assembly of Dymaxion House

to provide bracing against twisting and resistance to uplift. Instead of a typical floor framing system with members in bending, Fuller invented pneumatic bladders (whose membrane was held in tension by air pressure) sandwiched between layers of horizontal wire mesh stretched between the perimeter compression ring and the central mast. Prefabricated "pods" for mechanical services (kitchen/bath) and storage were the only solid wall-like elements. (Figure 1.2.19)

These strategic choices allowed Fuller to use the smallest, lightest, most efficient structural members. Most of the house's components would have weighed less than 10lbs (4.5kg) each, light enough for one person to carry. As a result, he calculated the final projected weight of the house to be only 6,000lbs (2721kg)—around 1/50th of the weight of a conventionally framed, smaller, single-family residence. Fuller argued that if the entire assembly weighed less, it would cost less, and become easier to package, ship, and assemble.

Unfortunately, after twenty years of effort, no Dymaxion Houses with this design were built; the prototypes significantly modified the structural strategy. The larger story of the project's developmental woes is a fascinating reflection of industry, economics, and design hubris. But the project demonstrated that "lightness" and efficiency could be valid technological and societal goals, and that tension structures need not be used only for long span bridges.

SECTION: Various projects (1951–1974), Louis Kahn

Louis Kahn (1901–1974) aspired to bring order and clarity to his designs. This organizational strategy, which he called "servant and served," used building structure to define the arrangement of spaces in both plan and section, and to enhance their functions and aesthetics. For Kahn, the selection of a structural system, its development, and its integration into the design were primary considerations inseparable from other architectural priorities. As he explained, "Design is not making beauty, beauty emerges from selection, affinities, integration, love."

Kahn looked for ways to use structures as more than just support—they became three-dimensional expressions of organization and aesthetics. He embraced beams' and slabs' potential, since his designs needed to be well ordered, regular, repeating, and deep enough to refract light and help integrate building systems. Kahn expected his structures to also embody an aesthetic expression, so he modified conventional cross-sectional shapes to "serve" larger purposes. This intention repeats throughout his work, but a few examples show the evolution of these efforts to combine "effectiveness and expression."

In his first project, the Yale Art Gallery (1951), Kahn (or more accurately Anne Tyng) developed a two-way concrete slab system that spanned the gallery space. Its pyramidal forms were cast up into the slab (some solid, some open), allowing the slab to work as a modified waffle slab. It also hid conduits, lights, and sprinklers from being seen from below. Aesthetic and structural performance are entrenched and inseparable. (Figure 1.2.20)

Ten years later, at the University of Pennsylvania, Kahn and his structural engineer, August Komendant, developed an innovative design for the Richards Medical Research Labs. The lab towers' structure follows a plan diagram of "servant/served" spaces, separating open labs from adjacent spaces for stairs, pipes and equipment. Instead of hiding the structure, it was integrated into the overall design. The structural scheme used more than 1,000 pre-stressed concrete columns, beams, and vierendeel trusses. These elements assembled together on-site like a massive three-dimensional puzzle that was tied together with post-tensioning cables. Although it was maligned for functional lapses, at the time of its completion the Museum of Modern Art held an exhibit dedicated to the building and hailed it as "probably the most consequential building constructed in the United States."

In his final project, the Yale Center for British Arts (1974), the structural grid's regularity is fully

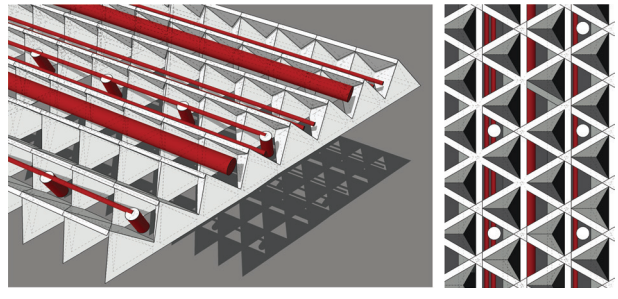
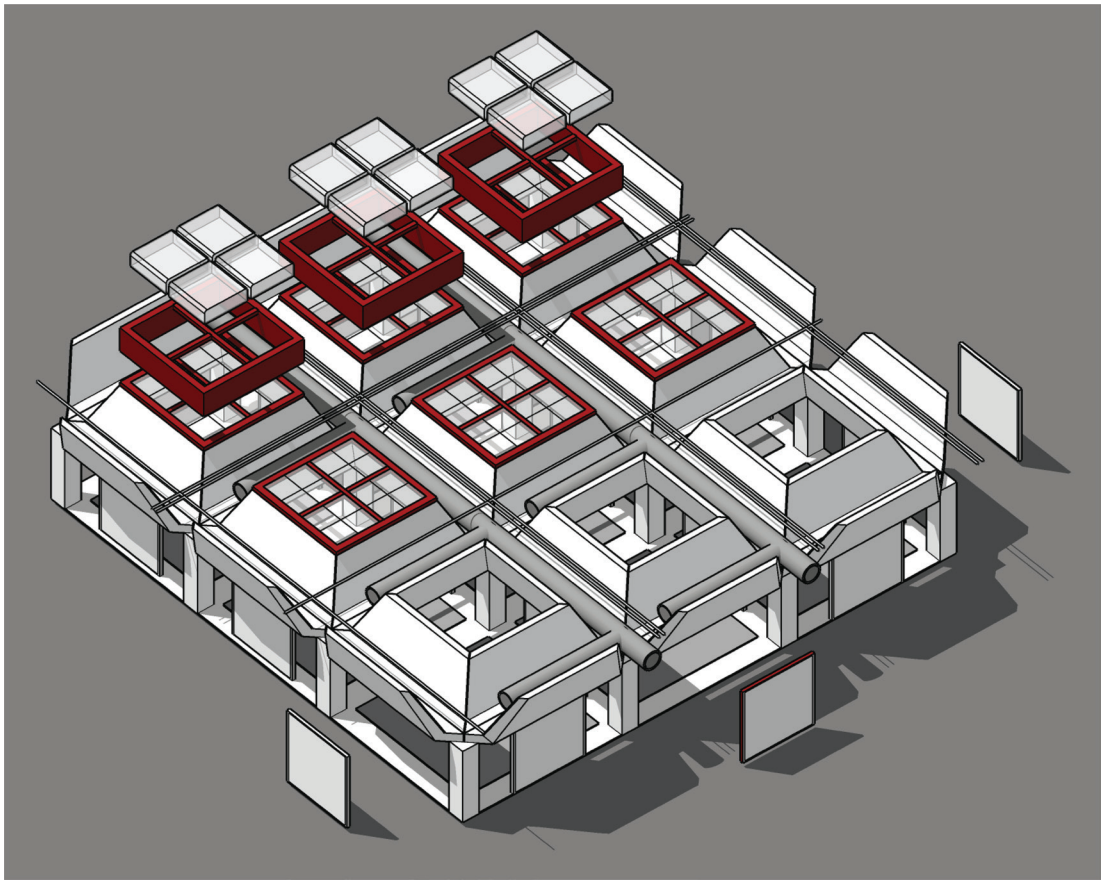
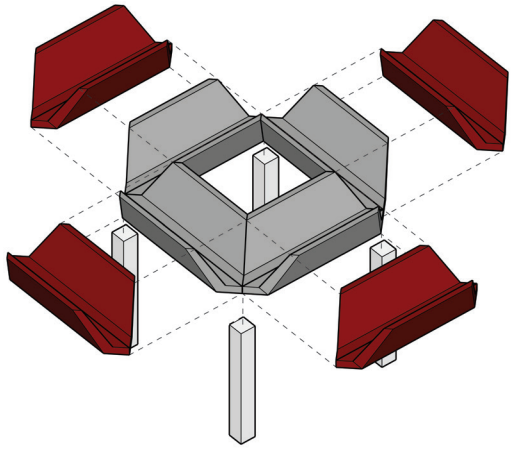


Figure 1.2.20 Integrated design for the Yale Art Museum floor assembly: structural principles, geometry, construction, aesthetics, and system integration

expressed and integrated with the building in plan and section. The defining expression of this grid is the skylight system. The entire roof is a square grid of V-shaped, precast concrete beams with large square skylights in void above—the roof structure is the source of light and structural expression, and this module defines the columns and gallery walls below. It is remarkable technically as well. The cross-sectional shape of the concrete makes an efficient beam, and the void in the V-shape allows for lights and duct work to be hidden within the structure, nearly out of sight. (Figure 1.2.21)





Figures 1.2.21a and 1.2.21b By organizing the spaces on a grid, precasting the structure, and repeating the same elements over and over, Kahn finds an economic benefit in the organizational and aesthetic clarity

VECTORS: Crystal Palace (1851–1852), Joseph Paxton

> *Those who will take the trouble thus to investigate the building will soon discover that, although vast in extent, in design it is exceedingly simple.*
>
> (Fox and Paxton, 1851)

Although trusses had been used sporadically for centuries, fabrication capabilities and early engineering design methods exposed the broader potential for their use in the 1850s. By resisting bending through lightweight linear triangular elements, instead of a mass of material like beams, engineers found that trusses could span farther with less material. They could be mass-produced from iron pieces, and they became reliable options for longer spans. They were used primarily in industrial buildings until constraints of time, budget, and span required their use in the Crystal Palace in 1851 in Hyde Park, London.

When Joseph Paxton's scheme (with the engineering company, Fox, Henderson, and Co.) was approved, there were only *eight months* to design and construct this 990,000ft^2 (92,000m^2) high-profile exhibition space. In an era in which construction of major civic buildings took years or even centuries, this was cumbersome. The Crystal Palace needed to be efficiently designed and rapidly constructed. These restrictions inspired structural innovations.

Paxton's solution was a simple and thoroughly contemporary idea for an incremental open system with long spans and repetitive elements. In his first sketch Paxton proposed that the entire building be constructed from the same square module—a unit that could be repetitively constructed horizontally and vertically. Resolving a single structural bay provided all necessary information to complete the rest of the building. (Figure 1.2.22)

They established a planning module, 24ft × 24ft (7.3m × 7.3m), based on the largest available plate-glass panels so the entire building could be clad using identical panes and reasonably sized and spaced structural elements. The considerable economies of scale gained from hundreds of nearly-identical modules allowed for affordable and easily assembled bays constructed from prefabricated iron trusses and columns around the perimeter, and supplemental wooden

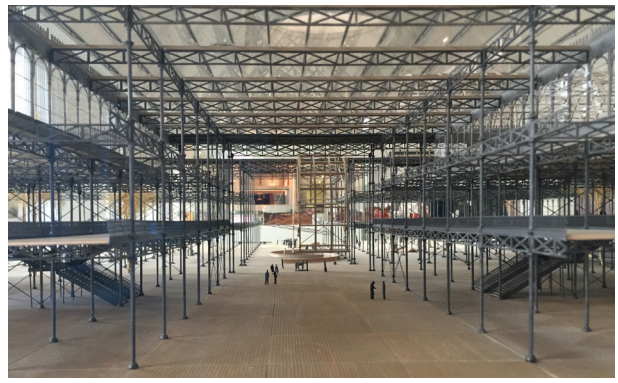


Figure 1.2.22 Model recreation of Crystal Palace (Architecture and Heritage City museum, Paris)

floor and roof beams, which were modified with post-tensioned wires to behave like trusses. All of the structural pieces were mass-produced, and many parts served multiple functions, which reduced the overall cost. Some sections of the structural frame were put in place less than one day after leaving the factory. Each module could be erected quickly. Because each

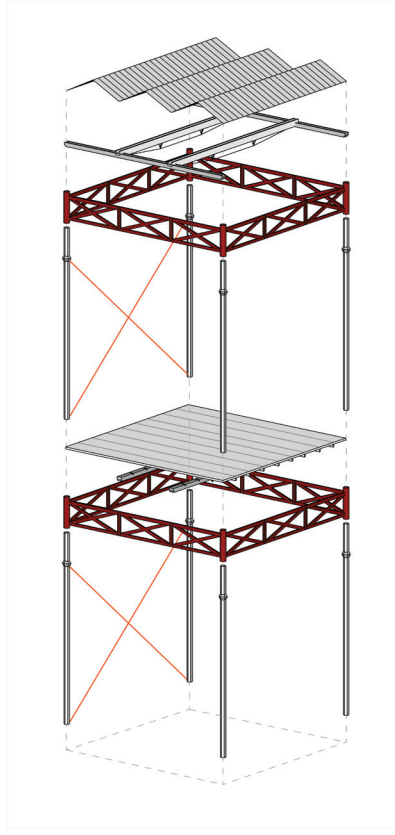


Figure 1.2.23 Axonometric, typical structural bay, Crystal Palace

bay was self-supporting, builders could assemble the building section-by-section, without having to wait for other parts to be finished. Because of the limits inherent in cast iron column and framing details, the frame wasn't stiff or stable on its own. Bracing had to be added to stabilize the structure. (Figure 1.2.23)

Most decisions about the building's design focused on efficiency and assembly—a good match for trusses. Trusses allowed the project to be lighter, allowed the bay sizes to be bigger, and made the construction process easier. Trusses were also aesthetically important, as a structural expression that could be exposed and celebrated in a high-profile public building. Because the structure combined efficient design and construction principles, the Crystal Palace was completed on time and under the original budget. (Figure 1.2.24)

SURFACE: Various Projects (1884–1962), Guastavino Fireproof Construction Company

In the late 19th century, the high cost of material and labor resources made traditional construction methods for long spanning heavy vaults and domes untenable. If these methods could be improved, however, a new range of structures would be possible. In 1881, a Spanish architect and builder, Rafael Guastavino, came to the U.S. and patented a new "fireproof" construction system based on traditional Catalonian vaults that could span as far as traditional vaults but used less resources, took less time, and required less height. The vaults could be arrayed to make larger enclosures. These advantages were significant and attractive to clients. (Figure 1.2.25)

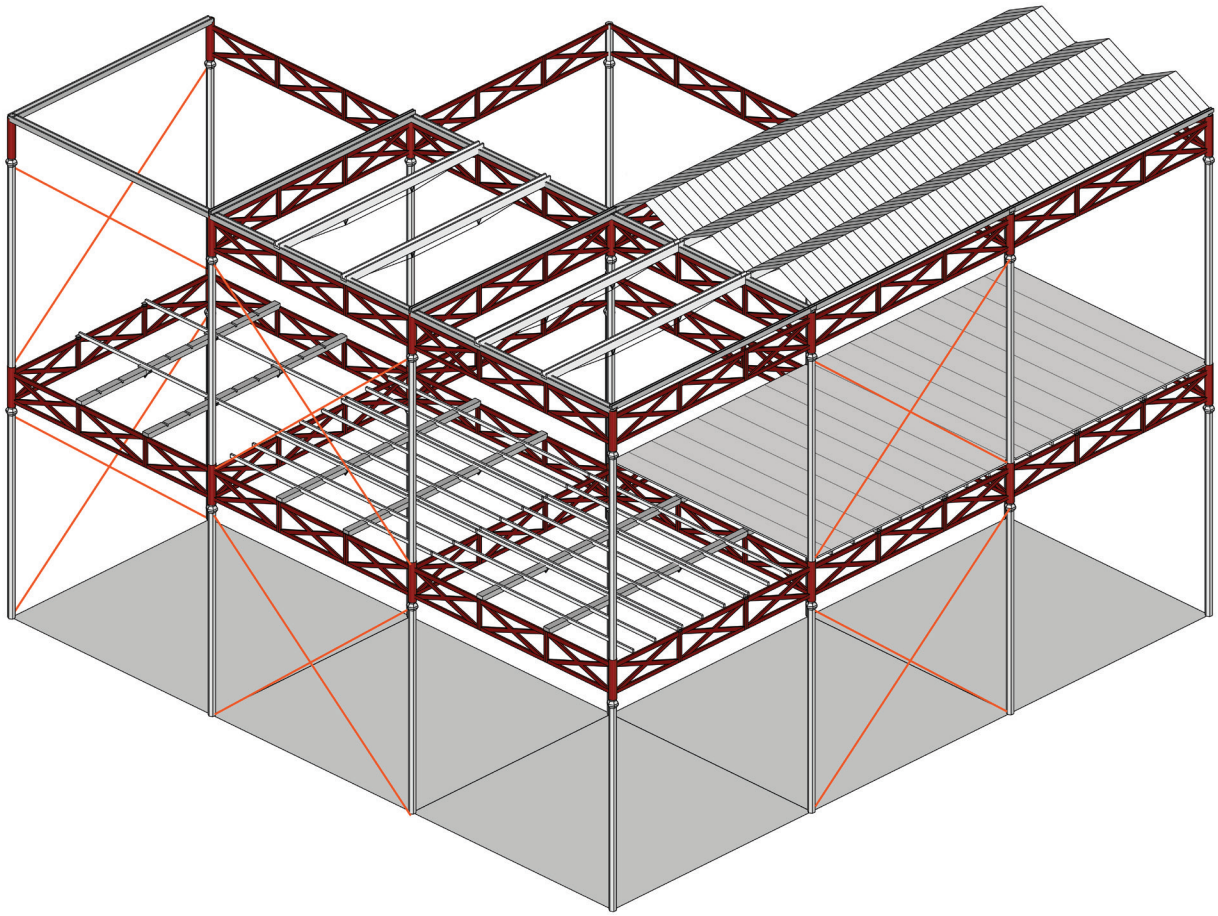
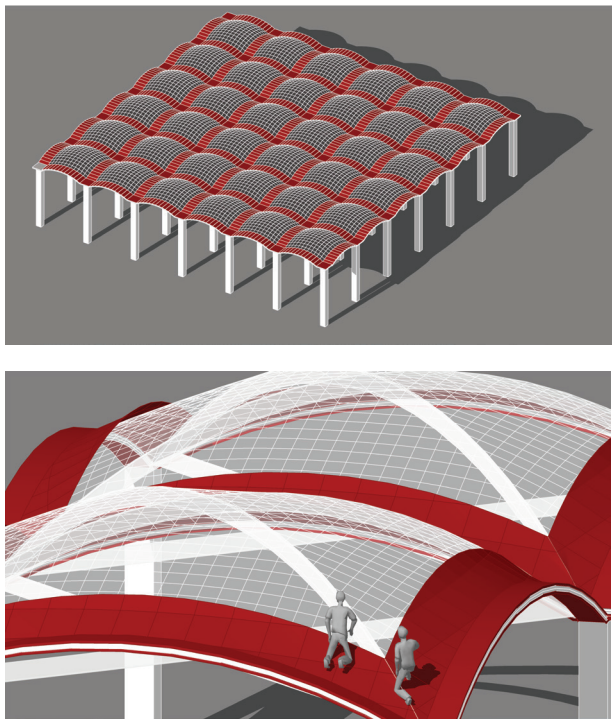


Figure 1.2.24 The bay-by-bay framing strategy and repetition of parts allowed for rapid assembly



Figures 1.2.25a and 1.2.25b The repetition of the geometry and construction methods made the vaults relatively affordable "fireproof" options for important large-scale buildings

The system exploited the efficiency of thin shell structural forms towards economic and functional advantages. "Guastavino vaults" were effective structurally, spanning up to 100ft (30.5m) with shallow rises and minimal thickness. The system's main advantage was its ability to build long spanning vaulted enclosures using small interlocked terracotta tiles layered with mortar. Instead of requiring scaffolding or "centering" like other vaults, this system was self-supporting—it was built up from the outside perimeter arches toward the vault's center. The mortar dried quickly so the vault became its own support. In its final form, the system produced a curved, thin shell surface made by stacking layers of tiles atop each other working from the edges in to the middle. (Figure 1.2.26)

Remarkably, the direction and magnitude of stresses in these vaults could be predicted using a graphic system. Because this was such an accurate predictor of form, only minimal materials were needed and most of the vaults have remained in-situ. From the 1880s to the early 20th century, this company's work was featured in *hundreds* of most of the celebrated civic buildings along the east-coast including the Boston Public Library, Grand Central Station, Ellis Island, Natural History Museum (New York City) and the New York City Central Public Library. (Figure 1.2.27)

FRAME: Sears Tower (now Willis Tower), Skidmore, Owens, & Merrill, 1969–1974

In 1969, when Sears, Roebuck and Company decided to relocate four million square feet of space into a new headquarters in downtown Chicago they didn't expect to end up in the world's tallest tower. For them, the functional and financial challenges were paramount—fast construction for the lowest possible cost. They knew the project was enormous and were worried about the premium cost of a high-rise design.

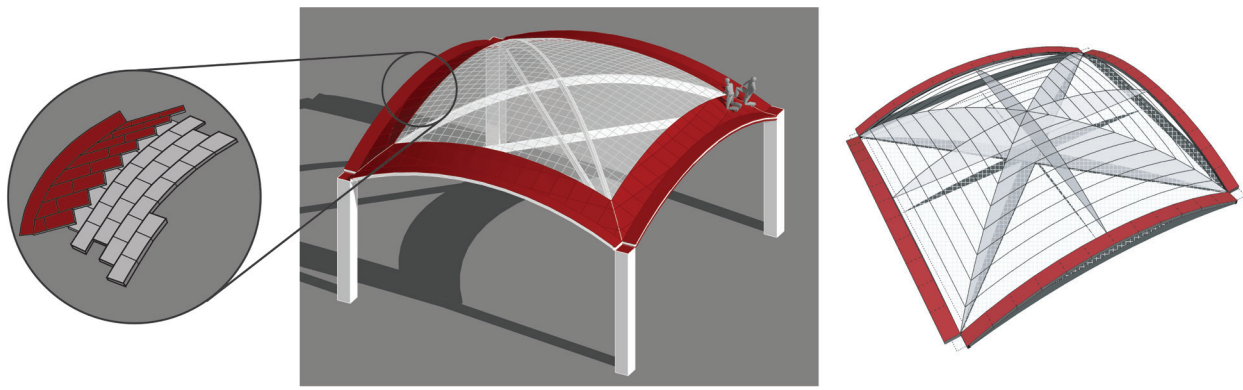


Figure 1.2.26 Vaults were built from the edges to the center—layer by layer. The formwork didn't support the shell, it guided the geometry of the curve so it was much lighter

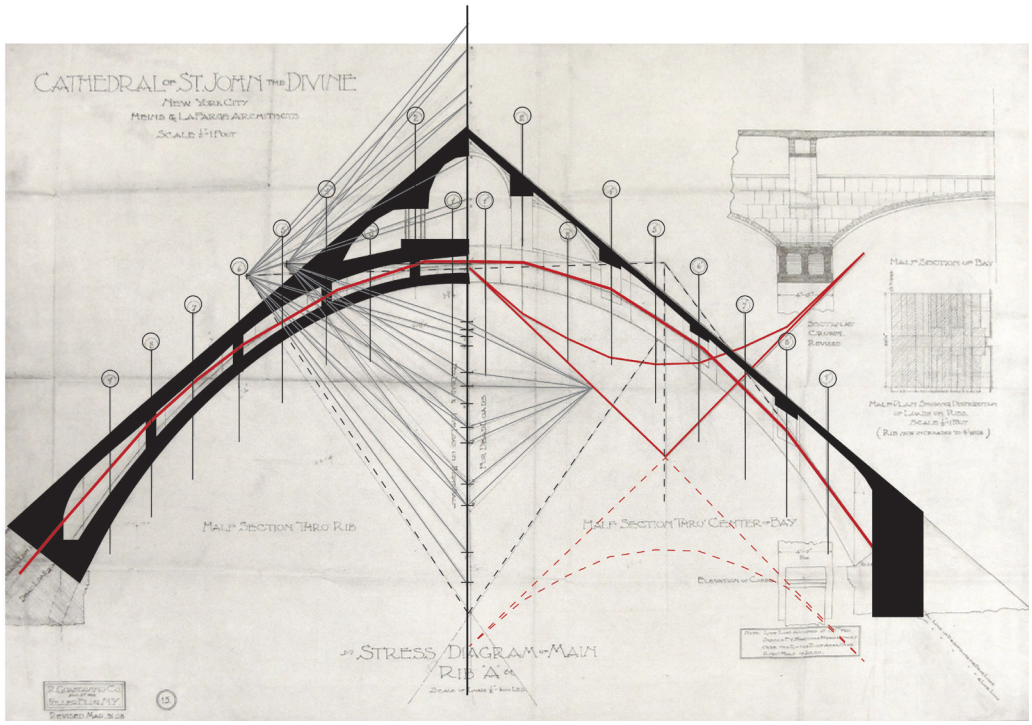


Figure 1.2.27 Enhanced illustration of Guastavino's thrust line drawing for the Cathedral of St. John the Divine

They turned to Bruce Graham (architect) and Fazlur Khan (engineer), of Skidmore, Owens, & Merrill (SOM) Chicago for their expertise. Because of their recent experience on the John Hancock Building (1969), Graham and Kahn assured Sears that an effective design strategy could be found.

The compressed construction schedule and tight budget requirements demanded an efficient structural form and innovations in how skyscrapers would be designed, analyzed, and constructed. Only few short years later, in 1974, when the 110-story building was completed on time and under budget, their uniquely effective structural proposal—a "bundled tube" form—became the tallest building in the world (1,729ft (527m) to the tip), a title it held for two decades.

As a building gets taller, it becomes subjected to ever-increasing lateral wind forces, which affect the building's structure and behavior. Because high-rises are essentially cantilevered structures sticking out of the ground, these wind forces cause even the stiffest buildings to sway at the top and bend sideways. Instead of relying on braced frames or stabilizing cores inside the building to provide adequate

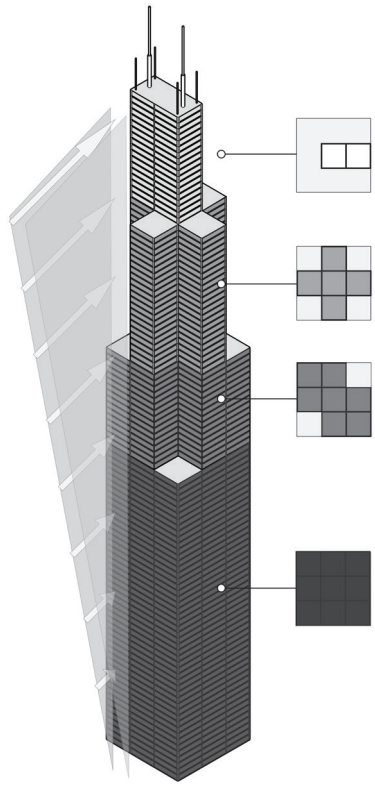


Figure 1.2.28 The building uses a structural and functional strategy based on a grouping of nine modules. The base is the widest and the bundled tubes recede as the building gets taller (and wind pressure increase)

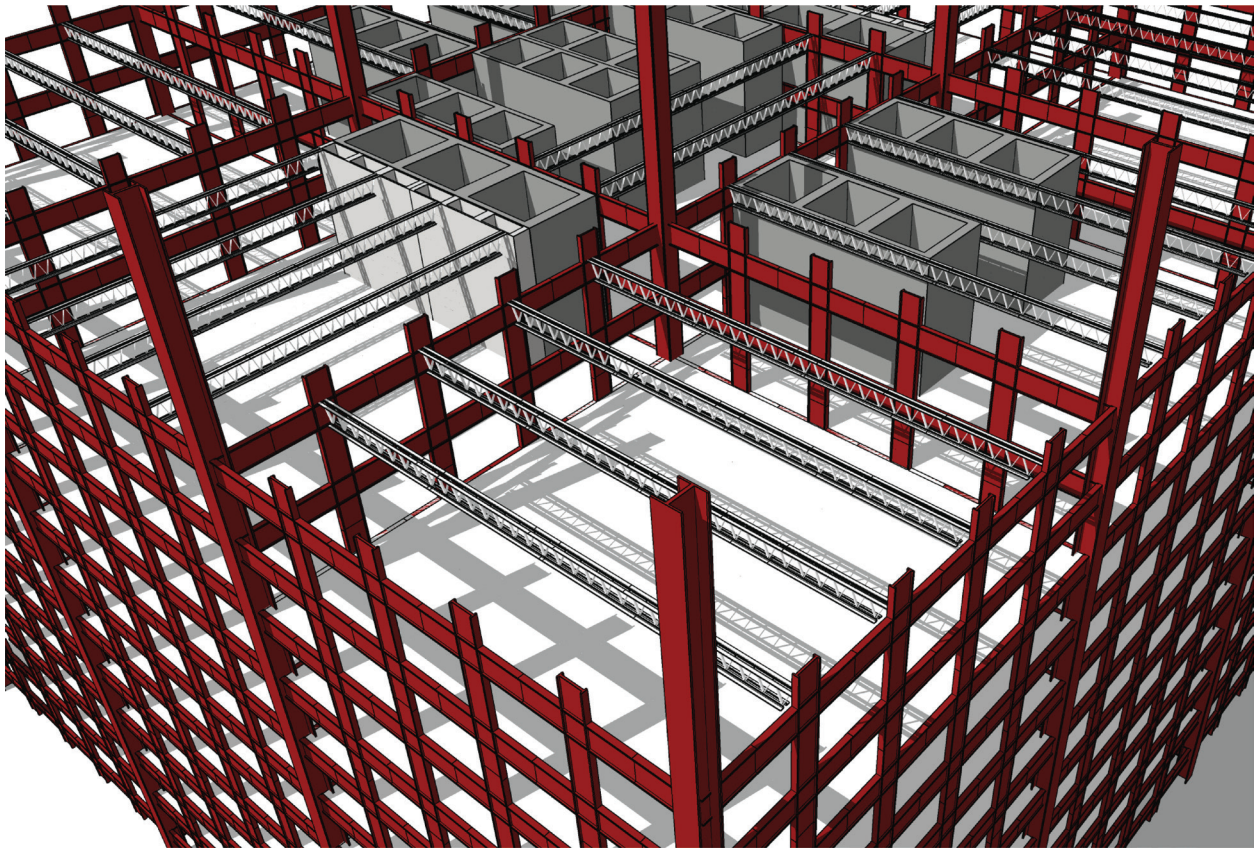
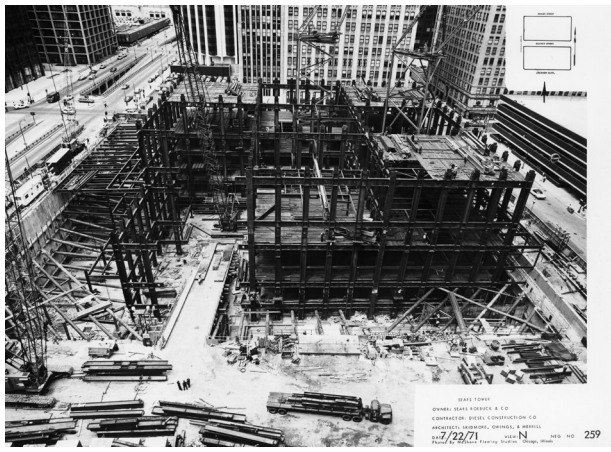


Figure 1.2.29 The regularity of the tube module added a uniformity to the structure, planning, and expression

stiffness and stability, which would take up functional space, Graham and Kahn developed a simple but effective solution: the "tubular brace," which placed wind bracing along the perimeter of the building in closely-spaced wide columns. (Figure 1.2.28)

Instead of designing the tower as a single tube, they broke the plan into nine smaller modules in a 3 × 3 plan matrix—each one of these 75ft (22.8m) × 75ft (22.8m) modules would be a separate structural tube with columns placed every 15ft (4.6m) along the perimeter. Steel truss floor supports spanned over the open office space inside. All nine tubes were bundled together for the lower fifty floors, above which certain tubes continued up while others ended at the 66th and 90th floors, leaving two tubes extending to the top floor. The building acts as a unified system of stiffened tubes. Regularly spaced belt trusses at mechanical levels tie the tubes together. (Figure 1.2.29)

This bundled arrangement stiffened the tower so much that less steel was needed to resist wind forces. Nearly every column in the building was providing both gravitational and lateral support. An economy of scale came from structural elements' repetition and a prefabrication process that allowed rapid construction. Most remarkably, the perimeter supports for *every* tube were all made from identical prefabricated structural elements—"Christmas Tree" units of two-story columns with four half-length beams welded to either side. The moment connections that stiffened these frames were created off-site so these units only needed to be bolted in place in the field—reducing the expense and effort of assembly. Not all of these pieces needed to resist the same amount of stress, but instead of changing the size of the elements, the design team changed the carbon content of the steel as needed. These units look



Figures 1.2.30a and 1.2.30b The tight confines of the site and constraints of schedule mandated a coordinated effort. Repetition in form, but variance in structural capacity of the steel was the solution

Figure 1.2.31 Models can demonstrate the possible associations between building forms and the methods of structural resistance provided. Shown: Historical precedent studies, SxD, Iowa State University

the same, but higher carbon elements have a higher structural capacity. (Figure 1.2.30)

Frequently Asked Questions

Do designers have to choose a certain structural classification (form, section, vector, surface, frame)? Wouldn't this selection restrict design exploration? There is never a good answer about *when* certain choices should be made in a design process, because each designer approaches problem-solving differently. However, the examples above show how these selections were logical extensions of the designer's priorities and/or the physical needs for the structure. Will these selections lock a designer into a restricted range of possible solutions? Yes and no (and that's a good thing). Properly placed constraints are guides, not barriers. For example, it is helpful to know that shell structures won't work for multi-story buildings, or cables may not work for floors, etc. These categories help designers understand relationships between structural forms, forces, and materials. Within each category are innumerable options. (Figure 1.2.31)

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PART 2

Resistance and Form

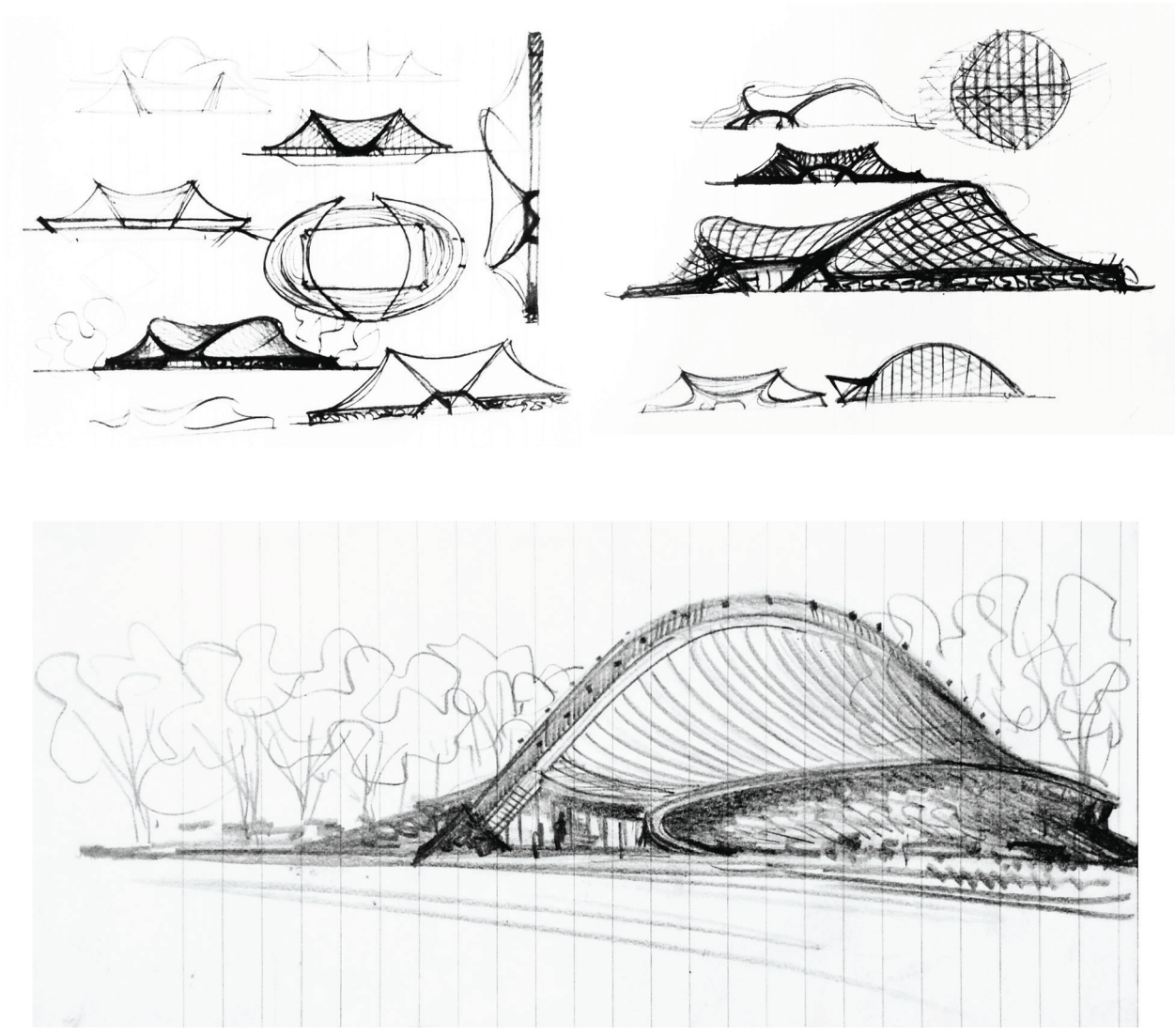


Figure 2.0.0 Sketches for Ingalls Hockey Rink by Eero Saarinen, (Yale University, 1958)

CHAPTER 2.0

FORM

Structures of Compression and Tension

Structures resist forces through a combination of their form and materials, but this relationship is adjustable. In this section, we will be exploring the structural system with the most direct and easily understandable relationship between form, material, and forces: the aptly named, form-resistant structures. Unsurprisingly, these structures utilize their physical form as the primary means of redirecting and resisting the forces.

Resistance through Form

Structural form is always an important consideration in determining how stresses are resisted and transmitted, but under certain conditions, the structure's form can be ideally matched with the *statical* form of resistance. Form-resistant systems—cables, arches, tent/membranes, and pneumatic (air) structures—align their materials with forces, providing resistance by being pulled or pushed. While it seems odd to pair heavy and solid arches with lightweight cable and membrane structures, these share fundamental commonalities in how they resist forces and the way their forms are derived. (Figure 2.0.1)

Axially Loaded Structures: Advantages and Limitations

Form-resistant structures transmit their loads in parallel with the material axially by matching form to forces. This orientation restricts the stresses they have to resist to either compression or tension (*normal stresses*). Because these structural behaviors are predictable, designers can select responsive materials that are strong and stiff enough to resist being pushed or pulled. Since these elements are either directly pulled or pushed and not bent, their entire cross-sectional area resists these stresses, regardless of shape. This maximizes material utilization and creates an efficient structure. Of course, because structural form is central to these types of systems' behaviors, they are inherently expressive of their structural functions. (Figure 2.0.2)

Matching form to forces is theoretically predictable but this inherent efficiency comes with risks. Structures designed to resist only one type of stress with ideally matched profiles can be risky if conditions change. Suspension cables are small and relatively light. Unless they are well-stabilized, cables can adjust too easily to changing loads (they are "form-active."). Arches are generally massive and heavy and can't change their shape if the load

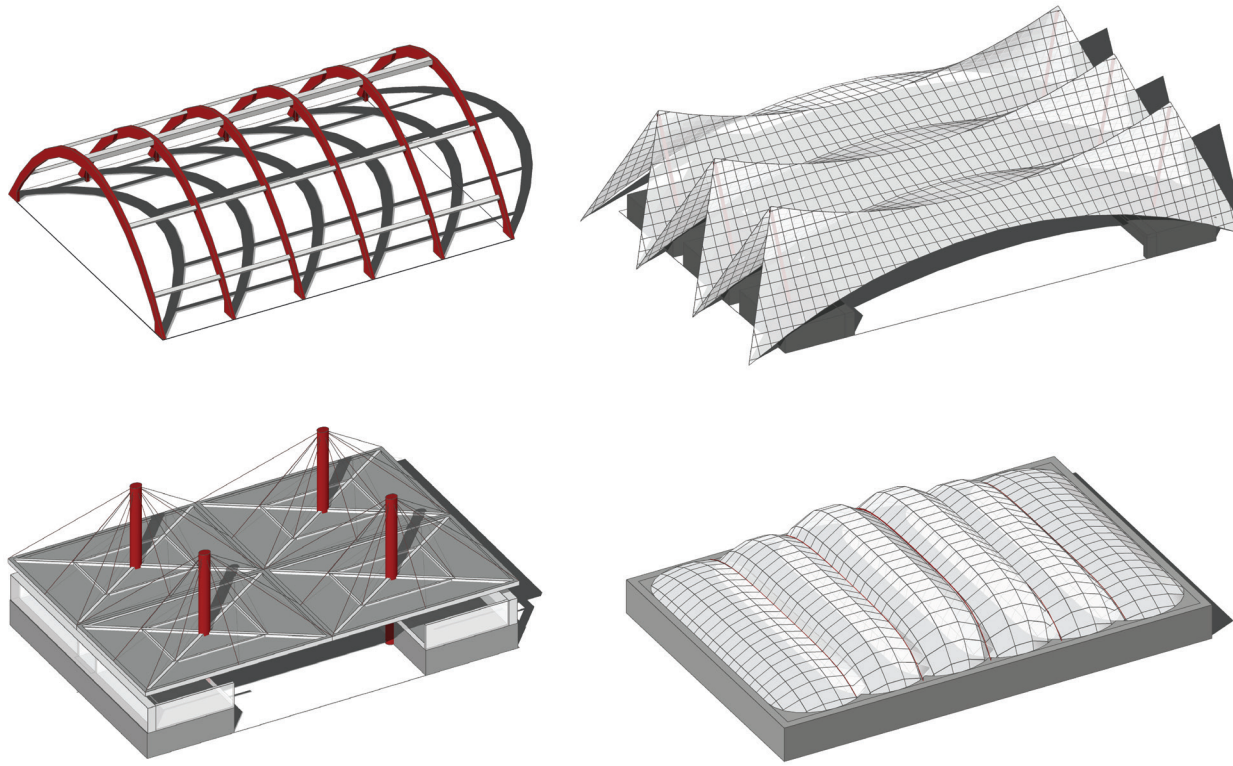


Figure 2.0.1a Resistance through form can be expressed as arches, cables, tents/membrane, air-inflated pneumatics, or hybrid combinations

Figure 2.0.1b The efficiency of resistance through form creates opportunities for long spanning buildings that express their means of resistance. Shown: Athens Olympic Velodrome Stadium (Santiago Calatrava, 2004)

location or magnitude varies from their ideal shape, so they are designed with extra material beyond what is needed to resist compression to stiffen them against these contingencies. This is why they are called "form-resistant." Without modification or additional elements, form-resistant structures aren't inherently stable to changing loads.

Some of the same factors that make form-resistant structures efficient and responsive can cause difficult construction challenges. Because they resist forces through a form with relatively minimal relative weight, they are suitable choices for extremely long spanning structures like bridges. But form-resistant structures consist of separate inherently unstable structural elements that don't achieve stability until they are complete—one half of an arch can't stand on its own and supporting cables will sway until they are loaded properly. This worsens the challenges of building across rivers or deep ravines since installing extensive scaffolding or framework across long spans is treacherous and difficult.

Because they resist loads through form and materials, the types of materials that are available for these structures is inherently limited. Certain materials do

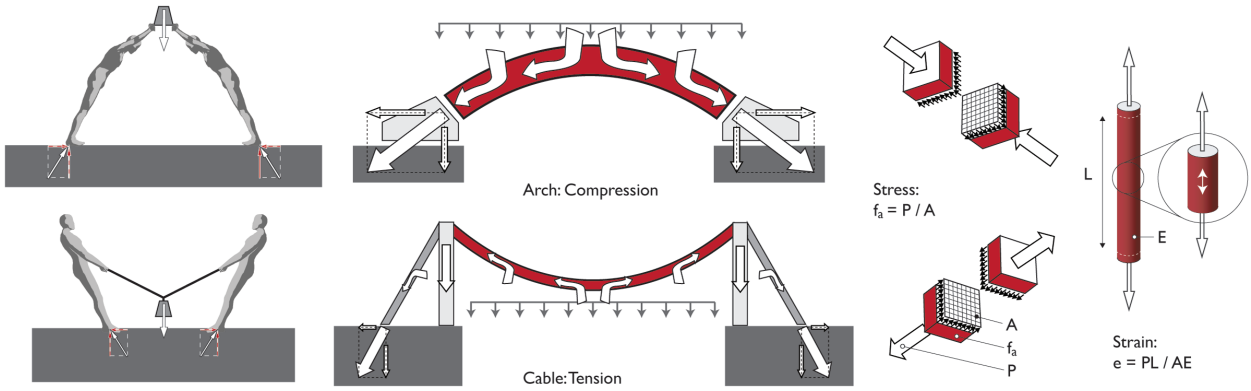


Figure 2.0.2 Fundamental behaviors, force flow, and formulas for axially loaded (compressive or tension) form-resistant structures visualized with bodies and diagrams

Figure 2.0.3 The Hypar Vault, by the New Fundamentals Research Group, is assembled from a series of smaller custom-cut limestone blocks that are post-tensioned together (Troyes, 2017)

Figures 2.0.4a and 2.0.4b Q'eswachaka Inca Bridge is one of the last hand-woven bridges in Peru—rebuilding it annually is part of a tradition that is hundreds of years old

well under compression (stone, brick), others do well under tension (rope), and some do well under both (steel, and wood). Compression connections between elements behave more predictably (thanks to friction) while tension connections are difficult in nearly any material except steel. (Figure 2.0.3)

Overall, the structural advantages of form-resistant structures are desirable, particularly their predictable structural behavior. For millennia these structures have appeared across many different cultures at various scales. The means of creating them was naturally inclined to the pre-scientific efforts of builders—after all, ropes automatically sag and change form as they are loaded and connected; an arch will only stand firm if its proportions and materials are viable. Outside smaller-scale nomadic structures and rope-bridges, tension structures weren't widely used due to the threat

of material failure. But arched forms, and the resulting vaults and domes based on their geometry, became the basis for the largest and sturdiest structures across many cultures and locations. (Figure 2.0.4)

Tools for Finding Form: Precedents, Geometry, and Early Experiments

For centuries, geometry provided a rational basis for the design and construction of compressive arches; simple diagrams of arcs and rules of thumb were used in lieu of drawings. The earliest arch prototypes weren't based on mathematical principles of force flow, they were based primarily on geometry for predictability and ease of construction. The distinction between the two wasn't made.

This was advantageous to ways that arches (and vaults) were supported and constructed. The pervasive half-circle Roman arches aren't structurally accurate reflections of a proper compressive shape, but their regular radii made them easy to repeatedly form and build, particularly with remedial tools. This form was close enough to the ideal shape in most cases, due to the excess material used, but tension cracks can frequently be found in them. Because arched masonry structures were so resource-laden and time-intensive to create, and because their failure would be catastrophic, they were designed and built conservatively. Ensuing arches were designed based on the proportions of span and thickness established by previously constructed projects. (Figure 2.0.5)

How did building sciences move past trial and error towards a scientifically-based method of finding structural forms that only resist tension or compression? Developing scientifically predictable rules for how form-resistant forces could be designed and built needed to come from an understanding of forces and material behavior—an apt task for the Rational Age and a scientific process of testing. (Figure 2.0.6)

Mathematics of Material Behavior: Stress and Strain

Eventually science sought ways to explain how physical forces interacted with masses to cause movement or to sustain equilibrium. Through drawings, physical

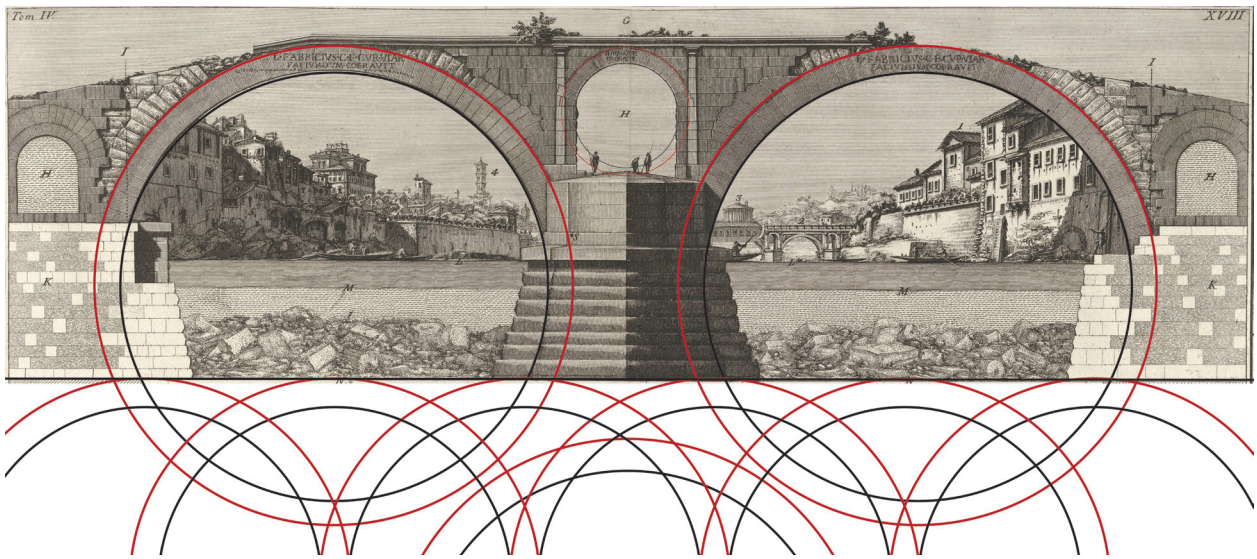


Figure 2.0.5 Drawing of the Ponte dei Quattro Capi, Rome's oldest bridge (62 BCE), showing the use of circular geometries for the form, supports, and foundations below. Original engraving by Piranesi, 1756

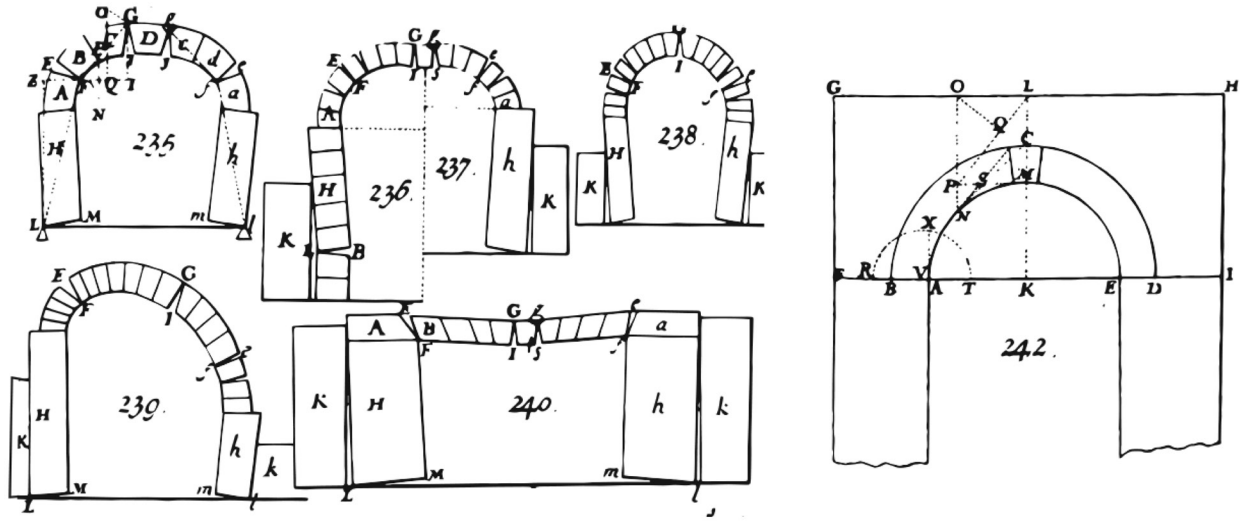
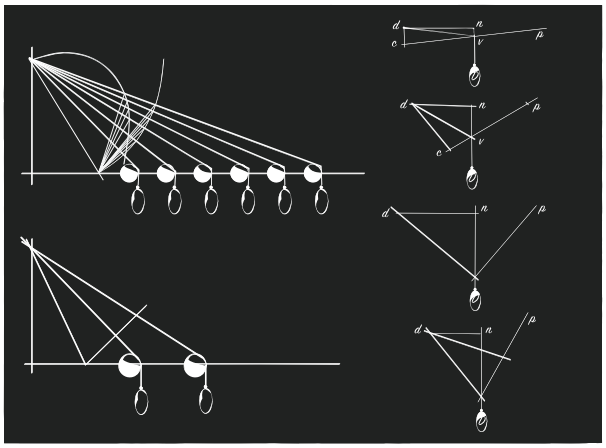
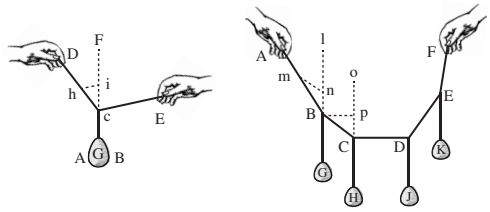


Figure 2.0.6 August Danyzy's drawings analyzing failures observed in plaster arches under testing, 1732





Figures 2.0.7a, and 2.0.7b Recreation of Da Vinci's sketches studying the relationship between cables, loads, and deflection (Codex Leicester 1510) and recreation of Simon Stevin's similar analysis of cable form and weights, 1586

models, and mathematical theories, noted scholars including Leonardo da Vinci (1452–1519), Galileo Galilei (1564–1642), Robert Hooke (1635–1702), and Isaac Newton (1643–1727) explored physical sciences critical to building structures. While Da Vinci and Galileo speculated on the inner-workings of spanning and cantilevered structures, Hooke looked for ways to understand the forces in arches and cables and established foundational theories of form and material behavior with such clarity that they are still in use today. (Figure 2.0.7)

As we demonstrated in Chapter 1.1, form-resistant structures (Strings and Stones) have a straightforward set of behaviors under stress. When they are pushed or pulled axially by a force (P), they rely on a combination of their material qualities (*allowable stress* (f_a), and cross-sectional area (A)) to provide resistance. Their mathematical relationship can be described in the equations:

$$\mathbf{f_a = P / A} \quad \textbf{or} \quad \mathbf{A = P / f_a} \quad \textbf{or} \quad \mathbf{P = A * f_a}$$

See Appendix Table A.4 for allowable stress properties of various materials.

MAKING: THE FEEL OF THINGS

There is a natural stress line called the *funicular* line (from the Latin word for "rope") that shows where tensile or compressive forces will naturally flow given a particular height and span. Although the mathematical derivation is complicated, finding the form is not. *Simply hold an unloaded string between your hands and let it sag*. Doing this allows you to feel how these structures behave and to see how different loads can change the form. Even without any additional weight applied to the string, your hands have to apply resistance both upwards and outwards (a horizontal stress known as thrust). (Figure 2.0.8)

The sagging string (called a catenary curve) is in pure tension, and the amount of distance it sags down is related to the length of string, the weight of the string and the location of your hands. Changing any of these factors immediately alters the string's shape. When weight is added to the cable, the structure's form changes as the internal tension straightens the two sides of the cable—off-setting the weight off-sets the profile. Adding more weight continues to change the form and the internal stresses—the form will change depending on the amount of weight and where it is added to the cable.

Load the string asymmetrically with a heavy weight on one side to see how the shape changes and how the support resistance of vertical and horizontal reactions you feel in your hands also changes. Each time the shape shifts, it shows the ideal profile to resist these new loads and it changes the amount of tension in each side of the string. The more regularly the load is added (towards a uniform distribution), the closer the cable will come to a parabolic form nearly identical to the unloaded catenary curve.

There is a clear relationship that you can feel between the string's height (or sag) and thrust. If you pull your hands apart from their original position, decreasing the sag on the cable, you'll have to increase the amount of outward thrust you provide. This relationship between sag/height and thrust is a fundamental principle of form-resistant structural behavior. Although there are clear advantages in structural efficiency created by matching the form to the forces, this inextricably conjoined combination of factors, *height and thrust*, can be clear disadvantages for certain architectural problems. (Figure 2.0.9)

To Do: Put at least two weights on a string and hold it apart at a certain distance and sag. Use calculations to determine the overall amount of stress within the rope and amount of thrust on the ends. Modify one factor (weight location or magnitude) and recalculate. Determine a cross-sectional area for the rope if its allowable stress capacity was 250lb./in^2 (17.6kg/cm^2)?

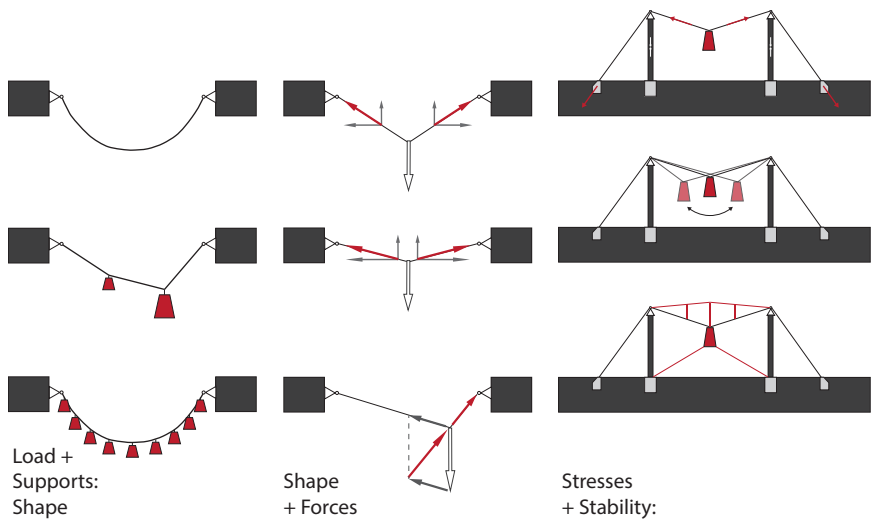


Figure 2.0.8a Basic concepts of cable behavior under loading

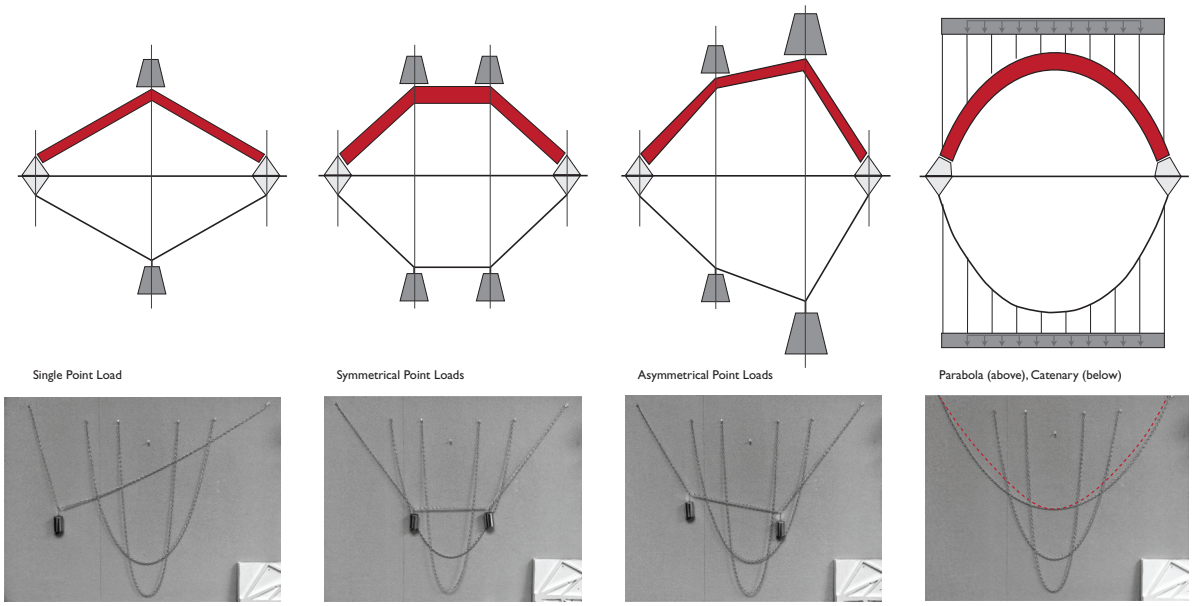


Figure 2.0.8b Demonstrations and diagrams showing how changes in the location and magnitude of forces also change the form. Note how the same form can be inverted for arches

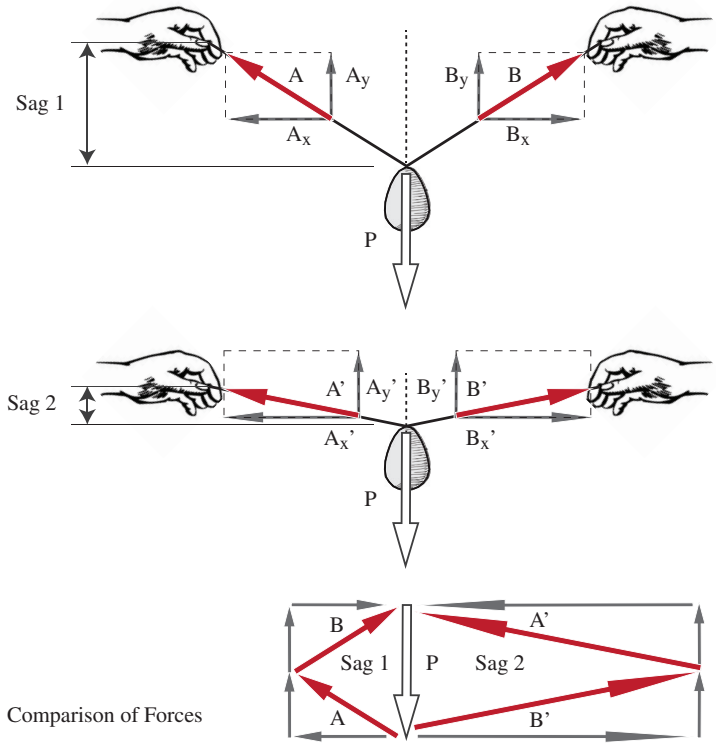


Figure 2.0.9 The inverse relationship between sag and thrust (i.e., lower sag = higher thrust) can be experienced through a simple experiment and diagrammed for confirmation. Free body diagramming can be developed to understand vertical and horizontal force magnitudes

Hooke's work established the connection between strength and stiffness. He discovered that *all* material was naturally springy under loading and that these qualities could be measured—most accurately if the material was pushed or pulled axially like form-resistant structures. He discovered, through experiments, that the strain that a material undergoes was directly proportional to the axial load applied (e.g., if a string elongates 1inch under 10 pounds of loading, it will elongate 2 inches under 20 pounds of loading), and that material returns to the same shape when the load is removed—up to a certain point (elastic range).

Newton and other 18th century scientists (Euler and Young) expanded on Hooke's work with insights into material behavior. They confirmed that all materials change shape under loading (i.e., they strain) and that some materials behave differently under compression than they do under tension. They discovered that under a certain amount of load a material would deform permanently and begin to act erratically, until it eventually failed (ultimate limit). This led to the stress/strain diagrams of material behavior we use today and established a viable formula for calculating the elongation of an element based on its material.

Strain is defined as the change in an object's length (*e*) from its original length (L) under loading. Stiffness is a ratio of stress (*f*) to strain (*s*)— also known as the material's Modulus of Elasticity (E) or Young's Modulus, as defined by the formula E = f/s. This stress/strain ratio can be plotted on graphs to demonstrate a material's stiffness. (Figure 2.0.10)

If we substitute equivalent formulas for stress (f = P/A) and strain (s = e/L) into this formula to get a more complete definition of a material's modulus of elasticity:

$$\mathbf{E = f / s \rightarrow E = (P / A) * (L / e) \rightarrow E = P * L / A * e}$$

By adjusting the formula one last time, we can isolate the value for an object's deflection (*e*) under axial loading:

$$\boldsymbol{e} = \mathbf{(PL) / (AE)}$$

See Appendix Table A.4 for degrees of stiffness (E) for various materials.

Form-Finding: Funicular Forms in Tension and Compression

The practical challenges of balancing height and thrust in suspension structures had been understood intuitively for centuries due to the challenges of construction with ropes and chains. But another significant discovery that Hooke made in the 1670s was that this method of form-finding for tension-based structures could also be used for arches. Because tension and compression are opposite stresses, he argued, their optimal structural profiles are also

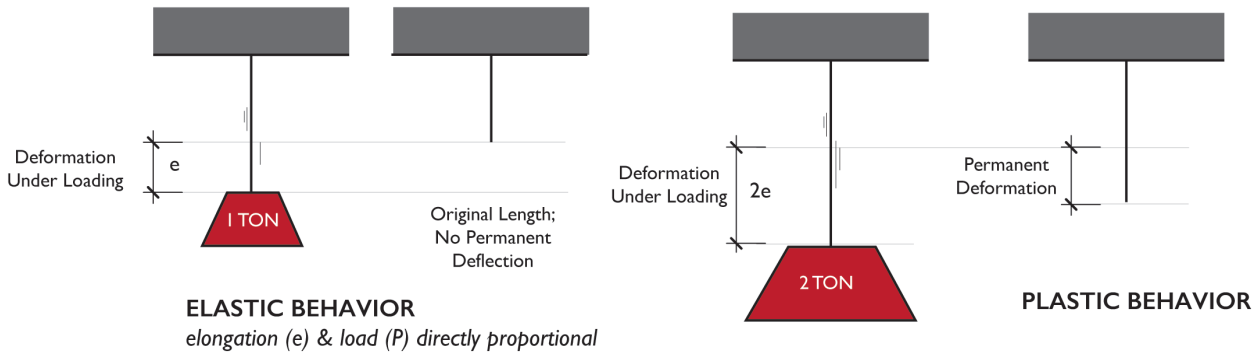


Figure 2.0.10 Material behavior under loading: Elastic versus plastic behavior and the direct relationship between load and elongation in the elastic range

mirror-images. He described this with the phrase, "as hangs a flexible cable, so inverted, stand the touching pieces of an arch."

Hooke's discovery was applied dramatically during his collaboration with Sir Christopher Wren on the domes of St. Paul's Cathedral in London (1675). St. Paul's three domes each have different profiles that can be seen in section. The outside dome was designed for aesthetic expression, and the inside dome was designed for spatial experience. But the middle, load-bearing dome, matches the profile of a hanging chain. (Figure 2.0.11)

Linking form and forces is fundamental to understanding structural behavior. It can be extrapolated to understand complex structural typologies like unreinforced masonry vaults, or applied to traditional construction techniques, like hay bale construction, using simple tools for form-finding and construction. (Figure 2.0.12)

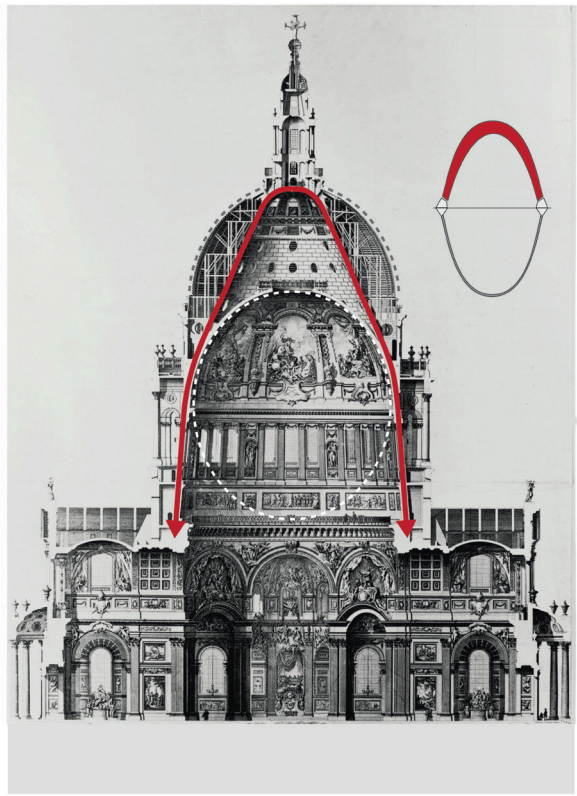


Figure 2.0.11 Section of St. Paul's showing the funicular form of the loading bearing dome (in red) and thrust line support

Form and Forces: Graphics Statics and Form-Finding Models

How did new knowledge about forces and the behavior of arches and cables under stress change the way these types of structures were designed and built? As always, it was a process of experimentation and gradual improvements.

One of the important tools that developed from the application of science to building was the ability to represent physical forces graphically. We accept that the direction and length of vector arrows can represent the direction and magnitude of a force, but imagine how difficult it was to describe and analyze structural behavior before this convention was established. Instead of speculating about these forces, or risking trial and error experiments, this

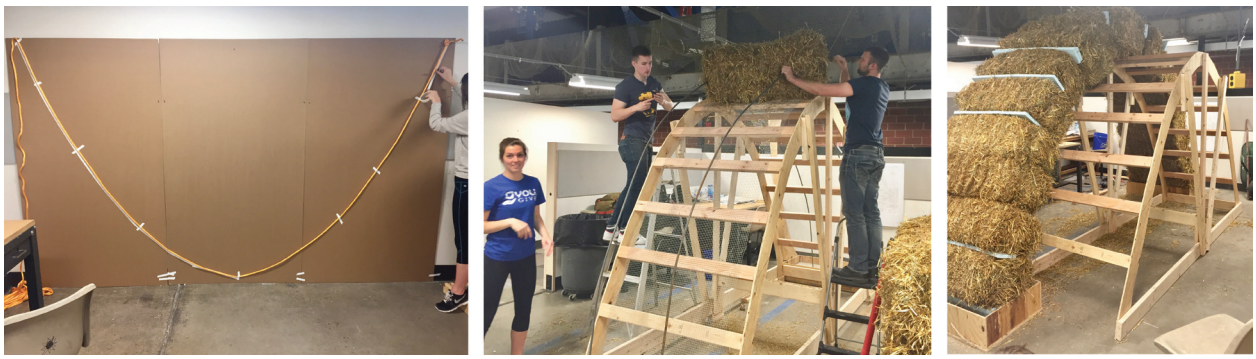
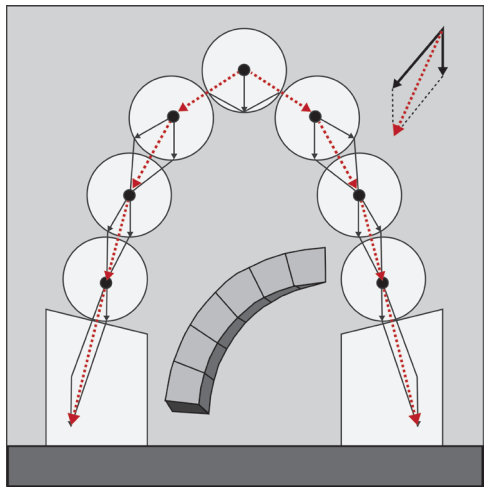
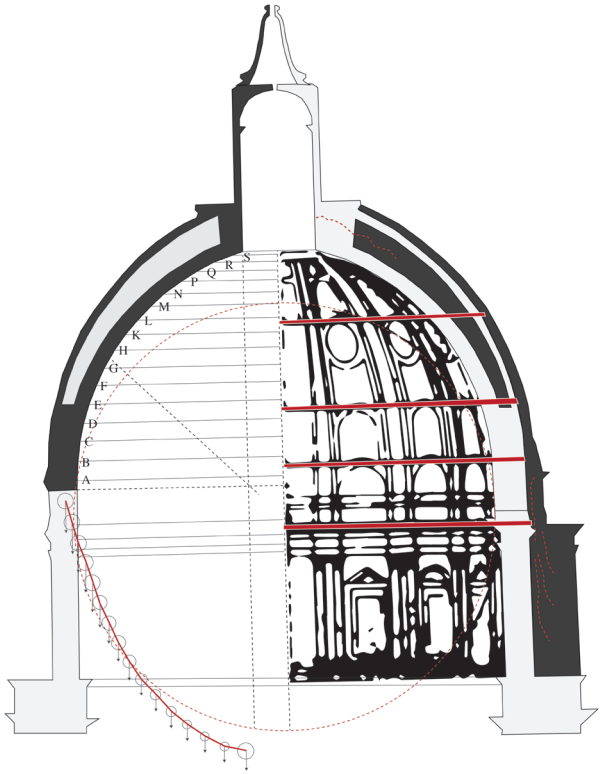


Figure 2.0.12 Students developed a design and construction technique for hay bale construction for remote locations (Design for Relief and Resiliency studio, Iowa State University, College of Design)

innovation allowed forces to be graphically "modeled" to test equilibrium and to derive calculations. As this method developed, it inspired detailed drawings of arches that explained their behavior and failures through force vectors. (Figure 2.0.13)





Figures 2.0.13a and 2.0.13b Giovanni Poleni's diagram of the forces and the resulting geometry of a thrust line in an arch (1748). When cracking was discovered in St. Peter's dome in the Vatican, Poleni compared the section to the hanging chain thrust line to identify locations where modifications needed to be made (1748)

By 1840, *graphic statics* evolved into an important educational and confirmative method of drawing that linked force flow with physical form. This method, which could be applied to structures with axial stress (arches, cables, and trusses) linked force polygons with force diagrams. Longer lines indicate more force than shorter lines and the direction of the lines from a reference point indicates the type of stress (tension or compression). This method's usefulness comes from the fast feedback it provides between physical form and magnitude of forces, especially if the designer is evaluating the effect of formal changes on the design. We will use this system to understand how arches accumulate their resultant thrust forces (based on their geometry and weight) and to determine stresses in cables of different configurations. (Figure 2.0.14)

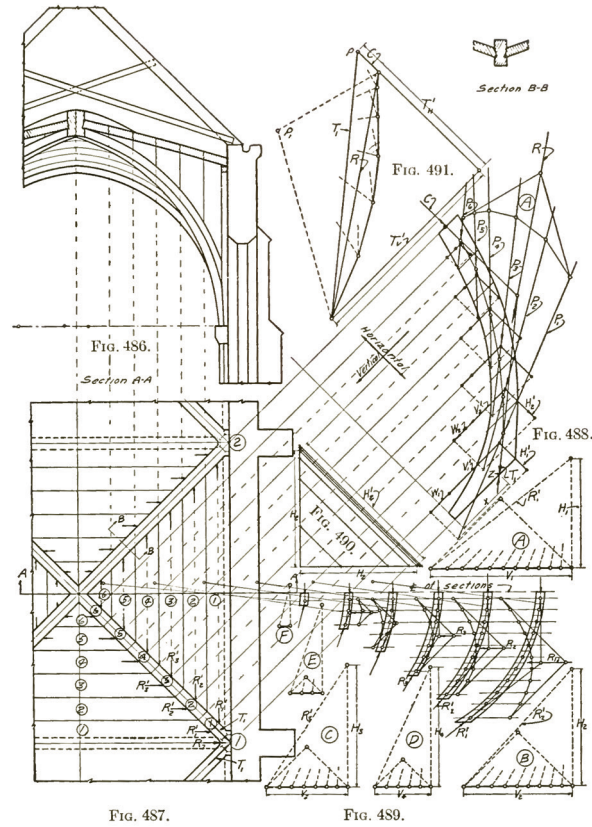
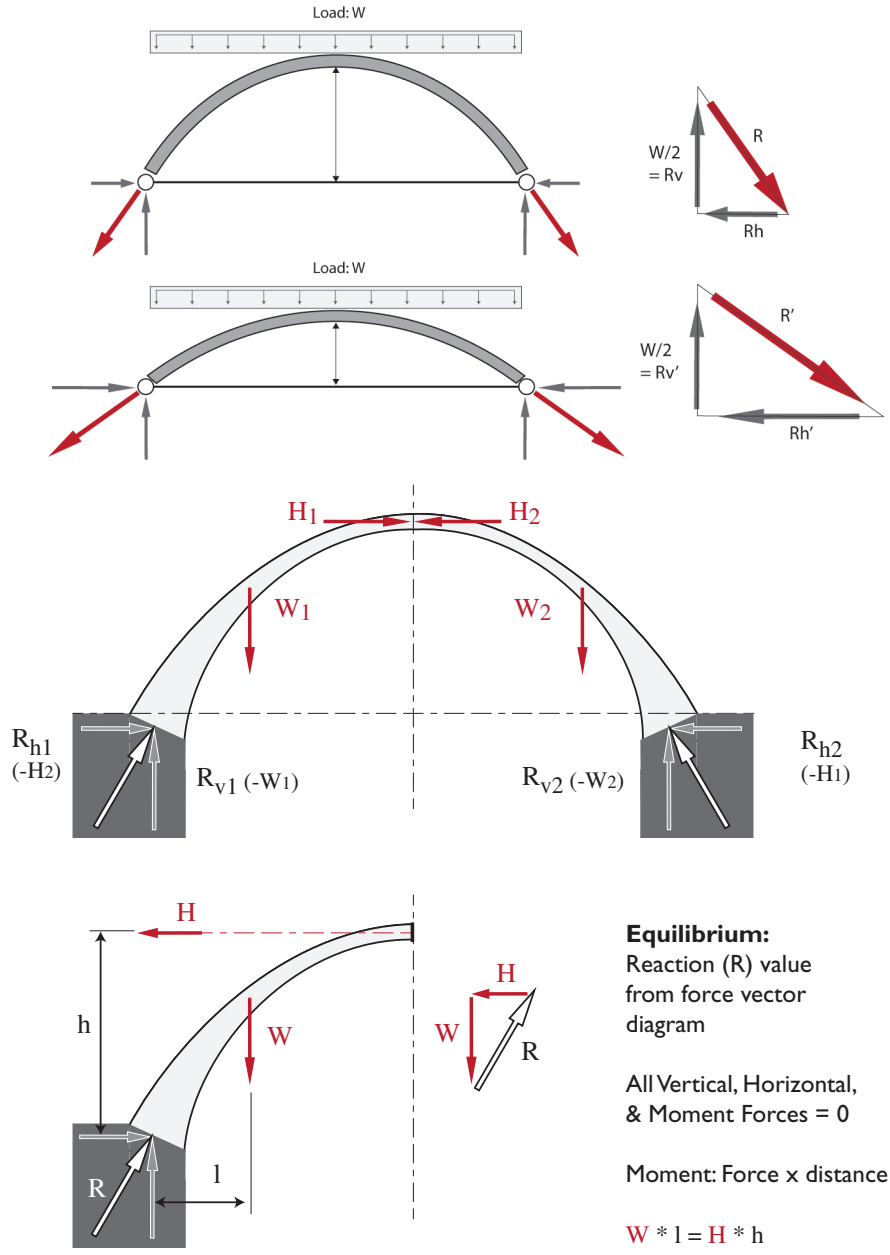


Figure 2.0.14 W.S. Wolfe's demonstration of how graphic analysis could be used to analyze forces in a masonry vault (as well as beams, trusses, and vaults)

Consequences and Confirmations: Calculating Thrust

Arches and cables, as we learned in our early experiment in holding the string, are complicated by the problem of thrust. Forces at the ends of arches and cables (that either push out or pull in) are typically quite large. Thrusts can have adverse consequences on building support systems. The amount of thrust is related to a structure's weight, height, and span. Because height and thrust are inversely proportional, this creates a challenge



Figures 2.0.15a and 2.0.15b The inverse relationship between arch height and thrust and overall state of equilibrium of forces between the arch and supports

for structures that need lower and longer spans. (Figure 2.0.15)

Using assumptions of equilibrium between the loading and the supports ($F_y = 0$) and horizontal (and moment) forces ($F_H = 0$), we can solve for the forces in the reactions (R_H and R_V). These components allow us to find the angle and magnitude of the thrust at the supports. We can determine the vertical and horizontal forces in a cable by applying principles of vertical and horizontal equilibrium. With a uniformly loaded cable, the vertical forces are equal to one-half of the total load (and equal to each other):

$$\mathbf{Rv_1 = Rv_2 = w * l}$$

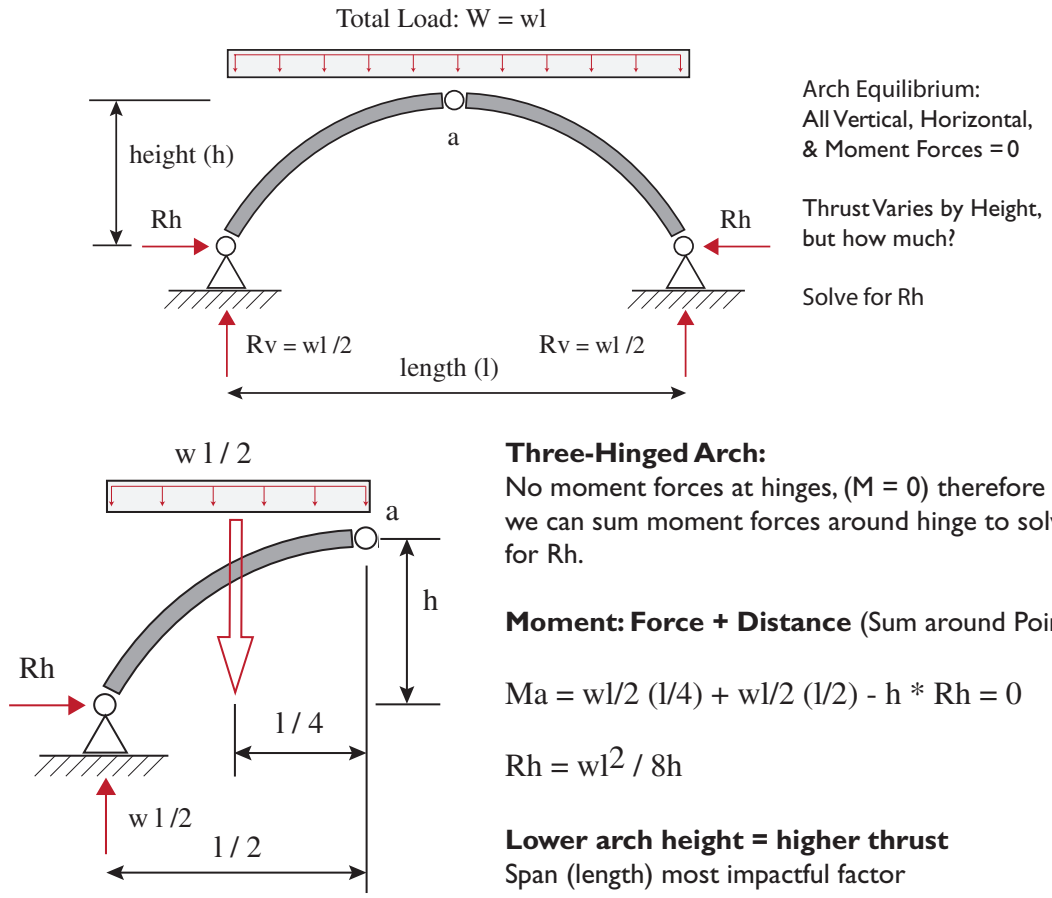
The mathematical relationship between a uniform load (w), span (l), sag (h), and the horizontal forces in the arch or cable (Rh) is:

$$\mathbf{Rh_1 = Rh_2 = w * l^2 / 8 * h}$$

Once we know the vertical and horizontal forces, we can find the diagonal forces acting in the cable (or arch) using the Pythagorean formula to solve for the diagonal force P (or hypotenuse in the force triangle):

$$\mathbf{a^2 + b^2 = c^2} \quad \textbf{OR} \quad \mathbf{(Rh)^2 + (Rv)^2 = (P)^2}$$

Resisting this thrust either requires massive foundations or some type of horizontal tie-back; either could



Figures 2.0.16a and 2.0.16b Compare the geometric and mathematical relationship between the loading, supports, span, and height of the arch

be difficult to integrate into certain designs. Even when thrust is resisted, these structures are still liable to movement and instability. (Figure 2.0.16)

Advancements in Materials, Assemblies, and Physical Modeling

Knowing how to design and analyze these form-resistant structures was only one challenge—advancements in construction were also needed. New materials, manufacturing methods, and machines emerged from the Industrial Revolution, increasing the ability to design and build reliable long span structures. Although masonry arches weren't widely used for long span structures after 1850, their arched forms were pervasive in curved-beam and trussed structures made of different materials (like cast iron, wrought iron, and eventually steel). (Figure 2.0.17)

Other form-resistant structures that depended on material advancements also emerged. Arched masonry bridges that once needed to be built over complicated timber framework could use new material technologies of reinforced concrete and temporary/movable formwork to achieve longer spans. Simultaneous advancements in material technology, engineering knowledge, and construction methods enabled very long spans and a dramatic expressiveness that corresponded with structural strategies, including suspension bridges. (Figure 2.0.18)

Figure 2.0.17 Expressive iron arches at the Sainte-Geneviève Library in Paris (LaBrouste, 1850)

Despite these advances in the physical and material sciences, design and construction of form-resistant structures relied on traditional methods, geometric rules of thumb, and constructed precedents until the late 19th century. Reliance on previous construction favored the physical models of full-scale prototypes. After all, although these graphic representations were helpful in assessing and testing structural forms, they weren't helpful in directly generating viable forms. Physical models, however, allowed designers to conjoin form-finding and testing.

An illustrative example is the massive 1:10 scale *stereotactic* (or poly-funicular) physical model of string and weighted bags of pellets that determined the proper compressive forms for columns and arches in the Crypt Güell in Barcelona, built by the Catalonian architect Antonio Gaudi between 1898 and 1908. Although generating, testing, and translating the physical model into drawings was tedious and time-consuming, Gaudi claimed that "with two rules and a ball of string, all the architecture is generated." Sandbags represented the anticipated dead load weight and the resulting string forms demonstrate the proper thrust line angles for pure compressive resistance in its supports. Gaudi's design model for the Cathedral Sagrada Familia in 1882 is on display below the now nearly-completed structure. (Figure 2.0.19)

Figure 2.0.18 The Brooklyn Bridge (Roebling family, 1883, New York), used steel (described by Roebling as "the metal of the future") for its bundled wire configuration of the main cables, allowing it to span a notable distance (1,600 ft (486m))

Sixty years later, Frei Otto relied on physical models as form-generating methods for new lightweight, tension-based structural systems. Instead of relying on cables to find the form of heavy compressive arches, like Hooke or Gaudi, Otto wanted to develop a structural system based on tensioned surfaces. He tested ways to generate physical forms using hanging string, inflated pillows, and soap bubbles. For the next half-century, Otto would continue to use physical models to create many of the 20th century's most iconic and influential cable and membrane structures. Even though the engineering of air-inflated pneumatic structures was well-understood by the early 20th century, it would be several decades until a reliable fabric was developed that could resist the stresses at the connection points and avoid UV degradation. When these membranes became reliable, an entire new range of form-resistant structures became available. (Figure 2.0.20)

Because cable structures vary their overall forms based on how they are loaded and the amount of tension in each cable, they're difficult to accurately document and assess, particularly when their nodal points are spread across double-curved surfaces or when their loads are potentially shifting. Digital models have simplified the process of documenting and analyzing these complicated structures. Interestingly, the earliest digital models, in which a three-dimensional surface was "mapped," were derived directly from physical models.

This connection between the digital and physical prototype continues to be an essential exploration. Contemporary digital modeling programs have returned to graphic statics for quickly analyzing form and forces under changing conditions. Because form-finding and calculations are more advanced, the resulting structures can move beyond conventional two-dimensional or planar arches (or cables) to consider how forces flow across three-dimensional surfaces. Fusing graphic visualization with digital analysis can be used for educational, generative, or confirmative feedback.

Contemporary Considerations

Form-resistant structures aren't vestiges of history. They are relevant and important structural options for even the most contemporary considerations. There are inherent efficiencies of resisting and transmitting forces primarily through form and materials. These projects can become incredibly expressive, particularly when developed using contemporary tools. (Figure 2.0.21)

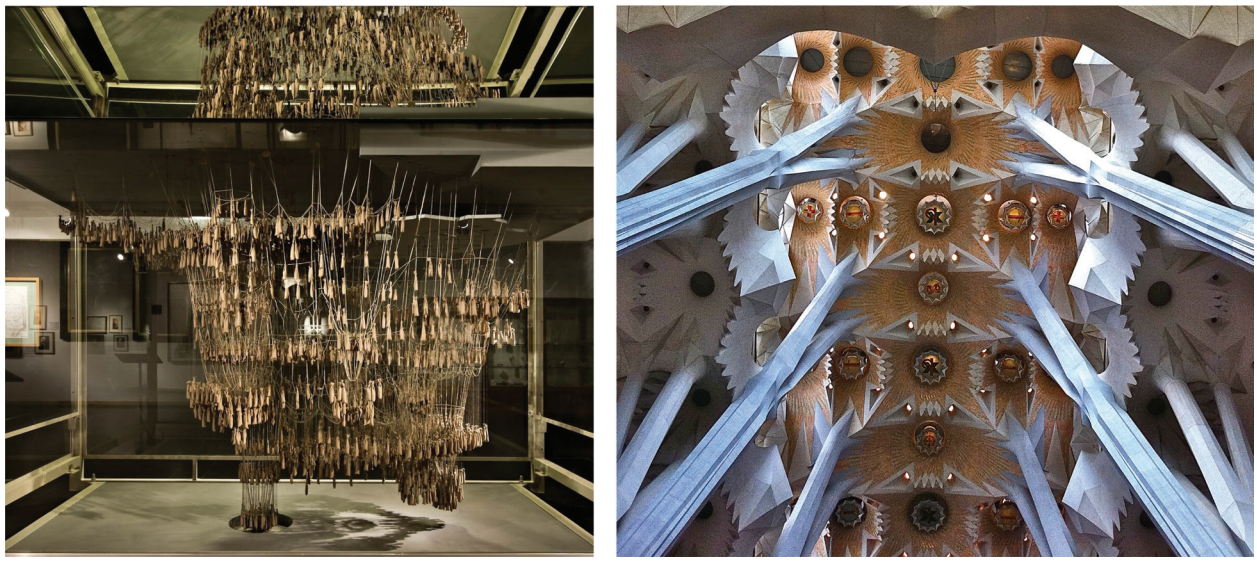


Figure 2.0.19 The Sagrada Familia model (Gaudi, 1882) and the resulting interior geometry of vaults and supports

Figure 2.0.20 The unreliable nature of membrane materials led Frei Otto to use layered acrylic panels for the 1972 Munich Olympic stadium surface

Figure 2.0.21 Tama Art University Hochioji Library (Toyo Ito & Associates, 2007)

Not every building form is aptly enclosed by arched volumes or suspended by cables. Because the form of the structure is the primary determinant to its effectiveness as a structural element, it often comes before function. This creates a limited range of acceptable forms and materials. To transmit forces effectively across long spans with minimal thrust, these forms often require very large heights to achieve their desired geometry—a strategy that isn't helpful for spaces that need to be compactly sized or for functions that don't require this volume. These structural types aren't suitable for multi-story buildings with flat-stacked floors and ceilings, and they may be too complicated and expensive for moderately sized proposals. Frequently form-resistant elements are used in long span, single volume buildings or infrastructural projects that require the elevated levels of efficiency and performance. (Figure 2.0.22)

However, there are certain projects that require this type of volume/height and embrace the inherent expression of spaces created by their structural resistance. This can be advantageous in cases where

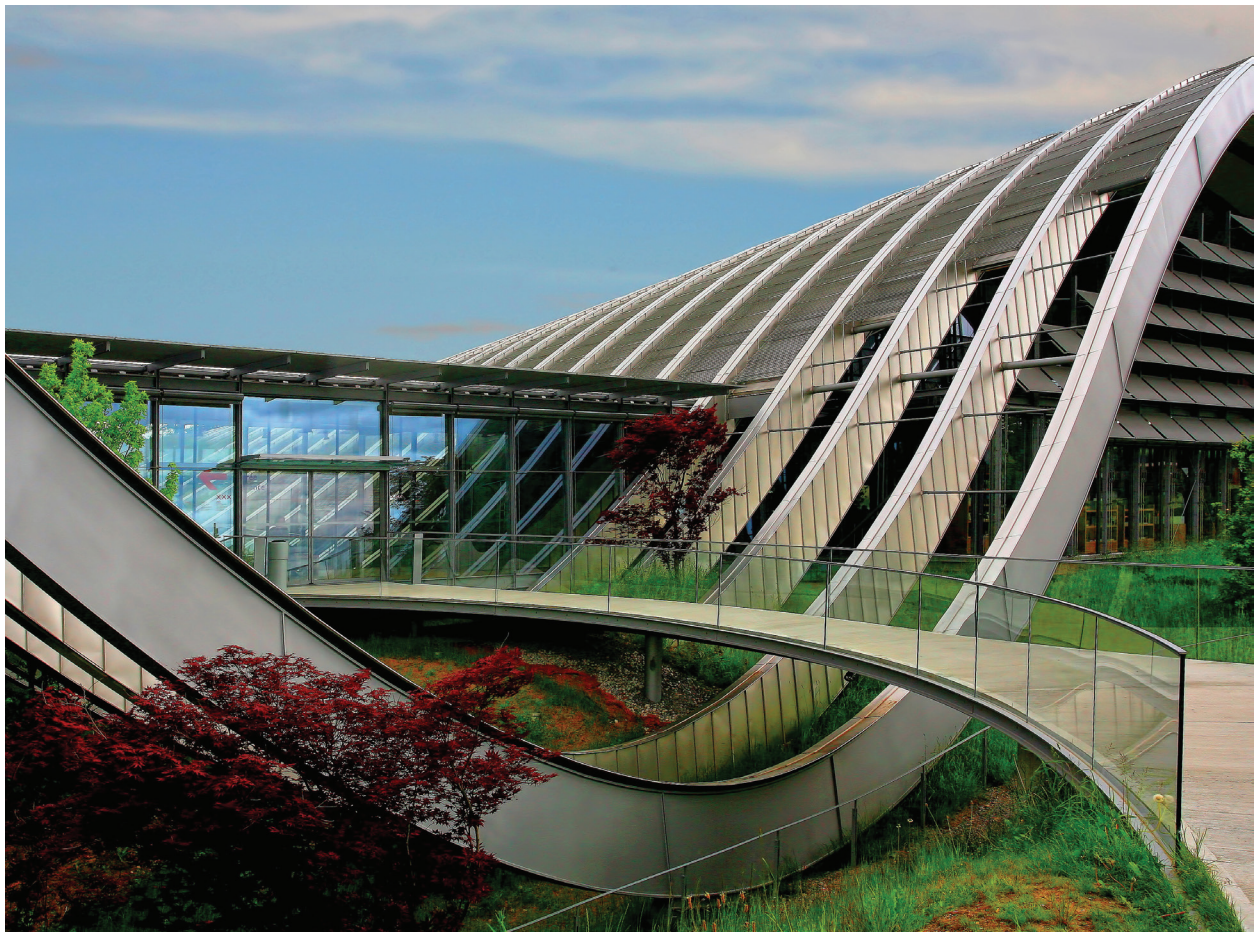


Figure 2.0.22 Zentrum Paul Klee, Bern (Renzo Piano Building Workshop, 2005)

Figure 2.0.23 The tallest bridge in the world (1,125ft (342m), the Millau Viaduct, France, uses a cable-stayed structural strategy to traverse (8,071ft (2460m) across the gorge (Foster + Partners, 2001)

uniting structural expression and spatial experience is desirable. (Figure 2.0.23)

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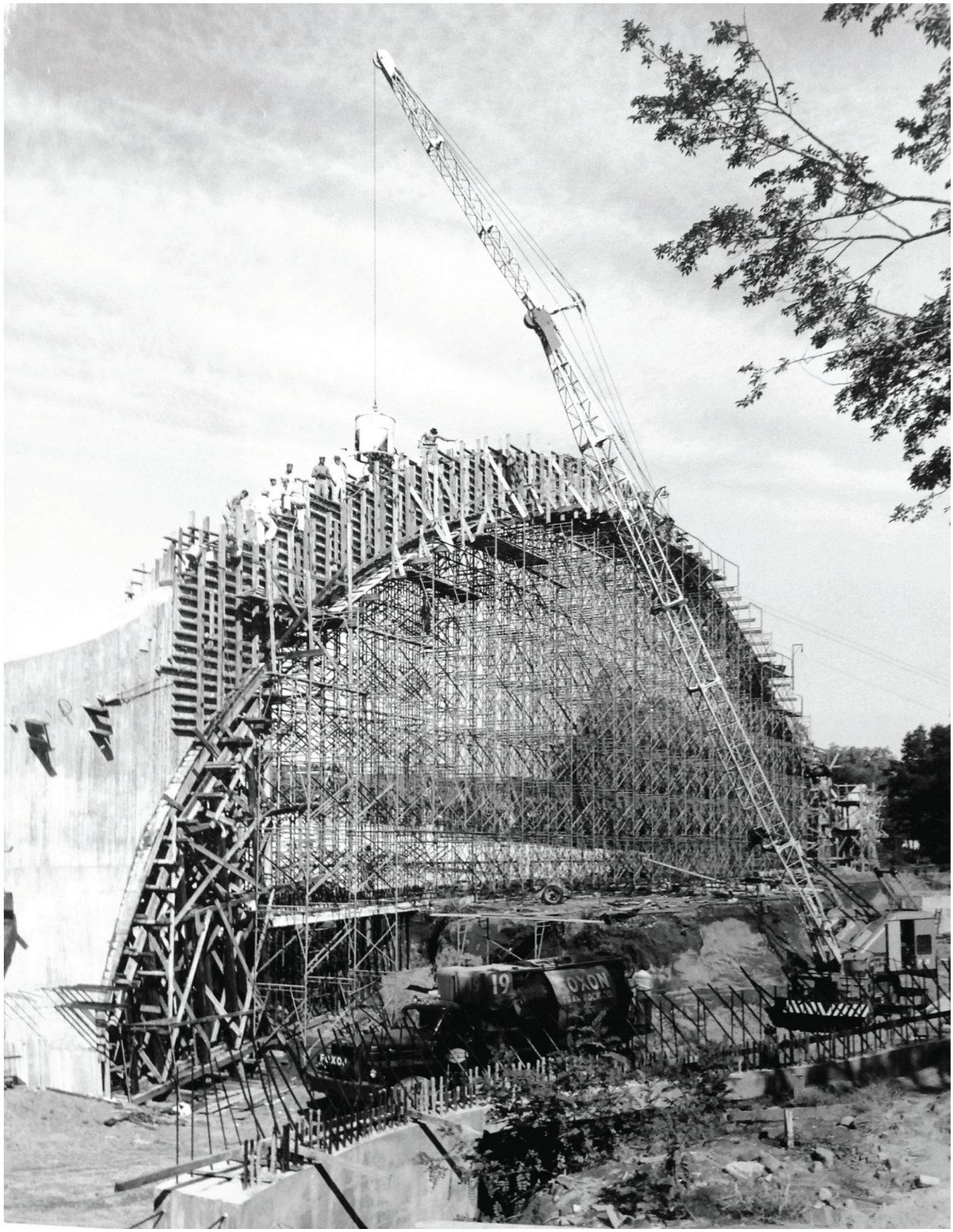


Figure 2.1.0 Construction photo of the central arch, Ingalls Hockey Rink, Yale University (Saarinen, 1958)

CHAPTER 2.1

Compressive Arches

Past the Breaking Point

For centuries, arched masonry structures were the most reliable and efficient options for long spanning buildings and bridges. The design and construction of arches followed established precedents of geometry and proportions. When exceptions to these traditional forms need to be made based on functional concerns and/or site conditions, designers rely on technical insights and persistence to create new solutions. Sometimes the structures don't behave as intended and formative lessons can be learned as a result.

Learning from Failures: the Old Bridge, Pontypridd

Predictable structural behavior of arches depends, in large part, on their forms and materials. Pre-scientific use of arches as structural mechanisms favored a reliance on the proportions and dimensions of successfully completed arches and eschewed experimentation.

This makes sense. There is an inherent risk in designing innovative structures that are longer, thinner, and/or lighter than conventional ones. Even though the threat of potential failure is always eminent, circumstances compel designers to take risks, to confront failure as part of their design process, or to learn from the failures they've personally experienced. If design experiments don't work and the project fails to perform as intended—or "breaks"—there is still an instructive value. This argument is of little comfort when buildings collapse, so innovative designers look for ways to find failures in the design process. Failure is important. As Samuel Smiles explains "he who never made a mistake never made a discovery."

Throughout this book, we will look at "instructive failures" to understand how/why something fails and to reveal strategies for improvement. The Old Bridge near Pontypridd, Wales, is a particularly instructive case that demonstrates the thin line between failure and innovation as well as the value of learning from failures. This unique single-span arched masonry bridge, completed in 1756, spans 140ft (43m) across the River Taff. Upon completion, it was the longest single-span masonry arch bridge in the UK, a distinction it held for eighty years. The final form and details of the bridge, including its steep slope and distinctive circular openings in the haunches, were the results of ten years of design and construction failures, including *three collapses* and reconstructions, that were suffered and primarily financed by the bridge designer and builder—the young and inexperienced stonemason, engineer, and Methodist minister, William Edwards. (Figure 2.1.1)

Figure 2.1.1 The Old Bridge, Pontypridd, (1756) as it currently stands

The Old Bridge, Pontypridd, is instructive because of its curious set of circumstances and solutions. The final solution seems so unconventional that it is worth considering why, of all the solutions that an inexperienced designer could propose for the fourth version of the bridge, did Edwards choose a bold, record-breaking, single-span arch? Why not a more conservative and conventional proposal? Was this final design a result of lessons he'd learned from the previous schemes? If so, what were these lessons? By looking at the circumstances behind the failures of previous schemes, we'll gain insight into the critical relationship between arched forms, materials, structural behavior, and construction challenges Edwards faced.

FOLLOW THE FAILURES

What are the greatest risks associated with the design and construction of compressive structures? Make a sketch and notations of concerns you'd anticipate for both design and construction activities. By what means would you try to address and resolve these concerns?

To Discuss: Are designers obligated to help resolve potential construction complications as part of their design decisions? Why or why not?

Compression Arches: Basic Behaviors

Edwards chose load-bearing masonry arches as the structural system for the bridge because this had been a safe and predictable structural option for bridges and buildings for thousands of years. Despite being resource-intensive and difficult to construct, arches were worth the effort as they allowed societies to build farther, taller, and firmer than before and they provided durability against weather and fire that allowed these structures to last for centuries. (Figure 2.1.2)

Form-resistant structures like arches use their form as the primary means of diverting and resisting loads. The behavior of arches is easy to intuit. The curved form of the arch collects and diverts vertical forces into the curved sides. Loads accumulate from the top of the arch to the bottom, so arches are typically thicker at the base. Masonry acts predictably, compressing blocks together and transferring loads to the ground below, generating vertical and horizontal forces (thrust).

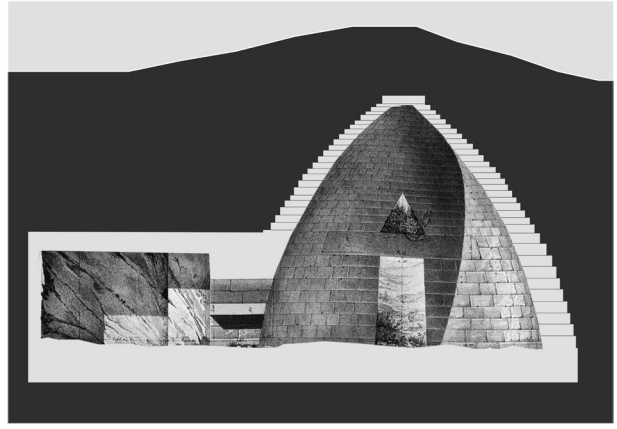
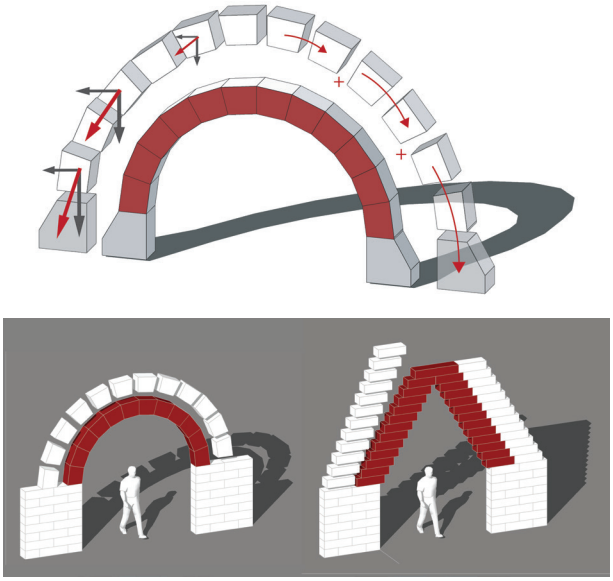


Figure 2.1.2 Section through the Treasury of Atreus tholos tomb (Mycenae, 1250 BCE)

The arch may appear to be a single element, but it is actually comprised of smaller elements. The size of the blocks, and how they are stacked, affects the resultant angle of force transfer. It is easier, and more accurate, to shape a larger curved structure from smaller, portable and stackable wedge-shaped modular units, called



Figures 2.1.3a and 2.1.3b How arches work: Diagram of the basic force transfer behavior in arches and two options for how elements can be stacked to create an arched opening. Left: Voussoirs, right: Corbeling

voussoirs or, alternatively, progressively cantilevering blocks in a *corbeling* form. When bound together with mortar, these structures stiffen up and these discrete elements will behave as a single, monolithic structural element—transmitting compression forces across the entire cross-sectional area. (Figure 2.1.3)

Arch Shapes and Materials

Although it seems strange to use such heavy materials to span long distances, these strong and stiff materials have the qualities needed to resist the compression forces caused by the arched form. Stone and brick are resistant to compressive stresses, which along with friction and a binding mortar actually hold them together. Stone and brick have high allowable stress (f_a) values, so they won't likely be broken or crushed under loading. Arches rarely exceed $\frac{1}{100}$th their allowable load in typical conditions.

Figure 2.1.4 The Roman Colosseum's (AD 80) structure consists of a series of concentric rings of arched walls connected by perpendicular arched walls that rise up to 157ft (48m)

The arched form is useful structurally *and* functionally. As long as the arch follows a particular geometric proportion of height and span—and resists the outward loads that try to spread it apart—the opening of the arch can be used for a wide range of applications. Stacking small pieces together (whether by corbeling or arching) allows the structure to take on larger, more complex, and aesthetically variant spanning forms. But because of the risk for failure, historic buildings that used arches favored heavy and even redundant massive load-bearing wall structures. Rules of thumb from previous eras (e.g., span: height: thickness ratios, etc.) were passed along and became the basis for new designs. (Figure 2.1.4)

MATERIAL MATTERS: BRICK AND STONE

Masonry arches rely on the predictable behavior of certain materials under compression to transmit loads and to serve as the literal building blocks of the structure.

To Do: Hypothesize about why bricks and stones were frequently used (besides their advantages as structural materials (strength and stiffness)). Are there advantages in how the materials are "sourced" (either fabricated or collected)? Is the size of the units beneficial (or disadvantageous) to assembly?

To Discuss: Most arches are held together with mortar—these joints may seem like a weak link in the system, but are they? What are the important chemical qualities of mortar that help it bond to masonry? (Figure 2.1.5)

Figure 2.1.5 Eask Tower (Dingle Town, Ireland, 1847)

Thrust and Foundations

Because height and thrust are inversely proportional, the form of the arch will determine the value of thrust—tall arches generate less thrust than flatter arches. Arches accommodate this outward thrust in four ways: Foundational arches transfer thrust directly to the ground. Tie-back arches resist the outward movement with a tensioned element across the bottom interior of the arch. Buttressed arches add additional bracing and weight to the outside of the arch (as buttresses or spandrels). And series arches counteract thrusts by other adjacent, connecting arches with the opposite thrust direction. (Figure 2.1.6)

When foundation arches, like those at Pontypridd, meet the ground, it may seem like an obvious solution to rigidly connect them to their foundations. Doing so creates a *fixed arch*—one that is bound in-place on both ends. This common historical solution is, however, risky. Although the bridge form and materials seem rigid and fixed, the materials are still "springy." The arch can still move due to changes in thermal conditions (expanding with heat, shrinking in cold) or shifting loads. When arches move, they bend—and bending adds new stresses that threaten the arch's stability. If it is fixed, the bridge may crack because it wants to rotate on its supports or internally. As we will see in more developed bridge designs, introducing hinges into an arch frame (two-hinged or three-hinged arch) reduces bending stresses by allowing for some rotational movement without large bending moments. (Figure 2.1.7)

During the 18th century, masonry arches, like at Pontypridd, would have been fixed into the side supporting walls; this seems structurally logical, but it

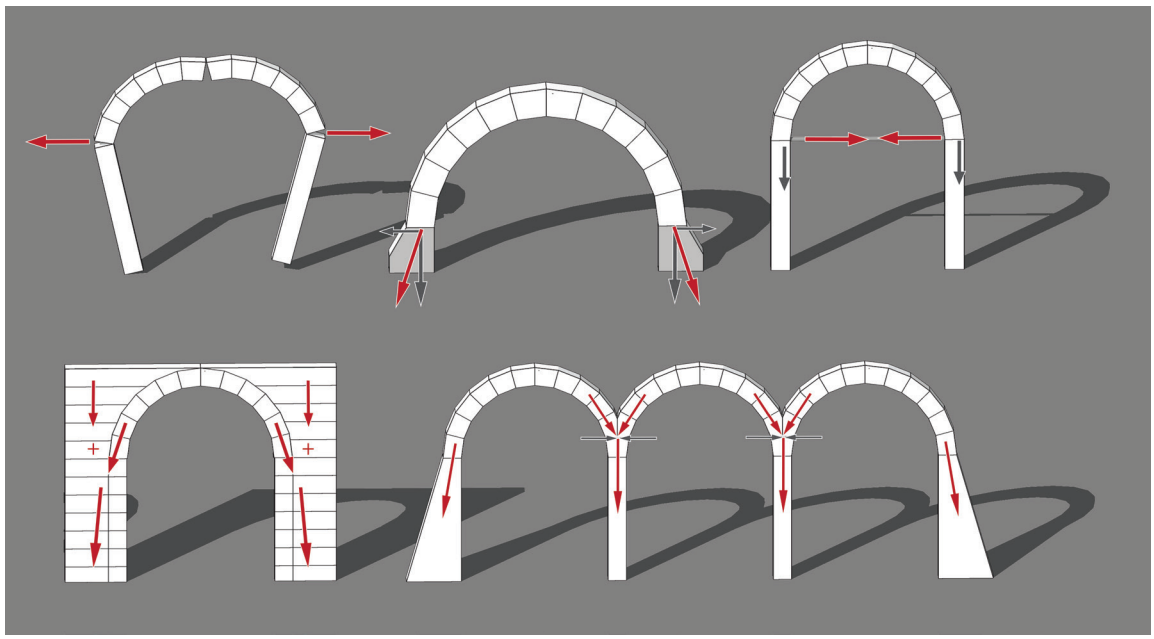
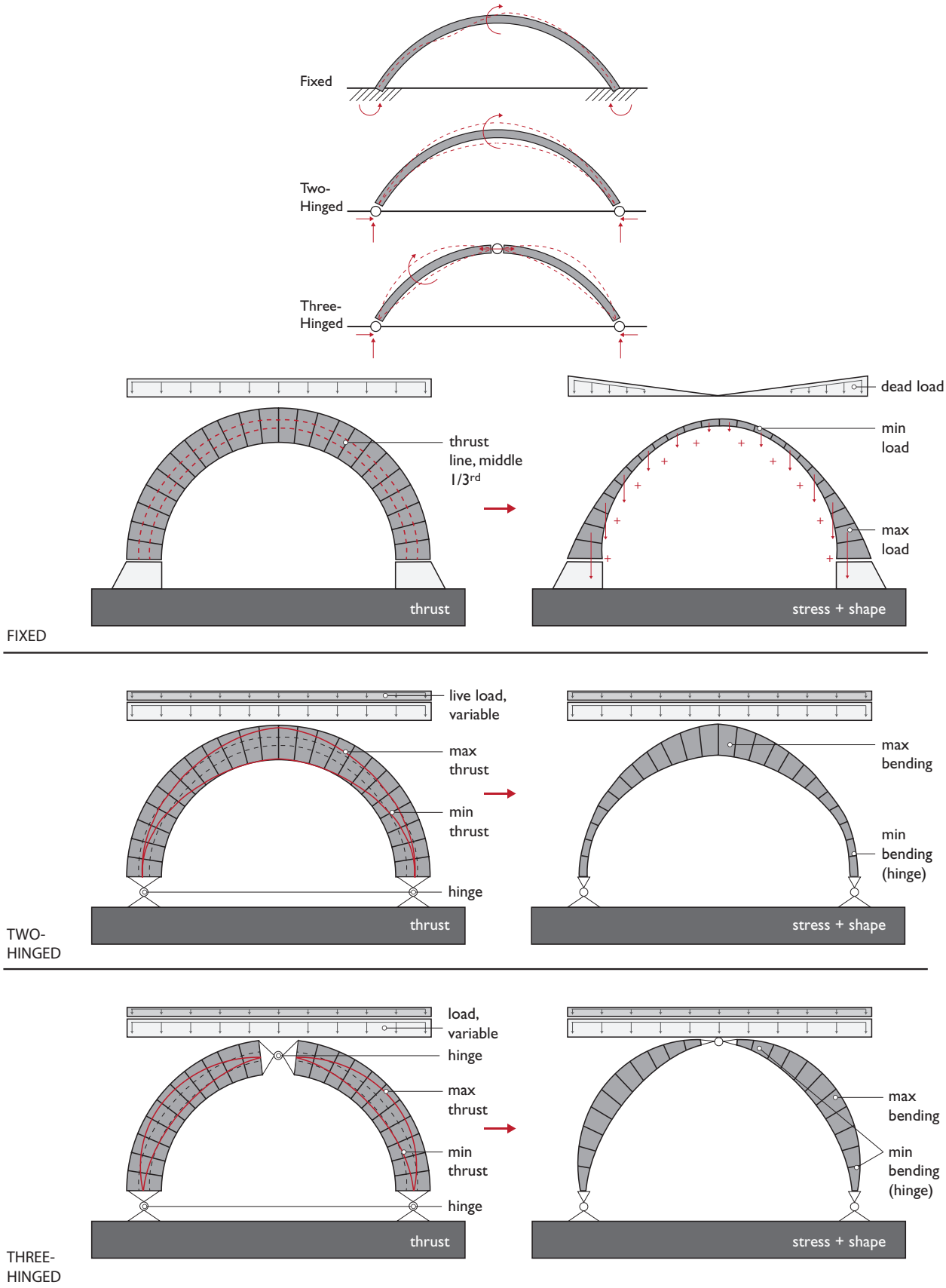


Figure 2.1.6 Strategies to resist thrust (left to right, top/down): Unbraced, foundation arch, tie-back, buttresses, and arches in series. The goal is to either counter the horizontal force or direct it down vertically



Figures 2.1.7a and 2.1.7b Three types of hinges. The different hinge options affect arch movements, thrust lines, and suggest modified profiles

also made sense because it followed the process of constructing bridges from the outside edges towards the center keystone.

Construction Challenges for Arches

Designing and constructing arches followed centuries of trial-and-error experience and used an easily replicated, oft-repeated process for form-finding and construction. Arch geometry can have one-centered, two-centered, segmental, or even pointed profiles. These forms aren't necessarily optimal for redirecting forces. They were often shaped for aesthetic and construction purposes. When building with string and hand-tools, establishing predictable geometries for curved elements was essential, and an art. And yet even if the geometry is predictable, arches have always been complicated structures to design and build.

The shape of the arch is its source of strength, but also its greatest liability. Arches are susceptible to four common and interdependent liabilities: Mismatched form to forces (that incorrectly anticipate structural load paths), outward thrust, shifting loads that cause bending stresses in the arch, and/or difficulties with construction. With these basic arch behaviors and potential problems in mind, we'll turn back to Edwards and his challenge at Pontypridd, where we can see the real-world consequences of each. (Figure 2.1.8)

The Old Bridge, Pontypridd, Attempt #1: Arches in a Series

In 1746, Edwards agreed to build a bridge across the Taff for GB£500. Although he was a young man of 27 years old with little formal masonry training and no experience designing or building bridges, he agreed to a "Guarantee Clause" that held him liable to create a bridge that would stand for seven years. Because the only material choice was stone, the only plausible form was arches. Perhaps unsurprisingly, his first scheme was conventional.

No drawings exist, but based on a description from the local archives, this scheme featured three

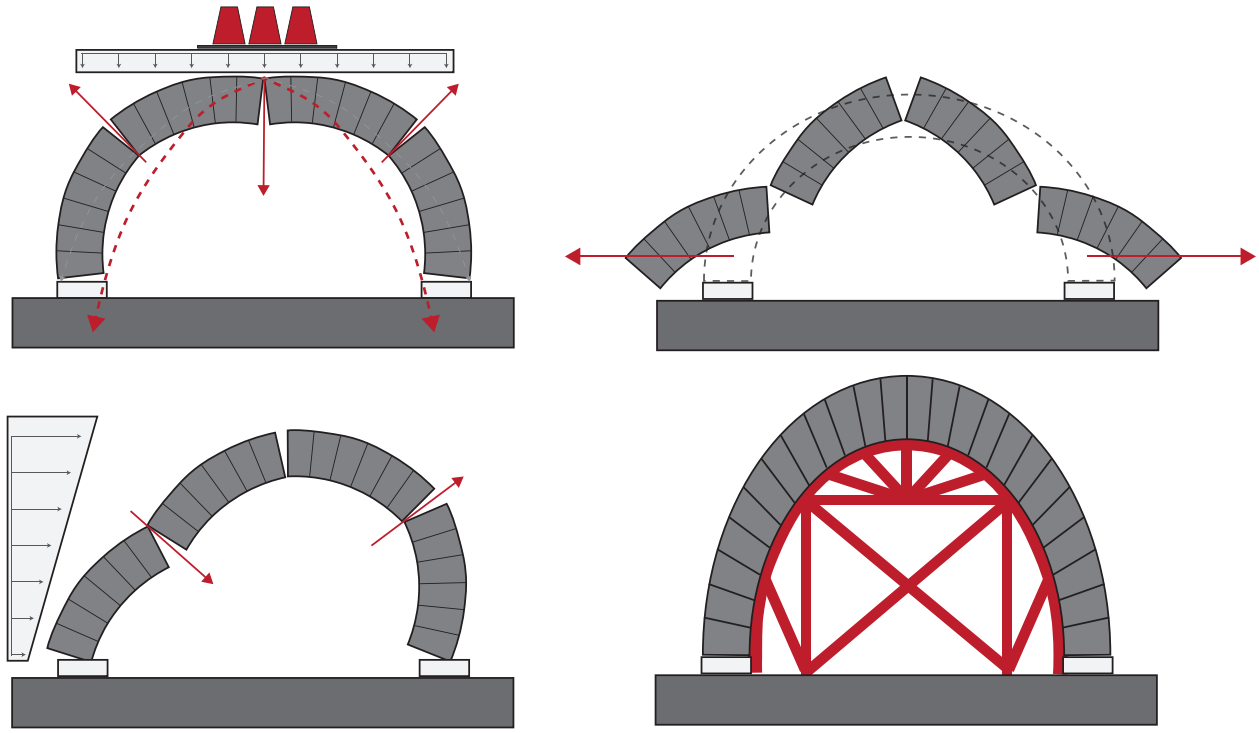


Figure 2.1.8 Four primary threats to arches (left to right, top/down): Mismatched form to loading, outward thrust, shifting/sideways loads, and construction challenges

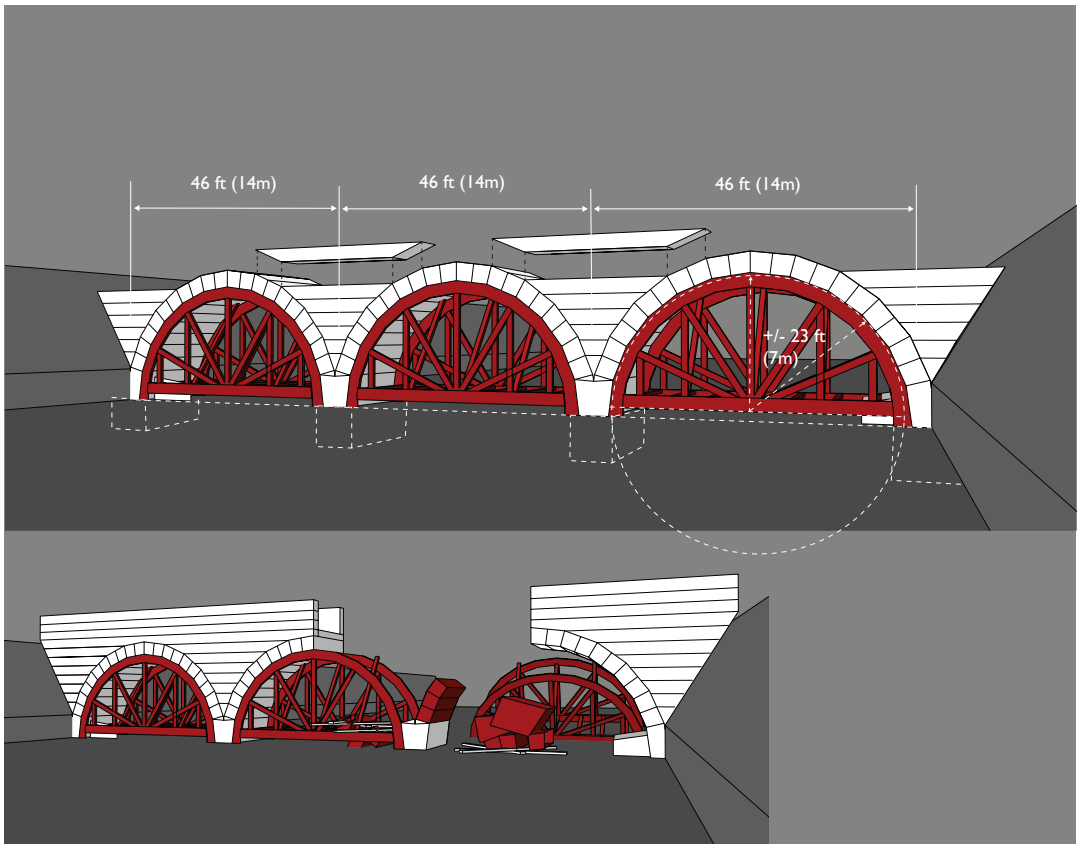


Figure 2.1.9 The Old Bridge, Pontypridd, Attempt #1 (Estimated)

small arches springing from two intermediate piers in the river. Extrapolating dimensions for span and height, we can estimate the arch profile. This scheme minimized thrust by reducing overall span and placing the center arches in series. Because the top of the bridge was described as flat with the arches spanning below it, the space between the walking surface and arch (the *spandrels*) were filled with stone. This added weight, which diminished horizontal stresses in favor of more vertical forces. The arches were probably segmental, not semi-circular, as this was common practice at the time. (Figure 2.1.9)

This bridge stood for two years, until the River Taff flooded and flushed a great deal of debris downriver. This debris crashed against the pier supports, breaking them and collapsing the bridge. Although there was nothing wrong with this scheme structurally, Edwards decided not to risk further potential failures from floods. His ensuing designs would all use a single-span arch.

The Old Bridge, Pontypridd, Attempt #2: Arch Geometry and Centering

We think of an arch as a finished element, but it is best considered the result of a complicated process. Leonardo da Vinci described an arch as, "two weaknesses which, leaning one against the other, make a strength." Its strength comes from its completed form—an arch is most susceptible to failure during construction. To construct an arch requires a falsework/scaffolding called *centering* below it. This temporary structure, typically made of heavy timber, establishes the profile for the arch and is strong enough to support the stones being laid upon it during construction. Upon completion, the centering is removed, leaving the arch to stand on its own. (Figure 2.1.10)

The challenge of centering affects the geometry of the proposal. Segmental hemispheric arches were chosen for their ease of construction, not for their structural profile. Because centering is typically made of straight-line sticks, the geometry of the curves

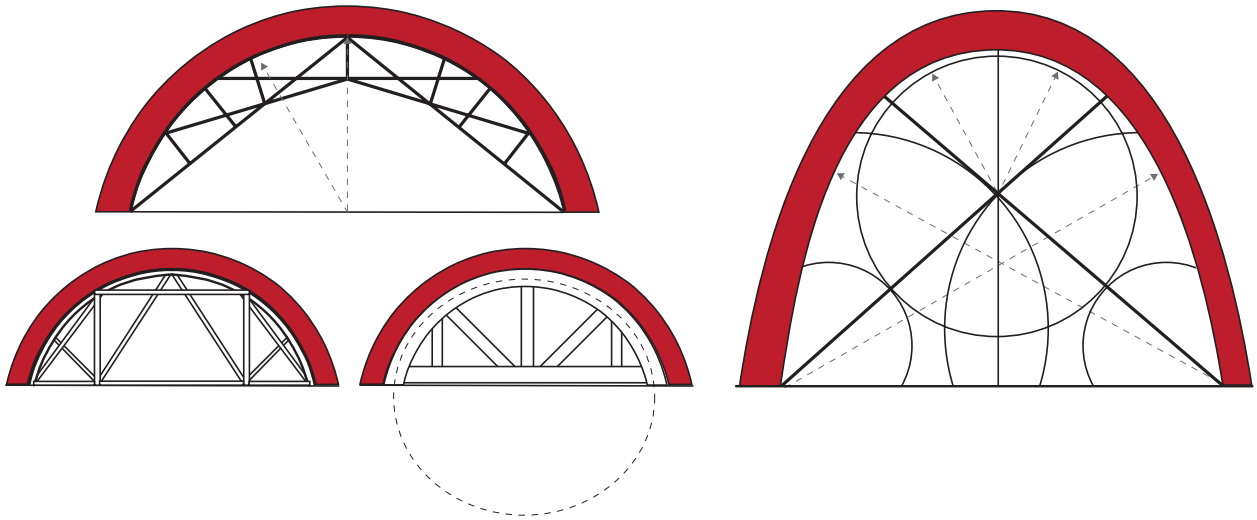


Figure 2.1.10 The relationship between the arch geometry and centering is critical. Difficult geometries are complicated to construct. The triangulated framing of centering were the earliest experiments in truss-like elements

was based on defined foci from which straight lines would radiate. Although traditional centering looks like contemporary truss structures, with triangulated straight components, centering wasn't based on science, just experience. Sometimes these centerings failed—as they did at Pontypridd. (Figure 2.1.11)

There aren't any drawings of the second scheme either, but observers claim it was a single-span segmental arch with a single arc that was shallower than a semi-circle. It is worth wondering how Edwards, as an inexperienced designer, developed the geometry for this arch. Because the span was unprecedented in the UK there wouldn't be another example to mimic. It could have been based on the same proportions as another shorter spanning bridge. Most likely, a segmental arch was selected because it was easiest to construct.

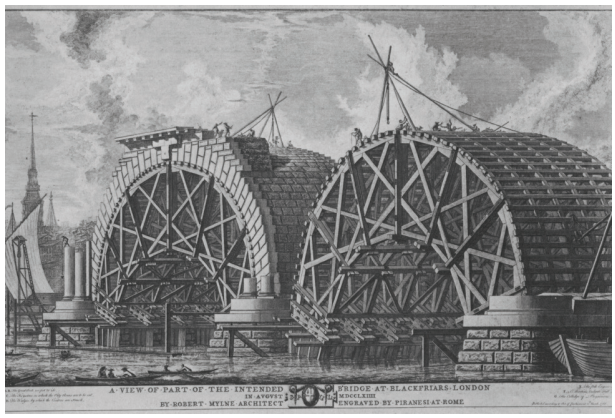


Figure 2.1.11 Engraving of the formwork at Blackfriar's Bridge, London, by Piranesi (1760)

Imagine this challenge for Edwards. He would have to design and construct a wooden frame to span 140ft across the river strong enough to support the weight of the bridge's stones.

There are two conflicting but related explanations for the bridge's collapse in 1748. One account describes a centering failure that caused the middle portion to "fall away" before construction was complete. Another account blames the failure on debris that had been swept down the river, becoming trapped against the centering, causing it to fail. It is unclear if this failure was a result of any impropriety or poor design by Edwards or simply the bad luck of flooding again. Nonetheless, because of the Guarantee Clause, he had to start again. (Figure 2.1.12)

The Old Bridge, Pontypridd, Attempt #3: Thrust Line and Thickness

Because Edwards had to personally finance the rebuilding of the arch for a third time, he wanted to be as materially efficient as possible, so he eschewed conventional arch proportions. The third scheme and its failure, provides the most instructive example of the relationship between arch geometry, weight, and the challenge of thrust lines, because he pushed these limits.

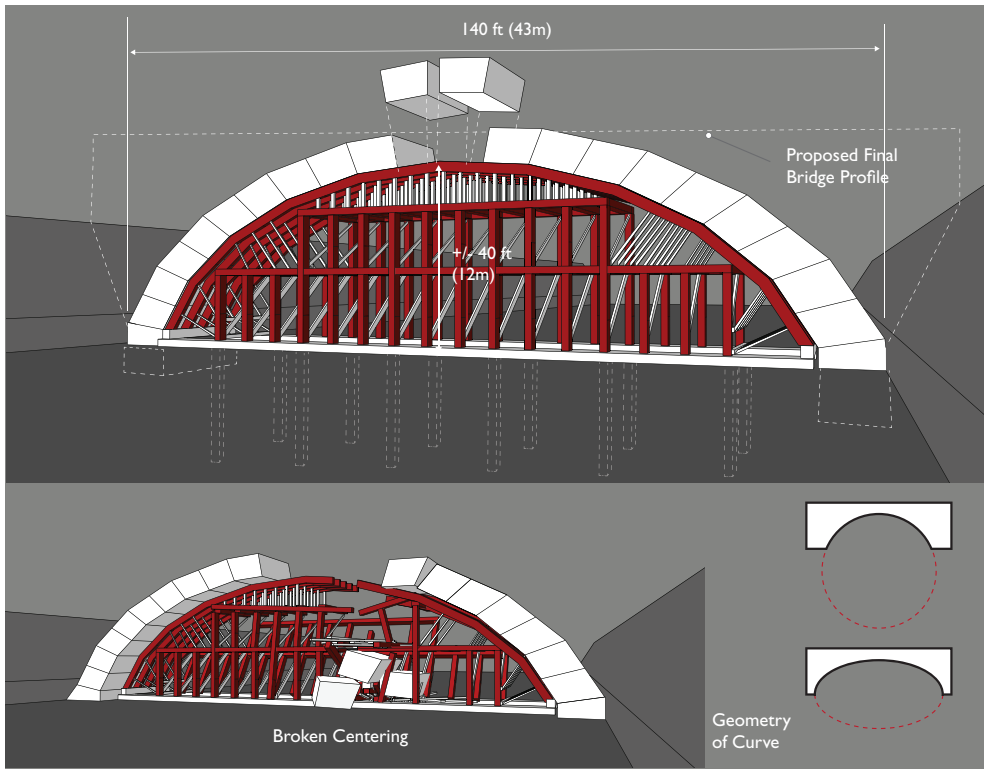


Figure 2.1.12 The Old Bridge, Pontypridd, Attempt #2 (Estimated)

The curved profile of this third proposal followed a rule of thumb for an arch span-to-rise ratio of 1:4. However, this curved geometry was based on a segmental arch, for construction purposes, and not the more structurally responsive parabolic arch. This may seem like a small deviation, but it is significant.

Form-resistant structures like arches should take their ideal form from the "hanging chain" load path explained by Hooke, which produces a diagonal resultant force called a *thrust line*. Like a hanging chain, this line's geometry is derived from the distribution of weight across its span. If arches closely follow this line, they can be made thinner and lighter, but this approach is risky. (Figure 2.1.13)

While it makes sense to match the profile of the arch with the ideal compressive thrust line, it can't match the line too closely or it will be susceptible to unanticipated or changing load conditions. Like the hanging chain, every time the load changes, the thrust line changes. An *ideal* thrust line is a myth; it is more of a range of lines. The thrust line should fall within the middle one-third of the arch's thicknesses, which

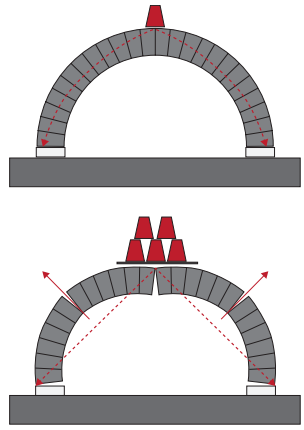


Figure 2.1.13 The thrust line geometry is a function of load magnitude (arch weight and/or external loads) and their location (uniformly distributed or point loads)—imagine the line as the inverse form of the hanging chain

allows a redundancy of materials to account for potential shifting loads. If the range of thrust lines falls outside of the arch's thickness—either from shifting loads or a mismatch between form and forces (like Edwards' scheme)—the geometry of the arch shifts as

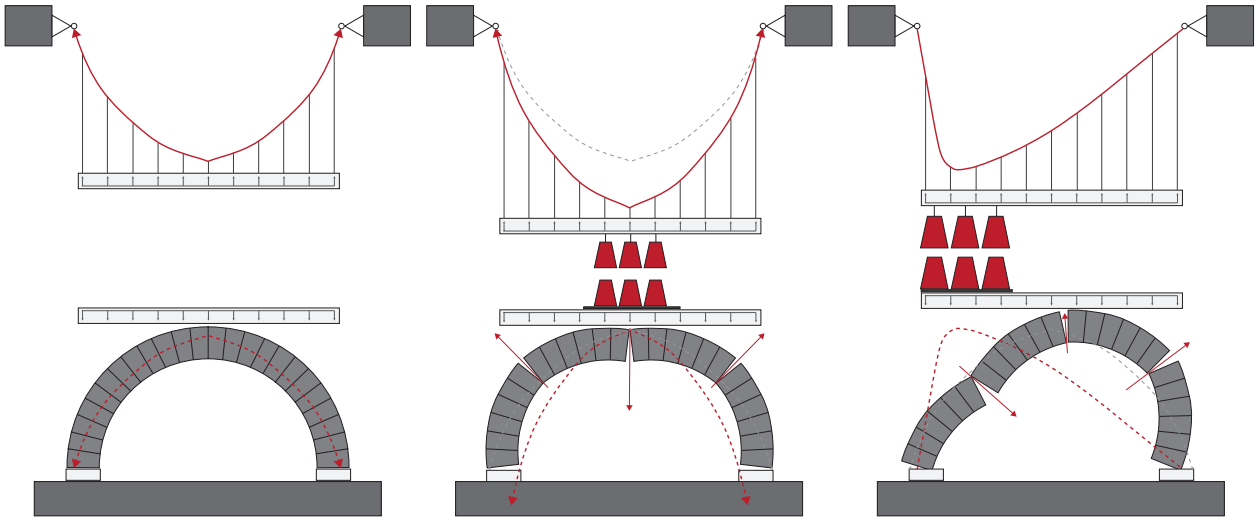


Figure 2.1.14 If the loading conditions change, the thrust line could fall outside the middle one-third of material to create bending stress in the arch—or in certain cases, instant failure

the stones get out of the way of the force path. Instead of pure compression, the arch also goes into bending, and it has to account for tension stresses. Masonry and its binding mortar have little reliable resistance to tension; if they are pulled apart or bent, these brittle materials are likely to break. (Figure 2.1.14)

Edwards looked for the thinnest and lightest profile of the arch in the middle (called the *arch ring*) and he deviated from traditional proportions in doing so. Centuries earlier, Palladio and Alberti each warned about maintaining a proper ratio between an arch's span and the arch ring; they suggested the thinnest stone be no less than $\frac{1}{10}$th–$\frac{1}{15}$th a long arch's overall span. Consider these rules of thumb as factors of safety in a pre-scientific age—it was safer, perhaps, to have more material in a more conservative scheme.

But there isn't a fixed ratio of thrust to geometry/size of stone used. The thrust line is a factor of the self-weight of the arch itself, so the thrust line changes depending on the size of the stone used across its span. Imagine the stones as weights hanging on a string—a heavier weight will pull the thrust line down more, particularly at the base of the arch. As loads accumulate, arch elements increase in mass, which helps push horizontal thrust forces down vertically. (Figure 2.1.15)

But what happens if the form and forces don't match? One of the primary loads on arches is the self-weight of the elements themselves. If the thrust line falls outside the proposed profile of the arch, the designer can either reconfigure the arch profile again to match the thrust line or modify the thrust line by either adding or subtracting weight. (Figure 2.1.16)

Edwards was trying to reduce materials, so the Old Bridge's thinnest section at the middle was only 3ft thick, or $\frac{1}{43}$rd the span—a risky endeavor. Six weeks after completion, the bridge's crown broke *upwards* and the bridge collapsed again. Why would a compressive arch break upwards? Edwards had failed to anticipate the range of thrust lines that would need to be resisted by his design. By visualizing the forces with arrows, we can see that there was too much weight on the side haunches and not enough in the middle. (Figure 2.1.17)

The Old Bridge, Pontypridd, Final Scheme: Advanced Understanding of Thrust Geometry

By 1756, ten years had passed since Edwards signed his initial contract. He was either encouraged or forced to try again, and he was given extra money to cover some of his losses. He could have changed the overall geometry of the bridge to a more parabolic arch with a profile that matched an ideal hanging

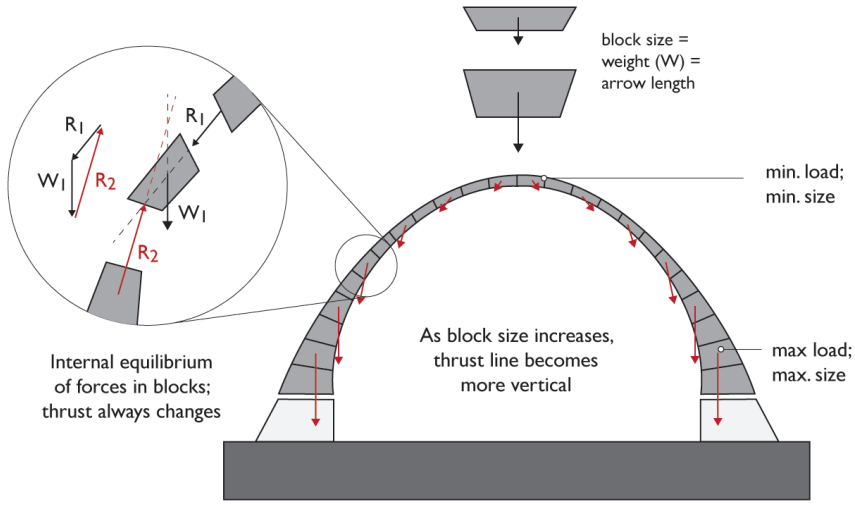


Figure 2.1.15 Up-close view of the relationship between the thrust line and the size (and mass) of arched elements

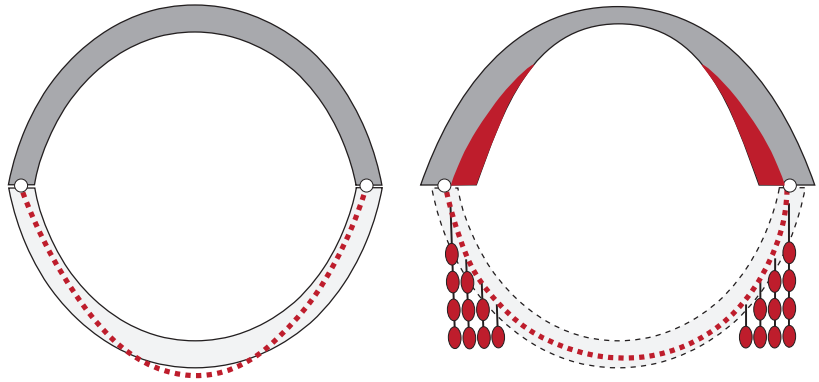


Figure 2.1.16 A simple graphic demonstration (or physical simulation) can illustrate how adding weight to certain parts of the arch can help pull the thrust line back into line with the arch thickness

chain, but this would have been difficult to construct and the results would have been unpredictable.

Edwards may have met with more experienced bridge designers who used a scientific basis for matching an arch's weight with the thrust line. Or, his eventual success may have been due to his perseverance and learning from his previous failures. Regardless, he made a small adjustment in the design to better align the form with the forces while maintaining the same segmental arch profile he had used for his previous scheme. This new scheme accurately anticipated the relationship between the flow of thrust forces and the size of stones.

Because compression structures are made of heavy materials, their size and weight dictate the flow of forces. With force vectors, we can diagram the way segmental blocks in an arch form create the vertical and horizontal component forces that make up a resultant force. In order to change the thrust line, either the geometry of the bridge or the weight of certain stones

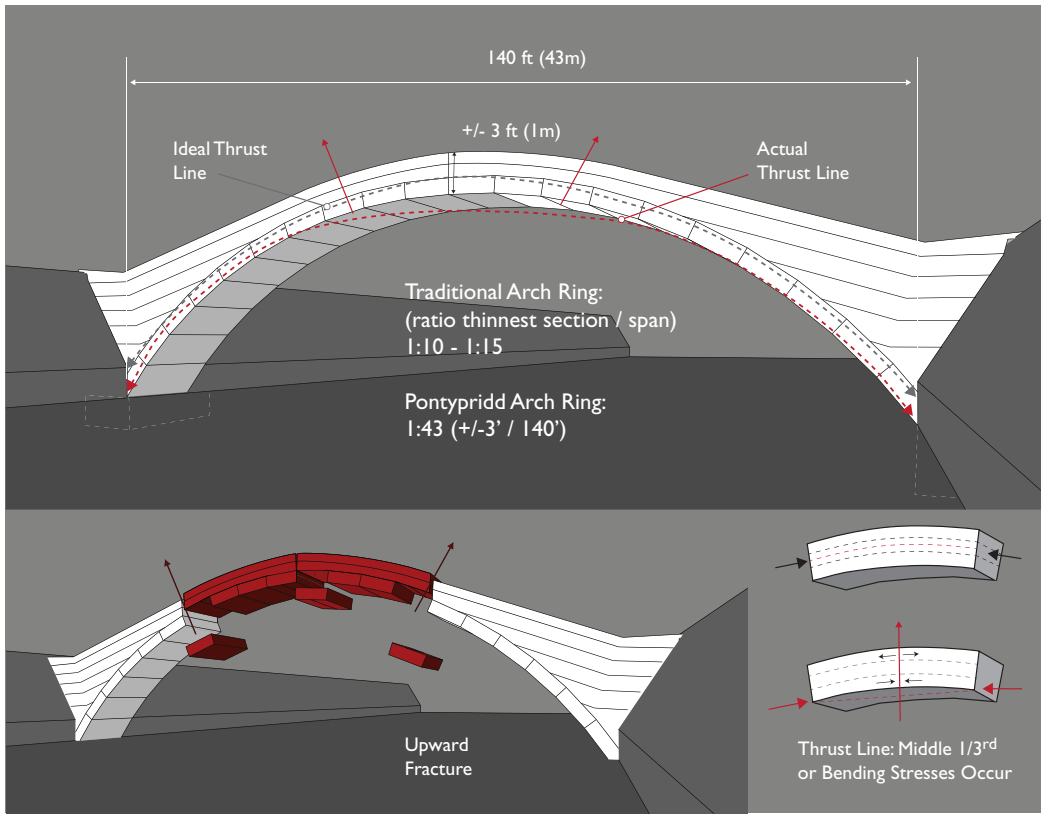


Figure 2.1.17 The Old Bridge, Pontypridd, Attempt #3, based on records of the design and graphic analysis of the thrust line location

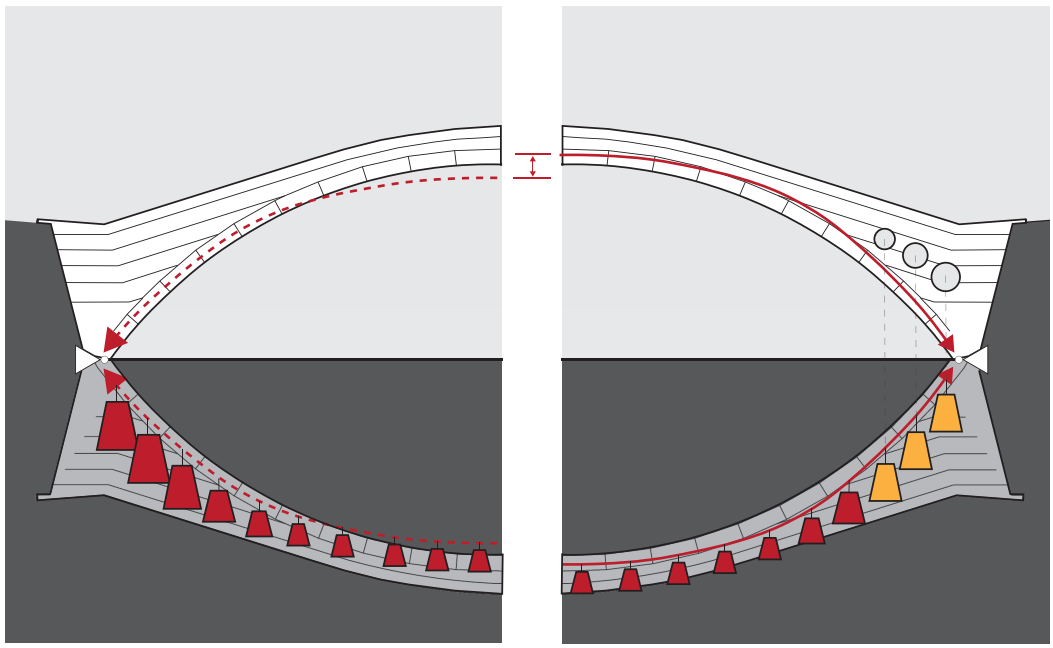
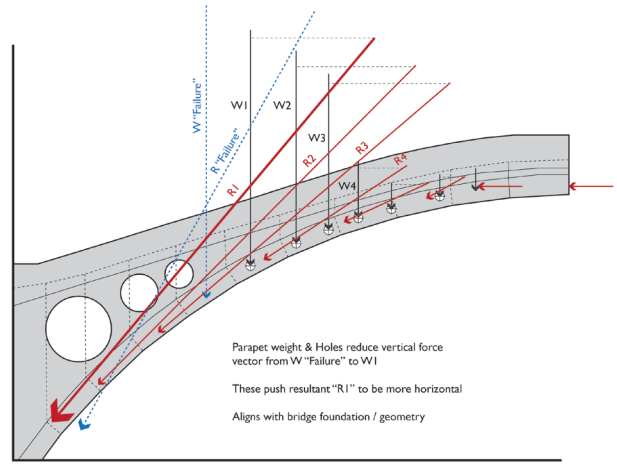


Figure 2.1.18 A before/after graphic analysis of the thrust line and weight of the bridge schemes #3 and #4. In the revised scheme (on the right), the thrust line was adjusted by reducing weight near the abutments and more weight near the top

have to change. We can understand this relationship between the weight and location of the stones with the resulting thrust direction by looking at physical models and graphic statics diagrams. (Figure 2.1.18)

Edwards changed the weight in two locations to correct the thrust line by adding a distinctive visual element to the design. First, he included a short parapet wall atop the arch to add weight to the center portion. Then he hollowed out the heavy haunches on the sides by creating three circular voids, with diameters of 9ft (2.7m), 6ft (1.8m), and 4ft (1.2m). Edwards may also have altered the weight of the haunches by infilling



Figures 2.1.19a and 2.1.19b Graphic analysis of the new thrust line is nearly ideal. It was the longest single-span masonry arch bridge in the UK for the next eighty years

lighter materials within the stone. Engineering analysis has shown that the line of thrust within the arch ring at the Old Bridge is almost perfectly positioned. By aligning the thrust line with the form of the arch, in 1756 it became the longest free-standing single arched bridge in the UK. (Figure 2.1.19)

Unfortunately, the Old Bridge at Pontypridd was a terrible bridge for anything but foot traffic. It was only 11ft wide between the parapets. Because the haunches sprung from the sides at such a steep slope, the top surface of the bridge was all stairs. According to *The Theory of Arches and Pontypridd*, wagons had to use a "chain and drag" system to descend from the crown and horse carts found it too difficult to navigate at all. Nearly one hundred years later, in 1857, a new flat-topped three-arched bridge (similar to the first scheme that Edwards had proposed) was completed directly adjacent to this "Old Bridge." Structural success alone is no measure of compliance and effectiveness. (Figure 2.1.20)

Masonry arches like at Pontypridd are useful and effective structures. For centuries, they spanned long distances with readily available materials. They were durable and relatively predictable. They were also heavy, materially intensive, difficult to build, and had inherently risky structural behavior if they were too thin. This combination of materials and form was also the only viable option for longer spans and these liabilities were accepted as part of the bargain. But with the emergence of new material options, these things were about to change.

Evolution of Arches: Beyond Masonry

Twenty-five years after the Old Bridge at Pontypridd, in the town of Ironbridge, Shropshire, one hundred miles north, the first long span cast iron arched bridge was completed—aptly named the *Iron Bridge*. Heavy boat traffic on the River Severn and the steep sided slopes of the gorge meant that only the thinnest and lightest structural solution was possible. Thomas Pritchard's design for the bridge used five thin cast iron ribs to span the 100ft across the river. His use of cast iron meant that the bridge could be designed and built as individually shaped components that didn't rely on segmental or semi-circular arch geometry to be constructible. Cast iron was lighter, thinner, made from standardized mass-produced elements, and easier to assemble. Perhaps the most influential change stemmed from iron's ability to resist both compression and moderate levels of tension stresses. (Figure 2.1.21)

As a result, the profile of these new arch structures didn't *have* to follow the thrust line exactly. When traditional masonry arch forms are closely aligned with funicular curves, any deviation puts unreinforced arches at risk for buckling due to wind or other unforeseen loads or lateral forces. Sideways forces create aberrant bending stresses within the arch and because masonry materials can't resist significant amount of tension stress caused by bending, the arches risked collapse. But with new building materials like iron and steel, arched structures could

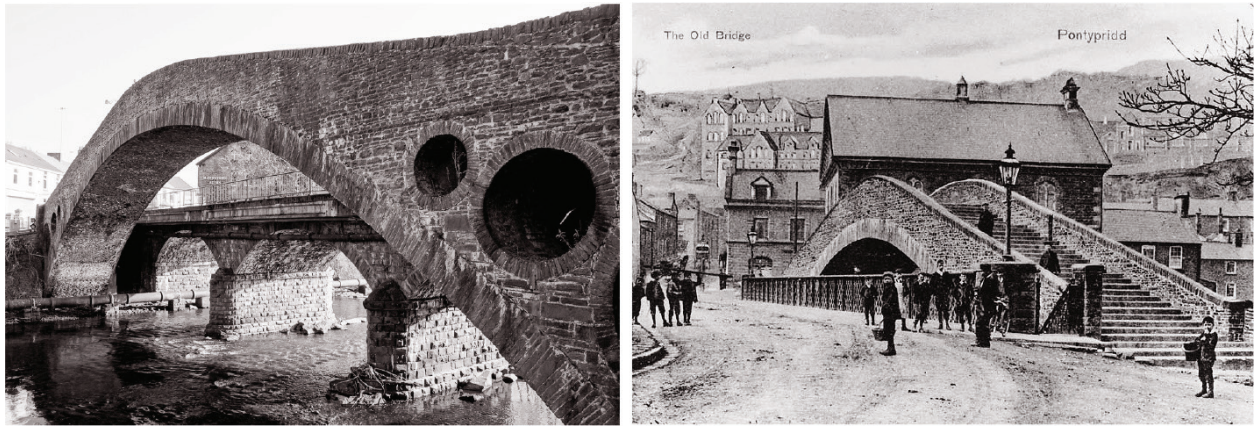


Figure 2.1.20 The steepness required to make the bridge span this distance required stairs along the length which adversely affected the serviceability of the bridge. A flat bridge was built adjacent to it 100 years later

Figure 2.1.21 The Iron Bridge (Ironbridge, Shropshire, 1781)

supplement the inherent efficiencies of load transfer allowed by the arched form with material qualities that could also resist some tension stresses.

By the middle of the 19th century, the Industrial Revolution had expanded the production, and use, of cast and wrought iron and, eventually, steel. The new profession of civil/structural engineering embraced the utility, economy, and effective structural behavior of these materials for infrastructural projects. New methods for accurately visualizing stresses in compressive structures, particularly the force vector (or graphic statics) method led to more predictability of form. (Figure 2.1.22)

Conservative and time-consuming arched timber and stone structures no longer sufficed. New ways of fabricating and assembling arches became possible with new materials, leading to the first use of "hinges" in arch designs. Because actual hinges that transferred vertical forces down but allowed rotation could be constructed, engineers started to

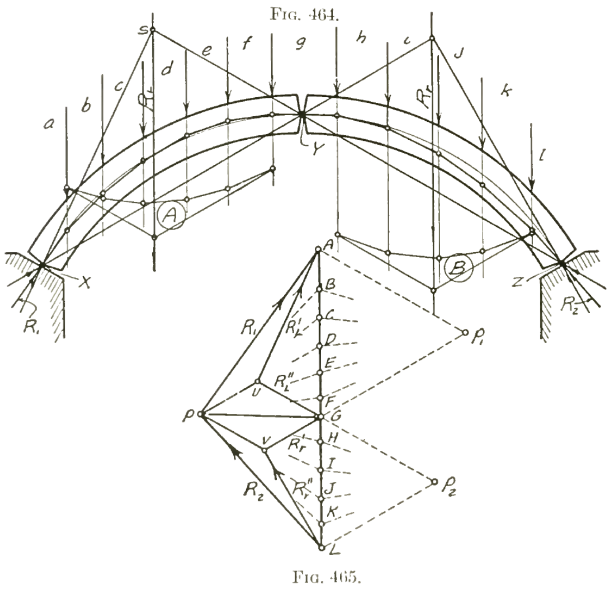


Figure 2.1.22 Wolfe's graphic evaluation of forces in a three-hinged arch confirms thrust line geometry with a resulting force polygon (1921)

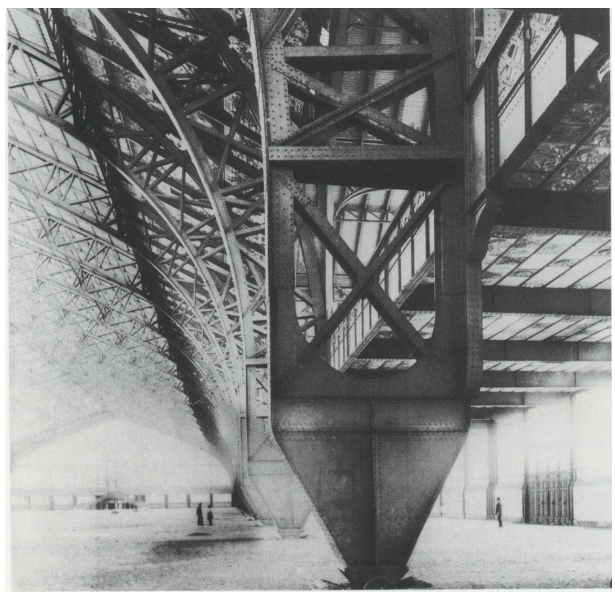


Figure 2.1.23 The three-pinned iron trussed arches in the Galerie des Machines (Paris, 1889)

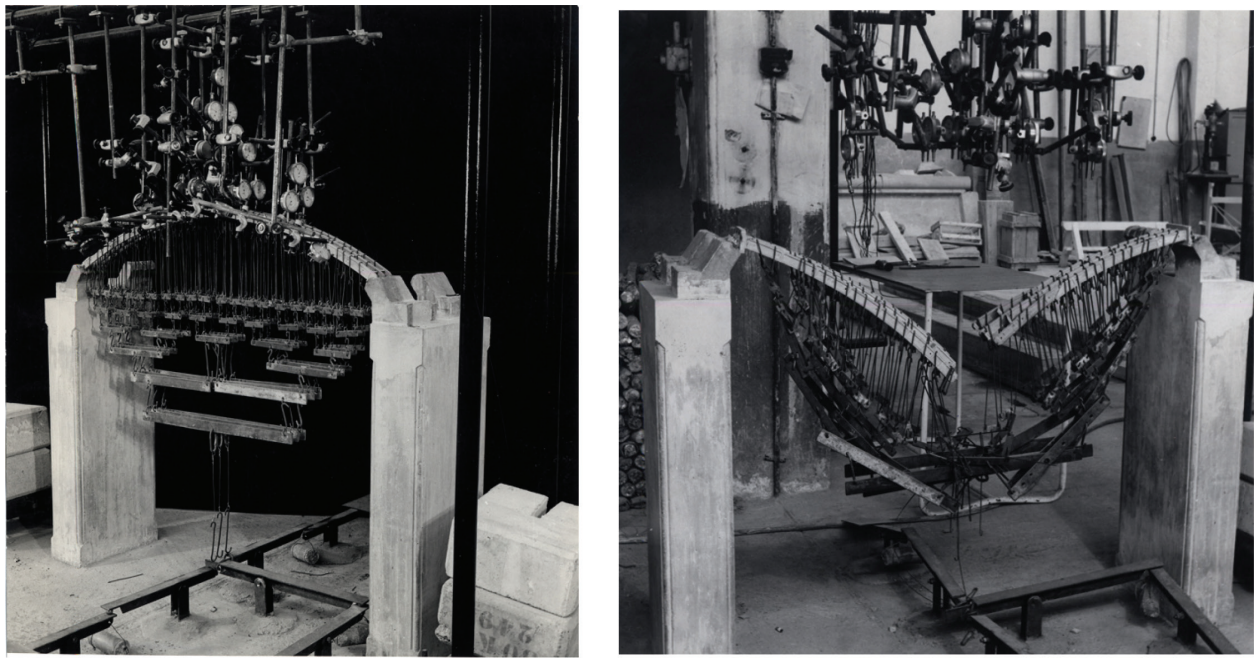


Figure 2.1.24 Model testing of a two-hinged arch by Eduardo Torroja (1959)

think of arches not as rigid bodies but as moving structural machines. If an arch can move with the weather and adjust to changing load conditions, some bending stress can be accommodated while still maintaining thin and lightweight members, since hinges allow movement.

When arches are hinged at each support, as *two-hinged arches*, they eliminate bending stresses near the supports but they bend at mid-span. By adding a third hinge in the middle of the span, a *three-hinged arch* is allowed to rotate more without bending. These hinges allowed arches to take on longer spans with lighter materials while maintaining predictable mathematical and statical behavior. Soon buildings with previously unimaginable spans, such as the three-hinged trussed arches of the Galarie des Machines at the 1889 Exposition Universelle in Paris (377ft (115m) span), and the two-hinged reinforced concrete Tavanasa Bridge by Robert Maillart in 1906 (167ft (51m) span). As hinges were integrated into arches and material was optimized, physical testing tools were needed to help understand and accommodate the influence of changing load locations on the arch. (Figures 2.1.23 and 2.1.24)

Today, thanks to advancements made in high-strength material science, manufacturing methods, and massive crane-based assembly processes, we are able to build arched structures that dwarf even the most ambitious earlier schemes. But even as impressive as these structures are in scale, they still follow the same basic structural concepts and behaviors found in the most remedial arched structures.

MAKING: SOME ASSEMBLY REQUIRED

Eero Saarinen's design for the Gateway Arch in St. Louis was based on a (nearly) catenary curve. What were all the advantages of the triangular plan shape of the arch (consider structural design, functional, and construction issues)? Both legs of the arch were constructed separately (and simultaneously) and eventually connected together at the keystone. The project was too large to use centering below, so how were the two legs constructed? What specific complications did the contractors have to address before placing the keystone? (Figure 2.1.25)

Figure 2.1.25 Gateway Arch (Saarinen, St. Louis, 1963)

Spatial Structures: Arches as Enclosure

Arches might be considered primarily planar or extruded forms, since their profiles are represented two-dimensionally, and arched bridges are linear. But arches' curves and their natural divergence and redirection of forces make the principle useful in multiple dimensions including domes, vaults, and shells. Arches need not be purely form-resistant masonry structures either—beams and trusses that are curved enjoy many of the structural benefits without the material liabilities of masonry. Many arch-like structures provide resistance with trusses or stiffened structural surfaces. Accordingly, we will return to the structural benefits of arches as a way to make certain structures behave more effectively. (Figure 2.1.26)

Design Assignment: Arched Pavilion

We can apply these lessons about form-finding and behavior of arches towards a small design problem. We'll design an arched pavilion made of lightweight material that curves through the site. The proposal is to span 48ft (14.6m) with a series of arches spaced 12ft (3.6m) apart, slightly adjusting the angle between them to create curvature in plan and to preserve a minimum interior volume (15ft w × 18ft h (4.5–5.5m)) throughout. In the sectional sketch you can see additional weight of hanging displays off to one side of the arch within the interior. We'll need a bearing material that is good under compression and stiff (stone, concrete, etc.). To make it lighter and easier to construct we'll use segmental arches assembled from smaller components. (Figure 2.1.27)

MAKING: TOOLS OF THE TRADE

At the beginning of the chapter you were asked to speculate on the greatest risks associated with the design and construction of compressive structures. Review that list before you start designing.

To Do: Consider how you might address those factors proactively with the tools you'll use in this design process, the "models" you'll make, and evaluation standards you'll use. For example, if finding the proper form is your top priority then determine the types of tools that would help you find this form and translate it into your designs (e.g., hanging cable or a digital model). Likewise, if you prioritized issues with foundation/thrust of even construction complications, make the same determination.

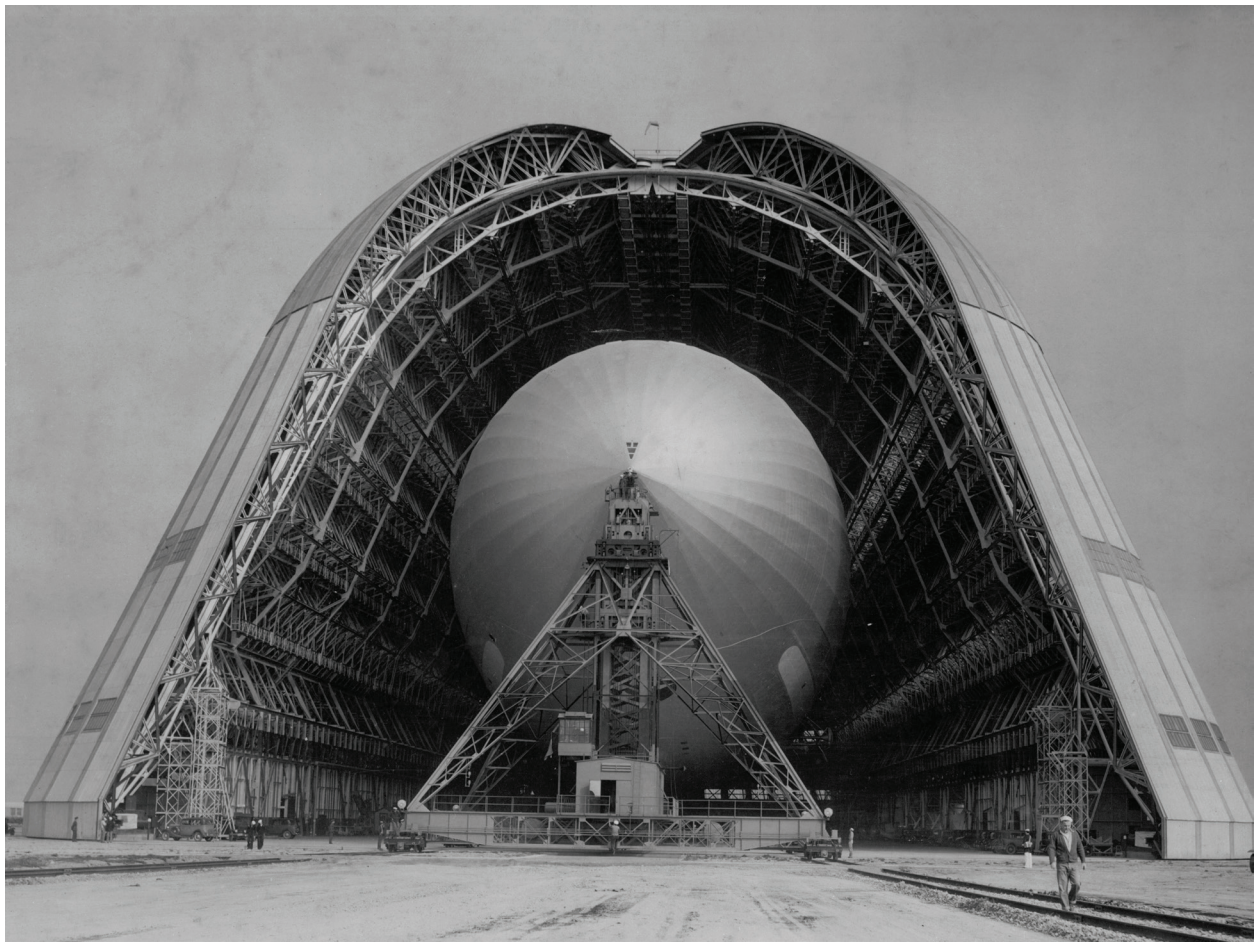


Figure 2.1.26 Hangar One Construction for Blimps, NASA (Sunnyvale c. 1931–1934)

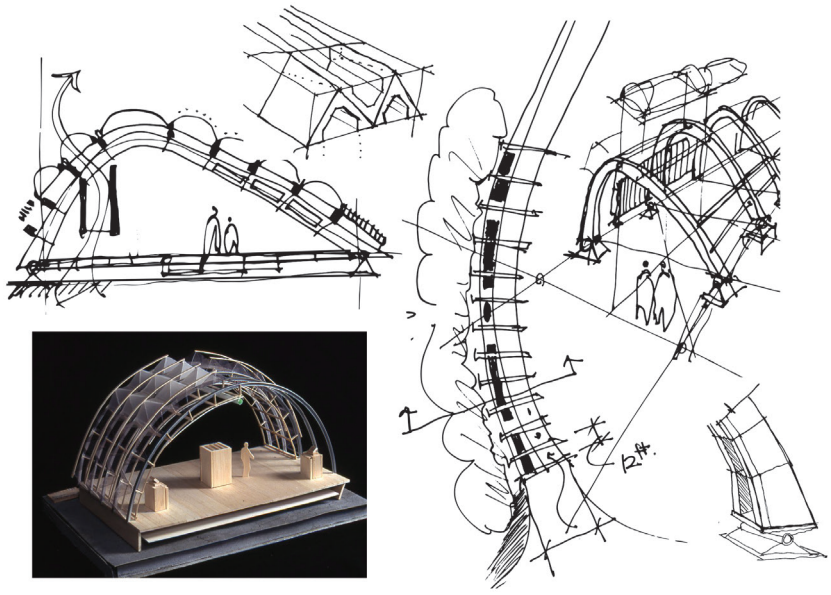


Figure 2.1.27 Initial sketches of plan, section, axon, and potential details for design problem. A study model of the IBM traveling pavilion (RPBW, 1983) shows potential inspiration

Form-Finding and Physical Models of Arches

A physical model is the most effective and informative way of testing our proposed form and understanding arch behavior. Although Hooke's method is centuries old, it is still effective, especially if we constrain and adjust the chain as needed. We can create a section drawing with critical "control points" of functional clearance (e.g., 10ft (3m) clear height 5ft (1.5m) from the ends, etc.) and hang the drawing upside-down. Next, we can hang a chain (or thin cable) against it to test the geometry. Because the hanging displays and desks are heavier than the uniform load by a significant factor, you'll need to hang additional weights on these points (e.g., additional links of a chain) to get the proper profile. (Figure 2.1.28)

Once we've established an idealized force line, we can translate this profile into an overall arched model. Accuracy in model making is important to ensure that it will behave as intended. One challenge is modeling the segmental nature of the arch accurately without creating small hinges that lead to a model's collapse (glue joints between won't work). Simply casting a solid arch flat and tilting it up misses the opportunity to address the challenges inherent in creating a complete thrust-resisting form from smaller elements.

As with all arches, resisting outward thrust is paramount. Gravity always finds the fastest way to equilibrium, and flattening the arch is the easiest way to achieve it. As the arch profile becomes complete, we'll want to represent the hinges at the base so the model moves accurately under loading, and to develop a way of resisting outward thrust (e.g., tying a string between the foundation points) so we will use a two-hinged arch. Unless we can affix the model permanently to a base, we should consider adding

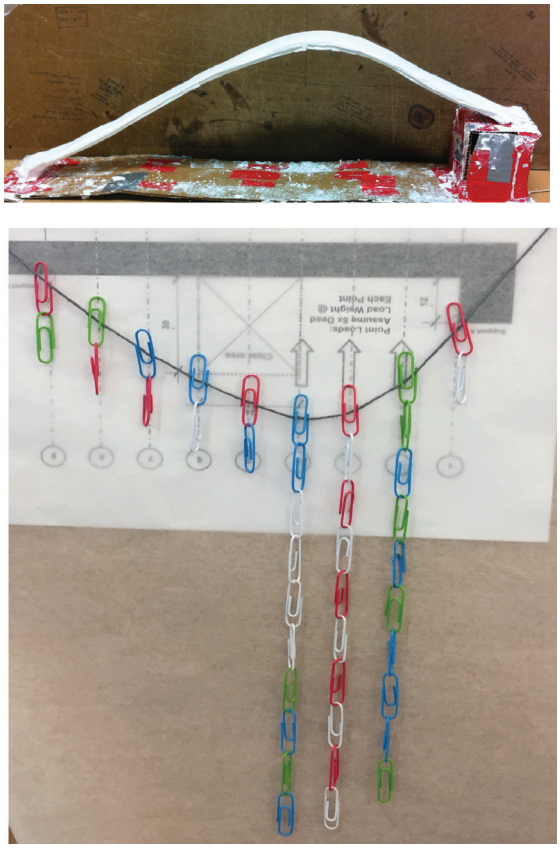


Figure 2.1.28 In this demonstration, paper clips are used to represent the weight of a typical unit of mass. The length of the string is adjusted as needed until the curve matches the interior volume required by the program

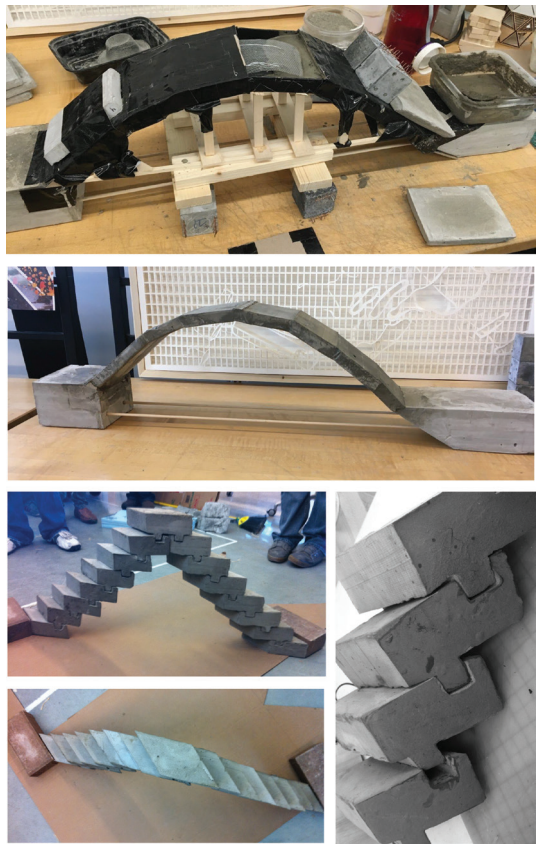


Figure 2.1.29 Experiments in model making that illuminated challenges found in the process of construction (SxD, Iowa State University)

a supplemental element, like a horizontal tension tie-back, between the two legs of the arch in the model mock-up. (Figure 2.1.29)

Testing the Arch Part I: Breaking and Re-Thinking

Before testing begins, record a hypothesis for how you think your structure will behave under loading. As testing happens, take photos, videos, and measurements (as needed) to test your hypothesis. Determine ahead of time, what you are looking for (movement, cracking, etc.). We can subject the model to different types of tests, depending on the type of feedback you need for schematic design.

A thrust line that falls outside the thickness of an arch in a drawing, won't get the attention that the sound of cracking and seeing it break under loading will. If an arch model starts to spread apart because the thrust isn't accounted for, you won't need to check your notes to understand how to fix it. Material qualities are better understood by connecting them together and then watching them move under loading. Even more abstract ideas, like "fixed versus hinged arches" becomes easier to understand because we can see and feel the differences in their behavior with a model.

Consider this model a test of the design's behavior, not as a confirmation model. We want it to reveal where stresses are consolidated and how it would deform. To do so, we'll need to subject this model to more loading than it would have in real-life. If you wanted to look at the general performance of the physical model under loading you would use a heavy uniformly distributed material (like Gaudi's sand bags) across the entire top of the structure and then apply other weights (placed atop or hung below the model) at the point load locations. But the intent here isn't to determine how much load this structure can hold—it is simply to determine how the model might behave under loading.

Figure 2.1.30 Testing process on arches built with different materials (SxD, Iowa State University)

Nearly all models should survive this basic load testing if they're built correctly, the materials are plenty strong. If you want to see clear evidence of how a changing thrust line affects the behavior of an arch you can asymmetrically load the arch or add weight where it wasn't expected. Eventually it will move and crack in interesting ways. (Figure 2.1.30)

Testing the Arch Part II: Calculations and Confirmations

Although we could confirm the bearing capacity of compressive materials and "scale-up" the test results to determine actual sizing, this would be unnecessary, cumbersome, and inaccurate. Confirmation for a three-hinged arch can instead be found with mathematics and/or graphic statics. There are two main formulae to consider: First, we can find the reactions at the base (both vertical (R_v)) and horizontal (R_H)) and the forces in the cable (P) by assuming equilibrium (Chapter 2.0). We can then solve for thrust. Looking at the second formula we'll find that the length of the span is the controlling factor for the stress levels (M) and the height of the arch (h) is the determining factor in the amount of outward thrust. Second, once we know the amount of anticipated compression stress (P), we can use the allowable stress formula ($A = P / f_a$) to solve for the amount of cross-sectional area needed (assume f_a = 1,550 psi for wood). We could also draw a simple graphic statics diagram to determine how much force is in each section of the arch. (Figure 2.1.31)

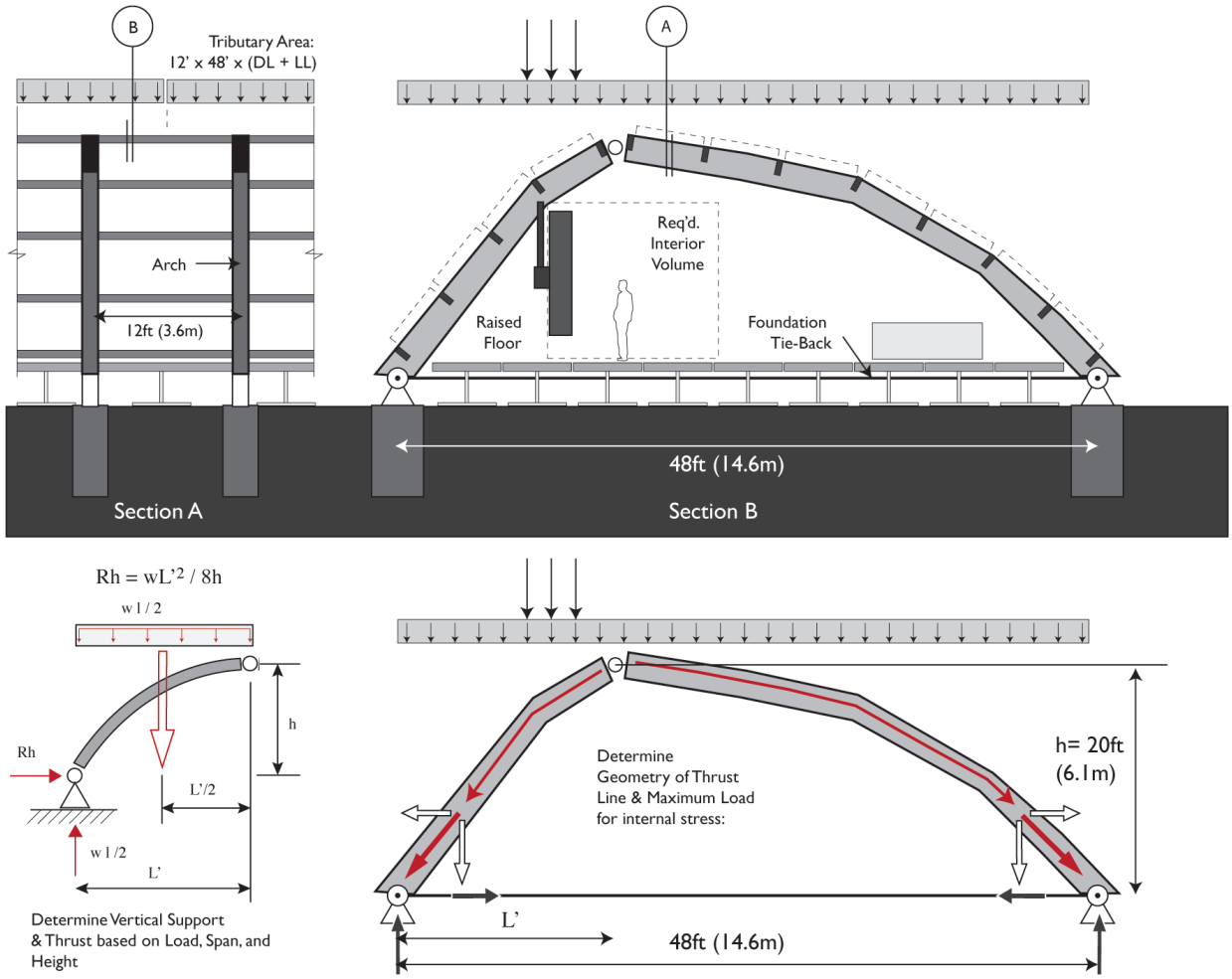


Figure 2.1.31 Establish final dimensions/geometry, collect data about loading, and establish confirmatory models (graphic or mathematical)

The evaluation process of the model testing tells us one set of information, while the mathematics tells us another. Because the model "arches" are made from smaller curved elements, we can recognize the threat of bending stresses at these "hinges" and see how any discontinuity in the material risks failure. The mathematics tells us how much outward thrust needs to be resisted while the model helps us visualize it. Our calculations allow us to quickly consider the consequences of other adjustments (e.g., would it help reduce compression if the height of the arch was increased (or reduced)?). This back-and-forth between behavioral and confirmative testing is what we need to better assess our options.

Frequently Asked Questions

Aren't arches an outdated type of structural system? Not at all. Think of arches as a structural idea that matches form to load transmittance and not as a historical archetype. Because they are fundamentally efficient, they'll always have utility and relevance. Today, because arches can be constructed from many different contemporary materials, they are integrated within a wide variety of situations including the longest spans of bridges and stadia. In many parts of the world, load-bearing walls are still used as a primary method for structuring modestly sized buildings, and understanding how to form and construct arches for openings and/or vaulted enclosures is important. (Figure 2.1.32)

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Figure 2.1.32 Arched structures can meet a variety of functional, formal, economic, and aesthetic criteria when critically integrated into a larger design strategy

1902, Courtesy of Royal Commission on the Ancient and Historical Monuments of Wales (RCAHMW).

2.1.20 Courtesy of Royal Commission on the Ancient and Historical Monuments of Wales (RCAHMW).

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2.1.26 Courtesy of Ames Research Center of the U.S. National Aeronautics and Space Administration (NASA), c. 1931.

2.1.27 Model by Renzo Piano Building Workshop.

2.1.32 By Matthew Henry (Unsplash).

Figure 2.2.0 Tomás Saraceno, "Stillness in Motion — Cloud Cities installation," San Francisco Museum of Modern Art, 2017

CHAPTER 2.2

Suspended Structures

Designing for Tension

Tension structures (cables, membranes, and pneumatics) are lightweight and expressive three-dimensional building elements that are advantageous for long span structures. Their litheness also presents critical challenges when enclosing spaces because of their relative instability. Finding and developing building forms, materials, and details that are responsive to these challenges creates a uniquely expressive set of building structures.

Suspended Structures

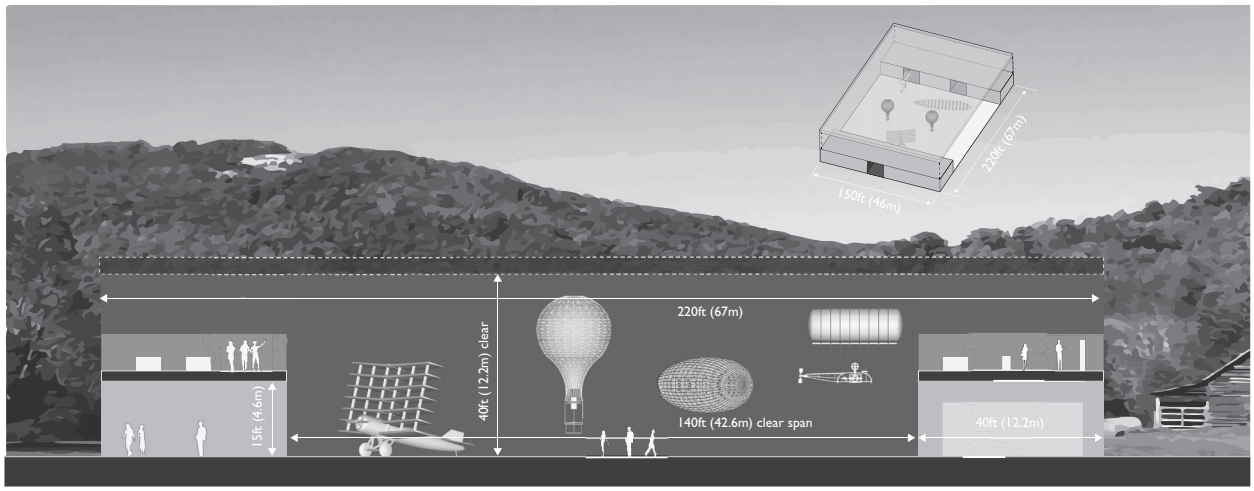
Unlike heavy form-resistant compression structures, tension structures are assemblies of relatively flexible structural elements including cables, rods, and membranes. These elements resist tension stresses as load-bearing and/or stabilizing elements, and take their forms based on how they are loaded. Cables can't take compression or bending, only tension, so they can only resist axial loads applied parallel to the cable length, which makes them materially efficient.

Tension structures can take a variety of potential forms based on: The geometry of the cable (e.g., straight or curved cables), the type of supporting elements (e.g., masts, arches, walls), how they are stabilized (e.g., parallel cables, perpendicular cables, additional dead loading, etc.), and how enclosure is achieved (e.g., hanging membrane, pneumatics, etc.). In the book, *Structures: Or Why Things Don't Fall Down*, J.D. Gordon warns that, "Tension structures are treacherous traps for the unwary." We will identify the unique challenges associated with their use as well. Finally, we will look at several examples to help reveal insight into form-finding, analytical challenges, and construction issues.

Design Assignment

Flight without feathers is not easy.
(Plautus c. 200 BCE)

The Carolinas Aviation Museum has issued a call for proposals for a new free-standing museum dedicated to their pre-Wright Brothers experimental aircraft such as aerostats/blimps and gliders. The site is a remote location near Charlotte, North Carolina, in the Appalachian foothills of South Mountain. The museum's plan must be able to enclose all the aircraft on display, and accommodate the public non-collection spaces, and private support spaces (approximately 35,000ft^2 (3,250m^2)). A general layout is included in the call for proposals, alongside a sketch for interior clearance requirements. Understandably, your project team is interested in exploring a design that incorporates



Figures 2.2.1 Sketch diagram of required interior volumes and overall building massing

many of the lightweight, cable-based elements found in the early aircrafts of the collection. (Figure 2.2.1)

Although the form of cable structures is determined by the magnitude and placement of the loads and supports, there are still many possible options. Physical models for form-finding can help us understand the behavior and instability of cables and membranes under loading. For confirmation, we can use basic vector mechanics (including graphic standards) and basic calculations. The range of potential design options is broad but we can narrow our options down to different categories of tension-based systems. (Figure 2.2.2)

Early Cable and Membrane Experiments

Ropes are useful load-bearing elements because they are lightweight but strong, easy to make from a variety of fibrous materials, portable, deployable, and easily connected/spliced. Their behavior is easy to understand because they *only* work in tension, so their form can only follow the forces.

Ropes have been used in maritime settings (e.g., sails, nets, etc.), construction methods (e.g., pulleys, stabilizing ropes, guides, etc.), shading structures, suspended structural bridges, and even as stabilization cables in

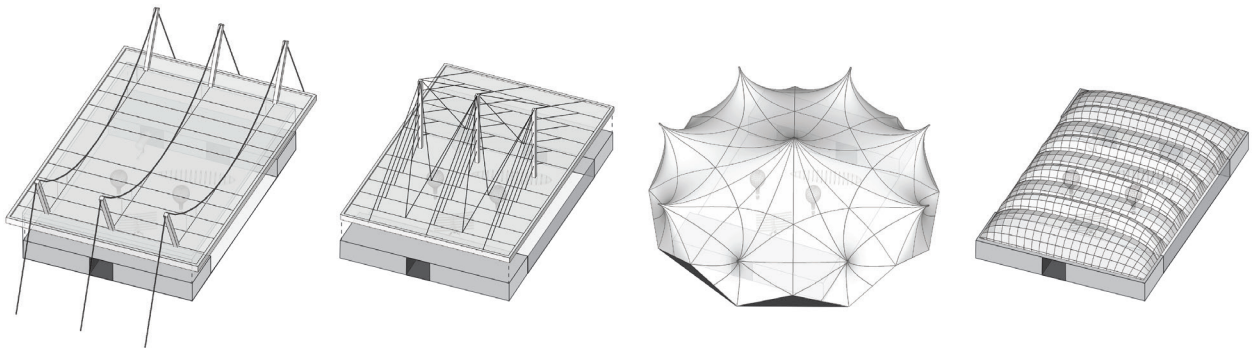


Figure 2.2.2 Four broad categories for tension systems (left to right): Suspended, cable-stayed, tent/membrane, and pneumatics

the lightweight aircraft featured in this building. The 18th century experiments in "lighter than air" aerostats were gas-filled membrane bags tethered together with rope mesh netting. Relying on ropes to resist tension stresses in a membrane was common thanks to centuries of sea-faring traditions, but the first experimental balloons exerted an unknown amount of force on the membranes, so ropes were needed to restrain the balloon's expansion and to connect the balloon to the passenger basket below. (Figure 2.2.3)

Suspension structures have several interconnected liabilities that discouraged their architectural application:

- The inherent instability of the supporting elements that can shift, sway, or flutter under changing loading conditions.
- The geometry of ropes is wrong for buildings; ropes sag but floors and roofs generally do not.
- The high horizontal thrust forces generated on the ends that need to be resisted or counteracted.
- The threat of catastrophic failure if a tension member breaks (especially at anchorage points).
- The material's lack of durability and reliability, including limitations in the strength and stiffness capacities of certain materials under tension. (Figure 2.2.4)

Although ropes could be made with a high enough load-bearing capacity to be useful in larger building structures, they generally weren't. Until the 19th century, there were virtually no durable construction materials that could reliably perform under tension. Tension elements were first used to stabilize building frames in greenhouses, factories, etc., that tended to rack side-to-side under lateral (sideways) stress. A similar bracing tactic is seen in the lightweight aircraft featured in the

Figure 2.2.3 Earliest experiments in aircraft rightly focused on reducing the overall weight (Morieau, 1885)

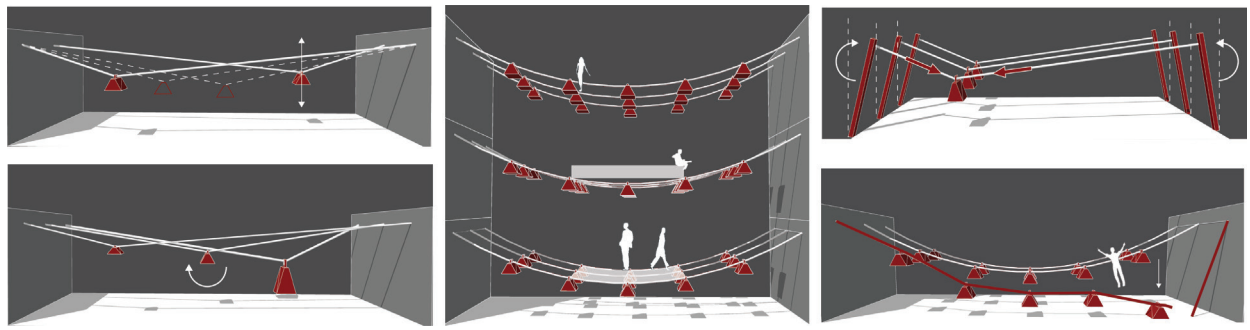


Figure 2.2.4 The common failures (or threats) of tension structures: Instability, changing form, thrust, and catastrophic failure

proposed museum's collection. It wasn't until the middle of the 20th century that architects and engineers began to experiment with cables as load-bearing members. This delay makes sense because the resulting three-dimensional forms were complicated to develop and document accurately and they were difficult to analyze because of their flexibility and elasticity. (Figure 2.2.5)

Many contemporary advancements came from a close-knit group of contemporaries and collaborators in the 1960s–1980s, including: Frei Otto, Jörg Schlaich, Yoshikatsu Tsuboi, Ove Arup, Fred Severud, and Edmond Happold. Their collective efforts included advancements in form-finding

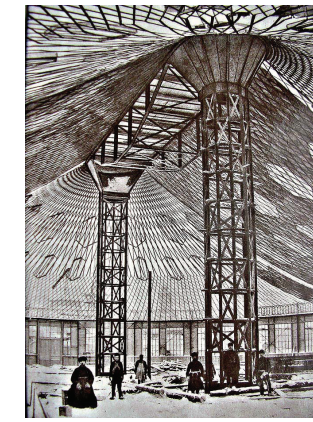


Figure 2.2.5 One of the first tension structures was the Exhibition Hall, Niznij Novgorod (Vladimir Shukhov, 1896)

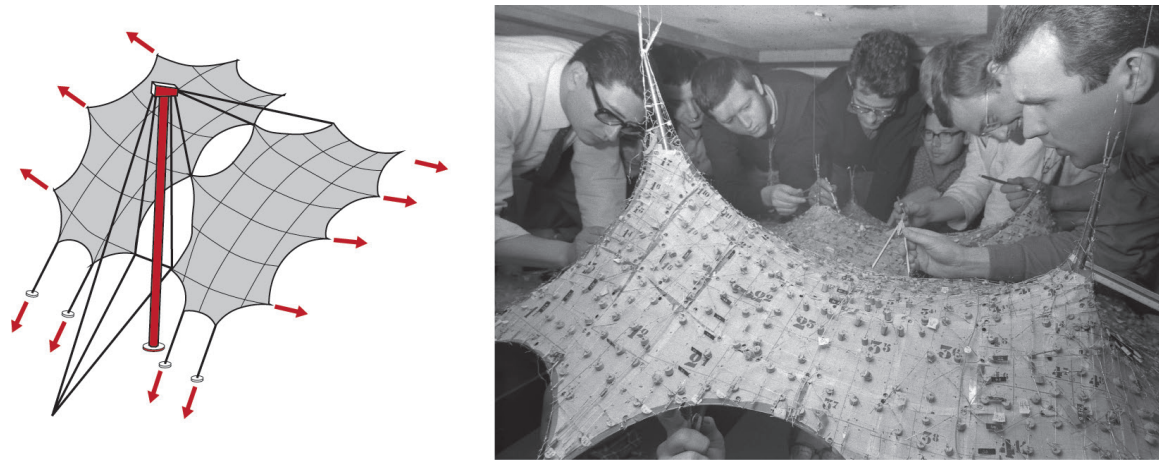


Figure 2.2.6 Otto's form-finding and confirmative models: (Left to right, top/down) illustration of Montreal Pavilion (1967), IGA Berlin model (1957), and German Pavilion (Montreal Expo) model measurements (1967)

(using physical models), analytical methods, construction techniques, and an expanded range of tensile materials and membranes (including plastics). Imagine the difficulty of calculating membrane pressure at a particular point, or the amount of displacement in a cable-mesh net without physical models. Their work spanned the pre- and post-computational era of design—a transition many of them help usher in. As Frei Otto explained, the work was, and continues to be, ambitious and hopeful: "My hope is that light, flexible architecture might bring about a new and open society." (Figure 2.2.6)

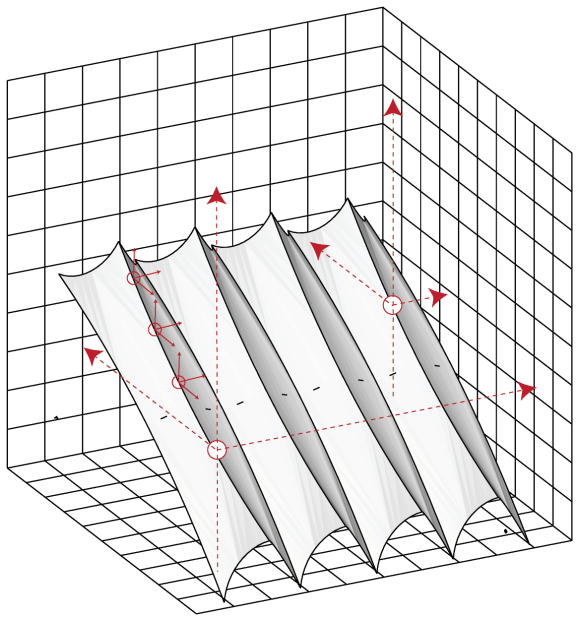
THINK: CONCEPTS > CALCULATIONS

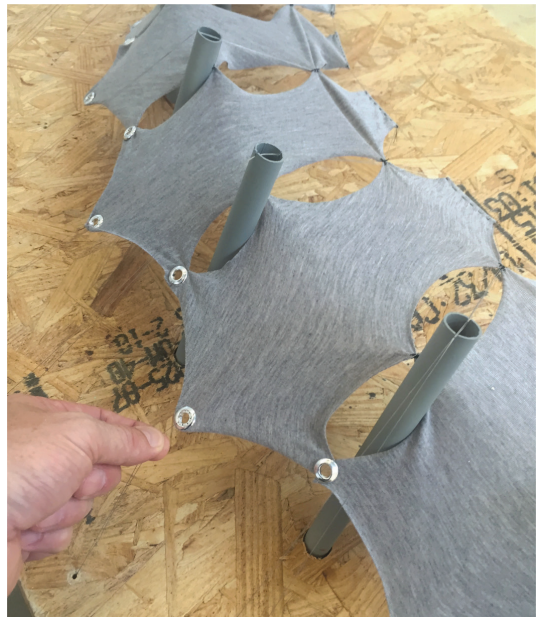
Free-formed tension structures were difficult to document, but they're easy to model.

Try It. Take a rectangular shaped flexible membrane material (e.g., nylon, cheese cloth, etc.); have one person pull down on two opposing corners while someone else pulls upward on the other two. A double-curved saddle shape emerges naturally as the surface is tensed. Change hand heights and locations and see how the membrane form and tension varies. Form will always follow tension.

To Do: Now try to document the forms that you've created so someone else could recreate the same form. The form always changes depending on how tightly it's tensed and where the supports are held!

To Discuss: What tools would you use to measure your model and how would you draw it so that others could build it? (Figure 2.2.7)





Figures 2.2.7a and 2.2.7b Differences in the support locations and the amount of tension pulling the membrane can drastically change the geometry of flexible structures. Imagine the difficulty of accurately defining and documenting the overall form in terms of x, y, and z axes

Suspended versus Cable-Stayed

Although tensile structures are recent developments for building structures, tension-based bridge structures have been a part of the vernacular in cultures worldwide. These can be classified as two types of tension-based strategies: The curved "hanging chain" form of a *suspended* system, and *cable-stayed* structures that have straight-lined tension members suspended from masts.

Suspended and cable-stayed structures support loads with tension-based members. Their differing forms result from the alternative ways they are loaded (i.e., uniformly or as a point load). In suspended structures, the loads collect and increase along their lengths up to the supports—they can be calculated like the forces in an arch. The loads in cable-stayed systems are dependent on the area being supported by each cable. (Figure 2.2.8)

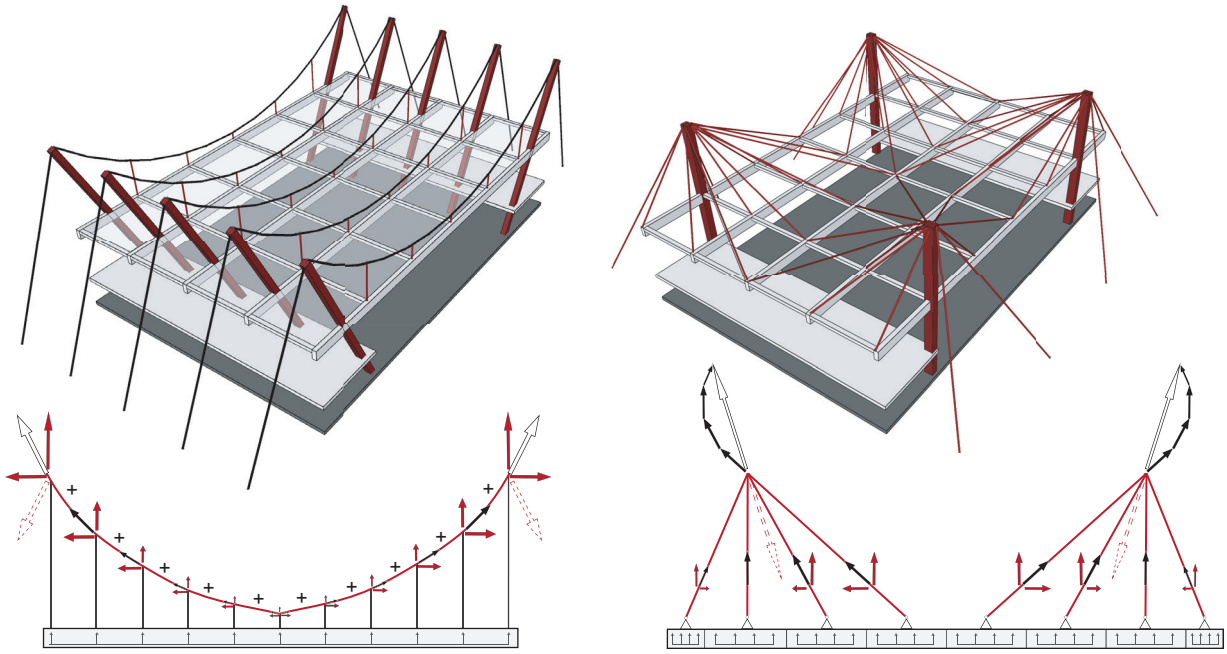


Figure 2.2.8 Two primary systems: Suspended cables (left) and cable-stayed systems (right). They collect loads differently based on their supports and geometry; this can be drawn as a series of vectors to find magnitude and direction of overall forces

FORM FOLLOWS FORCES: MODEL EXPERIMENT #1

Experiment with the ways that cables support loads and correspondingly change form. Using a physical model, set up wooden support posts at regular intervals to represent the boundaries of the design proposal. Connect the posts together with string to serve as "bearing" cables. Estimate the height of the posts and the sag of the string with a reasonable functional and structural proportion. Examine how the posts and cables are unstable without modification, especially as point loads are added to the string or when sideways forces are exerted upon the cables.

To Discuss: How would you modify the cables to stabilize the proposal? (Figure 2.2.9)

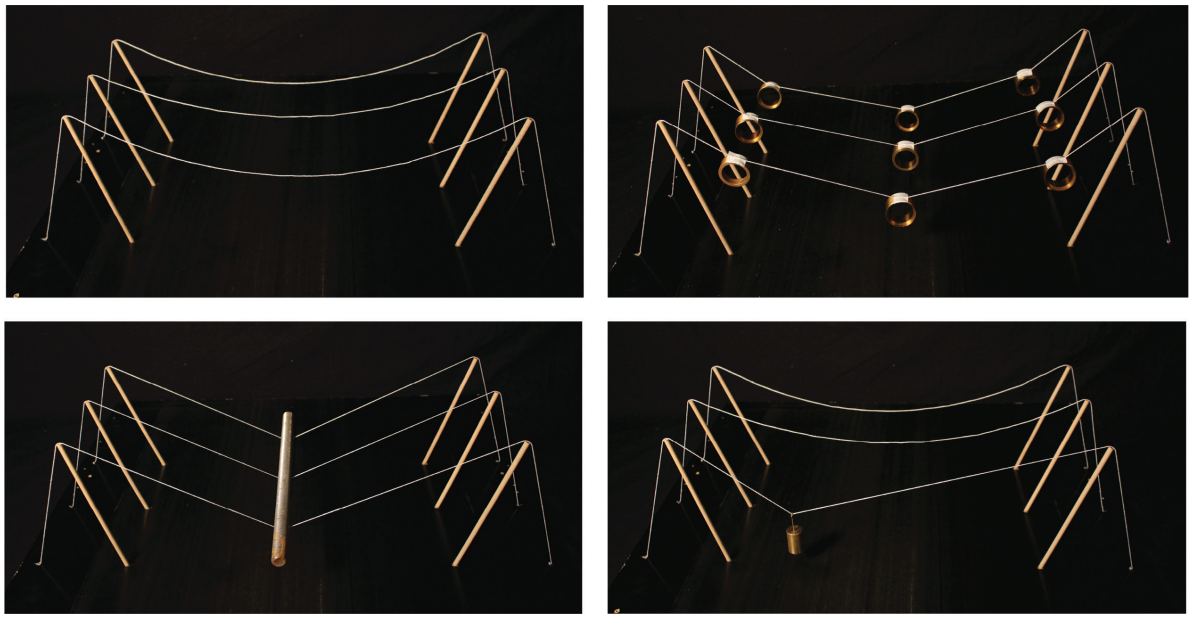


Figure 2.2.9 Physical model set-up and experiments showing how the form changes when load locations and magnitudes vary

Design Option: Cable-Stayed

In cable-stayed structures, load-bearing cables pull the forces upward diagonally (ideally between 45 and 60 degrees) in tension towards the top of the support tower (otherwise known as a *pylon*). If the cable stops atop the pylon, it exerts a horizontal thrust that pulls the posts over. If we extend the cables beyond the post and tie them into the ground, the thrust is now counteracted and only the vertical forces are transferred into the pylon (compressing it).

It is easy to imagine how a cable-stayed roof works. Instead of relying on regularly spaced columns to collect loads from roof beams and transferring these loads downward, cables connect to roof beams at similar intervals and pull the loads *upwards* to mast-like columns. Since cables can span farther than beams, there can be fewer masts in this system, allowing the interior space to be column-free without the penalty of deep long-spanning roof support members. (Figure 2.2.10)

This support strategy creates a unique and engaging clarity of expression between the supporting structural forms and interior space. At the Renault Distribution Center (Foster and Partners, 1982), columns are placed in four corners of each structural bay and the girders around the perimeter of this bay are supported by hanging cables (like large umbrellas). The Pat Center (Richard Rogers, 1984), relies on a tall A-framed central structure to support cables that drop towards the roof on each side. Four gigantic masts rise from each corner of the Alamodome (HOK Sport, 1993), supporting cables that support the long span roof trusses of the stadium below. All these projects are axially symmetric; cables' loads are counterbalanced with other cables spread across the structure or into the ground. (Figure 2.2.11)

Cable-Stayed Evaluation

To incorporate this strategy into our design, we'll look at the location of the supporting pylon/masts, the number of cables, and how these are distributed across the bearing surface. It is a long span so these will have to resist large forces, requiring large tendons and high levels of resistance to elongation.

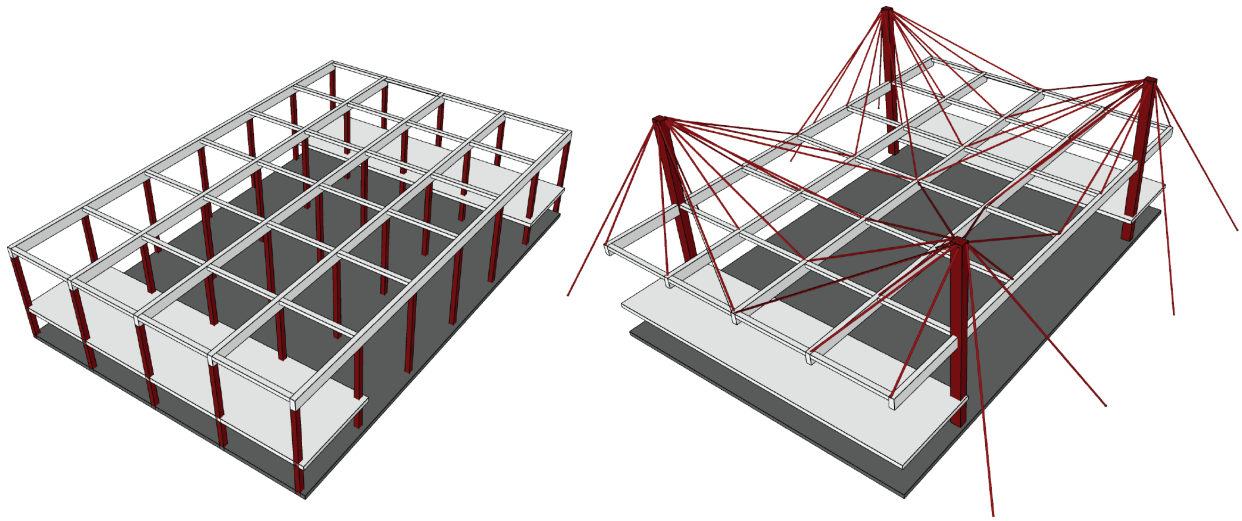


Figure 2.2.10 Comparing two options for vertical support: Columns or cable-stayed supports. Most columns can be eliminated if the cables pull the column loads up and concentrate them into fewer supporting masts

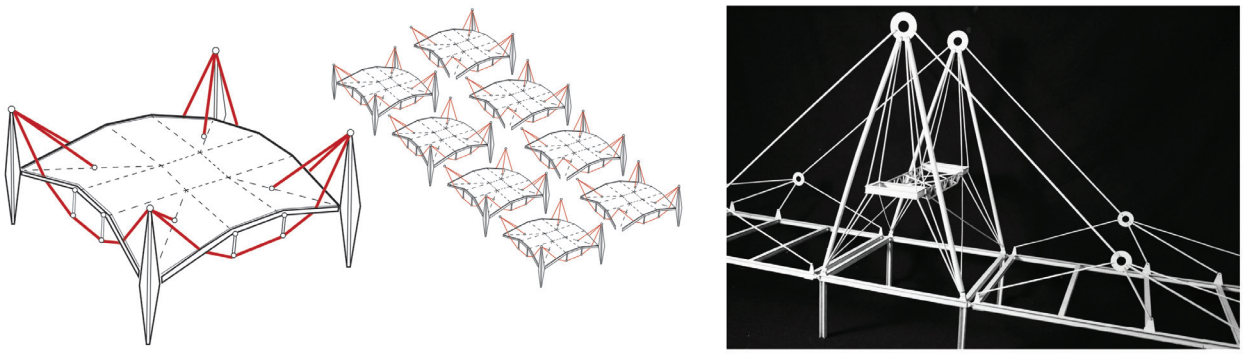


Figure 2.2.11 Examples of cable-stayed projects: Renault Center (left) and Pat Center (right)

We can make the structure more efficient by considering the construction and supports together. If the pylons are moved inward to one-fourth of the total span, the decking can symmetrically extend outward from the pylons, like a balanced double-cantilever, or an inverted umbrella. This is an elemental structural arrangement of tensed and compressed components, balanced by the form's arrangement and appropriately selected materials—unfortunately it places the columns further into the open exhibit space than originally intended. (Figure 2.2.12)

We can determine the stress in each cable and its elongation with the formulas we learned in Chapter 2.0. Once we know where the pylons are located, we can determine how much load each cable carries by calculating the *tributary area* and multiplying this area times the combined live and dead loads to find the total vertical downward force, (P). From there, we can use trigonometry and/or graphic statics to find the stresses in the diagonal cable. Once we know the allowable stresses and length, we can select a material to find the allowable stress capacity, (fa) and the modulus of elasticity, (E) to determine the cable size and elongation. But the exactness of these calculations is a bit premature, as we also need to assess the broader pros/cons of the cable-stayed strategy. (Figure 2.2.13)

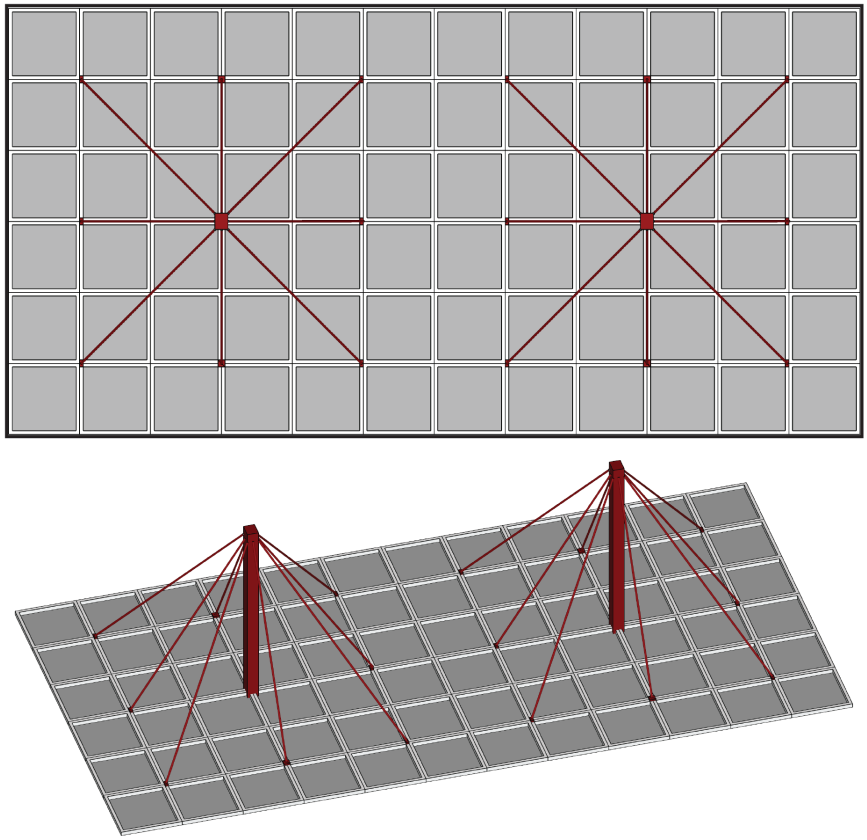
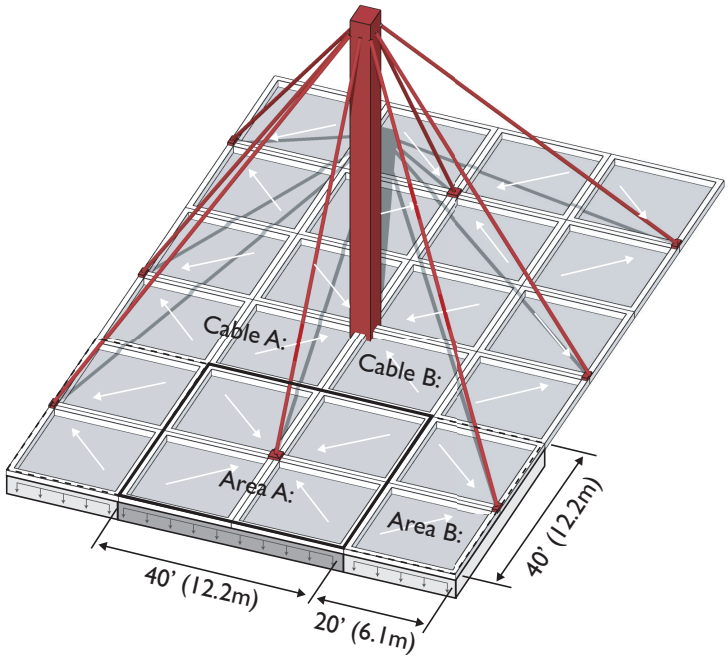


Figure 2.2.12 Centralizing the masts in plan equalizes the distribution of forces on the cables in all directions



Allowable Stress (f):
High Strength Steel Cable
=30,000lb/in² (2,110 kg/cm²)

Modulus of Elasticity (E):
=29,000,000 lb/in² (2,038,990 kg/cm²)

Loading Estimate, *(Dead Load + Live Load): 180lb/ft²* (7.6kg/m²)

Cable Length (L) = 900in (2,286cm)
Cable Angle = 60 degrees

$A = P / f_a$

$e = P\,L / A\,E$

Figure 2.2.13 Assessing loading and support conditions with calculations can determine values of stress and elongation in cables

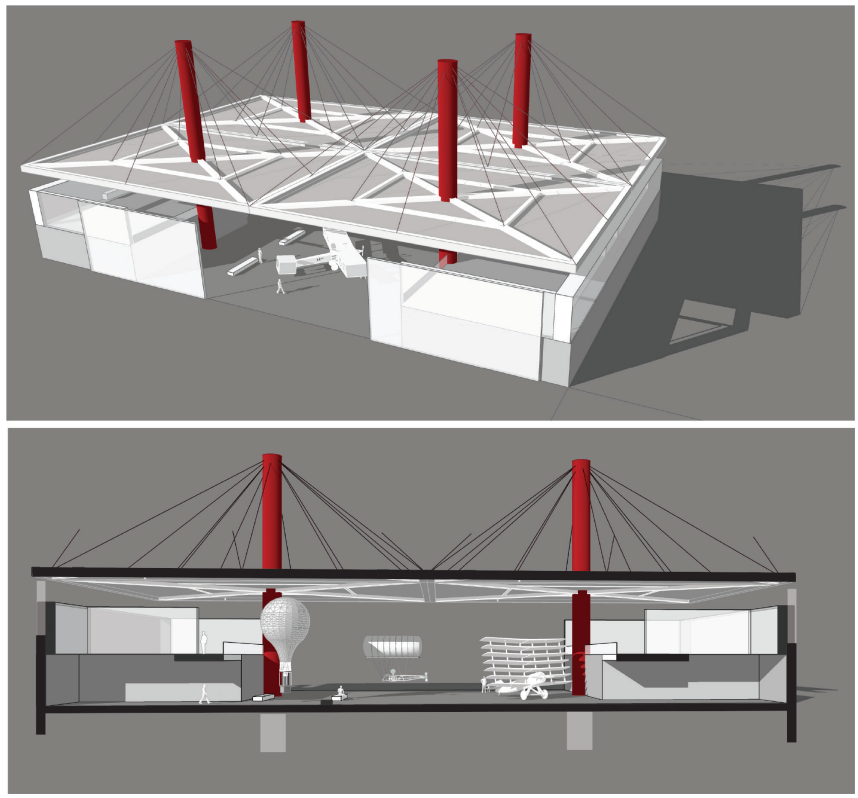


Figure 2.2.14 Scheme 1: Cable-stayed option

The final proposal for the cable-stayed system moves the large columns inward into the exhibition space but provides ample room for hanging exhibits from the ceiling and bringing indirect light through the perimeter. The roof plane itself can be relatively thin because of the all the cables supporting it from above. (Figure 2.2.14)

In these schemes, the cables are connected to the supports with pinned hinge connections that allow the structure to adjust to slight rotations and movements in the structure caused by wind or changing thermal conditions while maintaining tension. Their weak points aren't found in the material itself, instead they are always at connection points. Stresses concentrate at these points and the material itself is transformed or connected to another material or component—creating an opportunity for failure. Concurrent forces need to be cancelled out by the equal and opposite forces that are pulling at various angles, so when these come together it concentrates these forces in one location, and the connection point may become too large to accommodate all the fasteners. (Figure 2.2.15)

Figure 2.2.15 Pylon and cable connections at Millennium Dome (Rogers, 2000)

Design Option: Suspended Cables

Unlike the straight cables in "stayed" systems, suspended cables have easily recognizable curved profiles. The supported surface can either match the sagging supporting cable or, if the surface needs to be flat, it can be held by vertical cables that tie back to the primary support cables. As with cable-stayed systems, the load in the cables depends on the tributary area they support, but it also depends on the span and the sag. Because of the inverse relationship between the sag (height) and the thrust, each form needs to deal with thrust differently.

Traditional footbridges, supported with a cable at the walking surface, want to maintain a small sag for functional reasons, so they have to resist high thrusts on their ends. Alternatively, when cables are hung from taller supporting towers (or pylons), there is more sag and less thrust *but* this thrust creates a high moment force atop the tower (bending it inward) that needs to be counteracted. There aren't many examples of buildings that use suspended cables. It is useful for bridges that span hundreds of feet with no other support, but this is rarely needed for buildings. The added height of the pylons, difficulty of construction, and stability challenges outweigh the benefits. (Figure 2.2.16)

Suspended structures face threats of instability if they are too lightweight. Changes to the loading conditions, particularly wind uplift or concentrated point loads, can also make the structure change shape and become unstable. Suspension cables are very effective in holding a bridge up, but useless in trying to hold it down. The most famous example of how lightweight tension structures can be threatened by instability is the collapse of the Tacoma Narrows Bridge in Washington, in 1940.

Even before the bridge opened, the construction workers had nicknamed the bridge "Galloping Gertie" on account of the wave-like movement of the

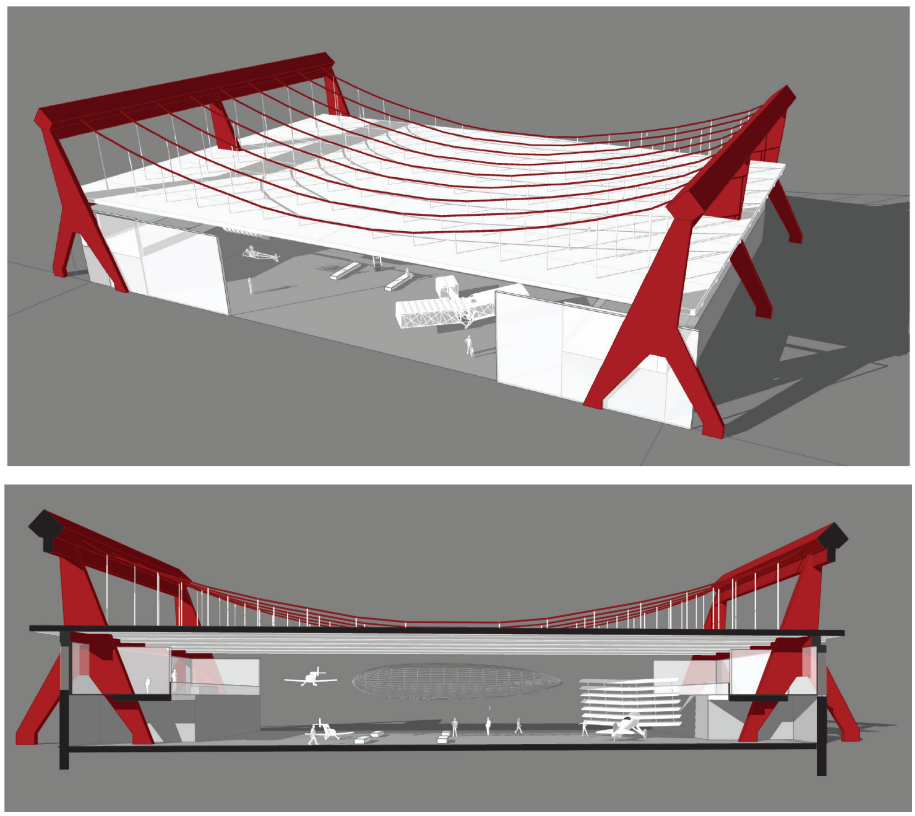


Figure 2.2.16 Scheme 2: Suspended cable option. Large body-like pylons pull the suspended cable outward while vertical hangers support the flat roof. Similar to Burgo Paper Mill (Nervi, 1963)

bridge decking in the wind. The bridge's shallow girders and its suspended support system weren't heavy or stiff enough to resist against the uplift. When the bridge opened, cable ties were added to the bottom of bridge to tie it down and stabilize the movement, but they couldn't stop the vertical oscillations. After only five months the bridge began to sway and became locked into a cycle of resonance which became so great that the bridge surface cracked and the cables broke, destroying the bridge. Although it was a significant loss of resources, from a design perspective it was a very important and instructive failure for future tension structures. Creating the lightest and most efficient structure was no longer advisable unless this structure could also maintain stability against dynamic loading. (Figure 2.2.17)

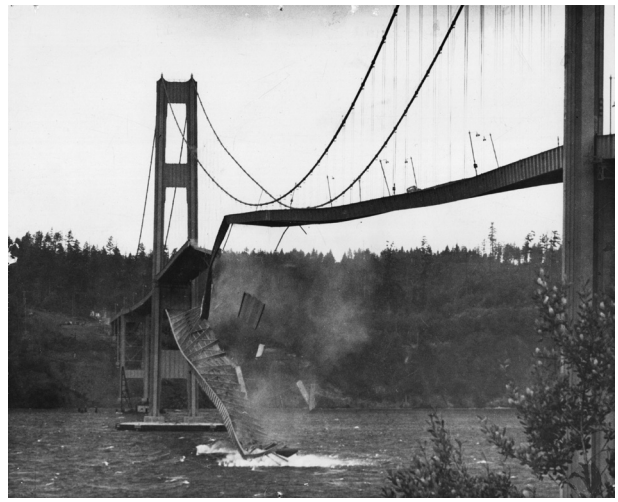


Figure 2.2.17 Tacoma Narrows bridge failure (Tacoma News Tribune, 1940)

FOLLOW THE FAILURES: MILLENNIUM BRIDGE, 1996–2000

Norman Foster & Associates, sculptor Anthony Caro, and Arup Engineering designed London's only pedestrian bridge across the Thames with an ambitious structural and functional goal to have a very shallow suspension bridge. They wanted to build a "uniquely thin profile" for a 1,050ft (320m) long cable bridge to allow pedestrians to have uninterrupted views to the city, so they limited the cable rise to no

Figure 2.2.18 Harmonic resonance dampers were added to the Millennium Bridge shortly after its opening to account for unexpected movement

more than 7.5ft (2.3m) above the deck. There are two large y-shaped concrete and steel armatures, placed one-quarter of the span inward from shore to support the cables. Each side of the large arm-like ends holds up four large bearing cables. At 26.2ft (8m) intervals along this cable, large steel support beams running transverse to the walking surface were secured to the cables. The weight of the bridge surface was designed to sufficiently resist against updrafts, but, during its opening weekend, it suffered a unique performance failure nonetheless.

To Discuss: What was the problem? What caused it? Were the total loads underestimated (or just the type of loads)? What changes were made? (Figure 2.2.18)

Evaluation, Stabilizing Strategies for Cables

Stability in suspension systems must be a central consideration. Sometimes, the steps needed to stiffen and stabilize these structures will either make the structure less "efficient" by adding extra weight or more "expressive" by adding new stabilizing elements to the basic cable form. Once we understand these options, we can critically integrate these load stabilization strategies into the overall design. Frequently these strategies transform simple two-dimensional hanging cable profiles into more complex three-dimensional forms. (Figure 2.2.19)

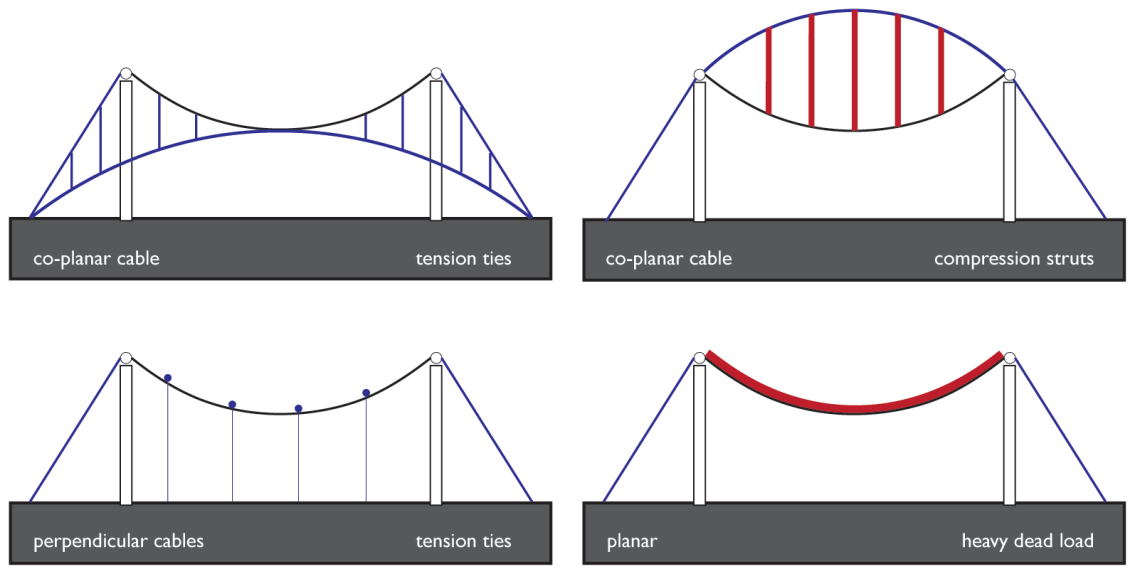


Figure 2.2.19 Various options for combining bearing cables and stabilizing cables together

FORM FOLLOWS FORCES: MODEL EXPERIMENT #2

In our physical models, it is easy to see how strategies used to stabilize the cables also change its form. Modify the same model used in model experiment #1, this time using suspended cables. Develop different strategies for stabilizing these cable supports. Construct each one of the options described below and evaluate the viability of these strategies architecturally and structurally against our proposed building program.

Tie-down or stabilizing cables: Suspend a bearing cable between two stabilized masts on the ends—notice how easily the wire moves with any changing loads. It can be stabilized by tying the cable straight down with perpendicular tie-down cables along its length, but this isn't viable if the cable structure was intended to run uninterrupted across a span. To maintain a clear span, we could add a parallel *stabilizing cable* in the same plane as the bearing cables. The two cables can be connected by ties, tension cables, or compressive struts. (Figure 2.2.20)

We could run a stabilizing cable below and tie them together, or we could add a second cable above the bearing cable and add a series of compression struts along the length of the cable, held in place by pulling the second top cable tight and tying it down to the ground. In both cases, the stabilizing cable is arch-shaped, although its internal forces remain in tension. The support cable can't move up or down because the stabilizing cable holds it in place (although it may rotate unless it is stabilized laterally). When the bearing and stabilizing cables are co-planar, it is stable up-and-down and side-to-side in two axes, but not front-to-back. It requires some type of lateral/diagonal bracing between supports.

If we apply bearing and stabilizing cables to our design proposal, we create a dynamic interior and exterior form. The roof can be arched by following the stabilizing cable on top, which will shed water and allow the interior to maintain its openness and high volume. (Figure 2.2.21)

Membrane: If we set up tall support pylons along the building's perimeter and hang bearing cables across the span, we'll need an enclosure that spans between these cables. If we use a flexible membrane in the model, we can see how stabilizing cables affect the form. If we try to use a lightweight tent/membrane roof, nothing prevents it from flapping up in the wind between our cables. We could pull stabilizing cables perpendicularly across the bearing cables (and membrane) to retain the original sagging cable form. Or we could run stabilization cables *parallel* to the bearing cables, shifting them over in-between the supports to get a wave-like form in the membrane. The membrane is supported by a sagging bearing cable but stabilized by another arching cable at mid-span. When stabilizing cables aren't co-planar with the bearing cables, we get other three-dimensional possibilities, including the possibility of a double-curved cable surface. (Figure 2.2.22)

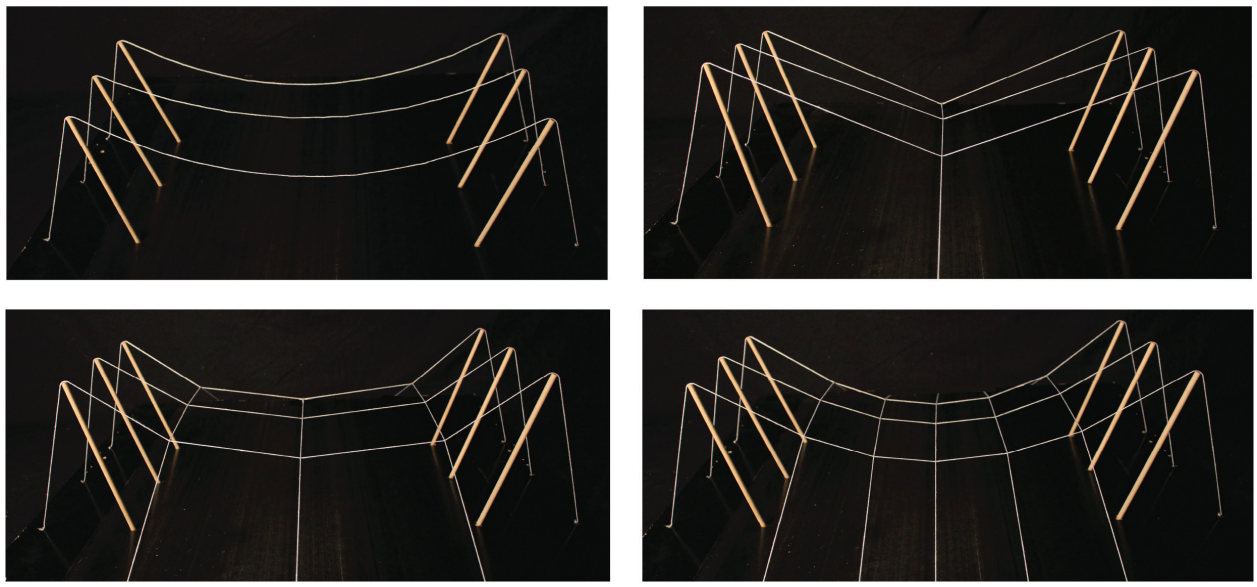


Figure 2.2.20 Model experiments showing tie-down cables and the transition from straight lines to the "double-curved" form that results from increased frequency of bracing cables

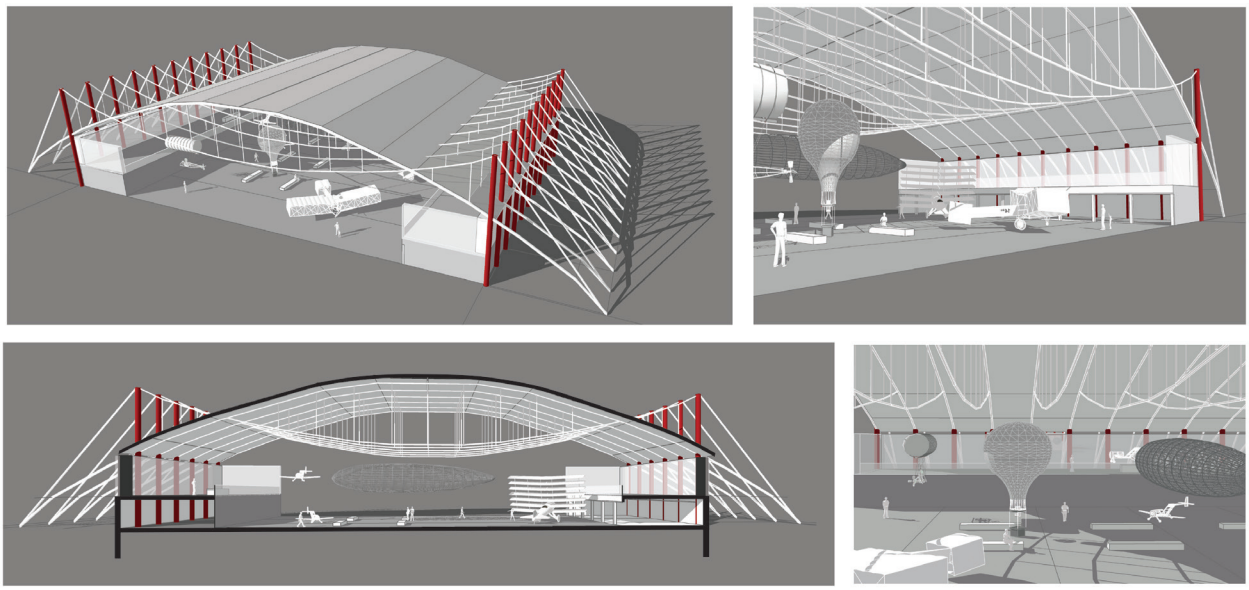


Figure 2.2.21 Scheme 3: Suspended roof with arched stabilizing cables

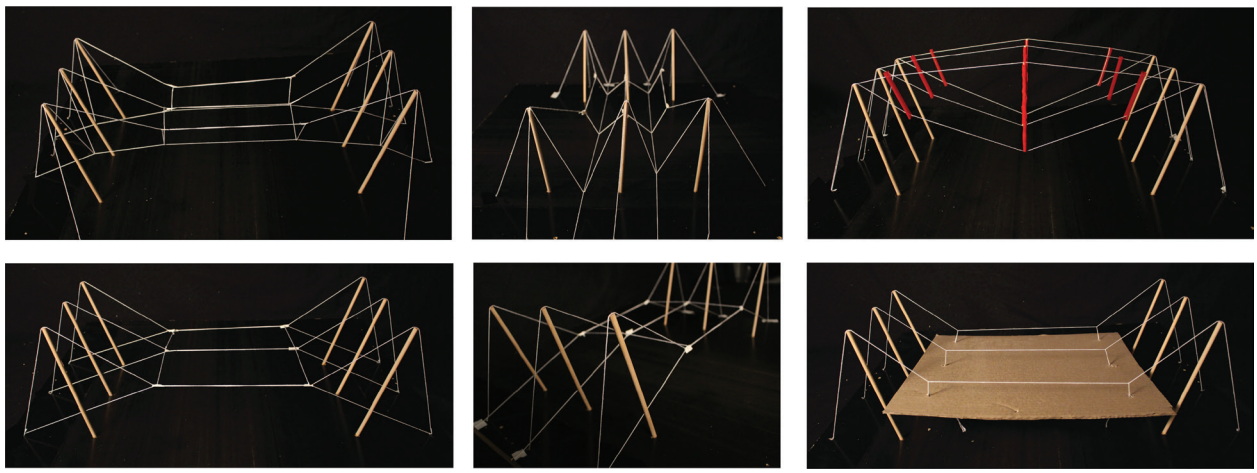


Figure 2.2.22 Model experiments with additional options for stability

Additional Load: The most basic way of resisting uplift and instability is to stiffen the deck and add more dead load, making the bridge (or roof) surface heavier than the uplifting force of wind and stiff enough to resist bending. We'll need to size supporting cables to hold up more load, but their overall form can remain the same. In this approach, the service parts of the museum program are housed in the book ends of the scheme, which also counteract the thrust from the sagging cable-supported roof. The roof is exposed on the interior but the interior volume is minimized. Direct and indirect light can enter the museum on the open sides. (Figure 2.2.23)

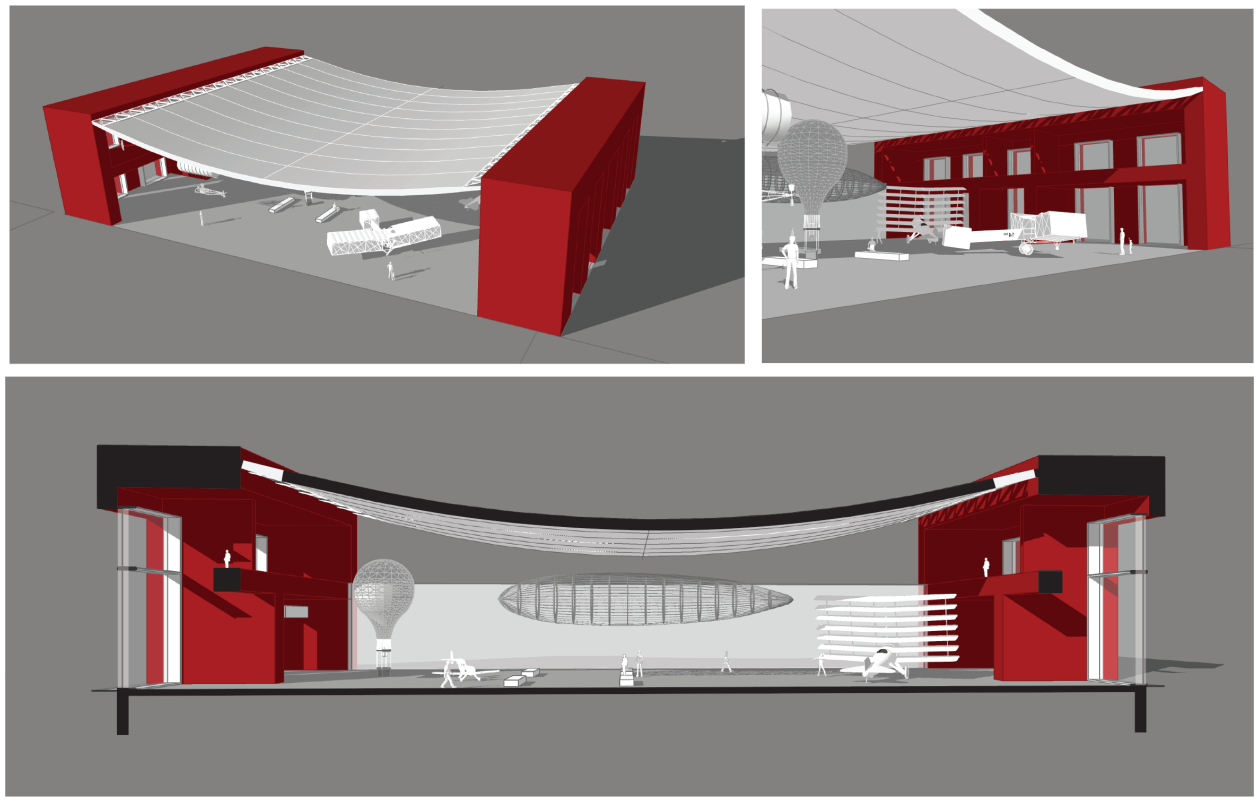


Figure 2.2.23 Scheme 4: Suspended roof with heavy roof

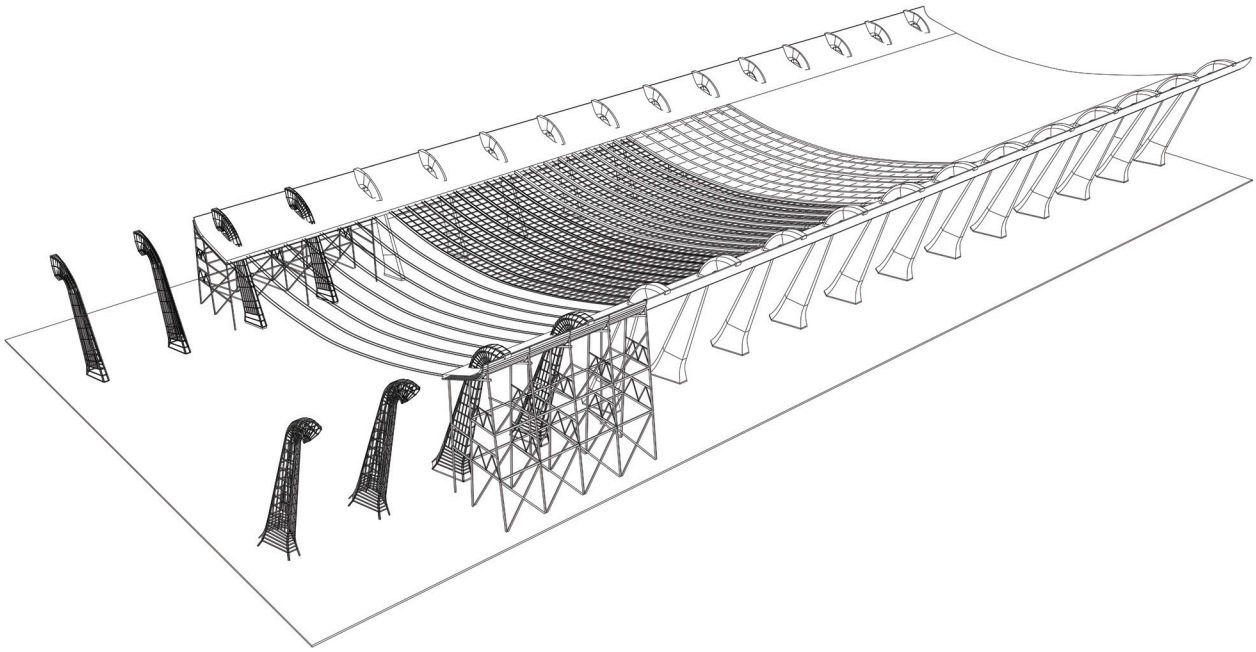
BREAKING: CONFIRMATIONS

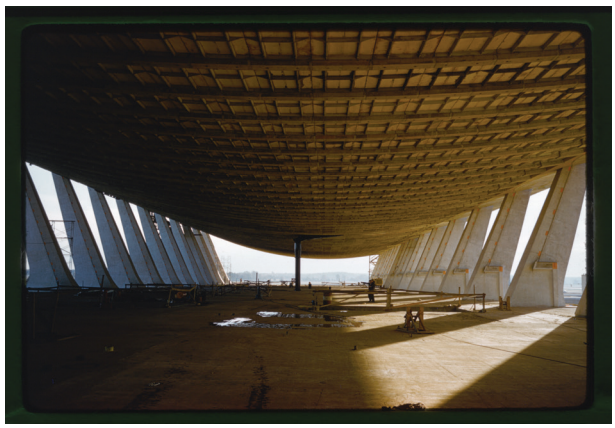
The Expo '98 Portuguese National Pavilion, by Àlvaro Siza and Cecil Balmond, was designed with a large "floating" concrete slab, suspended by supporting cables on the ends. The 230ft (70m) long, 8in (20cm) thick hanging concrete roof sags only 10ft (3m) at the center. Because concrete is heavy it exerts a tremendous amount of thrust to the sides (R_H).

To Do: Try to calculate the amount of stress on one typical cable using: $P = w * l^2 / 8 * h$. Estimate spacing of cable, weight of concrete per cubic foot, etc. Size the cable: $A = P / f_a$ (high-strength steel cable was used with higher f_a).

To Discuss: How was the section of the porticos on the ends designed to help resist thrust? Describe the construction process, particularly the coordination between pre-stressing of cables and concrete casting.

Membrane roofs are lightweight, but we can use a heavier roof material and still maintain the natural form of a hanging cable. For the Dulles Airport Terminal (1958–1963), Eero Saarinen & Associates (with Ammann & Whitney Engineers) envisioned a large open room with a solid parabolic roof that was higher at the entry and lower at the ticket gates. This hanging roof was suspended between the two upwardly curved, cast-in-place concrete slabs that connected to the expressive, outwardly leaning columns along the perimeter. The roof looks solid, like a cast-in-place slab, but it was actually made from 1,800 lightweight precast concrete roof panels that were hung between 1in steel cables, spaced 10ft apart





Figures 2.2.24a and 2.2.24b Dulles Terminal construction sequence. By suspending panels on cable and casting from above, no formwork was needed below the roof (Photo by B. Korab)

that sagged across the 120ft (36.5m) span. After the panels were slid into place, the cables were tightened upwards to the exact funicular geometry of the finished roof. The roof was then weighed down by sand bags until the cables and panels could be encased in lightweight poured concrete. The extra weight made the entire roof system structurally integral and resistant to uplift. (Figure 2.2.24)

MAKING MATTERS: WHAT IS PRE-STRESSING? POST-TENSIONING?

When modeling tension structures you may recognize how easy it is to find these forms but how difficult it is to keep equilibrium in the system as you tighten each cable in place. Tightening one cable too much pulls others out of equilibrium. Pulling all the cables up to a single point of support, and then fixing them in place

(continued)

(continued)

with the correct amount of tension is also difficult. Depending on the type of cable/string you are using for your mock-ups, you've discovered how much the cables move when they are loaded.

These issues reveal an inherent complication with constructing tension structures—they don't take their final form until all the construction and loading is completed. Tension structures will sag (or straighten) *and* elongate into new forms when they are loaded, but under construction loads are added incrementally as the structure is completed. How can the geometry of the cable be constructed accurately in anticipation of these changes to the form? The answer is to impose an intentional distortion on the cable's geometry. This process is called *pre-tensioning or pre-stressing*. During this process, hydraulic jacks or turnbuckles pull the cables at the ends tightly in anticipation of the sagging that will occur when the final load is applied.

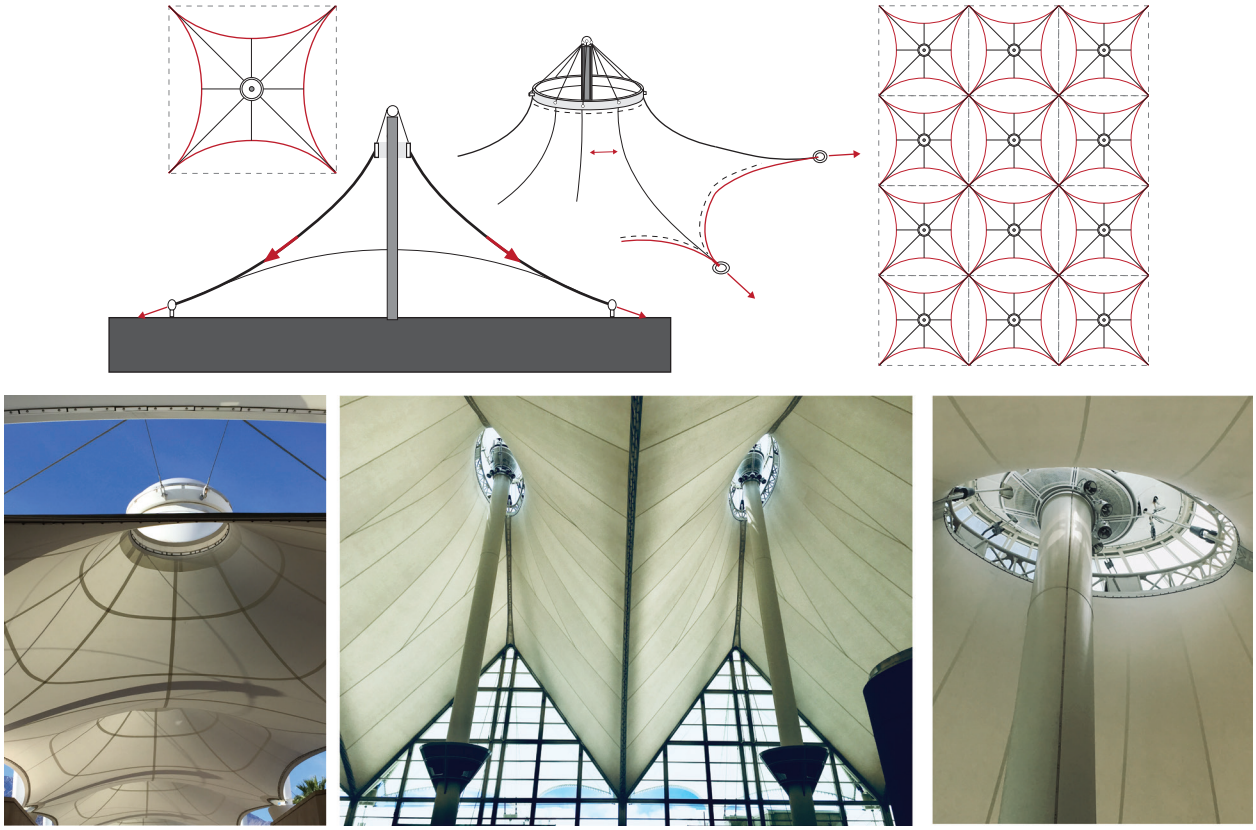
Alternatively, cables can be adjusted after their loads are applied—a process called post-tensioning. As anyone that has ever put up a tent can attest, this post-tensioning process requires coordination in how and when the cables are tightened. Pulling cables on one side only distorts the overall form. Ideally post-tensioning is a coordinated process where cables on opposite ends are pulled at the same time. In large tension structures, like tension wire and strut tensegrity-like structure), post-tensioning is usually a step-by-step process; how the cables are secured at the ends needs to accommodate this in-field adjustment process. (Figure 2.2.25)

Figure 2.2.25 Pre-stressed cable-stayed bridge: Queensferry Crossing Bridge (South Queensferry, Scotland, 2017)

Design Option: Masts, Arches, and Membranes

Tension structures rely upon load-bearing compressive members (e.g., column, pylons, or walls) to transfer their loads to the ground. These elements should contribute to the spatial experience and overall building form. Some types of supports are straight single elements, like masts, but the compressive members can also be a spanning structure, like an arch.

When masts (or pylons) are integrated into a tent-like membrane structure, there are a few fundamental rules to remember: First, the placement of the masts, their angle, and their height dictate the "pattern" of the membrane that spans between them, like a dress pattern, so these elements should be



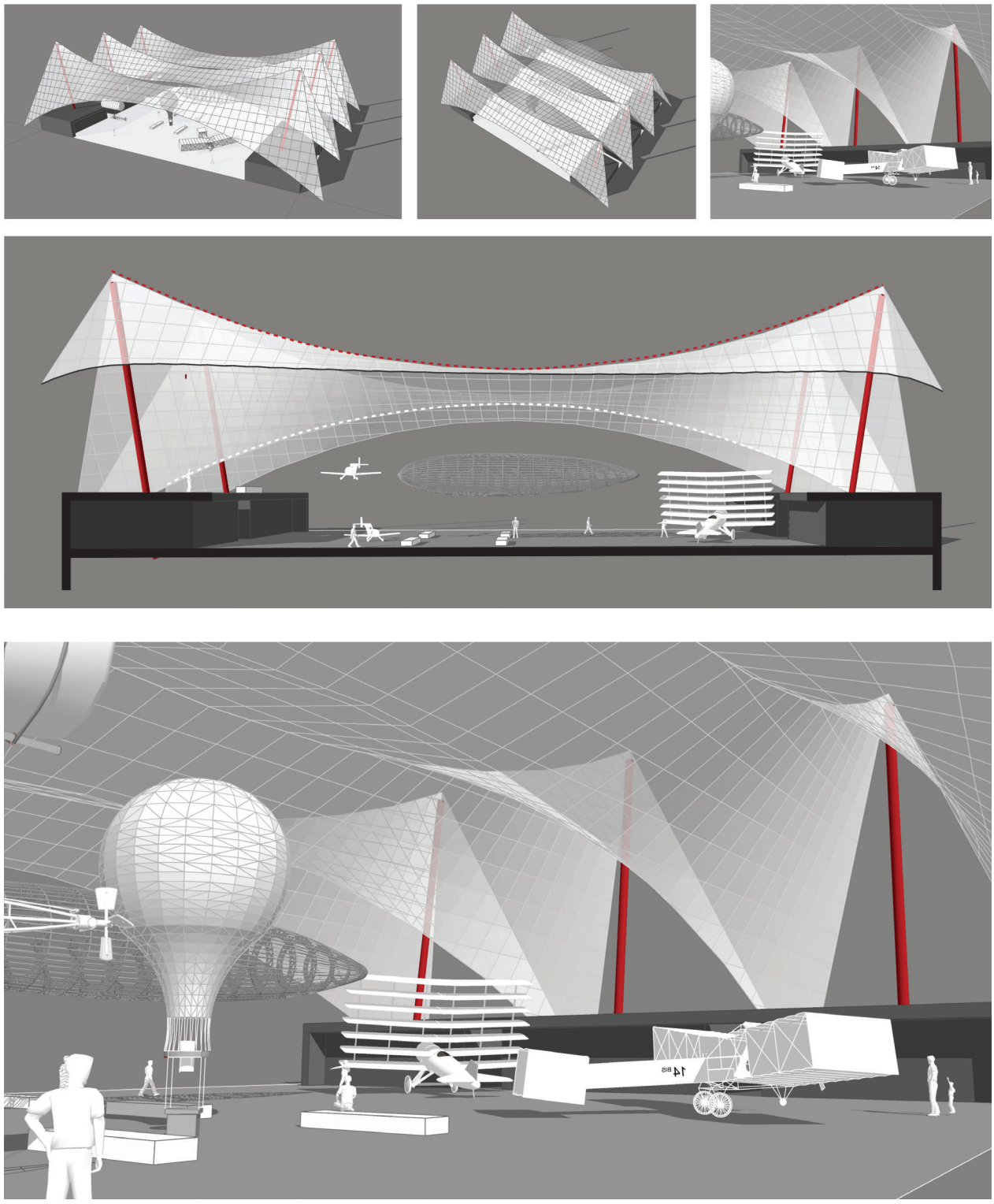
Figures 2.2.26a and 2.2.26b Diagram of basic membrane principles and geometry in plan and section. Shown: Palm Springs International Airport and Denver International Airport

carefully coordinated. Second, the membrane can't rest directly on the sharp top point of a mast, so some sort of tension ring collar should be provided to spread out the load and avoid "punching shear." Third, to keep a membrane surface in tension with the mast, there need to be cables around the bottom perimeter of the membrane in arched shapes to tie the surface down. (Figure 2.2.26)

We can create a mast and membrane scheme for our design proposal, establishing a number of bays and placing masts at their ends with sagging bearing cables running between. From our physical models, we know that we can add a stabilizing cable in between the bearing cables and connect these with a membrane. The interior volume is less than ideal—we have higher areas on the edges and the lowest volume in the middle. Further, hanging large exhibits from roof cables isn't ideal because of their lack of stiffness and the risk of movement. However, the membrane could allow light into the space. (Figure 2.2.27)

Arches provide a good alternative, since they avoid the concentrated stresses of connecting membrane surfaces at a mast. They can define the spatial volume and expression of the building (e.g., arches can be on the perimeter, or underneath, or both), while providing a continuous surface to secure the cables. Introducing an arch into the design, and combining it with cables, creates a logical and expressive language. Because the arches and the cables are both spanning structures, a larger area can be enclosed with fewer supports. By combining form-active cables and form-resistant arches into one structure, we can span incredibly long distances.

The resulting volume can enhance the program. We can create a higher space in the middle with two outward sloping high arches (held together through a tensioned surface), two smaller arches on the perimeter above the core service/mezzanine areas, and entries in the front and back where the arches narrow. The interior space could vary based on the arches' height, as long as they follow a proper funicular shape. (Figures 2.2.28 and 2.2.29)



Figures 2.2.27a and 2.2.27b Scheme 5: Mast and membrane option

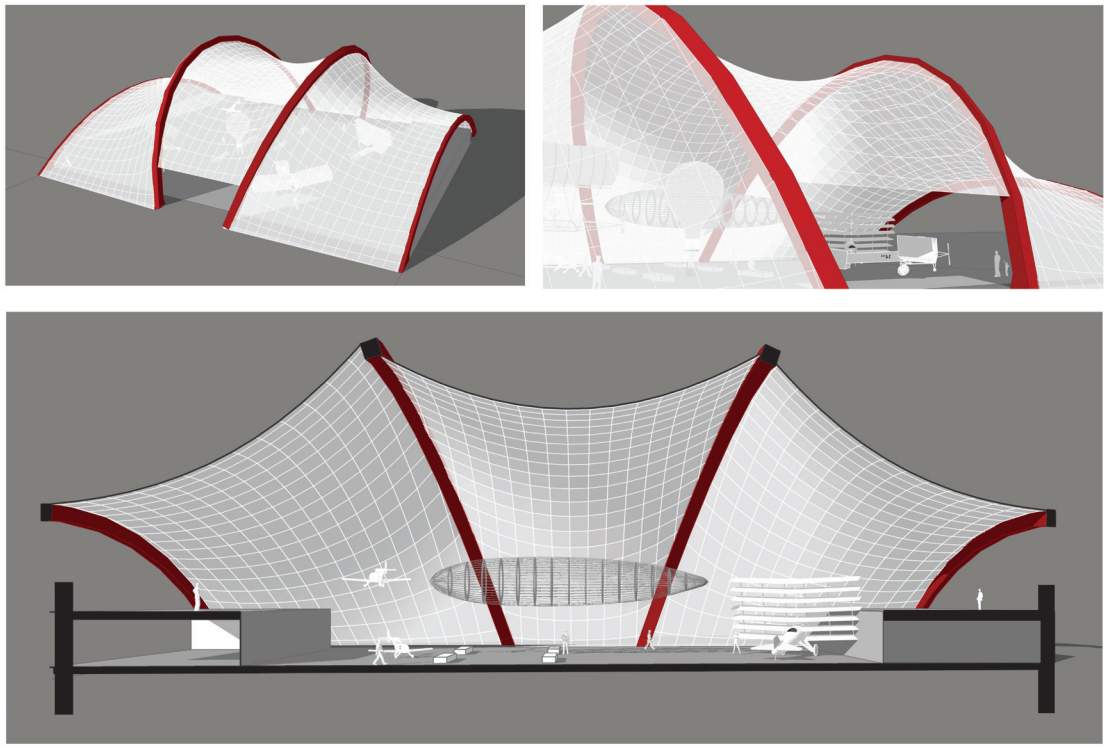


Figure 2.2.28 Scheme 6: Arch and membrane option

Figure 2.2.29 Double-curved cable mesh and slanted concrete arches at the Livestock Arena (Nowicki, Raleigh, 1952)

BREAKING: TOOLS OF THE TRADE, TESTING MODELS, AND TENSION STRUCTURES

When Kenzo Tange and his engineer Yoshikatsu Tsuboi proposed suspension cable structures for the two Yoyogi National Stadiums for the Tokyo 1964 Olympic Games, they did so because tension structures could produce functional, organic, and dynamic long span spaces in an innovative, structurally efficient, and expressive manner. But after major schedule delays in acquiring the properties, only *one month* remained for the project team to design *both* stadia. They assembled a collaborative project team, which agreed that models would be the most effective way to discuss and develop the designs. In the ensuing few weeks, they created more than ten models to test and review options.

Yoyogi Stadium #1 was similar to a suspension bridge, with large towers supporting four main bearing cables (spanning 126m) and bearing and stabilizing cables that formed a curved tension roof. Use of 1/30th scale models helped predict the roof members' deflection and the wind's effects. The smaller Stadium 2 was an unconventional snail-like design featuring a single off-set concrete wall column with a spiraling set of supports twisting out below from a central point.

One of the difficulties was the high flexibility of the roof (Stadium #1 sagged 2m in the center of the span) and the limitations for analyzing this work (this was pre-computer analysis). According to project engineer Mamoru Kawaguchi, "we had no reliable basis on which to calculate our roof structure." As a result, the initial form-finding models were rebuilt and tested to confirm the anticipated behavior. Because the roof needed to be stiff enough to resist bending stresses and wind-induced flutter, one breakthrough idea came from model testing. The roofs were redesigned as a stiffer "semi-rigid" laminar mesh, with the curved hanging steel members secured into the main cable with a custom connection joint called the "Saturn joint." This joint had a satellite ring that could freely rotate around the main cable support—absorbing deformations and making construction easier. (Figure 2.2.30)

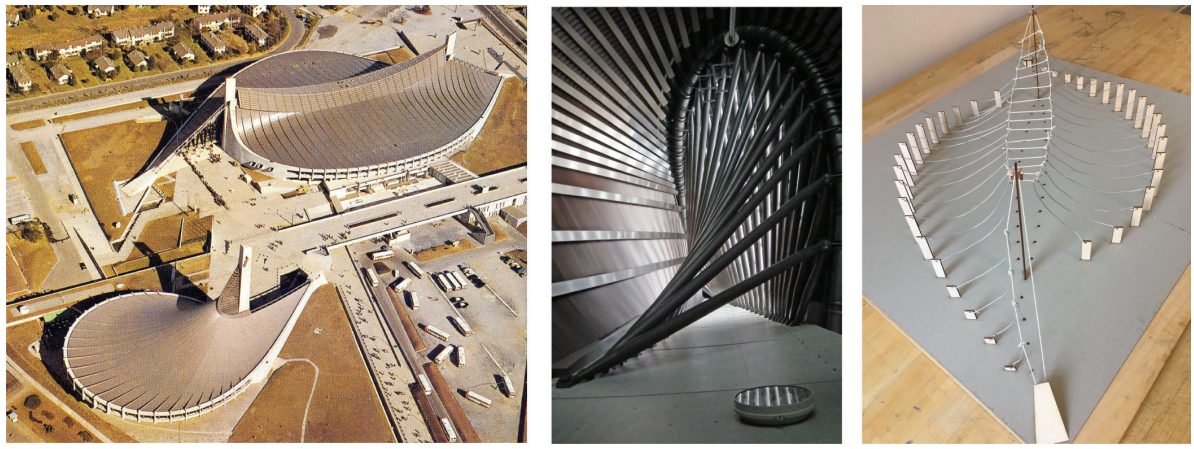


Figure 2.2.30 Aerial view of Yoyogi Stadiums, an interior view of the cable connections, and a student model demonstrating the structural geometry

Design Option: Tensegrity

To avoid large tall masts within the building or arches spanning across the space, we could use a cable system based on a lightweight system of tiered cables and compression struts. Such a *tensegrity system* is the lightest possible suspended cable roof option. It is based on structural principles developed by Buckminster Fuller and artist Kenneth Snelson beginning in 1948. Tensegrity combines bearing cables and stabilization

cables with compressive struts to make inventive three-dimensional forms. Typically, a net of continuous tension cables holds isolated compressive components apart from one another. These cables delineate the outer form of the structure.

If our building plan was circular, we could use the "Wire Wheel Dome" scheme invented by David Geiger in 1986. This is essentially a gigantic bicycle wheel, in which a group of bearing cables and stabilizing cables are run from a compression ring exterior to an internal tension ring, compressing radially-arranged vertical struts. Alternatively, we could create more of a domed roof using these same tensegrity principles, similar to the Georgia Dome (1992), in which compressive struts were hung between a series of tiered cables. Unlike the bicycle wheel, the Georgia Dome's radial arrangement of struts stepped up higher as the roof converged to the middle—the cable at the top of the outer strut became the bearing cable for the inward ring of struts and so on until the highest center tension ring "halo" was reached. This arrangement takes a great deal of coordination to construct, since all sides need to be tightened in-sync with each other, and the roof membrane needs to be custom fit to the tiered form. Tensegrity options are very spatial and well equipped to not only span, but also stabilize the roof. (Figures 2.2.31 and 2.2.32)

Design Option: Pneumatics

Instead of creating a tensed membrane by hanging it down and pulling it tight, we could hold it down and inflate it with air until it becomes tensed. *Pneumatic* structures rely on pressurized air to lift up an uninflated membrane to create an enclosed volume. These are by far the lightest types of structures possible because the membrane and tension cables are the only dead load weight the structure has to carry.

Like the first aerostat balloons, pneumatic membranes are stressed by air pressure, and wires that constrain the membrane are the main load-resisting elements. The building's form is defined by the pattern of the membrane and tension wires across the surface. The closer the cables are spaced, the less stress the membrane needs to resist—the cables are acting as de-facto "beams" that collect the loads from the membranes. In a confusing twist of geometry and stresses, the membrane and cables are both "arched" yet both are resisting tension, not compression.

Pneumatic Evaluation

There are two kinds of pneumatic structures, the air-inflated and the air-entrained. Both types require constant pressure to maintain their form, but they are both subjected to shifting forms from lateral wind forces or collected live loads (like snow) that are heavier than the uplift provided by the air. Because they require constant pressure, their entrances and exits must be pressurized, which may reduce their suitability for some building functions. They are easy to inflate and easy to deflate as needed which is useful for temporary structures. The risk of catastrophic

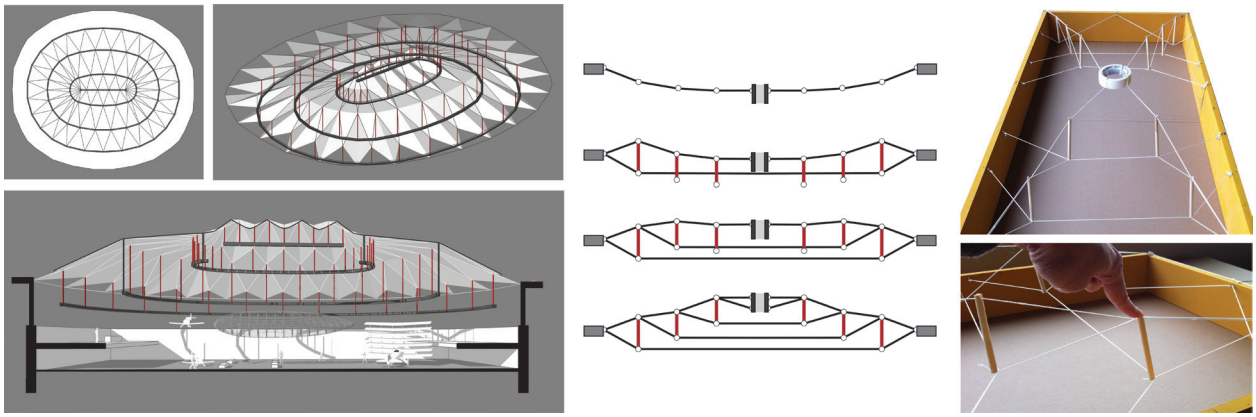


Figure 2.2.31 Scheme 7: Tensegrity design option: (Left to right, top/down) roof plan, worm's eye axonometric, building section, construction sequence staging, and demonstration model

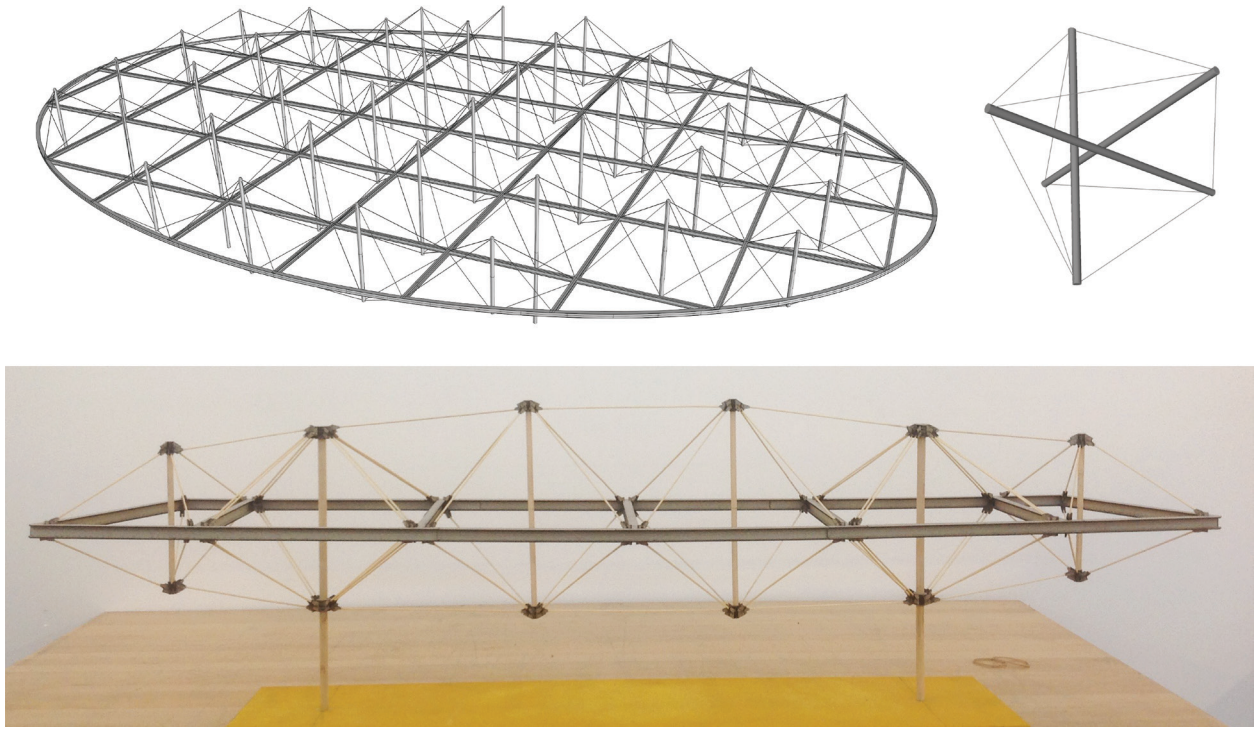


Figure 2.2.32 Student drawings and model of the Blur Building reveal the difficult nature of coordinating construction sequence and force equilibrium

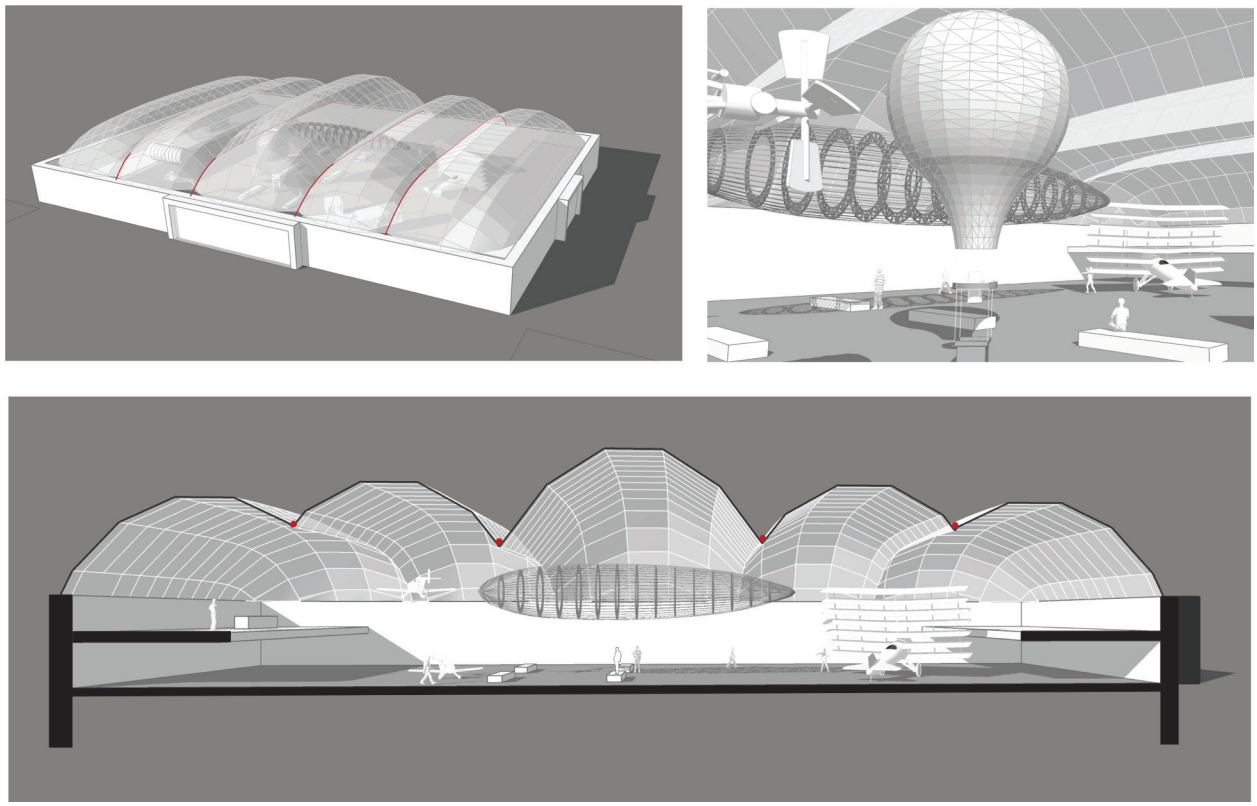


Figure 2.2.33 Scheme 8: Pneumatic design option. The cables help define the form, collect loads, and tie the forces back to supports

collapse is lessoned by the pressurized air—it may simply leak and slowly deflate.

There are pros and cons for air-inflated options. There needs to be a coordinated effort between the anticipated volume and the "pattern" of membrane. Bearing and stability cables are strung across the space to create a higher volume in the middle. The membrane can let light filter into the spaces from above. However, exhibits can't be hung off the membrane, and it won't allow any significant opening to the exterior because we have to keep it pressurized. (Figure 2.2.33)

FORM FOLLOWS FORCES: MODEL EXPERIMENT #3

Build a full-scale proposal for a pneumatic enclosure using lightweight plastic sheeting large enough to enclose four to six people. Devise a plan for providing air to inflate and retain this form. Note how difficult it is to splice the membrane together into a pattern without creating a leak (tape won't work, heat-welding will). To reduce membrane stress use twine or cables—you'll need to devise a way to keep those tied down to the ground. Large-scale proposals reveal a great deal of information about form-finding, construction, and limitations in use. (Figure 2.2.34)

Figure 2.2.34 Student-built experiments for pneumatic structures, both inflated and air-entrained (SxD, Iowa State University)

Design Outcomes: Suspended Structures

Ultimately, after evaluating these schemes a few revelations about tension structures seem clear:

- Their relationship between the building form and structural behavior is integral.
- They are lightweight and materially efficient, enclosing large volumes with minimal materials.
- Load-bearing tension elements are either straight-line (cable-stayed), hanging (suspended), or tensioned along an inflated surface.
- They rely upon compressive support structures to support their loads (e.g., masts, walls, arches, etc.).
- Load-bearing and stabilizing cables can be combined in a three-dimensional composition.
- They need a strategy to resolve the inevitable horizontal thrust forces.

Tension structures also come with risks:

- They are thin and lightweight and made of elements that can only resist tension stress, so they are inherently unstable to changing load locations and lateral forces.
- They need to be modified to resist this instability, either with added dead-load, parallel or perpendicularly run stabilizing cables.

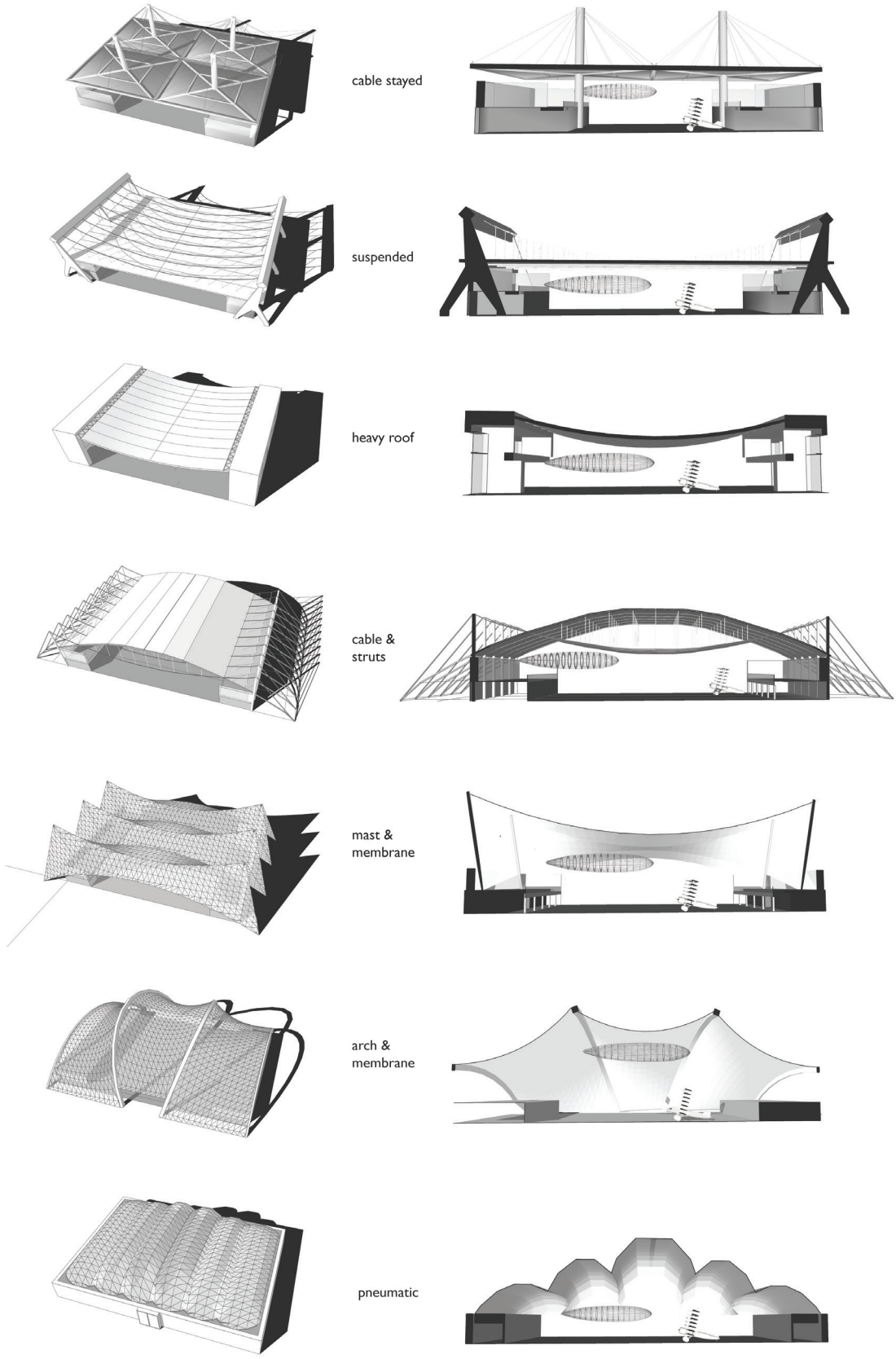


Figure 2.2.35 Summary of schemes. The variety of options reminds us that evaluative standards must go beyond basic compliance

- Membrane systems need to be geometrically matched with a mast, arched, or pneumatic support system which can create complicated three-dimensional forms.
- They can be difficult to design and document as form-finding drawings may be inaccurate representations of curved geometries and prototype models may become complicated.
- Their inherent instability during assembly and their complex geometries make them difficult to construct. They don't reach a state of static equilibrium until they are completed and they usually span long distances, sometimes across difficult terrain.

Despite these various complications, they are a good match for long span roofs. The diversity of tension systems available provides designers with formal options for how a building could be developed and refined. (Figure 2.2.35)

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PART 3

Resistance and Sections

Figure 3.0.0 Chicago Art Institute, (Renzo Piano Building Workshop, 2009)

CHAPTER 3.0

SECTIONS

Beams, Slabs, and Columns

A prism or solid cylinder of glass, steel, wood or other breakable material which is capable of sustaining a very heavy weight when applied longitudinally is . . . easily broken by the transverse application of a weight

(Galileo Galilei, Proposition I, Discoursi, 1638)

When loads are applied perpendicularly (or non-axially) to their span, beams resist bending largely by their cross-sectional mass. How these resistances are generated in section isn't visibly obvious, so it took centuries of theories and experiments to develop reliable ways of creating, testing, and integrating these elements in building designs. Each experiment attempted to gather and analyze the same information to answer the key question: How can we determine if a beam's material and cross-sectional area is sufficiently strong and stiff enough to resist applied loads across a given span? In other words, how can we be sure that beams, slabs, and columns will work?

Intuiting Beam Behavior

The Old English word for tree is "beam" so it is appropriate to begin our study of section-resistant structures, like beams, by first thinking about trees. In doing so, you may find that you may know more about beam behavior than you realize.

Imagine that you've found yourself facing a series of four river crossings. At each crossing there are two wooden, tree-like planks lying side-by-side. These planks each have different characteristics and sizes. For each scenario, you need to pick one of them to walk across. How would you assess their structural viability? (Figure 3.0.1)

Crossing #1: *Both beams are made of the same material and they have the same cross-sectional area, but the river widens quickly and so one beam has to span twice as far as the other.*

Crossing #2: *This crossing has a longer span across than Crossing #1. One beam has a smaller cross-sectional area than you've seen before and the other has four times the cross-sectional area.*

Crossing #3: *Both beams are the same size, but one feels stiff while the other is very flexible.*

Crossing #4: *These planks have the same cross-sectional area, but one is sitting upright vertically (with ample width to walk across) while the other is much wider and not very tall because it lies flat—which has better resistance against bending?*

It wouldn't be surprising if the answers came to you quickly: #1: Shorter span, #2: Bigger cross-section, #3: Stiffer, and #4: Upright orientation. But how can you be *sure* that some of the other answers couldn't

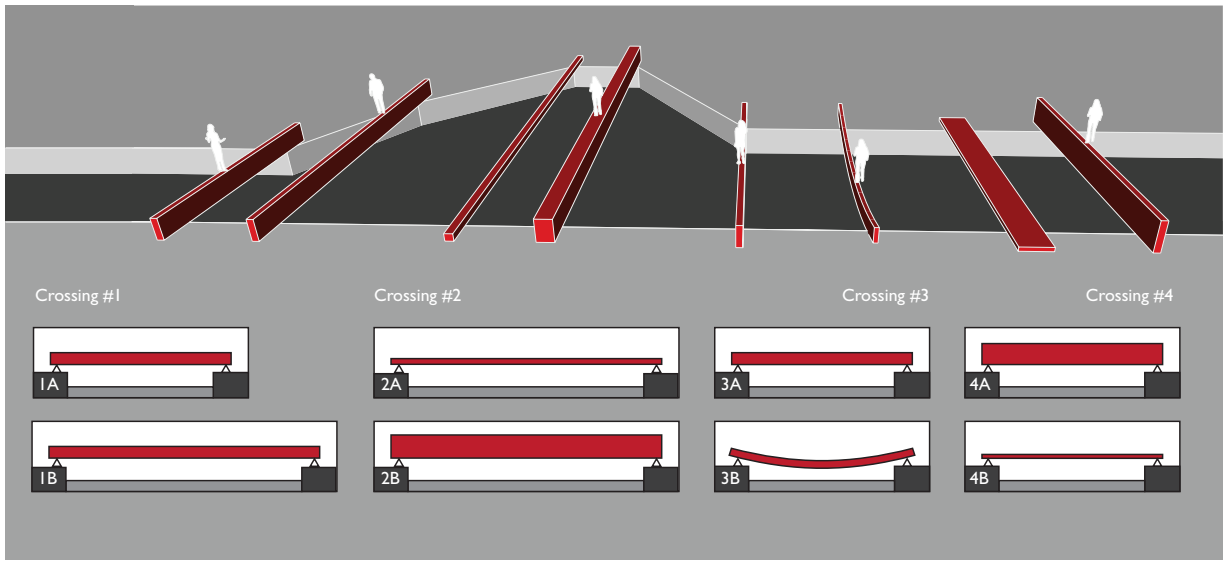


Figure 3.0.1 Options for spanning beam structures

also be correct? Would you be able to explain the logic behind your answers?

If you opted for the conservative answers for each, based on intuition, then you've chosen the same path as many of the builders and designers that have used beams in structures for millennia. Even if you don't know much about the science of beams beyond your intuition, you know about as much as the brightest minds of the 16th century. This chapter will retrace the history of how theorists and designers learned how beams work, what factors need to be considered when assessing them, and how the development of new materials and analytical methods led to the ubiquitous and predictable section-resistant beam systems we have today.

Form versus Sectional Resistance

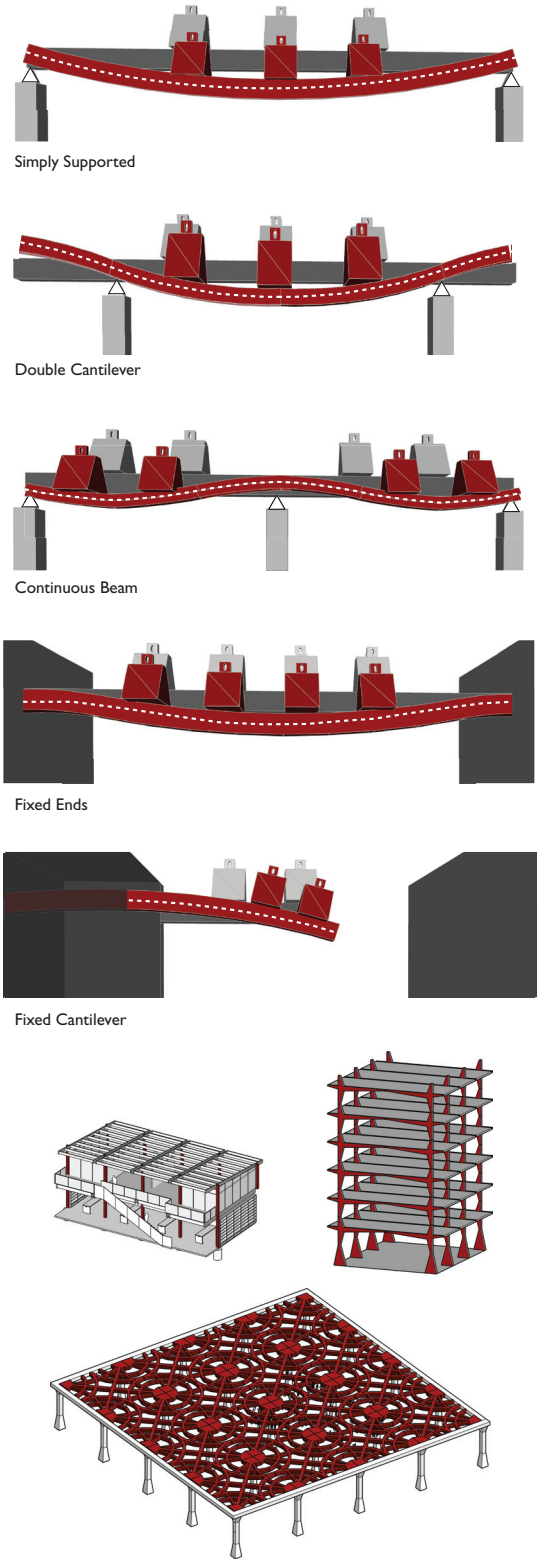
Arches and cables aren't always good architectural or structural solutions: only certain forms are viable, they require height, they generate thrust, they can be unstable, they can't be stacked into multi-story buildings, and they are difficult to construct and thus expensive. Although they aren't as materially efficient as arches or cables, beams don't share these liabilities: linear elements like beams are often functionally preferable, beams don't generate thrust at their supports, *and* they create flat surfaces that can be arrayed and stacked into functional arrangements. (Figure 3.0.2)

The utility of their horizontality has architectural benefits but it also has structural consequences. Beams' primary liability is caused by the different orientations of their spanning and loading. Beams are loaded perpendicularly to their span ("non-axially") so they bend, and bending is difficult to resist. When a structure bends, *all* of the five states of stresses (compression, tension, bending, shear, and torsion) occur simultaneously. Recall the stresses you felt in the body-structure plank pose where your torso spanned between your elbows and feet (Chapter 1.0). Because of bending, your back was compressed, your abs were tensed, your arms were fatigued from shear stress, and you had to fight to keep your body from twisting in torsion. Of these stresses, shear and bending are the two main causes of failures. (Figure 3.0.3)

Overall, beams work if they are able to generate an internal resistance to these bending stresses by their material and cross-sectional qualities. This is why they are called section-resistant.

Beam Basics: Hidden Resistance to Bending

Unlike arches that transmit axial loads somewhat vertically through their form, beams transmit loads horizontally, in defiance of gravity's natural downward orientation. *Gravity always wins*, so beams will deform



Figures 3.0.2a and 3.0.2b Options for beam support include: Simply supported, double-cantilevered, continuous, fixed ends, and single cantilever. Beams, columns, and slabs can be combined and expressed in a variety of ways

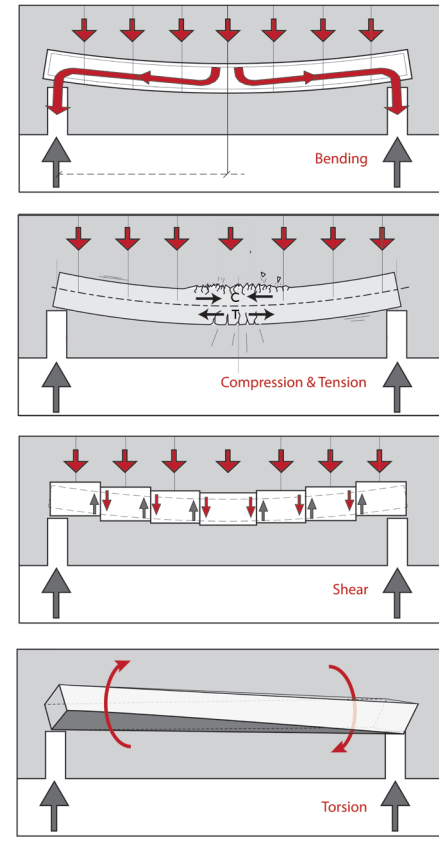


Figure 3.0.3 States of stresses in beams: Bending, compression, tension, shear, and torsion

under this loading, no matter the material. There are two primary stresses that beams need to resist through their cross-sectional area and their material qualities: Bending and shear. If the section stays stiff enough and has enough mass, the beam will resist bending enough to transmit the loads to the supports. It's hard to learn how a beam generates resistance to bending and shear simply by observing it, because beams internalize their resistance.

What goes on inside the beam is a largely hidden back-and-forth struggle to resist the pushing and pulling. Under loading, the material at the top of a beam is pushed together, in compression, while the bottom of the beam is pulled apart, in tension. The amount of compression and tension varies across the span. This behavior, and a beam's resistance to bending is best visualized as internal arches and cables that are laminated together—the arches push the compression stresses down to the supports while the cables also pull the tension stresses up. Their thrusts cancel each other out at the ends. (Figure 3.0.4)

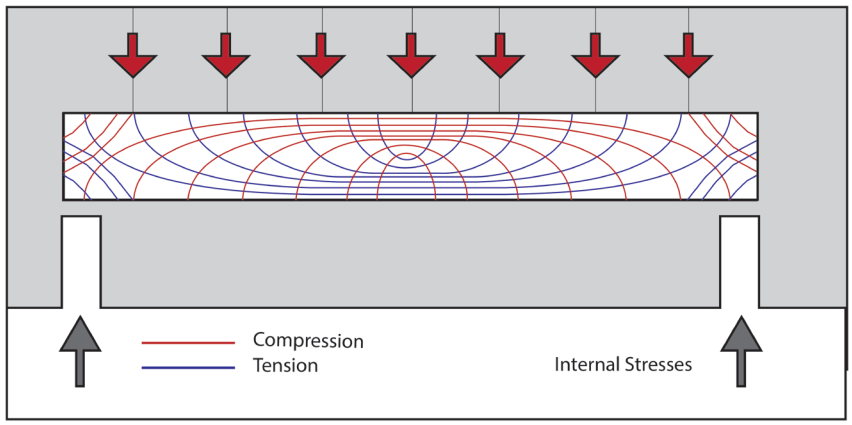


Figure 3.0.4 Visualizing the internal compression "arches" and tension "cables" stresses contained within the bounds of a supported beam. The intensity of stress is reflected in the closeness of line spacing

Early Use of Beams

Consider how vexing the behavior of beams must have been to pre-scientific era thinkers and builders by considering these questions:

- Why is stone "strong" as an arch but not "strong" as a beam? What's the difference?
- Why would certain beams fail near the supports while other would break in the middle? Which is more likely under which particular conditions?
- Why does changing the orientation of the same sized-beam (from horizontal to vertical) make it more or less resistant to bending?
- Why do columns buckle outward in unpredictable and seemingly random directions when they are loaded directly from above?

These questions were difficult to answer with scientific accuracy for much of our collective constructed history, so the use of linear and planar structural elements like columns and beams was limited to

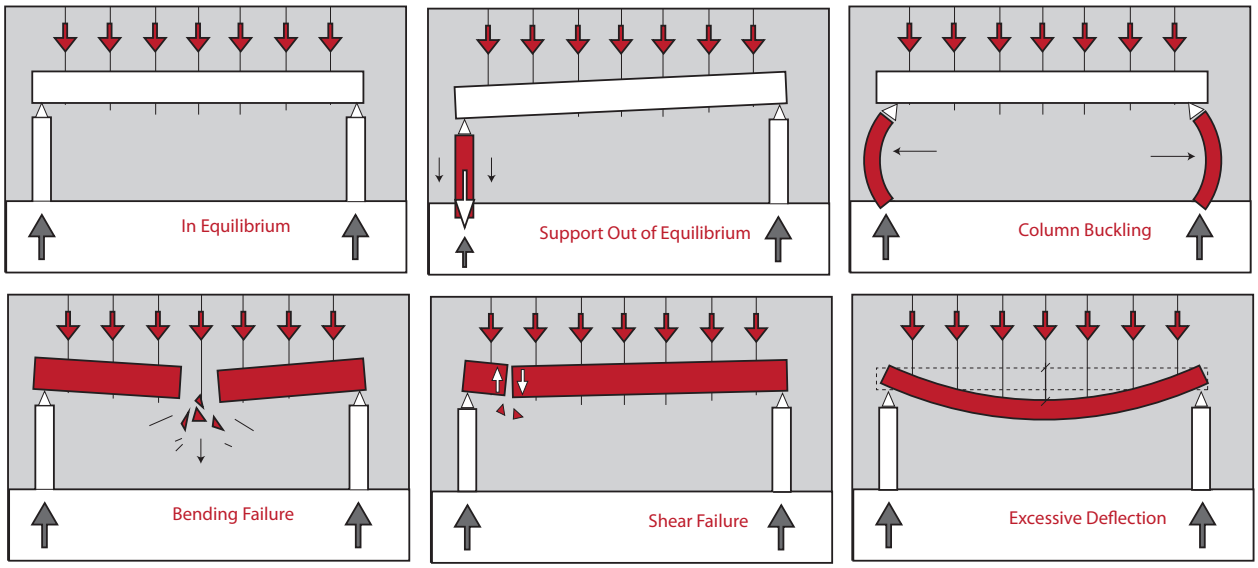


Figure 3.0.5 Understanding potential failures for beams and columns identifies areas to be designed and evaluated for compliance

smaller-scale buildings designed with "rule of thumb" guidelines. The real threat of structural failure came from the unpredictable nature of their behavior under loading. Beams could break under bending or shear, or deflect excessively, or fail if the supports weren't strong enough or stabilized to resist buckling. Decoding these threats was difficult. (Figure 3.0.5)

To off-set these uncertainties, builders opted for large and redundant cross-sections for spanning elements, using massive beams for short spans and closely-spaced, bulky columns. While it is conceptually correct to match bending resistance with a larger cross-section, this approach was also counter-productive. Massive materials like stone are often brittle and don't respond well to shear or bending stress. Simply making the beam larger adds more dead-load mass for the beam to resist, leading to the "weakness of giants" by which an element eventually fails under its own weight as it gets larger. (Figure 3.0.6)

Early beams weren't designed using any particular tool or scientific expertise about mechanics or materials, just trial and error. Although this was adequate for small or moderately-sized buildings, it limited the use of beams in larger and longer structures—an unhappy compromise for advancing cultures. To become viable structural elements, beams needed to be made smarter, not just bigger.

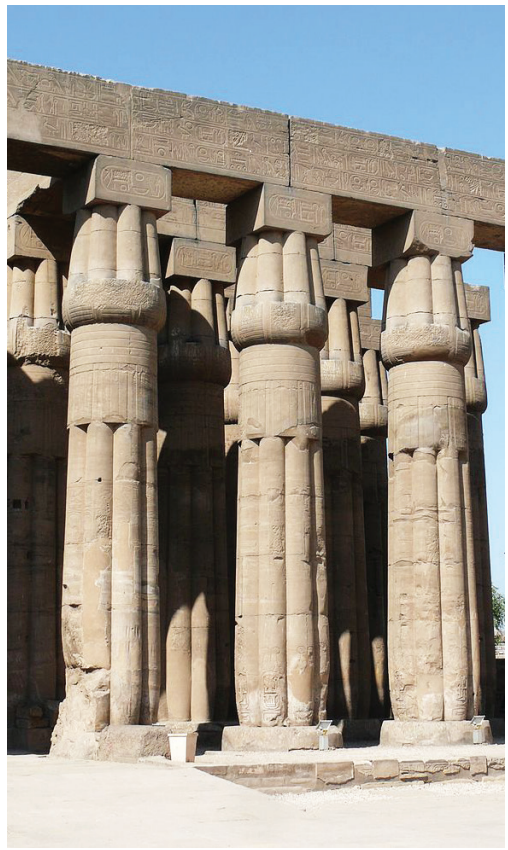


Figure 3.0.6 Court of Amonhotep III at the Luxor Temple with papyrus-inspired columns (52ft (16m) high), closely spaced together with 5ft (1.6m) deep architrave blocks spanning only approximately 10ft clear (3m)

Importance of Equilibrium

The most basic concepts of structural equilibrium were understood by even the earliest theorists and builders. Although they didn't have the means to communicate this understanding with vector arrows, they knew that when an object was adequately supported, it wouldn't move up/down, side/side, or rotate. Finding equilibrium in external conditions was sometimes easy to intuit (e.g., two people holding a heavy beam at each end have to provide half the weight in resistance), but finding these values in more complicated loading and support conditions (e.g., two people holding an asymmetrically loaded beam at different points along the length) required applied mathematics and an understanding of forces. Archimedes first developed an equation for moment forces on a lever (moment = force × distance) that was used to determine how much force would be applied to the supports—a process we still use to this day.

THINKING AND MAKING: UNDERSTANDING EQUILIBRIUM

Our goal is to find the balance between loads and supports using physical models. We can learn to calculate the external equilibrium between the loads and supports in beams by creating a demonstration model of a teeter-totter using a ruler, a pencil, and pennies. *Try it.*

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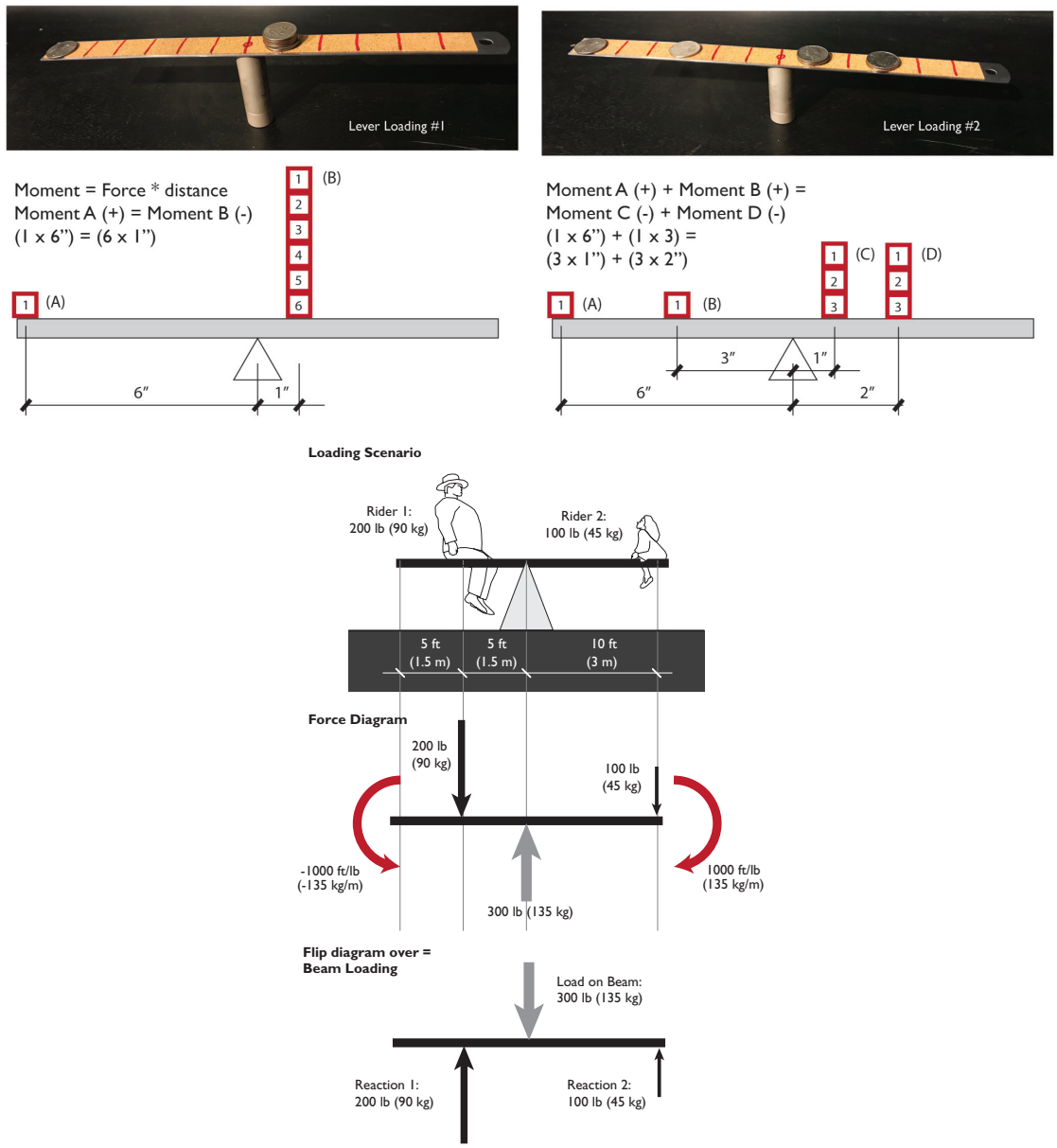
Center the ruler atop the pencil (as a fulcrum) and start stacking pennies on the two ends. As long as the weights of the pennies are balanced, and the distance from the pennies to the fulcrum is the same, the ruler will remain horizontal. Now move one set of pennies inward towards the supports and try to re-balance the beam by adding more pennies to the opposite side. Before you try different scenarios (moving loads, off-centering the fulcrum, etc.), draw it as a load diagram (loads and reactions) and predict the location and number of pennies mathematically.

Once the diagram is in equilibrium, flip it over. The loads become the reactions and the reaction is the load. You've created a loading/support diagram for a beam than can accurately predict how much resistance the supports will need to provide based on the loading conditions. (Figure 3.0.7)



Figures 3.0.7a and 3.0.7b Model experiments and diagrams describing rotational equilibrium

Theory and Experiments of Axial and Non-Axial Stresses

Even with a great deal of focused effort and experimentation, two of the world's most renowned thinkers of the 16th and 17th centuries, Leonardo da Vinci and Galileo Galilei struggled separately to find methods for explaining and determining the mathematical relationships that defined the behavior of beams (and their materials) under loading. Their efforts were rooted in the theories of material behavior and mathematics first developed by Archimedes and Aristotle. As pioneers in the Age of Reason, they developed a scientific method of hypothesizing, testing, observing, and evaluating their work. Using physical models, the behavior of these structures under loading was easy to observe—a rope with weights added would lengthen, a hanging rope with a weight applied in the middle would change form—and because of the direct relationship between form, materials, and forces these experiments led to insightful discoveries about mass, movement, and material behavior.

Through their work, they learned that materials that are bent behave differently than if they are just pushed or pulled. This created a body of scientifically-based information and analytical methods (e.g., testing of physical models, graphic force vector diagrams, and calculations) that could be applied to beams. They sought to explain beam behavior mathematically to be able to predict occurrences in the physical world. Although some of their conclusions needed refinement, several critical discoveries about beam behavior were advanced as a result of their work.

MAKING: REMEDIAL MODELS

One of the reasons beam theory and bending wasn't as quickly understood scientifically as cables and arches is related to the difficulty of translating physical experiments to reliable mathematical models (loads, stresses, and deflections aren't linear proportionally). However, if you are new to structures and are curious

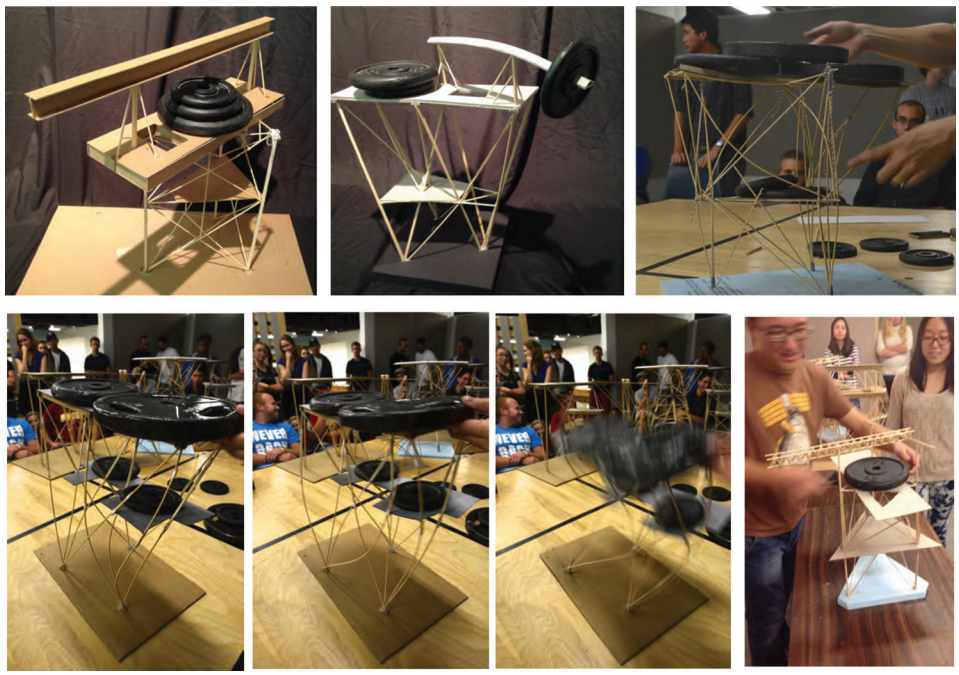


Figure 3.0.8 Model testing of a multi-story tower with irregular framing constraints. Failures reveal behavior (SxD, Iowa State University)

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about how beams, columns, slabs, and other section-resistant systems behave under loading, physical models and testing are quite useful.

To Do: Begin with basics: build a simply supported beam out of a relatively pliant material (e.g., chipboard); change the weights, the spans, the cross-sectional shape and depth. Look for patterns in how each factor affects the performance of the beam. Eventually include columns in your construction—again keep them small and lithe to see their behavior more clearly. An over-built structure won't break very easily and therefore won't reveal many lessons about how it works. (Figure 3.0.8)

Initial Explanations and Theories of Beam Behavior: Da Vinci and Galileo

Da Vinci demonstrated that beams under loading bent into an inverted arch shape, compressing together the top of the beam while pulling apart its bottom fibers. He noticed that bending varied depending on where the beam was loaded, how much it was loaded, and how far it spanned. He concluded that this "bending moment," (M) could be measured, anticipating how much internal resistance a beam would have to generate to resist bending. As DaVinci hypothesized, bending moment will vary based on three factors: *Span, loading, and support locations*. (Figure 3.0.9)

Da Vinci recognized the importance of sectional area in resisting this bending moment by showing that a bigger or deeper beam would deflect less over the same span, with the same loading, than a shallower one. He incorrectly assumed linear relationships between load and deflection and between depth and resistance—both relationships are actually exponential. But his observations established a foundational level of scientific understanding about beam behavior.

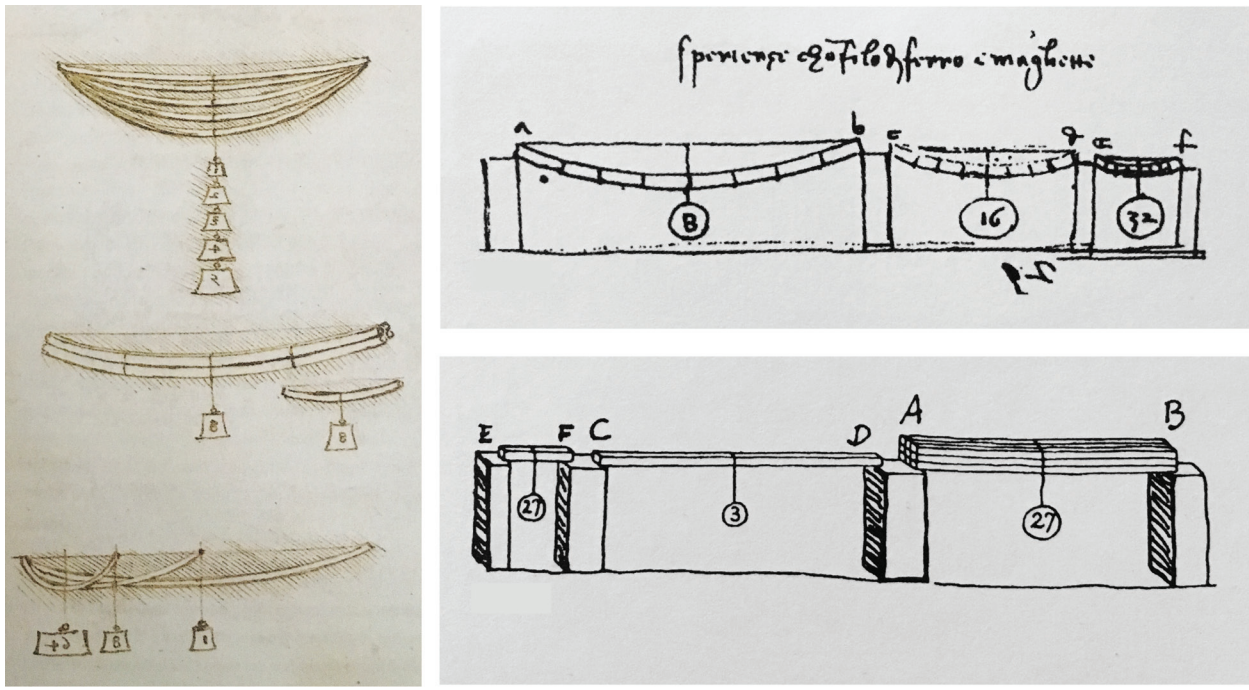


Figure 3.0.9 Da Vinci sketches describe beam/bending moment

Check Da Vinci's conclusions against your answers to the initial exercises at the beginning of this chapter—did you also consider load, span, and support factors in evaluating the viability of different options?

THINK: CONCEPTS > CALCULATIONS

This exercise demonstrates the relationship between bending and deformation—it starts with the assumption that the top of beams will compress under loading while the bottom edges pull apart because of tension. In order to visualize this, and to record the relationship between the amount of weight and distance between supports, we can use a physical model.

Find a somewhat flexible material, like a large rectangular sponge (or narrow foam cushion), and draw a series of equally spaced vertical lines across the face. Set this "beam" on two supports on the ends and take a photograph of it unloaded. Next, press down on the middle of the sponge, bending it like a beam and take another photograph (from the exact same location). Repeat as needed. Change the locations, amount, and number of loads you apply each time. Make a digital composite of these images and you'll find that the vertical lines of the unloaded beam change orientation under loading. The distance between the lines at the top of the beam narrow (indicating the material has compressed) and has lengthened at the bottom (tension)—accurate measurements can indicate the magnitude of bending stress at each location. (Figure 3.0.10)

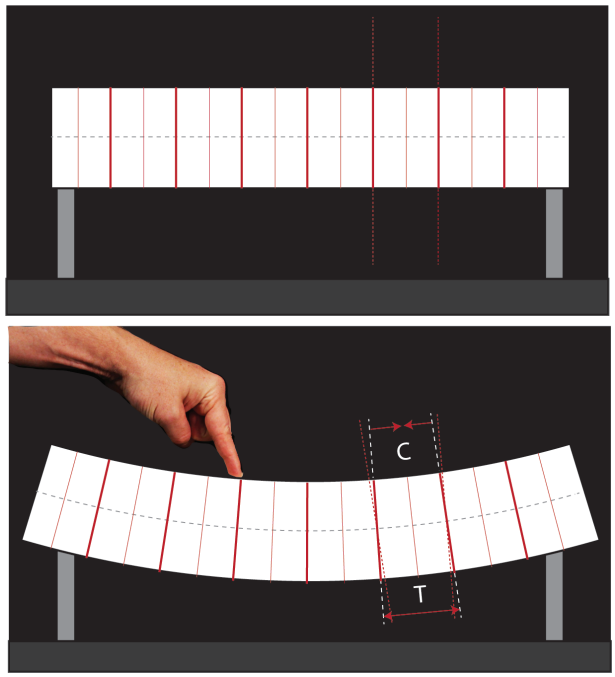


Figure 3.0.10 Photograph and diagrams demonstrating bending. Under bending, the top fibers get closer when they compress and bottom fibers get farther apart under tension

Galileo advanced these discoveries by focusing on the behavior of materials and how internal force vectors could visually explain their behavior. Importantly, he distinguished between strength of a material versus strength of object (i.e., the properties of stone must be understood in the context of the stone's properties *as a beam*). His drawings of a cantilever hanging from a wall, which also compared

a flat v. tall beam, hypothesized that bending resistance could be explained as an internal "force couple" of compression and tension forces. He was incorrect about the amount of force that would occur in this loading condition, but this finding paved the way for scientific and mathematic improvements in understanding beams.

Galileo correctly determined that the magnitude of these stresses acting in the beam would be different across the length of the span, which is why it is easiest to break a beam at its point of maximum moment, and that these stresses would affect deflection. He concluded that strength in a beam is proportional to the square of its length, which is why geometric similarity doesn't result in structural similarity in beams. Longer spans can't simply be proportionally "scaled-up" versions of a smaller beam. He also was able to confirm that the depth of a beam's cross-section mattered in providing resistance to bending. (Figure 3.0.11)

Beam Design: Euler's Calculations and Confirmations

Intuitively, a flat ruler is easier to bend than the same ruler rotated to stand upright—but why? What principle explains this? No one had mathematically resolved the question of why certain cross-sections with the same overall area could be more effective at resisting bending. This measurement, later known as the "second moment of area" or *moment of inertia*, (*I*) was resolved by a prolific and insightful mathematician, physicist, and engineer named, Leonhard Euler (1707–1783). Euler determined that a beam's resistance to bending was proportional to the product of its cross-sectional area times this area's average distance from the centroid (or neutral axis). (Figure 3.0.12)

The basis of this theory is that beams generate an internal resisting moment when subjected to bending. When a beam deforms under loading, the material at the top side of the beam shortens as it compresses while the bottom side elongates under tension. Because they bend, beams have to resist a combination of compression and tension stresses acting simultaneously in different directions at different locations in their cross-section. (Figure 3.0.13)

Having two opposite stresses within the same structural component pulling and pushing in opposite ways may seem like a problem, but this internal behavior is the key reason that beams are able to resist bending. This is how a beam generates internal resistance to the bending moment. If we look inside the beam, we can see why added depth is useful. Deeper beams essentially have more leverage to resist bending because, as Euler noted, they have cross-sectional mass separated farther away from the neutral axis. Essentially deeper beams have a longer lever arm (the distance between the compression and tension stresses within the beam) and therefore a higher moment capacity. (Figure 3.0.14)

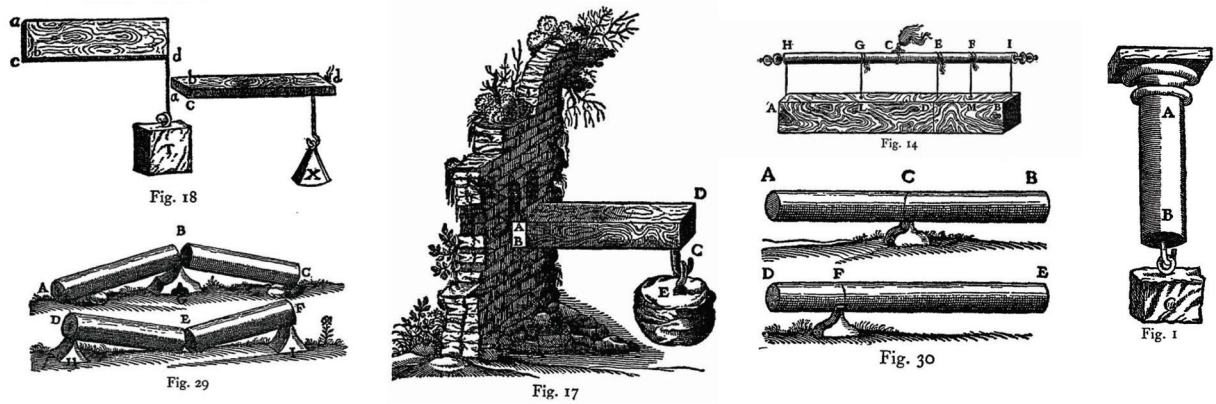
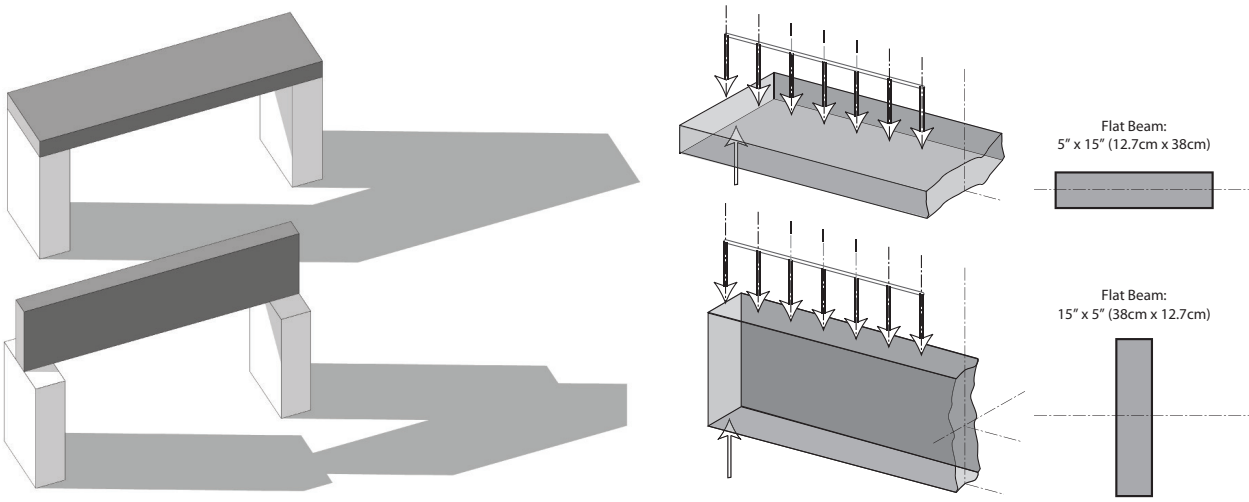


Figure 3.0.11 Sketches from Galileo's *Discourses and Mathematical Demonstrations Relating to Two New Sciences*, 1638



Figures 3.0.12a and 3.0.12b A taller beam resists bending more effectively than a flatter beam with the same cross-sectional area—bending effectiveness depends on *where* the area is located in the cross-section

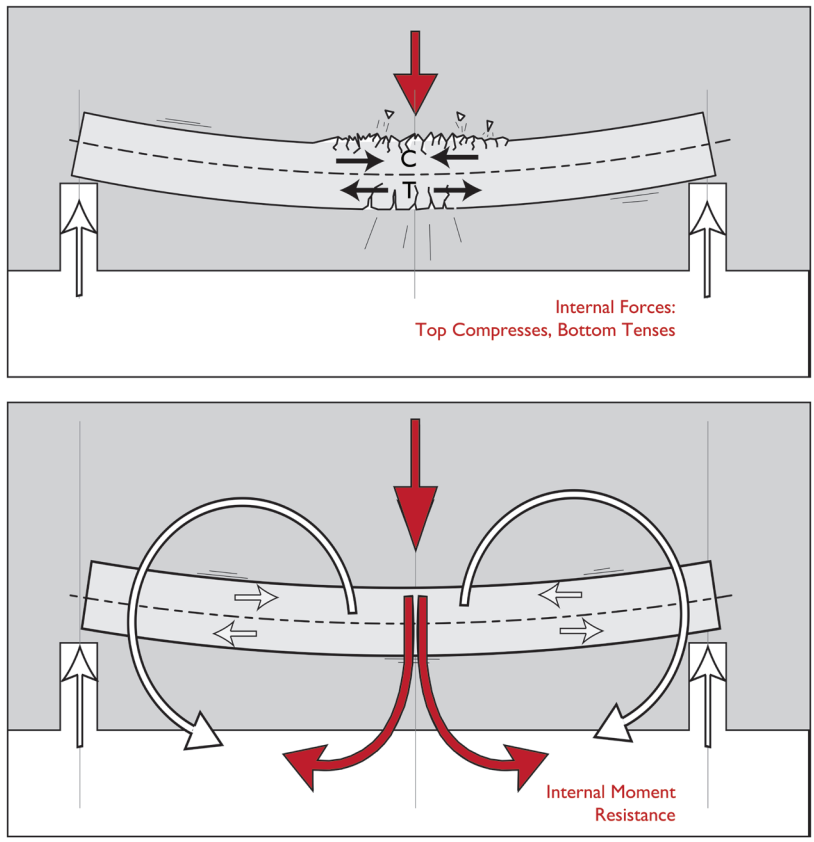


Figure 3.0.13 Applied forces create bending moments (red) in beams. Beams need to generate an internal moment of resistance equal and opposite to the bending moment from loading (white)

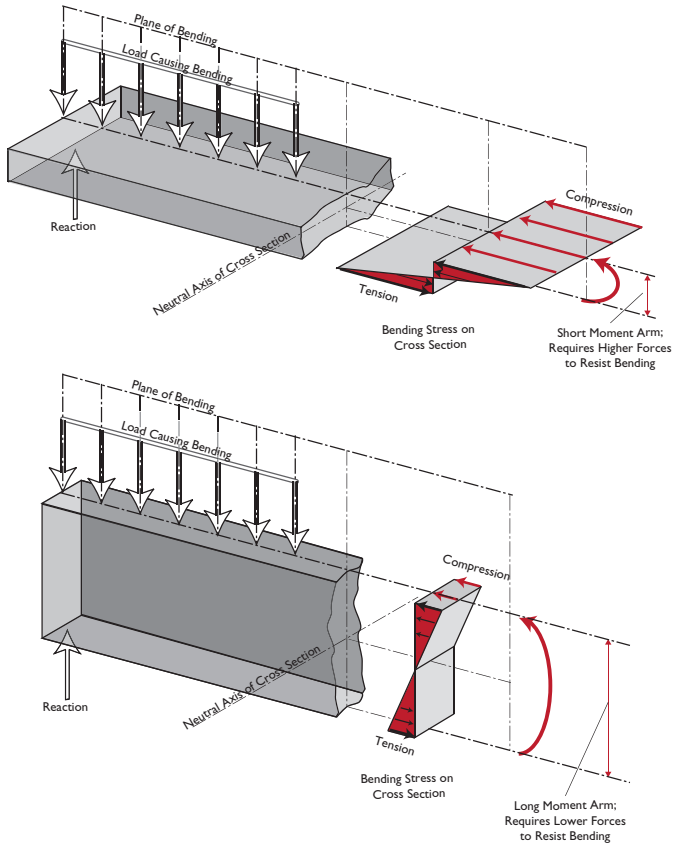


Figure 3.0.14 Taller beams distribute their stress across more sectional area and create a longer moment arm for the internal force couple to create internal moment resistance

Euler knew that cross-sectional area resists stresses, and therefore that beams should have the most cross-sectional area in the location with the highest stresses—in other words, at their top and bottom. A higher I-value (in^4), or the simplified factor known as the *section modulus*, S (in^3), indicates higher resistance to bending based on cross-sectional shape and area. This was one of the many influential contributions Euler would make. (Figure 3.0.15)

More than a hundred years had passed since Galileo's theories were published (and thanks to Newtonian calculus), until Euler and Daniel Bernoulli (1700–1782) developed an accurate mathematical model for understanding how beams carried load and deflected in 1750. This formula for bending resistance (known as the *flexure formula*) correctly quantified the mathematic relationships between the loading and support conditions (maximum bending moment, M), the beam's material behavior under loading (allowable bending stress, f_b), and the effectiveness of the cross-sectional shape to resist bending (section modulus, S) in a simple formula (presented here in its "non-engineering" forms):

$$\mathbf{f_b = M / S} \quad \text{or} \quad \mathbf{M = f_b * S} \quad \text{or} \quad \mathbf{S = M / f_b}$$

The formula is elegantly inclusive of many complicated variants. Any changes in the magnitude or location of supports and/or loading are reflected in maximum moment value (M). Allowable bending stress values (f_b) are an accurate reflection of how "strong" certain materials are under bending (allowing designers to compare the capacity of a material to resist bending under these conditions). The section modulus (S) is measurement of a beam's relative effectiveness to resist bending that considers the overall depth and the distribution of material together (i.e., it isn't just a measurement of depth, it

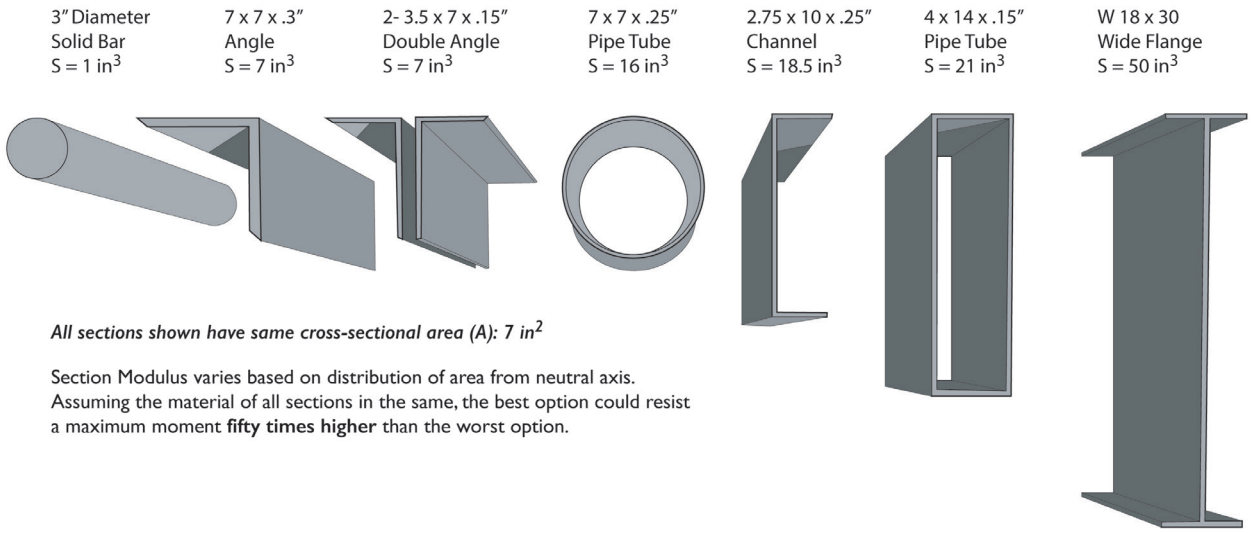


Figure 3.0.15 Depth and cross-sectional area are incomplete measures of bending resistance. The more important factor is where the area is located in cross-section (section modulus, S). Higher values indicate greater bending resistance

measures how much area is distributed towards the outer edges). This follows intuitive reasoning: if a beam's bending moment increases, it would either need to be made out of a different material with a higher resistance to bending or it would need to change its section (or both). (Figure 3.0.16)

Because a properly designed beam needs to be strong enough and stiff enough, their mathematical theory also described how to determine deflection in a beam under loading. This formula combines the load, span, material qualities of stiffness, and cross-sectional shape

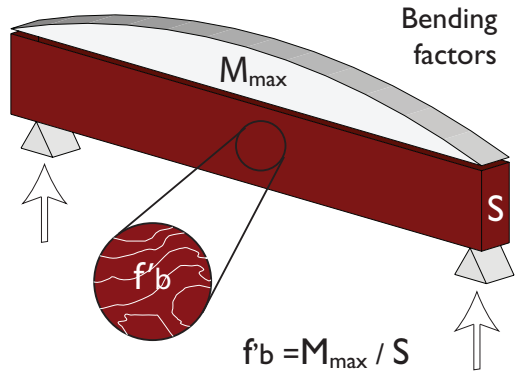


Figure 3.0.16 The factors contributing to bending in a beam can be concisely understood as a function of: Loading/support (M), materiality (f_b), and cross-sectional qualities (S)

together—of those factors, the span is the most influential. We can compare that value to given ratios for maximum deflection (l/240, l/360, etc.) that are usually dictated by code or building function.

$$\Delta \delta = 5 * w * l^3 / 384 * E * I$$

The factors included in this formula represent a fascinating amalgamation of centuries of work. First, equating the measure of deflection (δ) as directly proportionate to load (w) and length (l), is based on Hooke's discoveries. The distributed load value, w, was codified by Da Vinci and Aristotle, while Galileo determined that deflection was a cubed factor of the length. The two factors that are inversely proportional to the amount of deflection were the cross-sectional moment of inertia (I) (the "cube of the depth" value discovered by Galileo) and the material's stiffness (a stress/strain ratio Euler developed in 1727 based on the work of Robert Hooke (Chapter 2.0), known as the modulus of elasticity, E).

These factors make the formula easy to understand and evaluate without running any calculations. The length of the span is the controlling criteria—doubling the span increases the deflection by *eight times* the original value. Reduced load, reduced length, increased stiffness, or increased resistance to bending would all reduce deflection. (Figure 3.0.17)

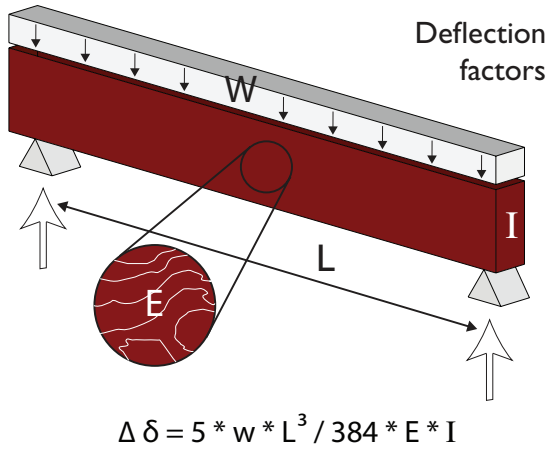


Figure 3.0.17 Beam deflection factors. Length of span affects deflection more than any other factor—thus deflection is the controlling factor for long span beams

See Appendix Figures A.7 and A.8 for general spanning estimates for various materials. Specific section modulus values for different beams are shown in Appendix Tables A.9 and A.10.

Shear Compliance

Understanding material behavior and visualizing the transfer of forces with vector arrows led to a greater understanding of shear stresses and compliance as well. When beams transmit loads to their supports, they consolidate and concentrate stresses at these points and create shear stress. Shear stresses (V, lbs or kg) make adjacent planes of material slide past each other vertically or horizontally. Shear resistance is dependent on the material qualities (its maximum allowable shear value, $f_{v,}$ in psi or kg/cm^2) and the cross-sectional area of the element (simplified as width (b) and depth (d) in in^2 or cm^2), described by the equation:

$$f_v = V / b * d$$

Vertical shear is the most prominent concern for most beams at or near its supports, particularly in beams with short spans that are subjected to high loading or beams made from a brittle or easily fractured or split material. (Stone, for example, has low shear resistance). Horizontal shear can be found in short, heavily-loaded, wood beams and joists as the shear force runs parallel to the wood grain and works to split the wood apart. Because wood is an anisotropic material with grains that run lengthwise across the beam, horizontal shear is its controlling concern. For that reason, we assume that the horizontal shear stress in wood is 1.5 times the average vertical shearing stress. Wood rectangular sections are checked for shear using the equation:

$$f_v = 3 * V / 2 * b * d$$

Shear seems easy to visualize, but, surprisingly, shear failure is rarely purely horizontal or vertical. Instead, diagonal fracture lines are created as a result of the internal force couple—a confusing occurrence for early beam builders. (Figure 3.0.18)

Appendix Table A.4 lists allowable shear levels for various materials.

Summary of Bending Resistance: Concepts and Calculations

By the middle of the 18th century we came to understand bending resistance this way:

Material matters: *A material's inherent qualities are the primary determinant of its resistance to bending and shear stresses. Materials can be defined by their allowable stress levels under compression, tension,*

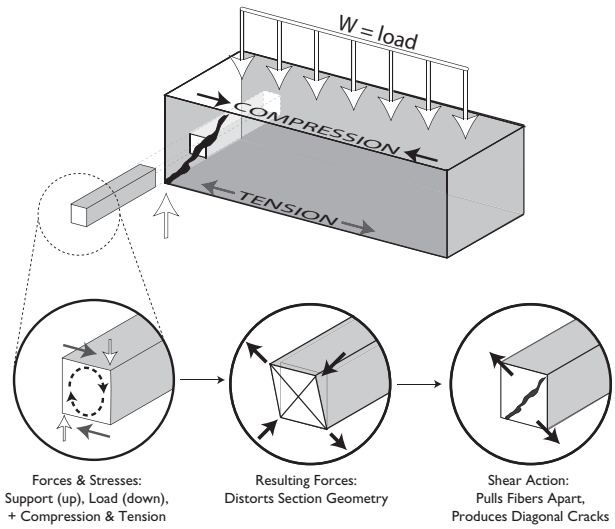


Figure 3.0.18 Shear stresses result in diagonal cracking as a function of the internal force couple

bending, and shear (f_b and f_v) and the ratio by which they strain under escalating stresses (E).

The magnitude of stresses will vary across a beam's span depending on the amount of load applied and the location of supports. The magnitude of these internal stresses can't be understood until equilibrium is established and calculated at supports.

Depth Matters: *Using free-body diagramming we can see that tension and compression stresses work together as a "force couple" to create internal moments of resistance. When these forces are separated further apart from each other (as a beam gets deeper) the "moment arm" between forces increases. Beams that have a deeper section create a greater distance between the stresses in the top and bottom of the beam, which means that deeper beams provide a greater internal resisting moment, and therefore a greater resistance to bending. A beam's strength to resist bending is proportional to the cube of the beam's depth.*

Shape Matters: *A beam's strength against bending isn't just a matter of overall area, or depth, it depends on where the area is located in terms of the beam's orientation. The farther away this cross-sectional area is from the neutral axis (within the plane of bending), the more effective the cross-section will be. Certain cross-sectional shapes (I-beams, tubes, etc.) are more effective at bending resistance because of where their cross-sectional area is located. A beam that is the same overall depth can resist more bending stress if additional area is added to the outer edges (or top/bottom flanges).*

With these points in mind, reconsider your initial answers for the river crossing scenarios. The most important lesson in how to use these formulae in assessing the behavior of beams is this: Sometimes the answer is, "it depends." When one factor changes, the other factors will have to be assessed and perhaps redesigned too. For example, in Crossing #1, increasing the span also increases the bending moment (M) that the beam needs to resist—this may be OK if it has sufficient material capacity to resist this bending (f_b) and/or if it has an effectively shaped cross-section (S). Or, in Crossing #3, we'd want to know how brittle

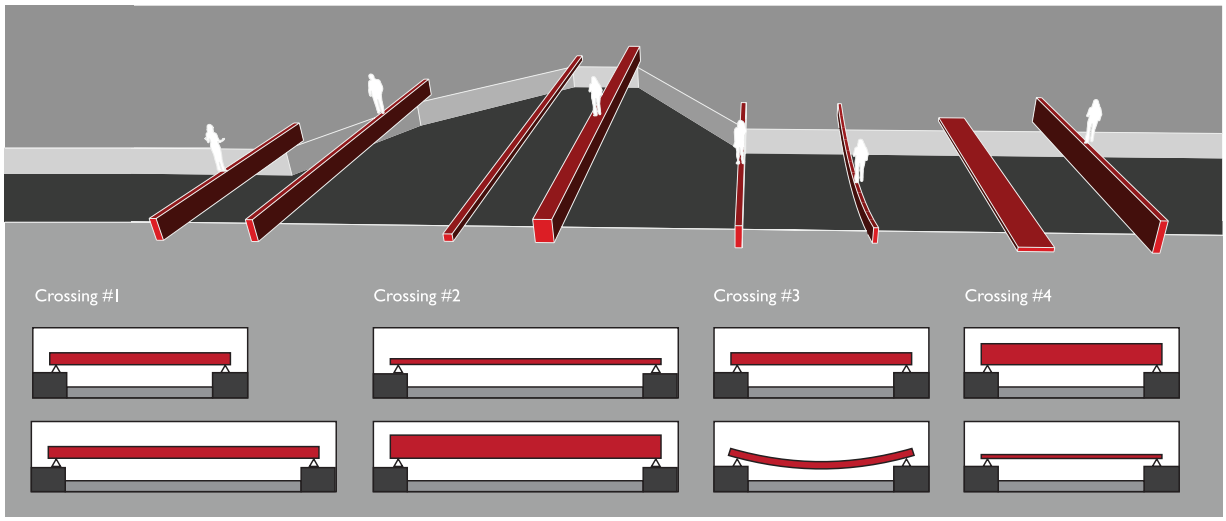
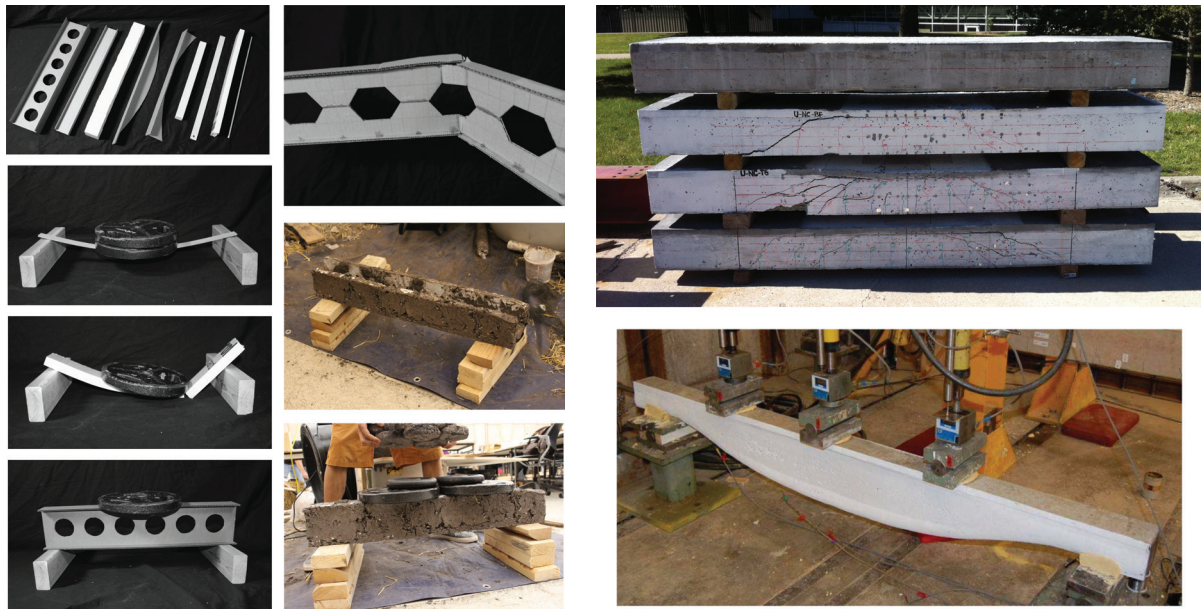


Figure 3.0.19 Revisiting the river crossing problem; consider which specific factors are involved in each of these evaluations

or flexible the tree is (its E value) *and* if these factors would be off-set by the length (*l*), load (*w*), or shape (*I*). In Crossings #2 and #4, we'd need to determine the required section modulus based on the other factors and check for compliance. We don't learn formulae for the sake of calculations only, we learn formulae to more accurately assess the interrelated aspects of structural behavior. (Figure 3.0.19)

FINDING FAILURES: BEAM TESTING

Beams are often tested for compliance according to two factors: the amount of internal stress (maximum shear and moment values and their corresponding location along the span), and how much they'll elongate/deflect under loading. These are primarily mathematically calculated, but in certain situations, particularly when the loading and supports aren't easily calculated (indeterminate beams with multiple supports, fixed connections, etc.), we use other tools. Under certain conditions, we might build and test large physical models to confirm the accuracy of the digital model—particularly for cast concrete beams. These aren't just efforts to break the beams. We're looking to test the compliance of a particular factor: relative ductility of concrete mix, ultimate strength of mix, effectiveness of rebar placement (particularly shear strips), etc. (Figure 3.0.20)



Figures 3.0.20a and 3.0.20b Examples of "beam lab" experiments (SxD, Iowa State University) and full-scale concrete testing (Iowa State University and University of Bath)

Column Design Basics

Column design had been a central consideration since the Ancient Greeks, as evidenced by the *entasis* of column shaping. For millennia, philosophers and builders understood that columns don't fail because of compression—they fail because of *buckling*. Buckling is caused by a combination of the column's height, the load applied, and its cross-sectional plan dimension (e.g., push down on a thin, long wooden dowel to see an example). (Figure 3.0.21)

To design a column, the first factor to determine is how much weight the column itself needs to hold up. This is based on the column spacing and the

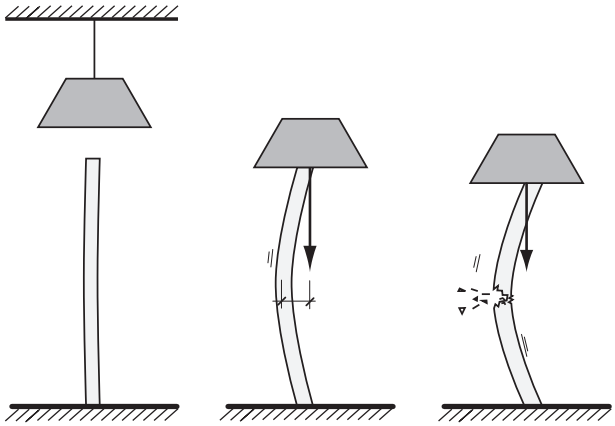


Figure 3.0.21 Buckling is the primary cause of failure in columns. Once buckling begins, the moment arm between load and resistance increases—often causing dramatic and sudden failure

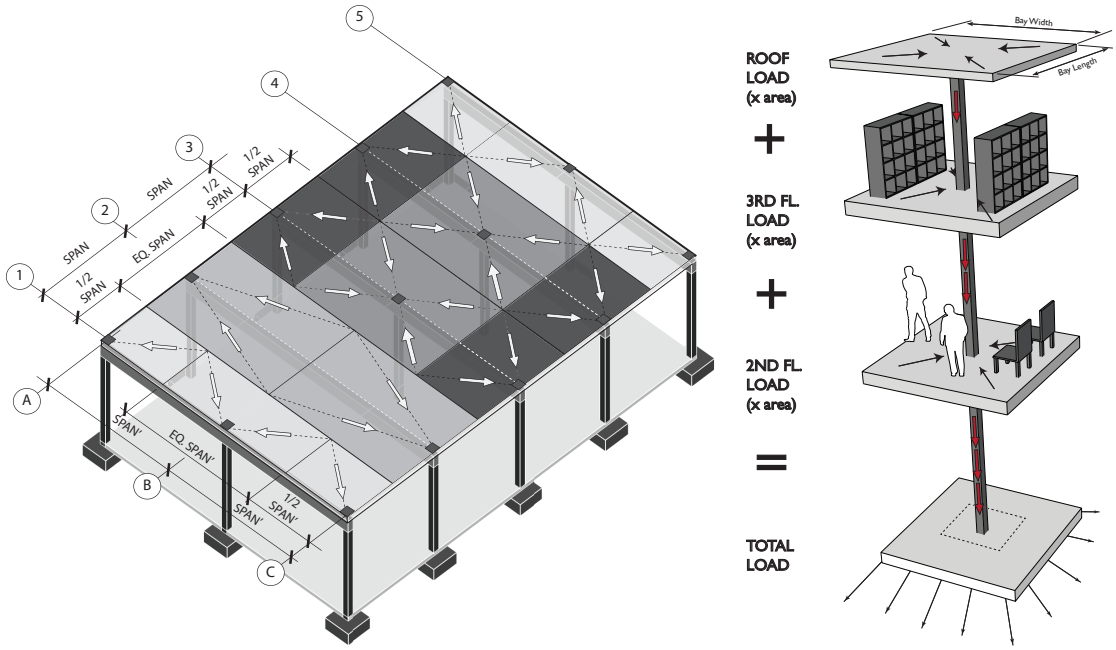
occupancy being supported. Columns gather loads from a larger area, concentrate them into a vertical force, and then spread this out again into a larger area at the foundation. (Figure 3.0.22)

Trying to predict the magnitude of factors that create buckling was elusive (e.g., which direction would it buckle, what was the maximum load that caused buckling, etc.). Again, it was Euler who provided the answers. He determined that the critical buckling load (P) depended upon the column's material's stiffness (E), its cross-sectional qualities (*I*), how it was connected on the ends that affected its potential for buckling (k), and its length (*L*):

$$\mathbf{P = k * \pi^2 * E * I / L^2}$$

In practice, cross-sectional shape and length are the two governing factors of column design—when combined they define the *slenderness ratio* for columns. The first factor, cross-sectional shape, resists buckling simply; if the column has a bi-axially symmetrical plan shape (square or circle) there is no "weak" axis for buckling. You can intuitively understand the strong versus weak buckling axis if you imagine pushing down on a wooden ruler—it would buckle side-to-side before it would buckle front-to-back.

Explaining this resistance can be quantified by a factor known as the *radius of gyration* (r). It measures the amount of cross-section area (A) and its distance from the centroid of a column's plan (measured by moment of inertia, (I)). The higher the value of r, the more resistant a column is to buckling. A tube is an effective column because it resists buckling by placing all of its material

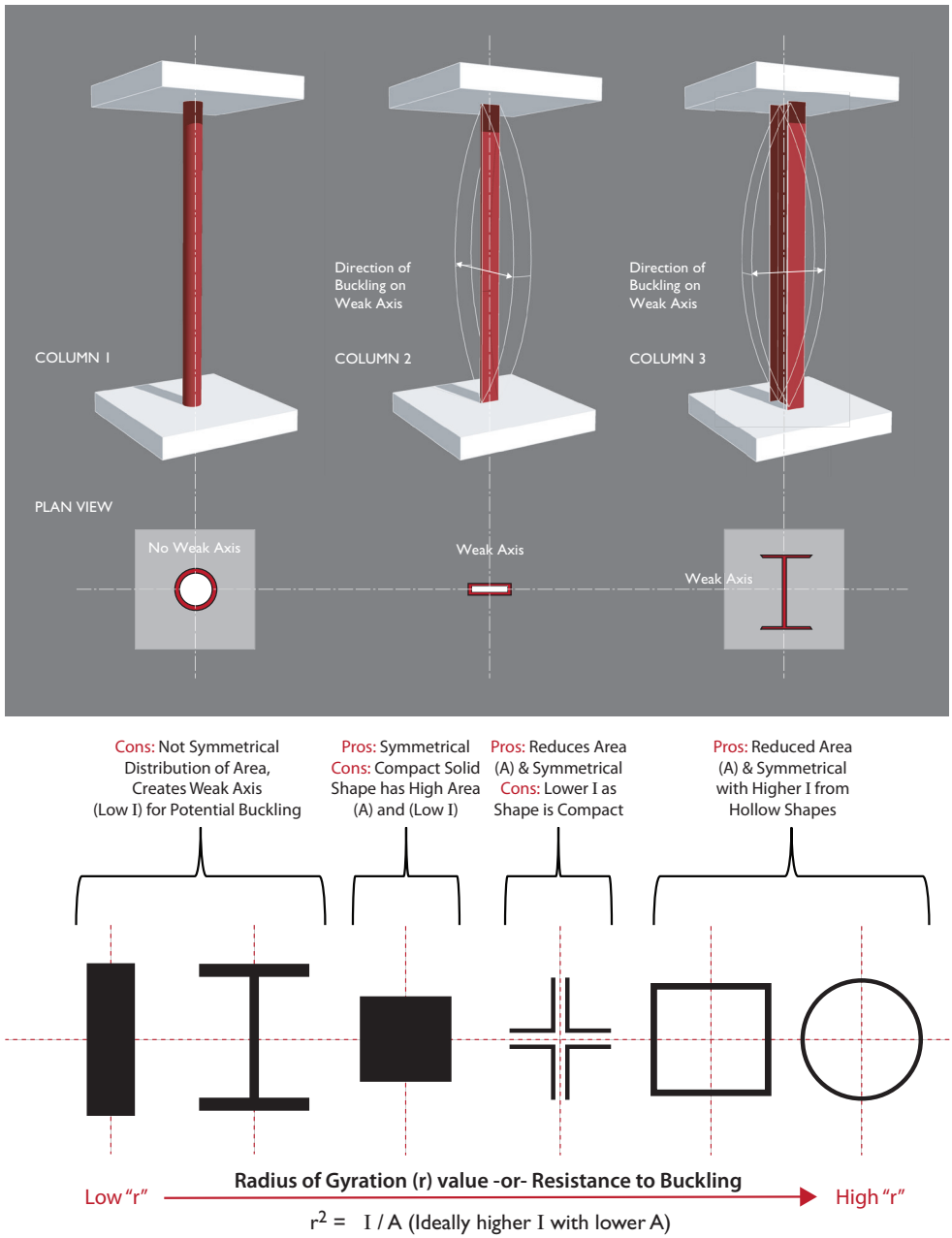


Figures 3.0.22a and 3.0.22b The column's vertical load is based on the amount of tributary area it supports from above. Columns in multi-story buildings also accumulate vertical loads from above

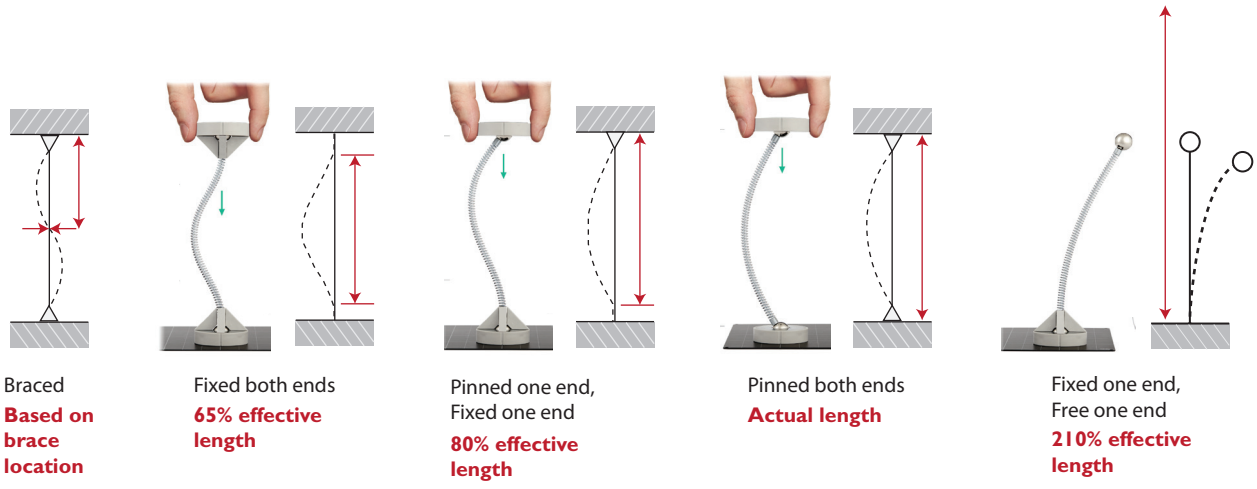
equally at its outer edges. This value is published in design manuals and product literature (Figure 3.0.23):

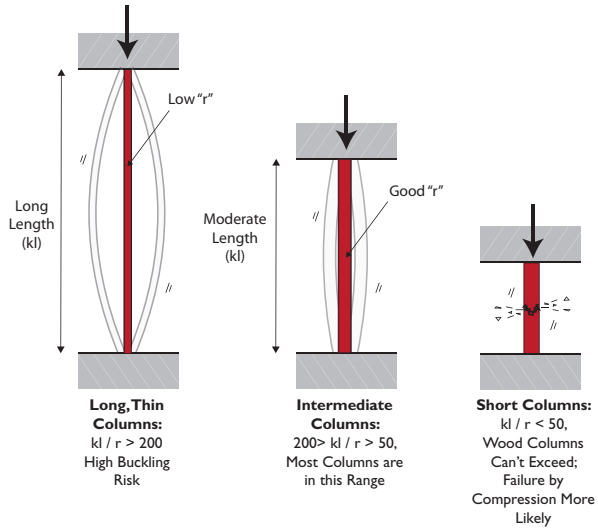
$$r^2 = \sqrt{I} / A$$

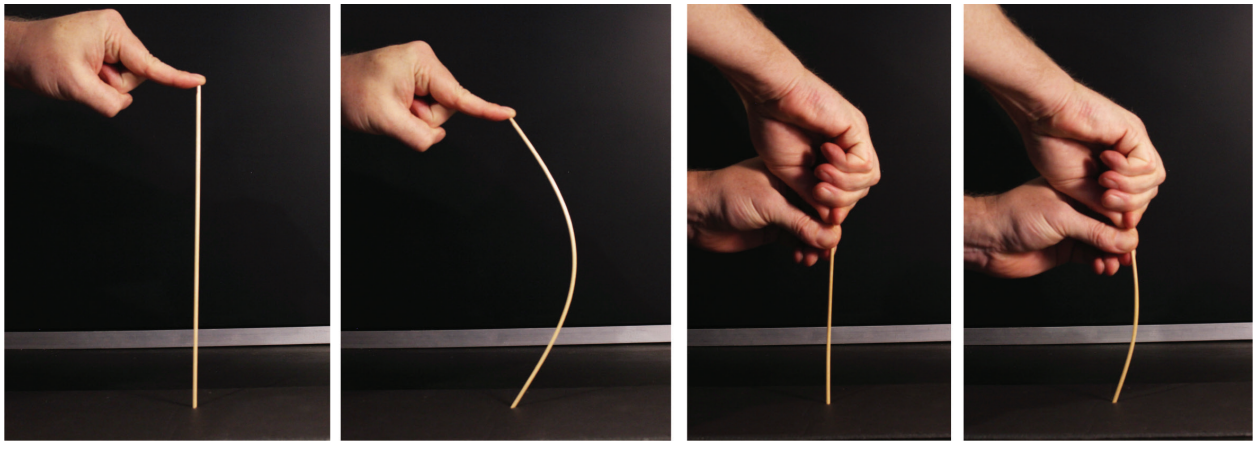
The second factor in slenderness ratio is the *effective length* of a column. Determining length seems like a simple thing ("how tall is it?"), but we are more interested in knowing if the column will behave differently than its actual length based on how it is connected at the top and base. A column with pinned connections will behave the same as its actual length—this is our base condition for assessment. But if the column isn't connected at the top (free), it will buckle more easily—it will buckle under the same load as a column twice as tall but pinned at both ends.



Figures 3.0.23a and 3.0.23b Asymmetrical plans have a weaker axis in buckling. A column's cross-sectional shape and distribution of area determines the value of its radius of gyration (r). Higher r values have better resistance







Figures 3.0.24a, 3.0.24b, and 3.0.24c Mola model demonstrations of how column connections determine effective length (kl). This slenderness ratio (kl/r) determines likely means of column failure. 3.0.24b: Slenderness ratio determines buckling risk. 3.0.24c: Try your own desktop demonstration of column slenderness and buckling.

Alternatively, a column that is fixed at both ends will act like a shorter column, which reduces its risk of buckling failure.

To accurately assume the effective length of columns we assign a mathematical modifier, called the k-factor, to column length based on the columns' end connections. When sizing a column, this k-factor is multiplied by the unbraced length to find the effective length. Ultimately, column design compares the ratio of the effective length (k * l) and radius of gyration, (r) back to rule of thumb/guidelines. Columns with a slenderness ratio over 200 are not recommended because of their extreme risk of buckling. (Figure 3.0.24)

The development and integration of stiffer materials such as iron and steel offered new possibilities for column design. These could be cast with effective

ALLOWABLE AXIAL LOADING FOR SELECTED A 36 STEEL COLUMN SHAPES

			EFFECTIVE UNSUPPORTED LENGTH (Kl) WITH RESPECT TO LEAST RADIUS OF GYRATION										
	COLUMNS, NOMINAL SIZE METRIC	COLUMNS, NOMINAL SIZE IMPERIAL	2.4m (8')		2.7m (9')		3.0m (10')		3.3m (11')		3.6m (12')		
			Allowable Load		Allowable Load		Allowable Load		Allowable Load		Allowable Load		
			kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	
	W 100 x 19	W 4 X 13	23	51	20	45	18	39	14	32	12	27	
	W 130 X 24	W 5 X 16	33	74	31	69	29	64	26	58	23	52	
	W 150 X 24	W 6 X 16	35	78	33	74	32	70	29	65	27	60	
	W 130 X 28	W 5 X 19	39	87	36	81	34	75	31	69	28	62	
B3	W 150 X 30	W 6 X 20	45	101	B2 43	96	41	91	38	85	36	80	
	W 150 X 37	W 6 X 25	57	126	54	120	51	114	48	107	45	100	
A3	W 200 X 46	W 8 X 31	76	169	74	164	72	159	69	154	67	148	A2
	W 250 X 49	W 10 X 33	81	179	78	173	75	167	72	161	70	155	
	W 200 X 52	W 8 X 35	86	191	84	186	81	180	78	174	76	168	

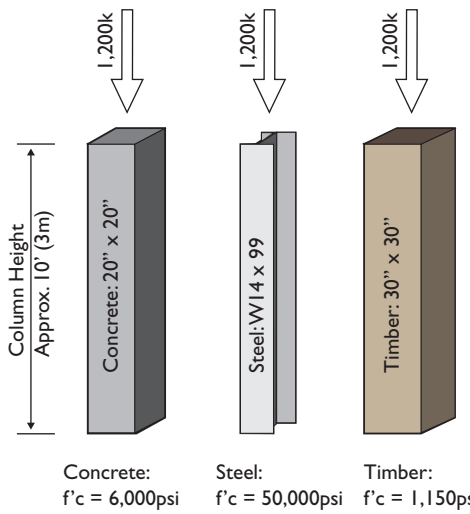
B1

A1

Using the Chart:
1.) Establish Effective Length (KL), 2.) Find Acceptable Allowable Load, 3.) Determine Column Size

Scenario A:
A1.) 12' column (3.6m) with pinned ends, (KL = 12'), A2.) supporting 100,500lbs. (45,586kg), A3.) W8 X 31 (W200 X 46)

Scenario B:
B1.) 12' column (3.6m) with fixed ends, (KL = 7'-10" (round to 8' (2.4.m)), B2.) supporting 125,000lbs. (56,700kg), B3.) W6 X 20 (W150 X 30)



Comparison of Material Axial Compression Strength

Figures 3.0.25a and 3.0.25b Column charts allow designers to compare effective length and loads to find acceptable column sizes. Different materials for columns will have their own unique column design charts

cross-sectional properties (I) and could be connected to other structural elements (k) in ways that would allow them to be taller and thinner. The ability to equate these factors to buckling load (P) allowed designers to determine how far apart these new columns could be placed. Applying a mathematical basis to column design avoided the unpredictability that had plagued designers for centuries. Column design today uses charts that compare allowable load and length with acceptable column sizes (in plan). These charts rely on load applied, length, material, and cross-sectional area. They can be used to size columns (if framing layout is finalized), determine bearing capacity (and therefore tributary area if layout is being determined), and/or determine column height (based on types of connections used). The charts are a fundamental design tool. (Figure 3.0.25)

Column design tables are listed by material in Appendix Tables A.11–A.13 and Figure A.14.

Foundation Design Basics

Foundations are the end point of a structure's force flow. They accept vertical loads from above, spread them out horizontally across a larger surface area, and release this distributed load to a stabilized base of earth below. Think of columns and foundations like funnels and pipes: A funnel on top collects the loads, concentrates them, and pours into the column. This concentrated load needs to be spread out into the ground so foundations act like another funnel that disperses the load across a larger area of supporting ground. Architects will work with geotechnical, civil, and structural engineers to determine the proper type and size of a building's foundations. (Figure 3.0.26)

Figure 3.0.26 Foundations being constructed below the Seattle Space Needle, 1961

Foundations need to be strong enough to resist point loads and stiff enough to resist bending stresses associated with dispersing this load, so they are typically made of reinforced concrete and set upon a crushed gravel base. Foundation shapes and sizes depend on several factors including: The type of load that is imparted on them (point versus distributed load), the type of soil that is below (e.g., gravel is great, clay is not), the threat of freezing soil (i.e., determines the "frost depth" to avoid heaving), and the types of forces it needs to resist (e.g., retaining wall with off-centered footing, pile cap with friction piles, etc.). Foundations are classified as shallow versus deep. (Figure 3.0.27)

See Appendix Table A.15 for allowable bearing capacity of various soil types.

Introduction of Beams in Buildings: Economics and Innovation

The most surprising result of the Euler–Bernoulli flexure formula was the enormous lag between its introduction and its implementation—nearly a hundred years! This was a result of the distinctions between science and practice (and design and construction), and a lack of the motivating factors that typically compel innovation.

Design and construction still relied on rules of thumb and constructed precedents, which were adequate for most moderately sized projects. Timber, of course, had been used (and improved) in ancient Chinese and Japanese buildings that pre-date most of the examples shown above by hundreds, and in some cases, thousands of years. But no matter the culture, beams were limited by the lack of materials with tensile capacity that could handle the high stresses in long spans and the deflection that wood allowed. Splicing beams together required difficult tension-resistant connections and was limited by material strength in heavier beams. Longer spans were thus still structured with arches or with timber trusses. But by the

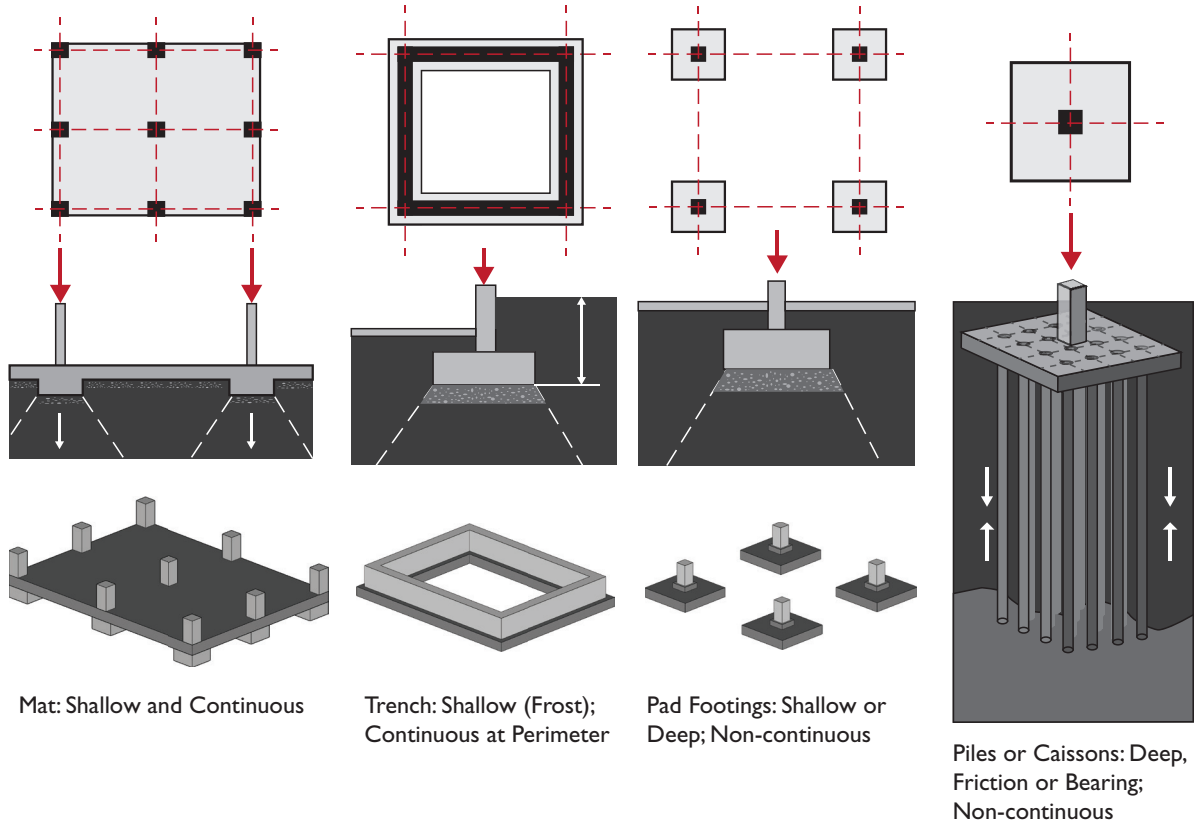


Figure 3.0.27 Types of foundations: Soil conditions, climate factors, framing strategies and design loads are contributing factors in determining the types of foundation required

mid-19th century changes in design, construction, and manufacturing altered the way section-resistant beam structures were conceived and constructed.

Developments in iron gave designers a compact and strong material that could resist moderate levels of tension stresses of bending. Unlike previous eras in which the beam design was plagued by unpredictable behavior these new materials were stronger, stiffer, and more reliable. Iron elements could be mass-produced with consistency in material and section that was impossible in wood or stone. Iron was also more fire-resistant than timber and it could carry heavy loads across long spans—essential characteristics for the new building structures found in production-oriented economies (e.g., flour-mills and warehouses).

Figure 3.0.28 Framing as a primary architectural expression seen in the Palacio de Cristal del Retiro (Madrid, 1887)

The designers and builders of these industrial buildings, including William Fairbairn (1789–1874), engaged in prototyping the shapes and cross-sections of the beams to see how these would affect structural behavior. They discovered that the key to resisting bending wasn't just a new material or a deeper beam, *but* an effective distribution of weight in its cross-section. These experiments produced the now familiar I-beam cross-section. (Figure 3.0.28)

Analyzing Discrete Elements: Expressions of Engineering

Advances in materials, connections, and mathematical systems of evaluation changed the way buildings were designed. Buildings could now be made as a framework of discrete elements (columns and beams) and freed from solid vertical bearing planar masses. Because beams collect horizontal loads and distribute these as vertical loads to their supports, the load path of an entire building's structure—from roof to foundation—could now be analyzed step-by-step. Each element could be designed to specifically resist the loads applied to it based on its location within a structure, and building codes were written to require this. As a result, iron and steel beams had members of different depths and cross-sections according to how much load they were carrying. Even anomalous loading conditions could be accommodated. *Buildings could now be built the way they were designed—as separate frames assembled from discrete elements*. (Figure 3.0.29)

The impact of this conjunction between science and practice on architecture and structures was profound. Because beams and columns could now be analyzed accurately and fabricated to be specifically responsive to loading situations, designers could rethink *how* buildings were composed, framed, and even constructed. Instead of thinking of structures as

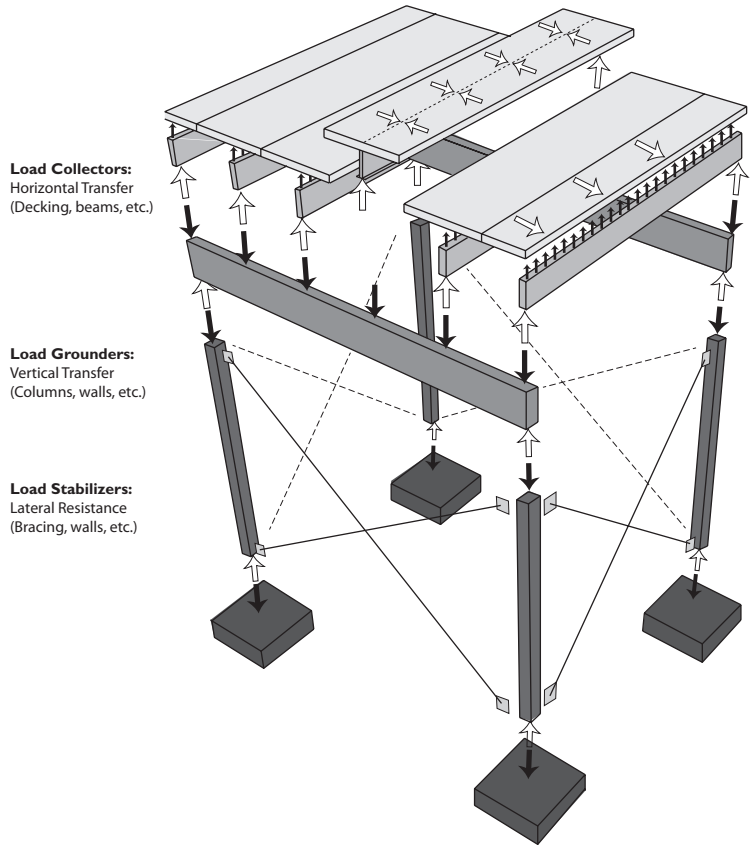


Figure 3.0.29 Classification of structural roles (collectors, grounders, and stabilizers) in conventional framing systems

monolithic forms (like arches and vaults), engineers could conceive and create buildings as assemblies of flat linear and planar elements (like floors, walls, and roofs) that could be stacked atop each other. Heavy load-bearing walls and conservative rules of thumb no longer needed to be the mode of practice. Consequently, as structures became "frames" of columns and beams, they became lighter and more predictable to design and construct—in other words, they became scientifically rational and efficient. Ultimately, these structures were able to span farther, stack higher, be built more easily, and perform more predictably, while also becoming more open to daylight and expressive of their structural form.

High-Rises, Indeterminate Structures and the Usefulness of Moment Diagrams

By the end of the 19th century, these structural benefits became the basis of a new structural typology: the "high-rise" building frames in Chicago and New York (Chapter 6.1). These innovations were motivated by larger economic and cultural factors, in this case, the need to consolidate many people in smaller areas by building taller. It's an over-simplification to think of these high-rise framed buildings as only individually designed elements that collect horizontal loads and transfer vertical loads to the load-grounding columns. High-rise structures developed more complicated framing solutions to better resist sideways (lateral) wind loads without using heavy and bulky walls. One of their innovative strategies was to create stability through "rigid frames," or connecting load collectors and load grounders together with rigid *moment connections* so both elements acted as load stabilizers. Taller buildings used moment connections instead of walls to create lighter buildings that could resist additional wind forces.

However, this rigidity meant that the behavior of beams and columns wouldn't be as easy to understand or calculate because the structure acted more monolithically. These structures became *indeterminate*, which required a new type of analytical "modeling." In 1930, Hardy Cross (1885–1959) introduced the *moment distribution method* for analyzing these structures. Whereas previous methods relied solely on calculations, this method produced visual representations of moment forces in the form of "moment diagrams." Cross created a way of visualizing *the location and intensity of bending moments* in beams with diagrams that illustrated the distribution of moment forces within each member. Because moment forces are mathematically related to the shear forces in a beam, this method also generated shear force diagrams. (Figure 3.0.30)

Thanks to this method, architects and engineers could now visualize locations in framing systems where moment and shear stresses were highest and how these varied along the length of the beam. They

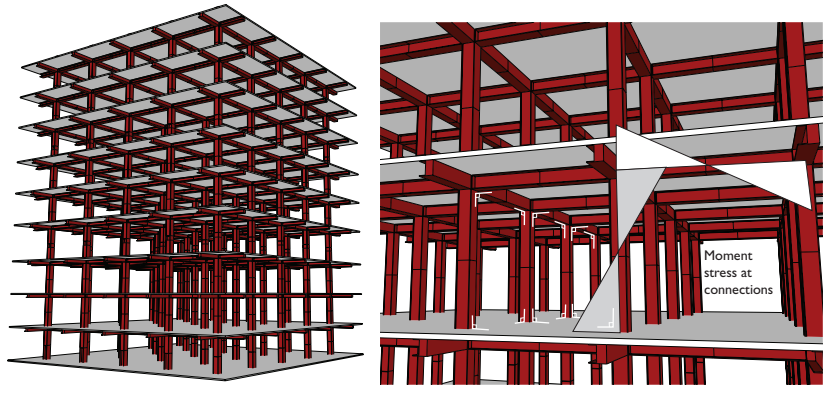
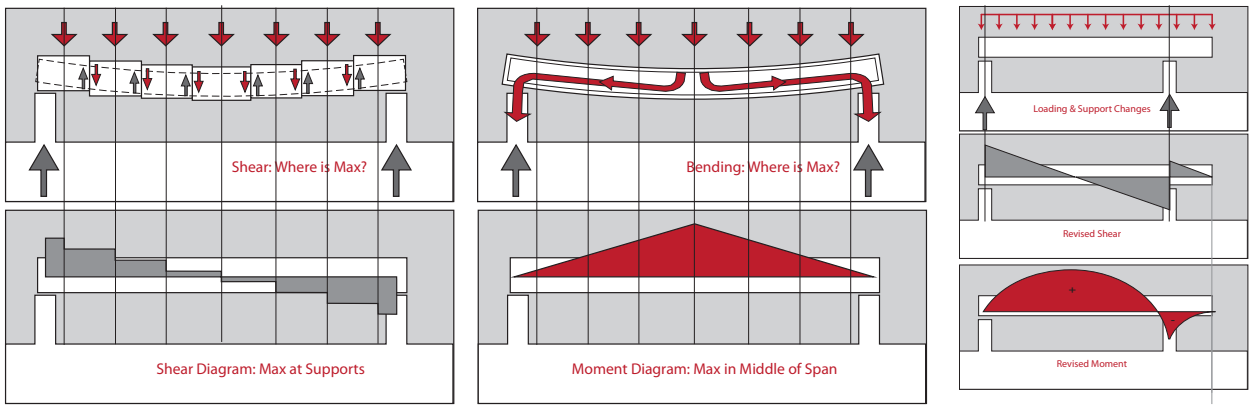


Figure 3.0.30 Moment connections in frames complicate the behavior of taller structures because of the shared bending resistance between columns and beams



Figures 3.0.31a and 3.0.31b Shear and moment stresses can be quantified and graphed. Changes in loading and support factors change the magnitude and location of maximum shear and moment values

could then make adjustments to the beam shape and depth to be more responsive (Chapter 3.2). Like the graphic statics method before it, this tool was initially intended for confirmation and analysis, but it also became useful for form-finding and instruction. Because this method combined visual and quantifiable information, it was immediately integrated into the education of engineers and architects as a way of understanding beam behavior. (Figure 3.0.31)

Moment forces vary across the length of the span (L) and are dependent on the location and magnitude of loads ("w" is a distributed force (lbs/ft or kg/m), "W" is total load (lbs or kg), or "P" (lbs or kg) for a point load). Moment diagrams are still the most pervasive method for representing beam behavior and they provide a way to determine the maximum bending moment (and maximum shear) using diagrams and mathematical short-cut formulas. (Figure 3.0.32)

Readers unfamiliar with shear and moment diagrams should consult the Additional Readings listed as references in this book, and seek out applied exercises. See Appendix Figures A.5 and A.6 for step-by-step examples of generating shear and moment diagrams for a particular loading and support condition.

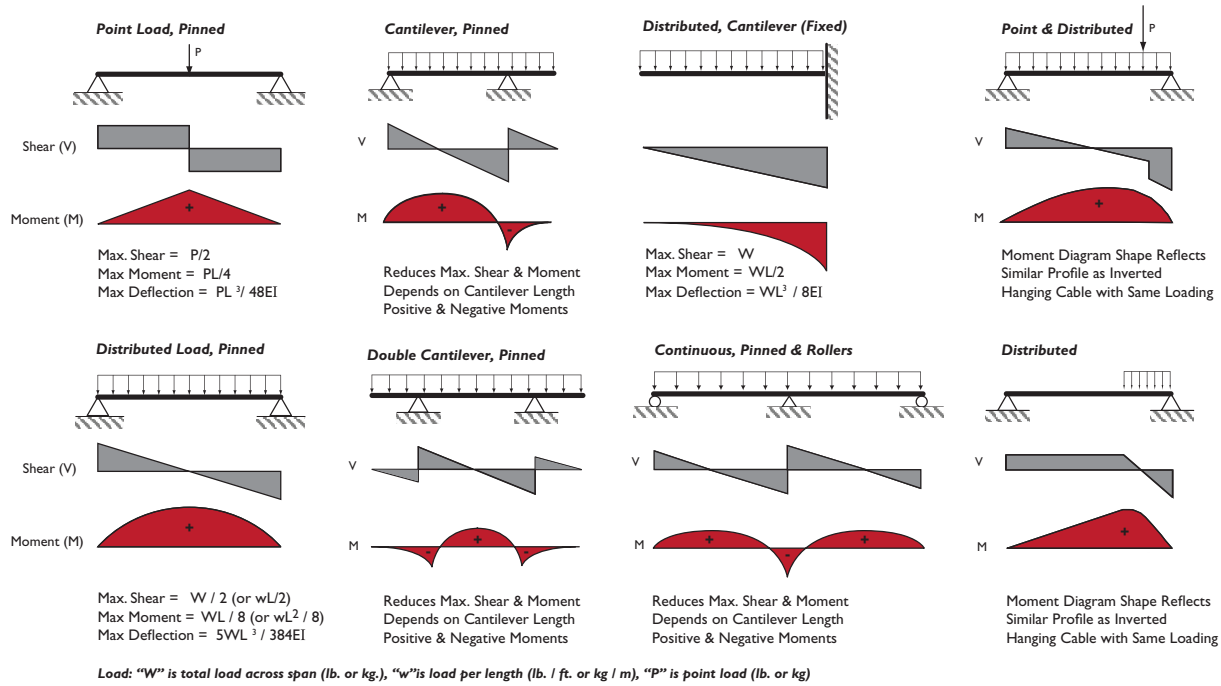
Beams Become Slabs: Concrete Integration

Once a reliable means of analyzing indeterminate structures was established, engineers could design beams made of reinforced masonry and cast-in-place concrete—advancements that were motivated by economic considerations of creating fireproofed structures that spanned long distances and carried heavy loads. Concrete behaves well in compression but poorly in tension. It requires steel reinforcing bars (rebar) to absorb bending's tensile components. But designers have to consider exactly where rebar should be located in the cross-section and along the length of the beam (i.e., the rebar should be located where tension occurs according to the force couple diagram of bending resistance) to resist both bending and shear stress. (Figure 3.0.33)

Once cast-in-place concrete was used for beams, it made sense to also use it as the structural material that spanned between beams—eventually morphing together to become a unitary load-collecting structural element called a "slab." Entire buildings could now be made of cast-in-place concrete slabs and columns. A new set of structural efficiencies resulted from this change—efficiencies that became new sources of expression (Chapter 3.2). Not long thereafter, the structural effectiveness found in this monolithic structural behavior would be harnessed and re-shaped into the highly efficient surface-resistant structural shells and plates (Chapter 5.0).

Digital Design and Fabrication for Beams and Slabs

In the mid and late 20th century, computers were integrated as an analytic tool for indeterminate structures. They allowed designers to complete the calculations on complicated proposals rapidly and more



Figures 3.0.32 Common loading and support scenarios and their corresponding shear and moment diagrams

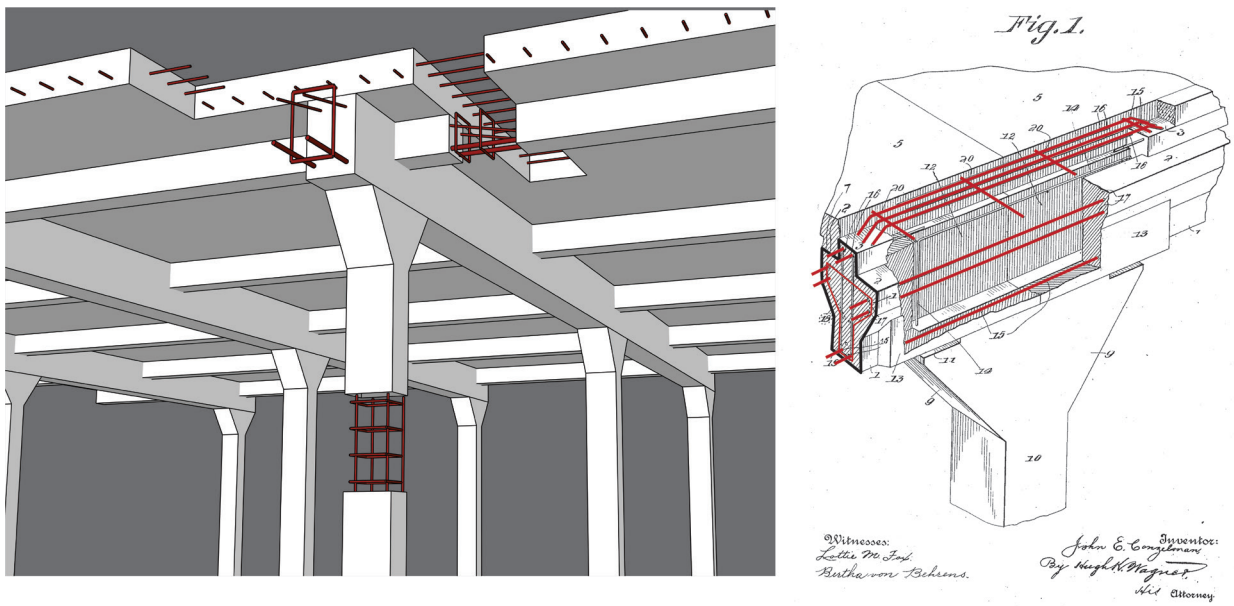


Figure 3.0.33 Drawings based on Francois Hennebique's patent drawing for reinforcing concrete (1892)

accurately. Compiling these calculations became known as the "model." As digital technology has advanced, it has become easier to calculate and visualize these stresses.

Digital design has been particularly useful for section-resistant structures like beams and slabs because it helps to visualize internal means of resistance and to estimate the concentration of stresses and deformations throughout the frame. In contemporary practice, Building Information Modeling (BIM) and parametric algorithms (Finite Element Analysis (FEA)) can run calculations and present visualizations of stresses and deflections in beams. These allow designers to evaluate the entire structure's behavior and to simultaneously evaluate changes in layout, material, sizing, and connections without redrawing the entire scheme. Because the model is digital, parameters can be adjusted as needed (e.g., type of connection, grade of material, depth of member, etc.) and designers can get revised feedback quickly by running a new model. (Figure 3.0.34)

These programs exploit the capacity of digital models to represent complex three-dimensional behavior by creating a color-coded language that describes the distribution of stresses in beams or the relative degree of displacement caused by applied loads. Digital models still rely upon a core competency of structural understanding from the person who defines boundary conditions and parametric attributes that the computer model will study. This is why the earliest programs that performed digital analysis were created, and programmed, by the engineers themselves and why digital designers continue make their own tools and craft their own codes to get more productive feedback.

Constructive Conclusions

Today, beams and slabs are used everywhere in our built environments. These section-resisting structures are convenient choices for designers with material options for beams that include wood, steel, concrete, and sometimes plastics and glass.

Although they may not be the most materially efficient structural systems, beams have become pervasive in our constructed environments because they are convenient and beneficial. They create linear and planar structural surfaces like floors, ceilings, roofs and walls that are functional necessities. This planar quality reduces overall structural height compared to arches and cables and allows beams and slabs to be stacked in multi-storymulti-story arrangements. Because beams resist shear and bending stresses, there isn't any external thrust exerted at their supports. Under normal conditions, beams can resist a variety of different load arrangements without changing their shapes. (Figure 3.0.35)

Mathematic analysis of beams may seem enigmatic, but it need not be—beam design for architects isn't solely about calculations. Remember that calculations aren't as important as the principles of behavior explained by the equations—equations define relationships between factors and help explain behavior. The behavior of beams follows straightforward principles.

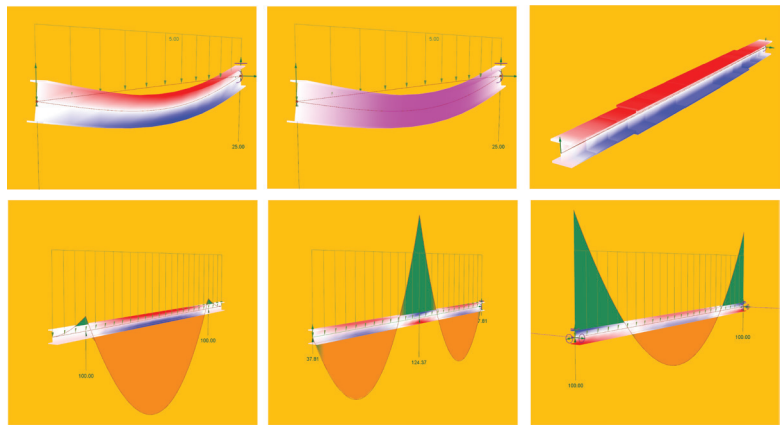


Figure 3.0.34 Examples of digital analysis methods for beam design

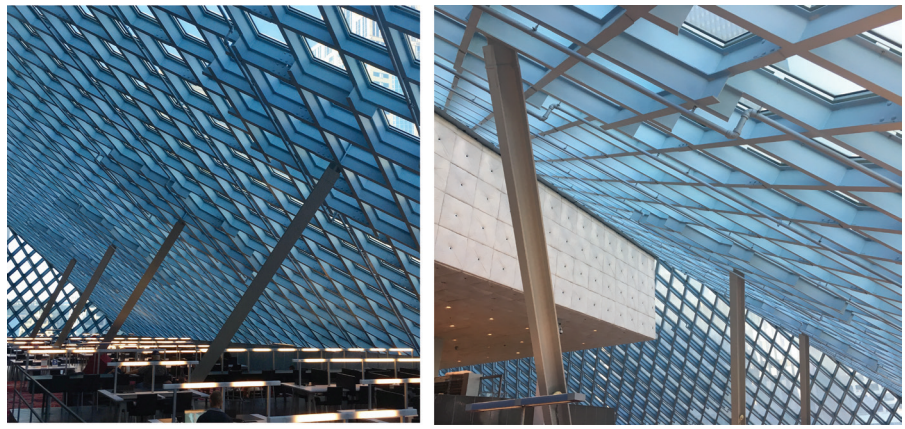


Figure 3.0.35 The Seattle Public Library's tilted volumes are created with curtainwalls that also span as structural planes between columns (OMA, Seattle, 2004)

Understanding these will create a productive feedback loop between design and analysis. (Figure 3.0.36)

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CONSIDERATIONS FOR DESIGNING SECTION-RESISTANT STRUCTURES

1. Structural Form and Layout: Combine the broad architectural and structural principles together to outline a form that seems suitable for continued development.

2. Material Matters: A material's qualities are included in every structural evaluation. Different materials determine various possibilities. Define these choices early and in correspondence with architectural priorities.

3. Determine Role: Section-resistant structures can be load collectors, grounders, or stabilizers. Their different roles will determine the structural principles affecting their behavior and their overall part in the structural frame.

4. Establish Equilibrium (Layout Matters): Align elements in a spatial organization as desired architecturally and structurally. Establish equilibrium between loads and supports, then solve for internal equilibrium within each element.

5. Flow of Forces: Because gravity works from the roof down, you should too. Start with the smallest load-collecting components and trace the flow of forces down to the foundation. Determine the resulting load on each element and how much load it will transmit onward.

6. Bending Moments & Shear: Determine maximum bending and shear values and locations through diagrams. Bending moment is the key design factor for most situations. Length of span is usually the determining factor, but the amount of load (and where the load is placed) matters too. Shear stress is the biggest concern with short beams with heavy loads (and fibrous materials prone to splitting).

7. Beam Shape: Select a cross-sectional shape that is naturally effective at reducing bending. Bending resistance can be improved by increasing the depth of the beam with additional cross-sectional area at the top / bottom.

8. Deflection: Beams can fail if they aren't stiff enough. Deflection is typically the controlling factor for long span beams—length of span is the design factor that matters most. A stiff material helps considerably, as does the cross-sectional shape.

9. Columns: Buckling is the greatest threat for column failure. Thin, tall columns with heavy loads aren't advisable. Because buckling is unpredictable, a bi-axially symmetrical column shape (like a pipe column) is most effective.

10. Frames & Stabilizers: Stabilization strategies and connections affect behavior; either increasing bending through shared resistance or by integrating structural elements dedicated to stabilizing the structure. See Part 6: Frames

11. Foundations: Foundations are designed to collect all the loads from the building and spread them out to the ground in a way that keeps the building stiff and stable.

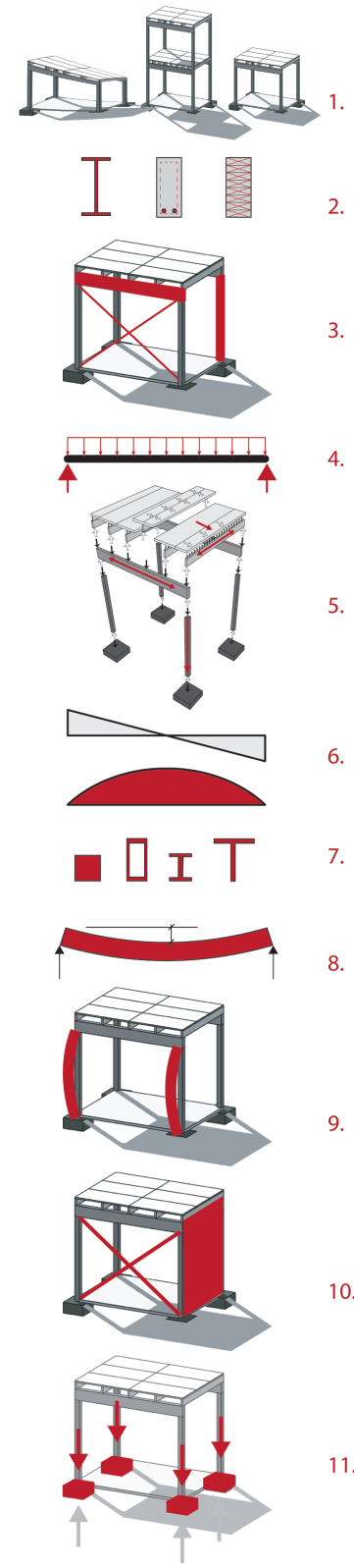


Figure 3.0.36 Summary of the design process for section-resistant systems

Figure 3.1.0 Louis Vuitton Foundation (Gehry Partners, Paris, 2006)

CHAPTER 3.1

Stacking and Spanning Timber

Fundamental Framing and Analysis

Most constructed environments are formed by the planar surfaces of walls and floors, and thus they rely on the integrity of the beams and columns. These provide functional and structural utility—thus their ubiquitous presence in structures of all scales (from tables to high-rises)—and myriad choices for designers including the type of element, its material qualities, its arrangement in a framing plan, and its size/shape.

Benefits and Challenges of Beams

Beams perform a seemingly simple task: they resist perpendicularly applied loads and transmit them horizontally to supports. They resist the resulting bending and deformation internally by generating a resistance to bending moments and shear forces. To do so, beams rely on sufficient cross-sectional area and the strength of their material to resist bending stresses.

Because beams are arranged into separate framing modules, variations in load allow each beam to be designed as a discrete element within a larger, more holistic framing strategy. Unsurprisingly, decisions about framing options are decided during initial meetings with clients and consultants. Understanding the spatial and constructed consequences of these choices is an important skill—one we will simulate in this chapter in the design of a wooden pavilion.

Design Challenge: Timber Pavilion

This pavilion, located in Cottonwood Canyon State Park in Oregon, will serve as a flagship educational, training, and exhibition center for the Oregon Outdoor Education Coalition (OOEC). This two-story, approximately 3,500 ft^2 (325.25 m^2) building is cut into a hillside, which allows the lower ground floor level to have a large display area along its rear retaining wall and a large open area used for exhibits, with shelving units and tables, in the middle of the space. The upper level features two enclosed classrooms on the ends and two open outdoor lab spaces in the middle that are accessible from the top of the grade. An open stair connects the two floors together. Architecturally, the building spaces are meant to be open to each other and to the landscape. It should be welcoming to mottled sun and passive ventilation, but protected from rain. (Figure 3.1.1)

Because the structural expression of columns and beams is such a predominant part of a pavilion's aesthetic, we will explore the impact of our structural decisions about layout and materials. Designers need to conceive and document basic structural framing strategies in a conversant and

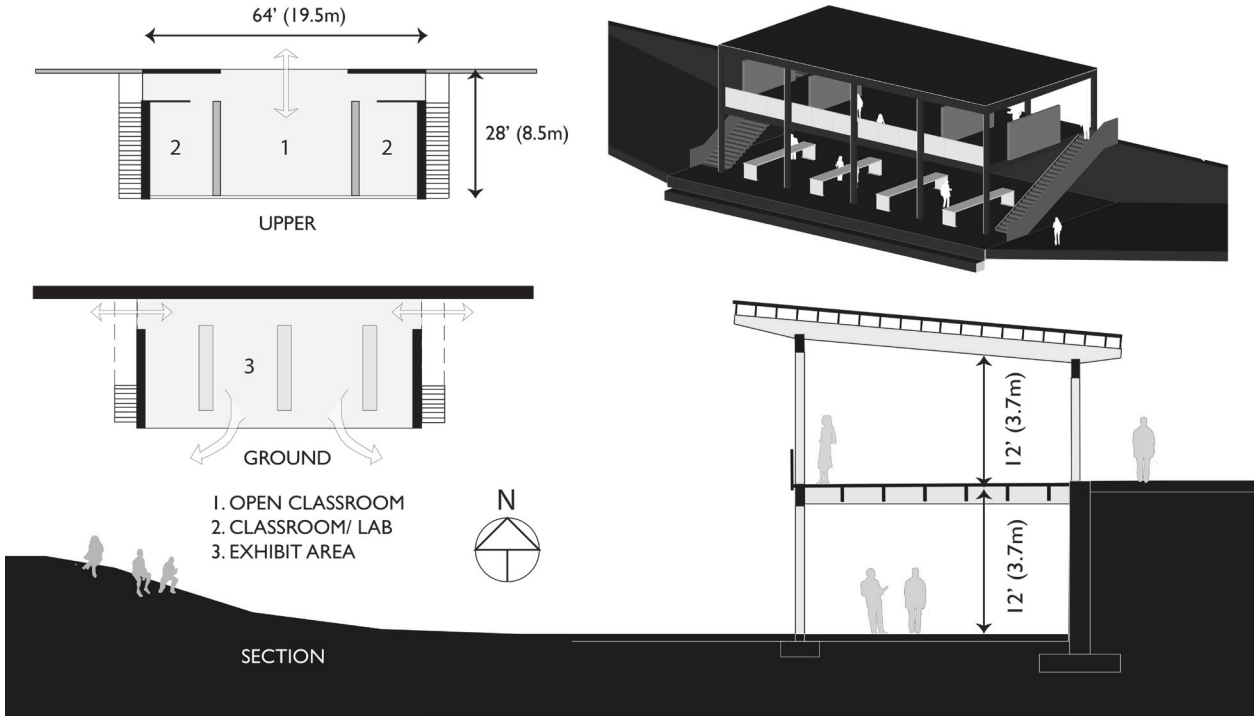


Figure 3.1.1 Massing diagram, section, and plan arrangements for proposed problem

collaborative design environment, which is often aided by three-dimensional representations of the framing. Our drawings will evolve throughout the chapter to include more information and details to assess our proposals.

But before we delve too deeply into the framing, we need to first understand our chosen material—timber. See Appendix Table A.4 for material properties and Tables A.9 and A.11 for specific beam and column charts for timber.

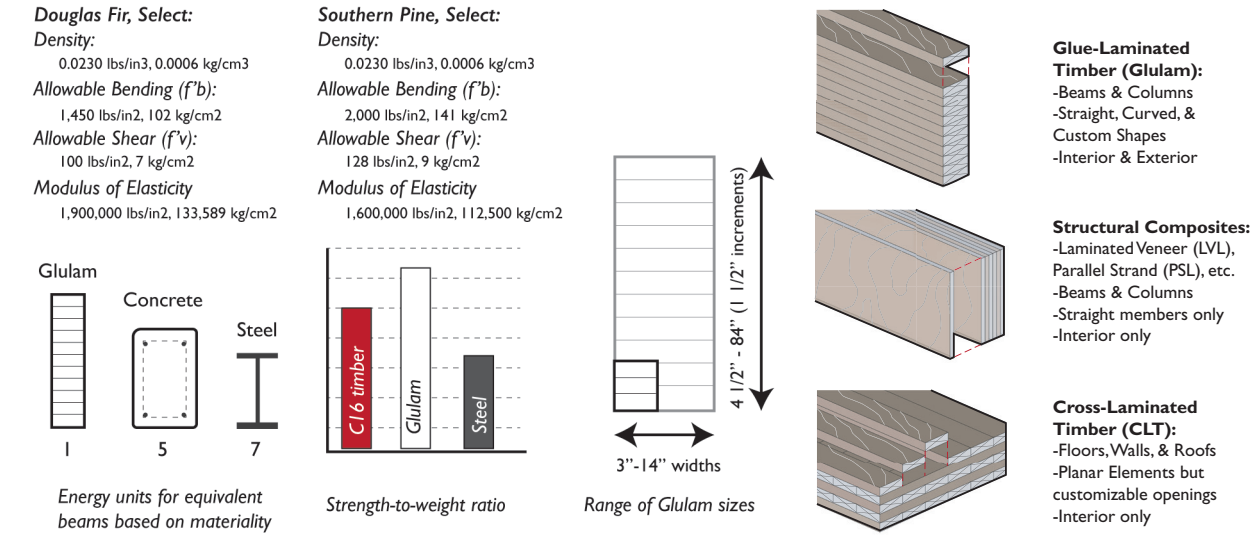
MATERIAL MATTERS: ENVIRONMENTAL PROFILE OF MATERIALS

Materials are the primary indicator of a structure's behavior, so establishing the materiality early on into design is essential. Material choices involve more than just structural performance. Environmentally responsible buildings require their design, construction, and operation to minimize the carbon footprint that causes global climate change. A building's material is also connected to ethical, economic, aesthetic, and technical considerations.

To Discuss: Compare and contrast the environmental profiles of the three most common structural materials: Timber, steel, and concrete. By what standards *should* materials be evaluated? Structural capacity? Longevity/durability? Embedded Energy? By what standards *are* they evaluated/discussed (and by whom)?

Timber Structures

Wood is the only primary building material that comes from a renewable resource and that naturally cleans the air in its natural state. We use nearly 100% of harvest wood as product. Its manufacturing has the lowest embodied energy and the least air and water emissions, and is reusable, recyclable, and biodegradable. Wood is a more effective insulator than steel, masonry, or concrete. And it is an affordable,



Figures 3.1.2a and 3.1.2b Options for heavy timber framing systems and structural information about wood

accessible, and versatile material that can be made into different types of structural and non-structural elements used in buildings.

Structurally, wood has a very good *specific-strength* (strength-to-weight ratio), which means that it's lightweight and strong. Being lighter has many benefits. Wood structures have less overall self-weight meaning less material used and smaller foundations. Lightweight materials are also easier to transport and construct. Wood is easy to work with because it doesn't require as much specialized construction training or assembly as concrete or steel. (Figure 3.1.2)

But wood has structural limitations. The natural inhomogeneous qualities of wood, particularly its grain and inherent imperfections, give it unique structural properties (it is markedly weaker in resisting stresses perpendicular to its grain). Timber's use is limited in some applications either because wood isn't as strong or stiff (compared to steel or concrete) and/or because of code restrictions (wood is combustible, even though mass timber burns slowly and predictably). Not all wood products have the same structural capacity or grade. They vary by species, density, moisture content, slope of grain, and percentage of knots and fissures. Each wood species and grade are tested to determine their allowable stress value under different conditions: Compression, tension, bending, and shear.

The most common concerns about using timber center on three topics: Combustibility, durability, and sustainability of timber as a resource (e.g., "won't it burn, rot, and deplete the forests?"). Timber is combustible but it burns slowly and predictably, maintaining structural strength for a relatively long time (thus the "heavy timber" code classification). Not all mass timber products need to be fully protected from moisture; glue laminated and heavy timber members can be made from a naturally durable species of heartwood or pressure-treated to ensure durability in exterior conditions. Finally, because of responsible harvesting, net reserves in timber have increased for decades in the U.S. with timber growth exceeding harvesting. With newer mass timber structural elements, old growth trees don't need to be cut down and harvested for long span options.

Timber's superior ecological environmental profile, its availability through locally-sourced manufacturers in Oregon, robustness and warmth, and pervasive use in the objects displayed within the building, all make

Figure 3.1.3 Examples of mass timber elements. Shown: Olver Design Building, University of Massachusetts Amherst, 2017

it an obvious choice. Before we begin, we'll need to collect structural information about mass timber options: span tables, available cross-sections, allowable stresses, stiffness, etc. (Figure 3.1.3)

Structure and Experience: Initial Ideas

We start with an unresolved diagram of two floors stacked atop each other. How can we evolve this into a more refined proposal for a framework? Purposeful structural design should consider technical and experiential qualities together. There are simultaneous implications for where structural elements are placed because they are both load-bearing and space-defining elements. Looking at different options for how spanning and support elements can be arranged will allow us to assess the qualities of the spaces that result from these decisions—even in the broadest terms. We don't need to know any specific sizes to assess these broader options. (Figure 3.1.4)

Structural Grids

Once a general approach is selected, we can diagram an organizational strategy by establishing grid lines. Grids establish the patterns and proportions for framing (orthogonal, radial, or irregular), and they define the outer boundaries of space. Grid crossings usually indicate the presence of a vertical support (e.g., column or wall). The patterns created by these grid lines are called "structural bays." These bays define a unit within which the structural forces will be resolved. Like any pattern, grids can be broken or abstracted. They can be altered with different spacing or volumes to emphasize compositional or functional hierarchies within a building. Cecil Balmond succinctly describes grids as "a template for action." (Figure 3.1.5)

Grids are usually labeled with numbers along one axis and letters along the other (e.g., 1, 2, 3 and A, B, C). Grid line intersections define the points where loads are transferred vertically, so they are drawn at the center line of the load-grounding elements, usually columns.

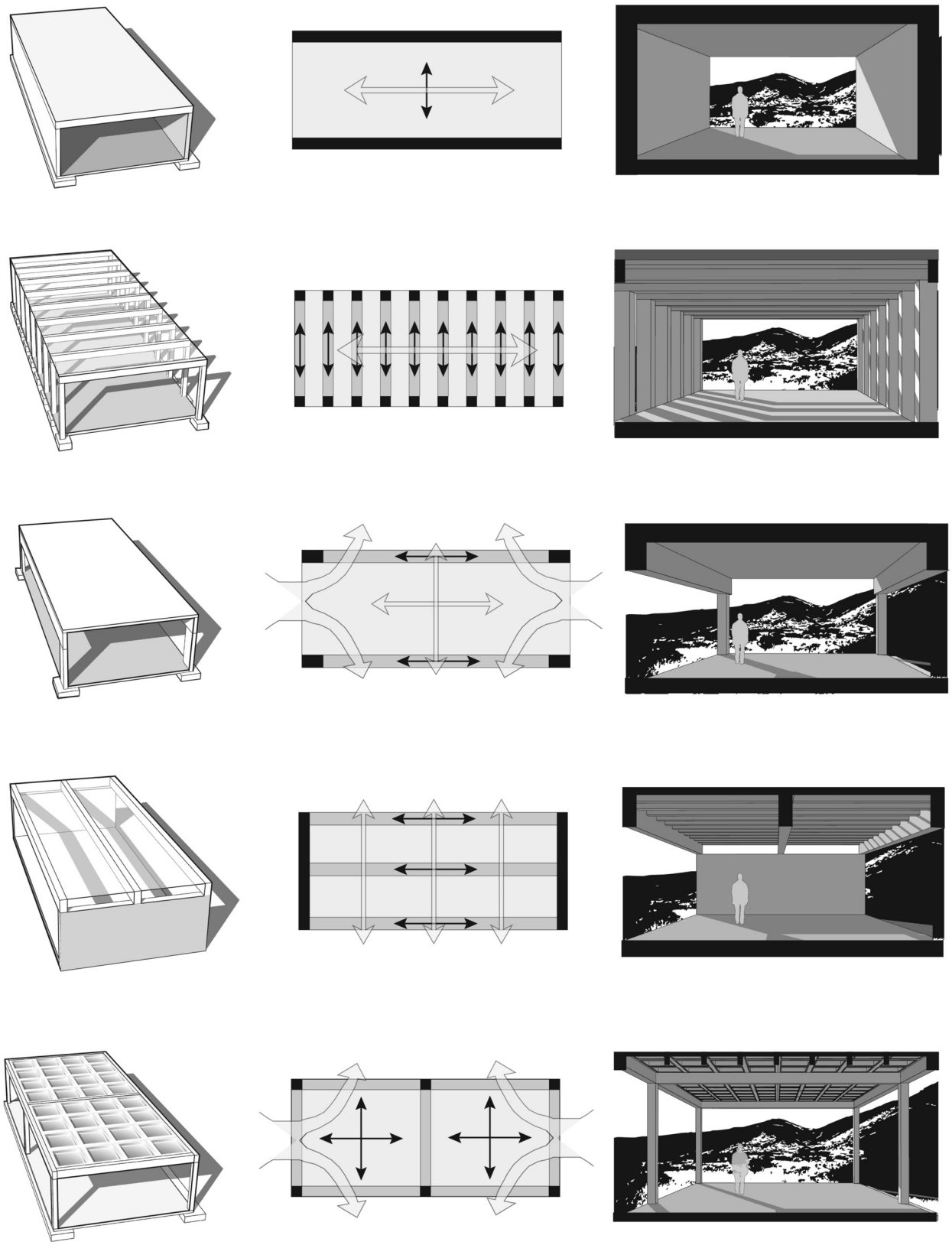


Figure 3.1.4 Selecting structural arrangements must first consider the potential experiential and spatial consequences and benefits

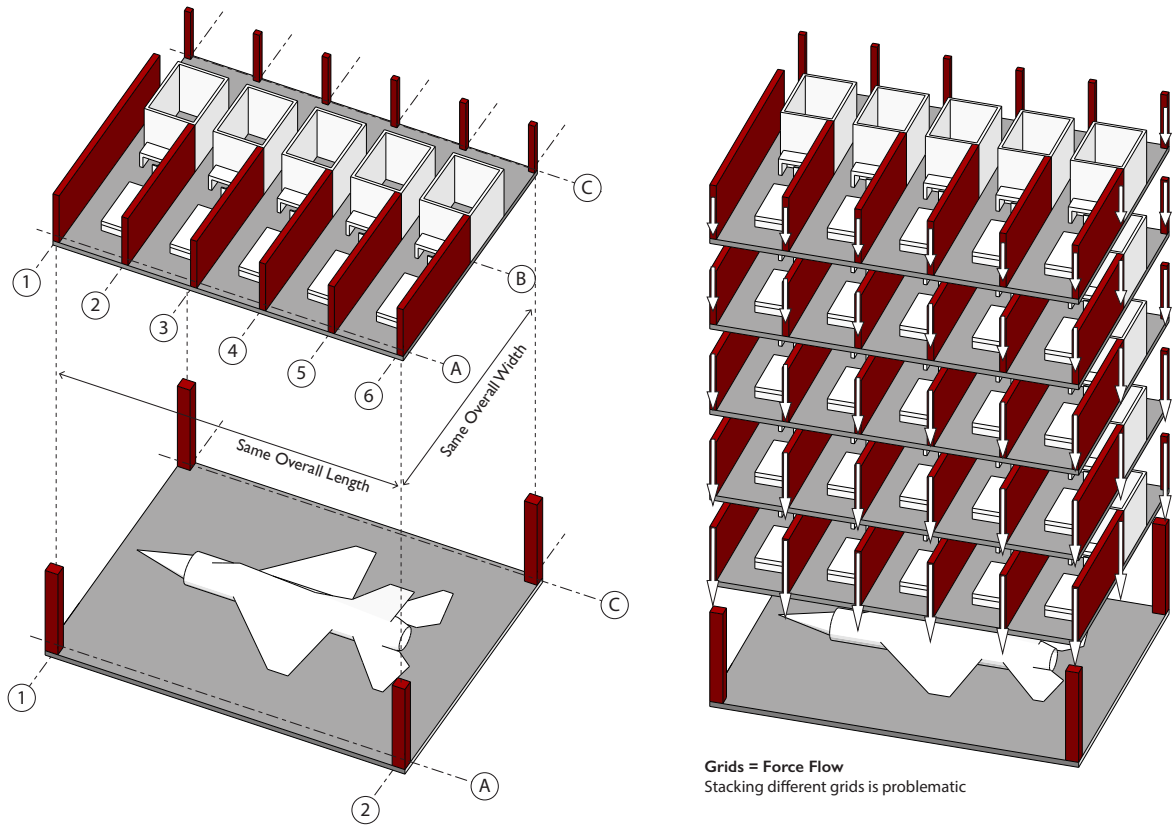


Figure 3.1.5 Grids should correspond with function, aesthetics, and force flow. Both floors shown have the same area, but need different grids; if they were stacked these differences in gravitational force transfer would need to be resolved

As these get larger or smaller (in plan) they are still centered on the grid intersection point so that loads are transferred vertically through their centers. Grid lines become the controlling dimensions for all other building systems, which is why defining the proportions of the grid is the most important decision to disciplined designers—engineers and architects at Skidmore, Owens, and Merrill jokingly claim the firm's name, SOM, means "stay on module." Grids establish bays, which in turn establish module sizes within them—the difference between a 29ft 8⅝in (9m) bay and a 30ft 0in (9.1m) bay isn't much structurally, but it makes a difference if the interior composition is based on a 3ft (0.91m) wide door (or skylight or wall panel system or all the above). Dimensional consistency across multiple bays creates repetition even in non-structural elements, which helps with manufacturing and assembly cost-efficiencies. (Figure 3.1.6)

Force Flow and Framing

Gridlines help us speculate on the size and frequency of the structural modules. We start with the assumption that column and wall supports should be located along the perimeter to avoid conflicts with interior spaces; Grid lines on the north are centered on the wall and across the south they are centered on the end of our "box." Perpendicular column lines are trickier because they establish forces' flow patterns through the framing and how spaces are defined by the structure.

There is an inextricable relationship between loading, span lengths, the bending moments these conditions produce, and the resulting required mass of the resisting elements. Closely spaced grid modules reduce span and member depth, but they add more supports, beams, and connections. Intuitively this makes sense.

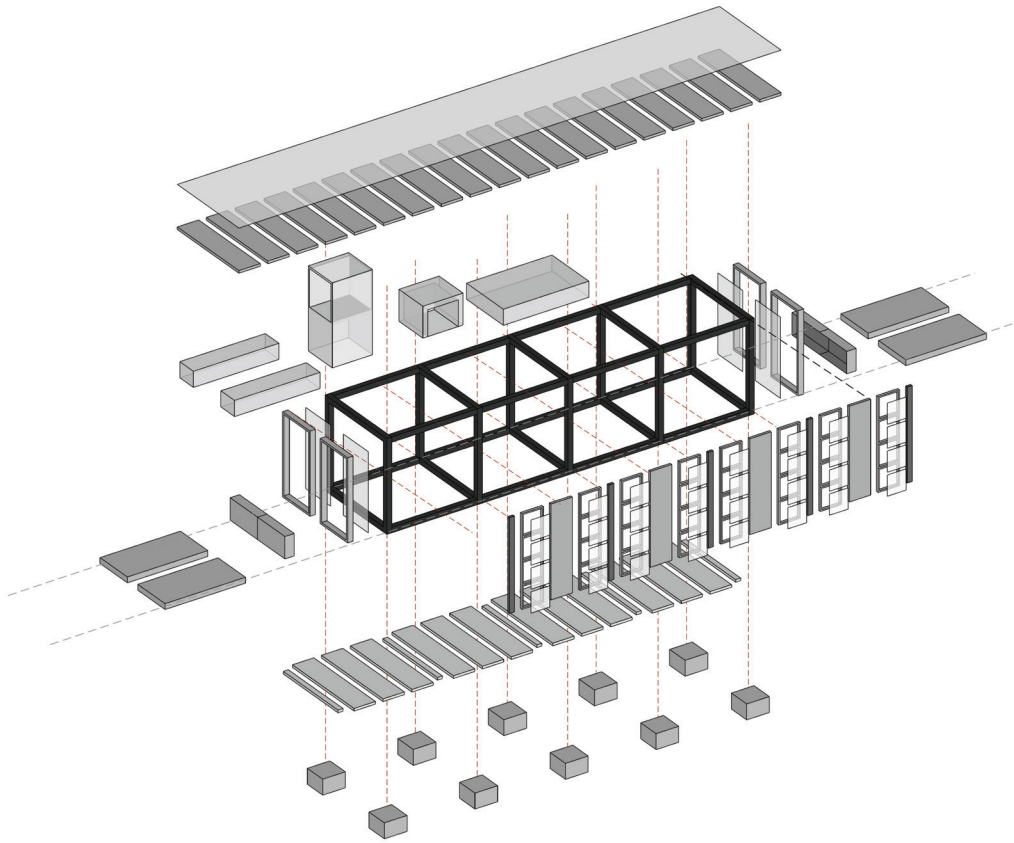


Figure 3.1.6 The composition of a multitude of architectural elements can (and should) be coordinated with the grid. Diligent designers will establish grid dimensions by considering more than structural issues

Elements with longer spans and/or fewer columns will have more load to carry (higher tributary area), thus higher bending moments, and more deflection/sag than structural bays with shorter spans and/or more supports. Start with rules of thumb for column spacing, which will vary by material, but generally for timber would range between 15ft and 40ft distance between column supports.

Once a structural bay size is established, we still have choices as to how to direct the flow of forces within that bay, mostly by changing the arrangement and spacing of supporting elements. We can stack bays on top of one another or place them side-by-side to create a larger structural framing system. This is the primary design benefit of section-resistant systems: If we get one bay of the framing well resolved, the other structural design choices will be easier to evaluate by repetition. (Figure 3.1.7)

Drawing the framing in 3D shows how greater spacing between elements results in higher loads, and thus larger supporting elements, based on *tributary area*. Load transfer in a linear direction, along the length of the spanning member, is called a *one-way framing system*. We can work layer by layer, diagramming these forces like flowing water. In doing so, we see how the forces flow from various load collectors until they consolidate at the load-grounding columns/walls where they are transferred to the ground. (Figure 3.1.8)

"Load collector" is a general term that includes structural elements such as decking, purlins, and girders—we'll resist calling them all "beams" from here on out in lieu of more descriptive terms. These rely upon their cross-sectional shape to provide resistance to bending. As the loads increase, the successive members get bigger, like larger pipes collecting more flow.

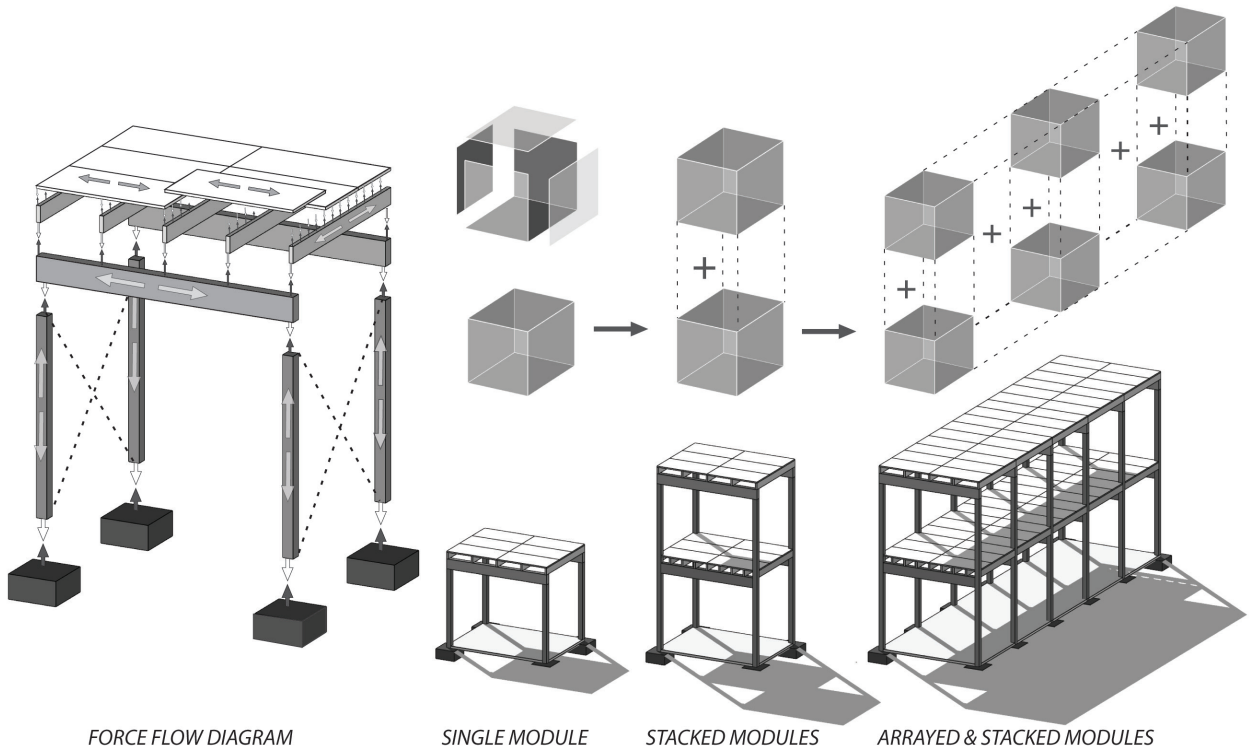
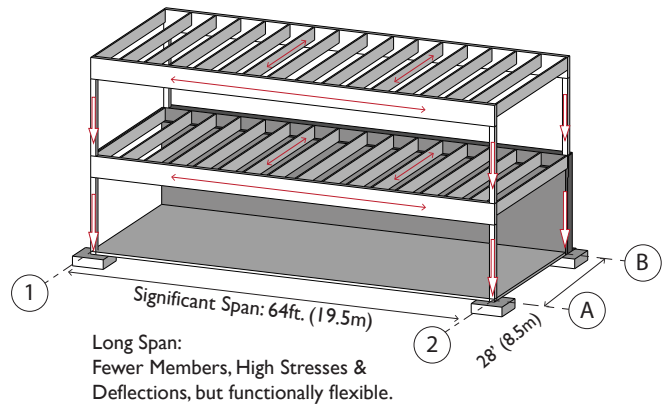


Figure 3.1.7 A single bay can become the organizational module for a larger structure. One bay, once resolved, can be stacked and arrayed



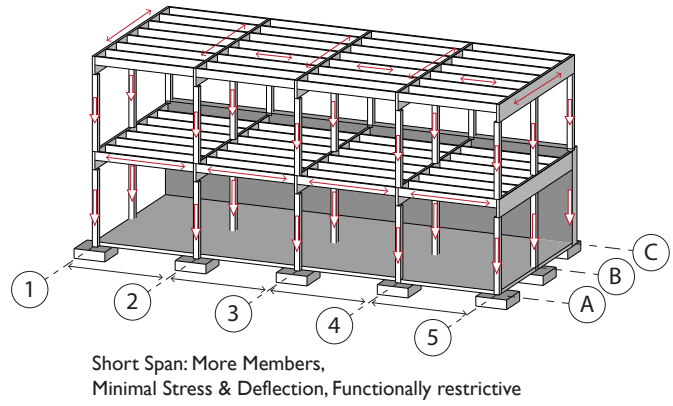


Figure 3.1.8 Two opposing framing options with corresponding consequences: one spans too far, the other (perhaps) not far enough

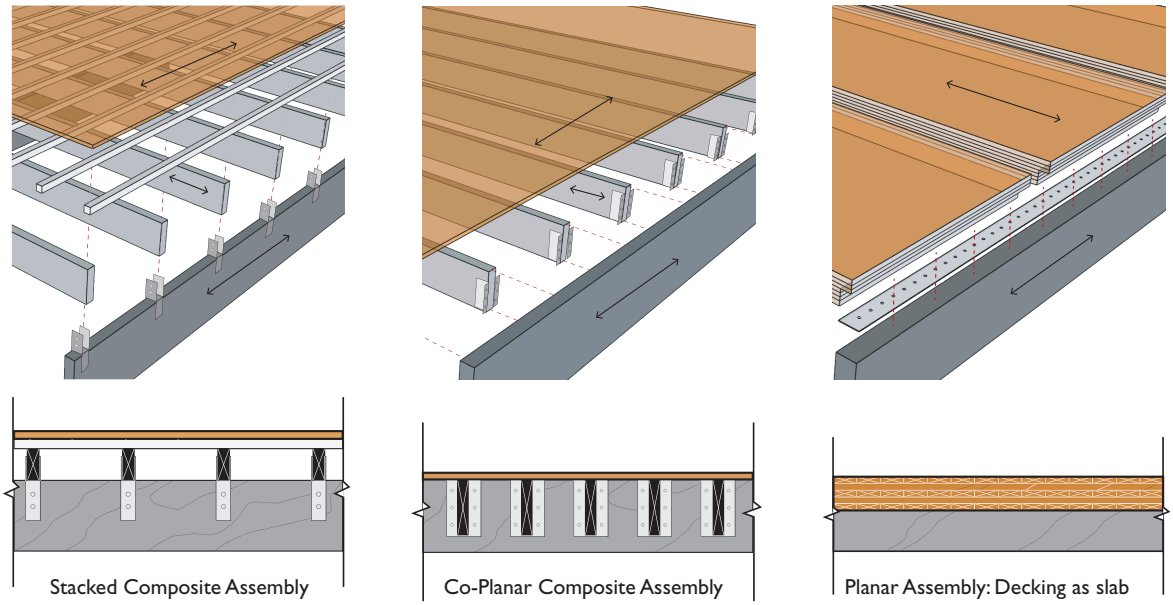


Figure 3.1.9 Examples of different composite floor assemblies

The load collectors at the highest level (floor or roof decking) usually span the least distance—accordingly, they typically have the least depth. Generally, as we work our way down the structure, each successive layer is spaced less frequently, spans farther, and gets deeper. Eventually one of the load collectors, typically called a *girder*, collects loads from the *purlins* above and transmits these into the load grounders (columns or walls). All the individual parts work together as a composite assembly.

The elements in a composite system are interrelated, so changes in loading or spanning length of one element affects another. For example, if we wanted to use the smallest decking possible in both schemes (such as plywood) we'd have to space the purlins closely together because plywood can only span short lengths. Alternatively, if we amend the schemes we've drawn by integrating decking that could span twice as far, then we'd double the distance between the purlins, and thus double the load each purlin has to resist). If we use decking that is deep enough, with enough load-bearing capacity, like cross-laminated timber (CLT) we could forgo purlins altogether and simply span the decking from girder to girder. (Figure 3.1.9)

THINKING: STRATEGIES AND SCHEMATICS - EXHIBIT TABLE, PART I

Sketch out designs for a wooden exhibit table to be located on the lower level. The exhibit tables' goal is to be thin, open, and elemental. The tables are 18ft long (5.5m) and 4ft wide (1.22m) to allow for multiple people to gather at or around each table at the same time to view the exhibits. Supports should only be provided on the ends to allow for participants to sit at the tables (use pinned supports). Some exhibits on the table will be small, while others may be massive and heavy.

Activities: What strategies would you use to start designing the table tops and supports? What information about loading and materials would you need? What tools would you need to draw or test it? Make a list. Work through initial sketches to develop the designs. (Figure 3.1.10)

(continued)

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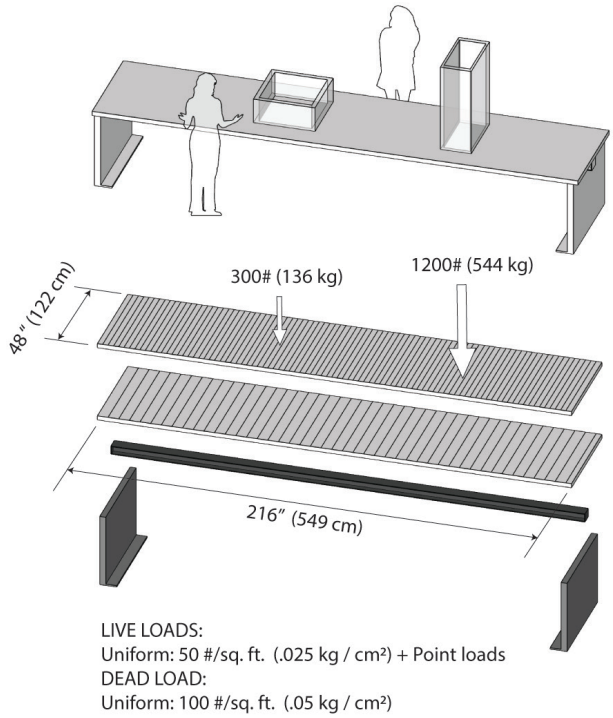


Figure 3.1.10 Sketch of exhibit tables framing

Load Collectors, Layout and Assessment Options

There are a few ways to begin assessing different schemes: One is to look at the consequences of the spans, sizing, and layout; a *qualitative* assessment. Another involves the use of more *quantitative* data to confirm these assumptions and sizes. We'll do both.

We can generate two basic framing options and compare them side-by-side: Option #1A has support girders running *parallel* to the long sides of the building supported by only a few columns (four bays), Option #1B has girders running *perpendicular* with shorter spans (eight bays). Our goal is to test these initial choices, establish basic sizing of the elements, determine compliance (or failure) of what we've designed, and suggest changes to improve the proposal. (Figure 3.1.11)

Option #1A has fewer elements, each of which will need to resist more load. Fewer columns and a larger bay size results in girders spanning 32ft (9.76m) on-center and purlins spanning the width of the space (approximately 28ft (8.53m)). Glue laminated beams could span these distances with deep members, but do we need to span this far? The increased spacing between elements was done out of a desire to have fewer elements and to be more efficient, but our efficiency would drop as we fight against ramifications of increasing the span length, especially for purlins.

In Option #1B, since the columns are spaced half as far apart, we've switched the orientation of the girder framing and purlins. This reduces the span of the purlins and the girders, creating smaller bending moments, less deflection, and shallower spanning members. Further, it provides a sound organizational proportion for the building design. Even if we keep the decking and purlin spacing the same for both options, Option #1A will have higher bending moments in its purlins and girders.

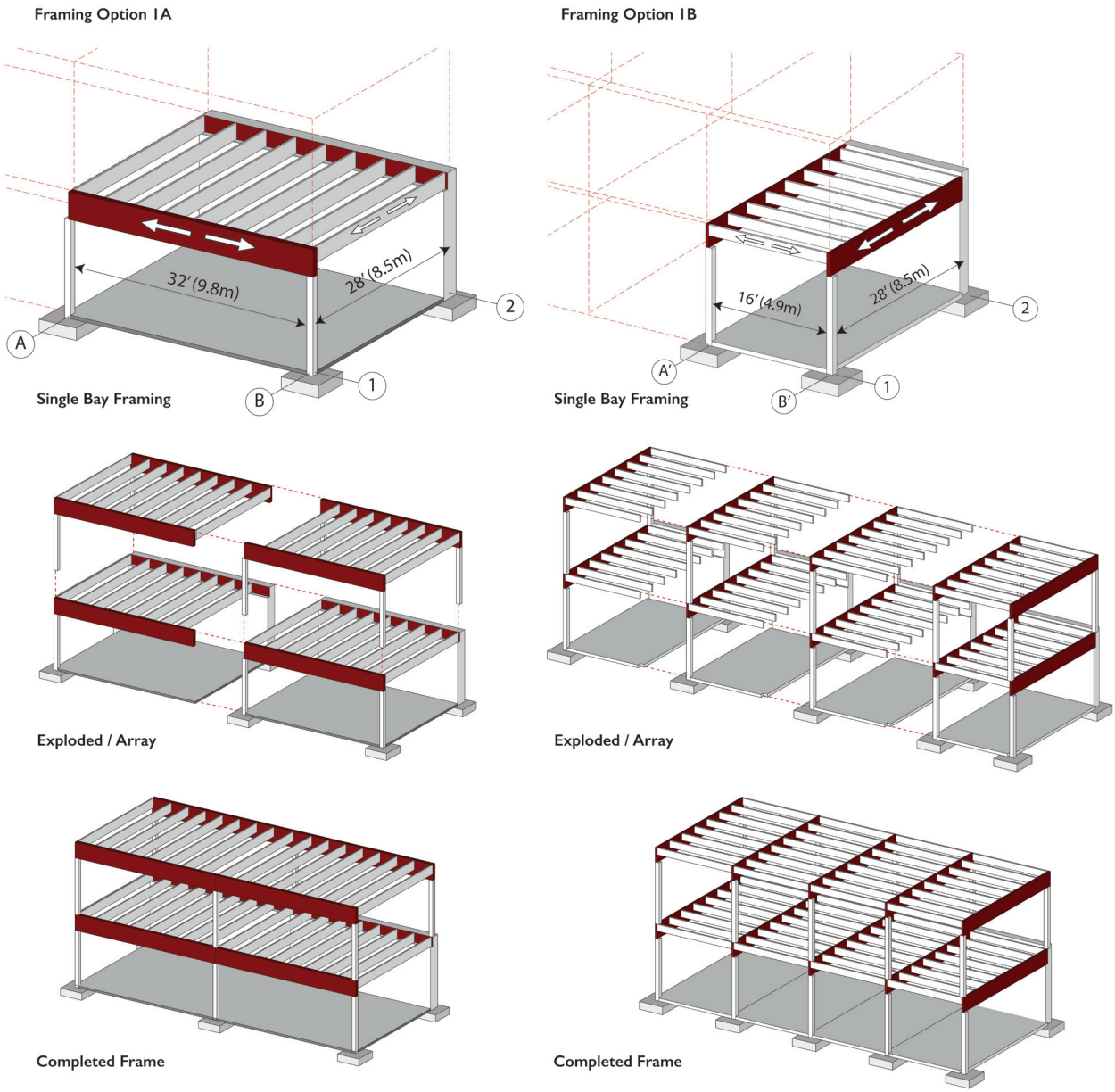


Figure 3.1.11 Two framing options. Axonometric drawing of framing, plans with tributary areas for girders and purlins, and a graphic summary of materials are shown for each

To compare these schemes beyond what these schematic drawings tell us, we'll also need a rigorous approach to the calculations and diagrams we use to analyze the work. Much of this analysis will be done with short-cuts, rules of thumb, and annotated graphics. These are not calculations that are ready for permitting or construction—they are schematic evaluations that help us understand what the design could become. (Figure 3.1.12)

Timber elements are available in numerous sizes and spans. One of their benefits is that they can be milled and manufactured in different sized and shaped elements (e.g., plywood sheets, thick decking, 2x light framing, glue-laminated and/or cross-laminated spanning elements). Our structure is too large for conventional 2x wood framing, so we will explore mass timber options including: glue-laminated timber and cross-laminated (CLT) elements.

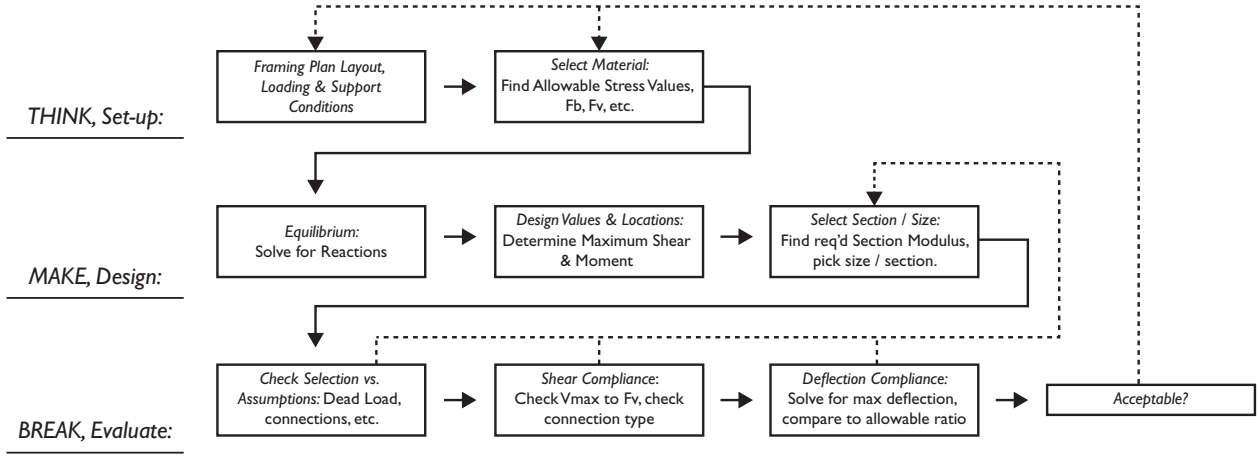


Figure 3.1.12 Workflow for designing and testing section-resistant framing

Using the framing drawings that we've already generated, we can use short-cuts to find tributary area, maximum moment values, and deflection values. We'd expect to see larger purlins in Option #1A than in Option #1B. (Figure 3.1.13)

Part I: Equilibrium and Loads

To determine how much support is needed at each reaction (support point), we need to add all the vertical forces: the dead load (e.g., an estimate of the

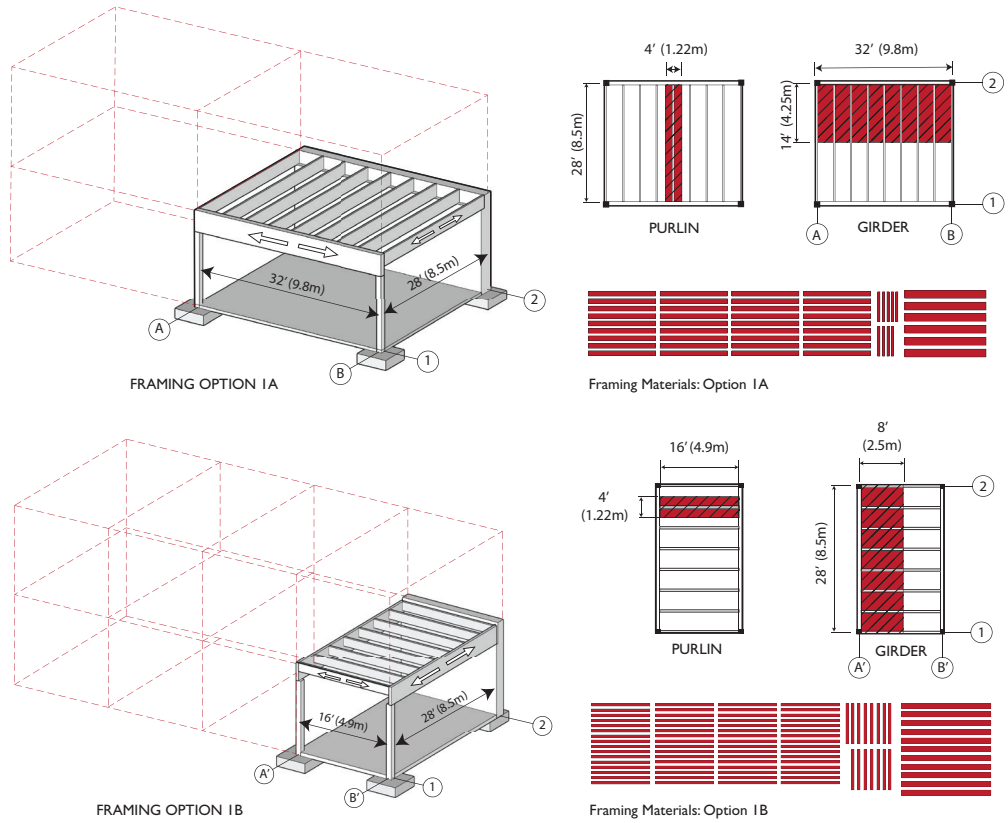


Figure 3.1.13 Side-by-side comparison of schemes Option #1A and Option #1B showing the single bay, total number of bays, and a view of the completed proposal

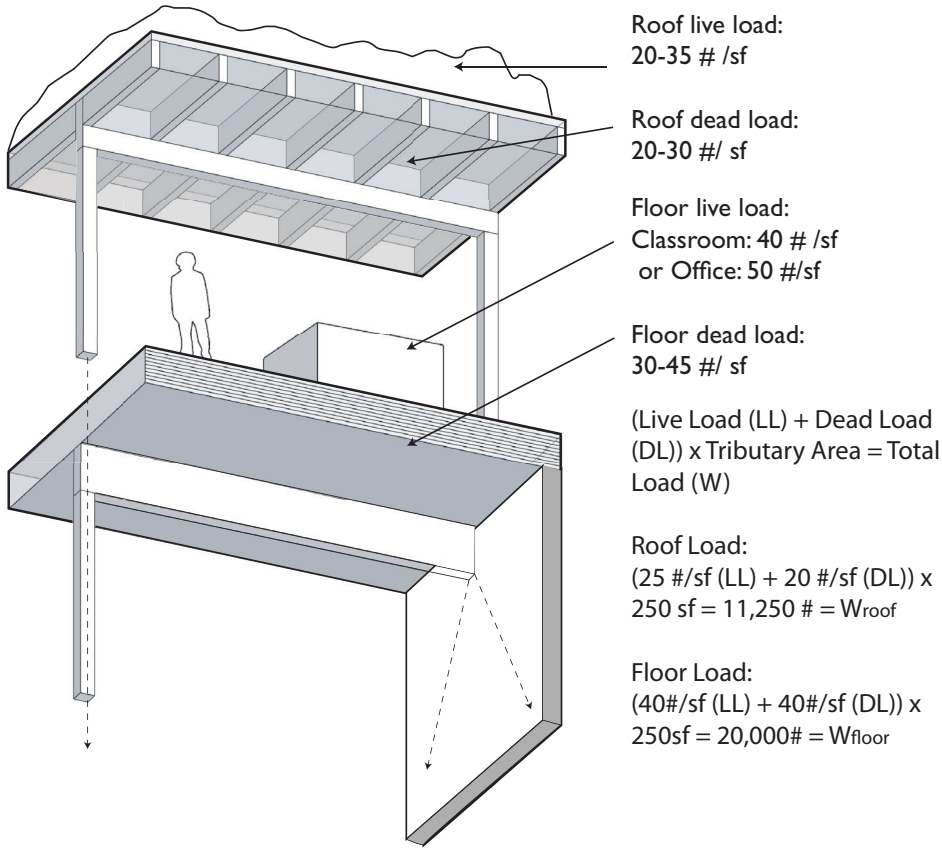


Figure 3.1.14 Three-dimensional drawing of tributary load anticipated for one bay of framing

structure's weight) and live loads (rain, snow, etc.). Although the tributary area will vary between the two options, local building codes will give us requirements for the live and dead load factors (including the factors of safety). The loads shown in the example are estimates only. The total load is the product of these loads times the *tributary area* being supported.

There are two common designations for weight in uniformly loaded beams, "W" (the capital letter) is the total load being supported by a beam (in lbs, kg, etc.), while "w" (lowercase) is the total load per unit of span (e.g., pounds per linear foot or kg/m). We'll assume that all loads are distributed loads and all connections are pinned connections, so that each support point will carry half the total load. It is common to designate these reactions as R_1 and R_2 or R_L and R_R. If the loads are asymmetrical in magnitude and/or location, the reactions will have different values and can be determined using the sum of moments around a point process (Chapter 3.0). (Figure 3.1.14)

See Appendix Tables A.1–A.3 for information about live and dead loading estimates for various construction assemblies and occupancies.

Part II: Maximum Shear and Moment Values (Diagrams or Shortcuts)

Once we know the magnitude of reaction forces (R_L & R_R), we can draw free-body diagrams of shear and moment forces to visualize where the maximum stresses occur along a beam (see Figure 3.0.32 for examples).

Shear (V) Diagrams: Maximum shear tells us where the beam is under the greatest threat to have fibers slide past each other (or shear)—this maximum value always occurs at the supports in a simple beam. Imagine the shear diagram as a map of the up and down forces between the reaction supports (pushing up) and the applied loads (pushing down). The value

of shear, and how it is represented in a diagram, will vary across a beam based on the type of loading (point loads versus distributed loads). In our scheme (working from left to right), the shear value starts at its highest value at the support, drops at a constant rate (equivalent to the uniform load) until it reaches the reaction support where it returns back to zero. There is no difference in the beam's behavior between positive or negative shear values.

Beams that are sized to resist bending and deflection are usually large enough to also resist shear, but there are a few exceptions that should always be checked for compliance. Vertical shear is a controlling design concern in deep, slender members, with short spans that are subjected to high levels of loading. These stresses can be exacerbated if notches or holes are cut into the end supports as these reduce area of resistance and create fracture points.

Moment (M) diagrams: Moment diagrams show the intensity of bending. Before drawing the moment diagram, ask yourself where you'd expect the highest bending moment to occur. You'll likely guess that it will happen at the heaviest point load in an area that's farthest away from the supports—which is correct. Moment values reflect the amount of shear forces multiplied by their distance from the supports. Therefore, we can mathematically connect the shear diagram (vertical values and length) with the resulting moment diagram. The pinned connections at the supports cannot take any moment forces, so our moment diagrams will begin and end at zero. The moment forces will rise and fall based on the magnitude and distance of shear forces as a straight or curved line (think of them as mathematical integrals). Where the shear diagram passes through zero, and its values become negative, the direction of the moment diagram curve changes. When a moment diagram becomes negative it signifies an important behavioral difference with the beam, namely that the beam is bending upwards instead of downwards. In areas with negative moment, such as in cantilevers, compression is at the bottom and tension is along the top.

Shear/moment diagrams can become cumbersome for schematic design. If all we are *really* trying to determine are the maximum values for shear and moment and where those occur on the beam, there is an easier way. Because these values are mathematical expressions of loading and support, when those conditions are standard, regular, or otherwise common (e.g., a simply supported beam with a uniformly distributed load), the mathematical solution for these values can also be standardized. All simply supported beams with uniformly distributed loads, no matter the span, can follow the same formula to determine reactions, maximum shear, and maximum moment.

See Appendix Figures A.5 and A.6 for examples of shear and moment diagrams for a particular condition.

We can use the short-cut formulas to determine maximum shear and bending moment for uniformly loaded beams: $\boldsymbol{V_{max} = W/2}$ **and** $\boldsymbol{M_{max} = W * L^2 / 8}$ and we can plot the diagrams accordingly (or again use short-cuts). We see that the maximum shear is at the supports and the maximum moment occurs in the middle of the span (gradually increasing from the supports to the middle). (Figure 3.1.15)

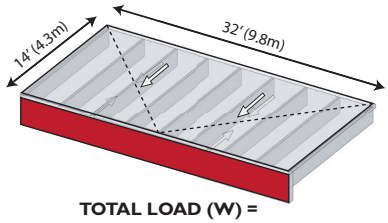
Part III: Beam Sizing/Compliance

Before we run calculations, it is helpful to use rules of thumb for estimating depths of purlins and girders to make sure our answer seems realistic. As a reminder, start with the length of the span, **L**, in inches or centimetres. For glulam members, purlins are **L/20**, girders are **L/15**.

To design a beam in detail, we'll need to know two of these three values: Maximum bending moment, **M**, section modulus, **S**, and allowable bending stress, $\mathbf{f_b}$ so that we can use the flexure formula: $\boldsymbol{M = f_b * S}$ (or $f_b = M/S$). The material for glulam and CLT products is structural timber, No. 1 grade pine with an allowable bending stress of 1,550lbs/in^2 (109 kg/cm^2). From our short-cuts and diagrams, we can find the maximum bending moment (in pound*inches or kilograms*centimetres). From there we can solve for the section modulus, **S**. Because we are using a solid rectangular cross-section for our wooden girder, the section modulus will be: $\mathbf{S = b * d^2 / 6}$ (in^3 or cm^3), b is the width and d is the depth. Before all the factors are equated together, we should make sure these units all match (inches or centimetres). We will use our estimated depth and available product data to find a width and depth that will suffice.

See Appendix Table A.9 for section modulus values for various heavy timber elements.

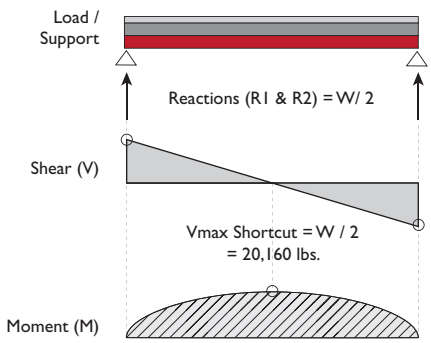
OPTION 1A: GIRDER SIZING



TOTAL LOAD (W) =
(LIVE LOAD + DEAD LOAD) * TOTAL AREA (A)

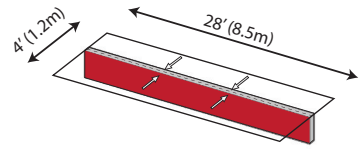
Dead Load (DL) = 40 lbs. / ft.^{2} & Live Load (LL) = 50 lb / ft.^{2}
Trib. Area (A) of Purlin: (14 ft. x 32 ft.) = 448 ft.^{2}

Total Load (W) = 40,320 lbs.



M max Shortcut = WL / 8 =
= 1,935,360 lbs. in.

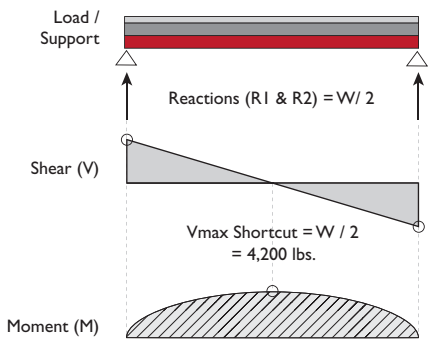
OPTION 1A: PURLIN SIZING



TOTAL LOAD (W) =
(LIVE LOAD + DEAD LOAD) * TOTAL AREA (A)

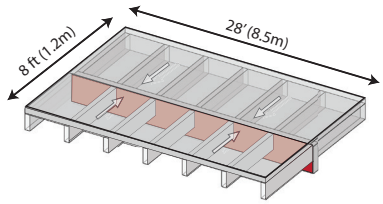
Dead Load (DL) = 25 lbs. / ft.^{2} & Live Load (LL) = 50 lb / ft.^{2}
Trib. Area (A) of Purlin: (4 ft. x 28 ft.) = 112 ft.^{2}

Total Load (W) = 8,400 lbs.



M max Shortcut = WL / 8 =
= 352,800 lbs. in.

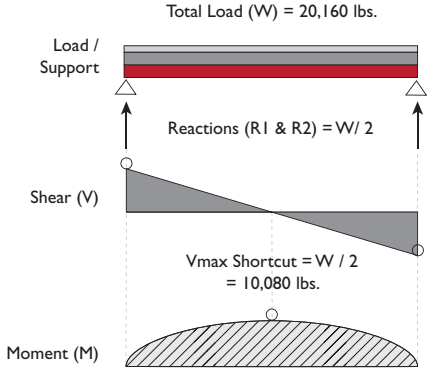
OPTION 1B: GIRDER SIZING



TOTAL LOAD (W) =
(LIVE LOAD + DEAD LOAD) * TOTAL AREA (A)

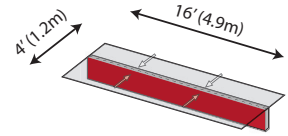
Dead Load (DL) = 40 lbs. / ft.^{2} & Live Load (LL) = 50 lb / ft.^{2}
Trib. Area (A) of Purlin: (8 ft. x 28 ft.) = 224 ft.^{2}

Total Load (W) = 20,160 lbs.



M max Shortcut = WL / 8 =
= 846,720 lbs. in.

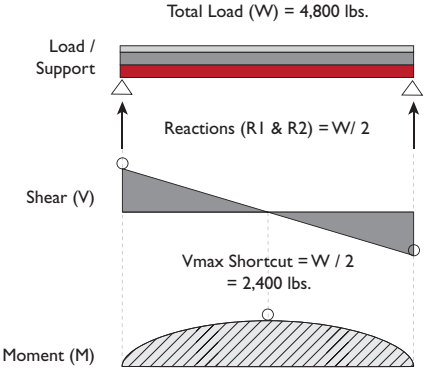
OPTION 1B: PURLIN SIZING



TOTAL LOAD (W) =
(LIVE LOAD + DEAD LOAD) * TOTAL AREA (A)

Dead Load (DL) = 25 lbs. / ft.^{2} & Live Load (LL) = 50 lb / ft.^{2}
Trib. Area (A) of Purlin: (4 ft. x 16 ft.) = 64 ft.^{2}

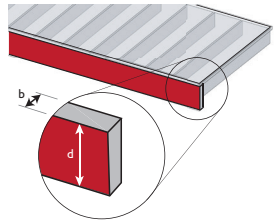
Total Load (W) = 4,800 lbs.



M max Shortcut = WL / 8 =
= 115,200 lbs. in.

Figures 3.1.15a and 3.1.15b Loading calculations, shear and moment diagrams for Option #1A and Option #1B purlins and girders

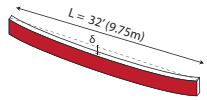
ESTIMATE SIZE OF FRAMING, OPTION 1A GIRDER:
RULE OF THUMB, GLULAM PURLIN: L / 15
= 32 ft. * (12 in./ft.) / 15 = 25.6 in. (.65m)



CALCULATE SIZING:
Section Modulus (S) =
Maximum Moment (M) / Allowable Bending Stress (fb)
=1,935,360 lbs. in. / 1,550 lbs. / in^2
S = 1,249 in^3

SECTION MODULUS (S) = b * d^2 / 6
Select depth (d) from available sizes and solve for width (b)
-OR-
Look for Section Modulus (S) properties of standard sized elements:

8 x 32 = 1,365 in^3
10 X 28 = 1,300 in^3 ***(Best choice)***
12 x 26 = 1,352 in^3

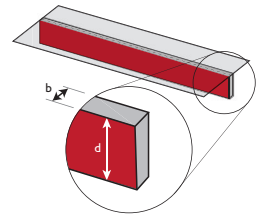


CHECK DEFLECTION (Δ δ) = 5 *W * L^3 / 384 * E * I
I = b * d^3 / 12 (Use 10 x 28 member)

= 5 * 40,320 lbs. * (32 ft. * 12 in. / ft)^3 /
384 * 1,600,000 lb. / in^2 * 18,293 in^4
= 1.0 in
Deflection Allowable: L / 240 = 1.4 in.

10 x 28 Glulam Girder is acceptable!

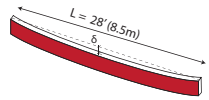
ESTIMATE SIZE OF FRAMING, OPTION 1A PURLIN:
RULE OF THUMB, TIMBER PURLIN: L / 20
= 28 ft. * (12 in./ft.) / 20 = 16.8 in. (.42m)



CALCULATE SIZING:
Section Modulus (S) =
Maximum Moment (M) / Allowable Bending Stress (fb)
=423,360 lbs. in. / 1,550 lbs. / in^2
S = 273 in^3

SECTION MODULUS (S) = b * d^2 / 6
Select depth (d) from available sizes and solve for width (b)
-OR-
Look for Section Modulus (S) properties of standard sized elements:

6 x 20 = 400 in^3
6 x 18 = 324 in^3 ***(Best choice)***
6 x 16 = 256 in^3

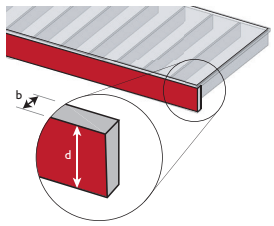


CHECK DEFLECTION (Δ δ) = 5 *W * L^3 / 384 * E * I
I = b * d^3 / 12 (Use 6 x 18 member)

= 5 * 10,080 lbs. * (28 ft. * 12 in. / ft)^3 /
384 * 1,600,000 lb. / in^2 * 2,916 in^4
= 1.06 in
Deflection Allowable: L / 240 = 1.4 in.

6 X 18 Glulam Purlin is acceptable!

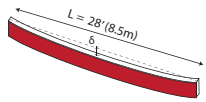
ESTIMATE SIZE OF FRAMING, OPTION 1B GIRDER:
RULE OF THUMB, TIMBER PURLIN: L / 15
= 28 ft. * (12 in./ft.) / 15 = 22.5 in. (.57m)



CALCULATE SIZING:
Section Modulus (S) =
Maximum Moment (M) / Allowable Bending Stress (fb)
=846,720 lbs. in. / 1,550 lbs. / in^2
S = 546 in^3

SECTION MODULUS (S) = b * d^2 / 6
Select depth (d) from available sizes and solve for width (b)
-OR-
Look for Section Modulus (S) properties of standard sized elements:

6 x 30 = 900 in^3
6 x 24 = 576 in^3 ***(Best choice)***
8 x 20 = 533 in^3

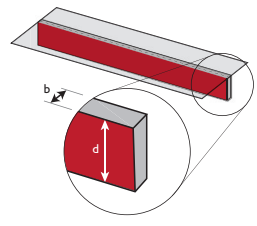


CHECK DEFLECTION (Δ δ) = 5 *W * L^3 / 384 * E * I
I = b * d^3 / 12 (Use 6 x 24 member)

= 5 * 20,160 lbs. * (28 ft. * 12 in. / ft)^3 /
384 * 1,600,000 lb. / in^2 * 6,912 in^4
= .9 in
Deflection Allowable: L / 240 = 1.4 in.

6 X 24 Girder is acceptable!

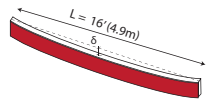
ESTIMATE SIZE OF FRAMING, OPTION 1B PURLIN:
RULE OF THUMB, TIMBER PURLIN: L / 20
= 16 ft. * (12 in./ft.) / 20 = 9.6 in. (.25m)



CALCULATE SIZING:
Section Modulus (S) =
Maximum Moment (M) / Allowable Bending Stress (fb)
=115,200 lbs. in. / 1,550 lbs. / in^2
S = 74 in^3

SECTION MODULUS (S) = b * d^2 / 6
Select depth (d) from available sizes and solve for width (b)
-OR-
Look for Section Modulus (S) properties of standard sized elements:

4 x 14 = 131 in^3
4 X 12 = 96 in^3 ***(Best choice)***
6 x 10 = 100 in^3



CHECK DEFLECTION (Δ δ) = 5 *W * L^3 / 384 * E * I
I = b * d^3 / 12 (Use 4 x 12 member)

= 5 * 4,800 lbs. * (16 ft. * 12 in. / ft)^3 /
384 * 1,600,000 lb. / in^2 * 576 in^4
= .48 in
Deflection Allowable: L / 240 = 1.4 in.

4 X 12 Purlin is acceptable!

Figures 3.1.16a and 3.1.16b Bending and deflection calculations for sizing Option #1A and Option #1B purlins and girders

Part IV: Deflection Compliance

Deflection is the controlling factor for most long span projects; ours is a medium span, but since wood isn't as stiff as other materials like steel or concrete it is more susceptible to bending deflection. We can use the deflection formula, $\Delta\,\delta = 5 * w * l^3 / 384 * E * I$, which combines load, span, material qualities of stiffness, and cross-sectional shape together, to compare our design to a given ratio (*l*//240, *l*//360, etc.) dictated by code or building function. We don't have strict clearances to adhere to or other surfaces that risk cracking under excessive deflection, so we'll compare our value of deflection to *l*//240. Note that length of span is the most influential factor in a beam's deflection. (Figure 3.1.16)

MAKING: MASS TIMBER DEVELOPMENTS

The benefits of heavy timber, improvements in their fabrication techniques, and changes in building codes, have made heavy timber construction a viable option for many mid-to-high rise buildings.

To Do: Find an example of a contemporary building that has recently been constructed using mass timber products—preferably multi-story. Collect information about the structural frame (including the structural bay sizes and the building sectional heights and depths). Find images of the construction process. Explain how the load collectors and load-grounding elements were combined to facilitate force flow.

To Discuss: How did the use of timber affect structural bay size, depths, and construction methods? What were the benefits of timber in these circumstances (structurally, aesthetically, environmentally, etc.). What were the liabilities or challenges? (Figure 3.1.17)

Figure 3.1.17 Mass timber (glulam and CLT) was used for the building frame, floors, walls, and trusses at the Olver Design Building at University of Massachusetts Amherst (Leers Weinzapfel Associates, 2017)

Load Grounders: Columns, Walls, and Layouts

The structural grid's layout and spacing determines the placement and number of vertical supports. In both of our schemes, we have walls and columns as the load-grounding elements (the retaining wall on the north side supports the upper floor).

There are obvious spatial consequences between using walls or columns, and they transmit loads differently, too. Load-bearing walls collect loads from a larger area, resist these loads through their plane of support, and transmit these loads to continuous linear foundations. Resistance and support are shared along the wall's length, so any difference in the magnitude of loads the wall collects won't have visible consequences. If the walls become too tall or thin, they are subject to potential buckling. (Figure 3.1.18)

Columns ground the consolidated loads through smaller, more concentrated cross-sections. Fewer columns means that greater loads will need to be resisted by each column, but the plan will be more open. Timber columns can be singular massive elements or they can be built up as a combination of other smaller pieces. Which option is best will depend on the aesthetic implications and/or the desired structural behaviors and the connection details to girders.

The primary role of load grounders is to transfer vertical loads to foundations. Foundation types will vary depending on soil conditions, amount of load, and type of load grounder. In our case, we will need two types of foundations, one for the wall and another for the columns. For the columns, we could use spread footings (also known as pads) which are essentially square concrete slabs with the columns centered in the middle. We could also use concrete

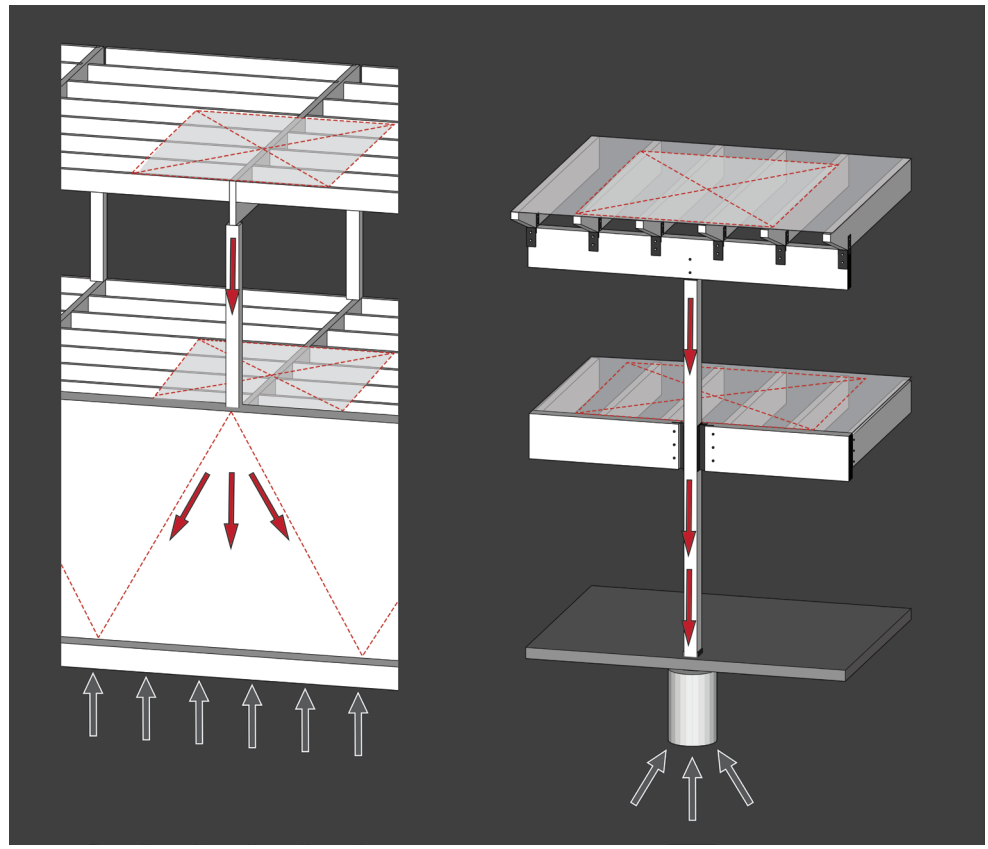
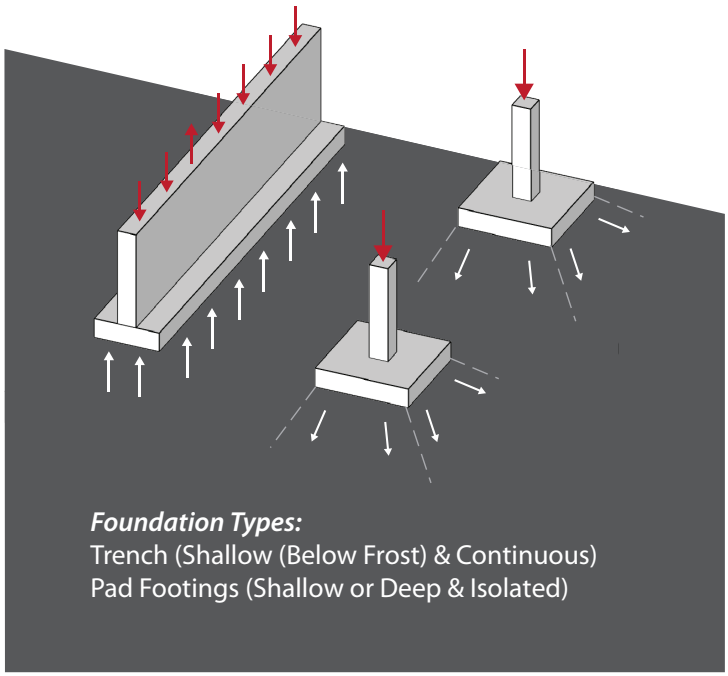
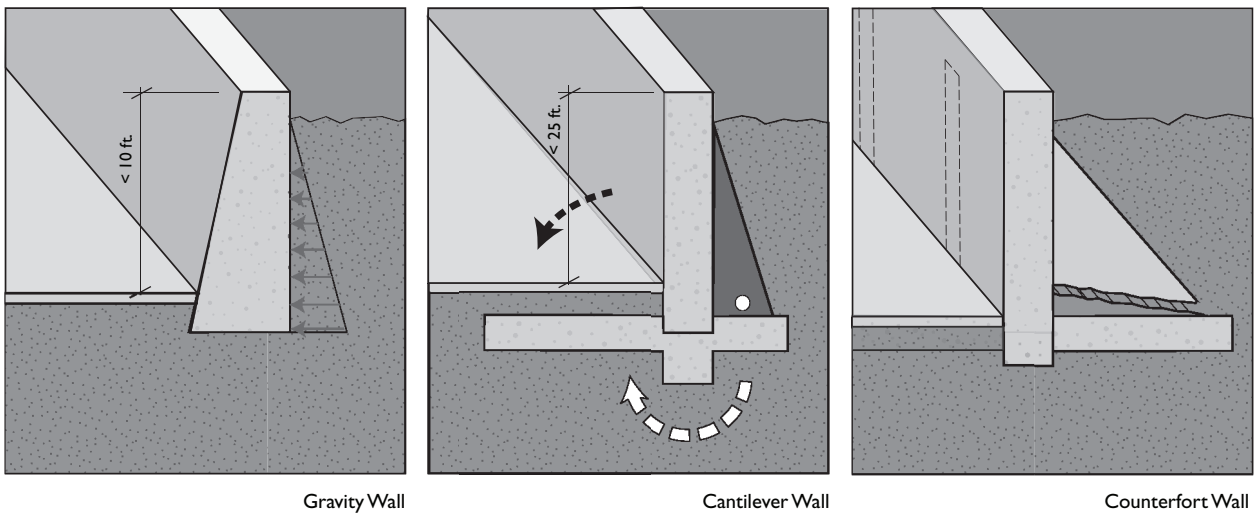


Figure 3.1.18 Concentrated vertical loads transferred into a wall spread out downwardly to the foundation. Columns collect the load, concentrate it vertically along the length, then distribute it across a larger area at the foundation

piers, which are solid cylindrical forms encasing the column and extending below ground, or small caissons (longer cylindrical elements that go down to bedrock). Because the wall is a continuous bearing surface, we'll need a continuous trench footing under its entire length. A retaining wall has special conditions; in addition to vertical loads, it has to resist horizontal loads from soil and water pushing against it from the side. To respond to these loads, retaining walls have specific articulations in their section and foundations.

We need to know what soil type and bearing capacity will support our foundation. Frequently, we place a bed of compacted gravel below the footing to





Figures 3.1.19a and 3.1.19b Our design incorporates two foundation types: Trench (for the wall) and pad footings (for columns). The retaining wall would require additional design as it also holds back the load of the soil

absorb its load without risking a compaction of the soil. Appendix Table A.15 lists soil bearing capacities. We'd need to determine the frost level to make sure our column won't heave upward if/when the soil freezes. Perhaps most importantly for our purposes, we can reduce the length of our columns and make more secure column-to-foundation connections if we embed the columns in the foundation to make "fixed" connections at their base. (Figure 3.1.19)

Behavior and Assessment of Load Grounders

We need to insure vertical continuity between the columns on both floors, so we will use a single column for both floors on the south, since splicing at the mid-point would be difficult, unsightly, and risky. We'll center the columns on the north side within the thickness of the retaining wall. Higher concentrated loads will require larger columns to support the roof and floor, but these columns aren't at risk from compression stress but instead from buckling. We know from Chapter 3.0 that cross-sectional shape (radius of gyration), unbraced length, and connections (k-factor) are the three governing factors of column behavior.

We can change the column's effective length with bracing or connections. In our proposal, we have two-story columns along one side, but we can brace each column at the intermediate floor and embed its base in the foundation. By bracing the columns at mid-length, we've made them less likely to buckle by reducing their length by half. We'll assume fixed connections at the column bases and pinned connections at the tops—therefore our columns will share their resistance to buckling with the foundation and act like shorter columns (k-factor of 0.8). When sizing a column, k-factor is multiplied by the unbraced length to find the *effective length* (or kl). This is the length used in column design charts. As we compile different designs, we can compare our conditions back to the slenderness ratio guidelines and requirements for wood structures (kl/r > 50).

Although the columns in Option #1A carry twice the total load as Option #1B, we shouldn't assume they'll be twice as large. Column design isn't just about compressive resistance to load; it is also about resistance to buckling. Both schemes have columns with the same unbraced length.

Although columns can be sized mathematically to find the maximum loads at which they buckle and the maximum compression load they can carry, we need a more direct answer, namely, what is an acceptable load to avoid *both* buckling and compression? For that, we can use column design charts. These charts list different sizes for the columns in plan, and the corresponding allowable loads based on the columns' effective lengths (their "kl"). (Figure 3.1.20)

Since mass timber columns have limited sizes, we'll choose a solid section with a square shape. Our preliminary calculations indicate that our columns will be between 8in × 8in – 12in × 12in (20 × 20cm – 30.5 × 30.5cm). We could also use a more rectangular shape in plan (with an increased depth in the north/south direction) because our columns are braced from buckling to the east/west at mid-length—perhaps 8in × 12in (20 × 30.5cm). One factor in the size of the columns is providing enough area to accommodate the depth of connections to floor/ceiling framing and the lateral bracing members. See Appendix Table A.11 for timber column sizing options.

Timber Connections and Load Stabilizers

To effectively transfer loads, all the building elements must be connected. Connections are locations where stresses are consolidated and the risk of failure is high. Crafting these connections with a structural strategy is necessary to ensure that beams behave as intended, that loads transfer effectively (from horizontal to vertical), and that shear stresses are adequately resisted. These connections will affect the structural behavior of the elements too (e.g., pinned versus fixed connections). Some of them can provide lateral stability.

Although some wood construction can rely on wood-to-wood connections like dowels, or modest steel fasteners (bolts, screws, lags, etc.), these connections cannot handle high stresses. As such, mass timber structures rely on steel splice plates and connectors to ensure elevated shear resistance.

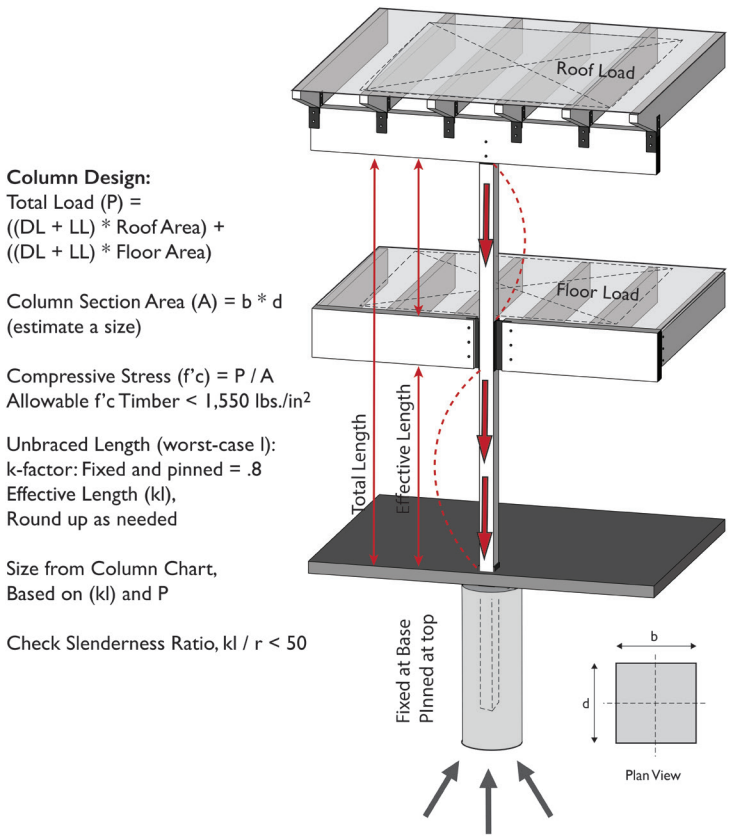


Figure 3.1.20 Column design and calculation factors. Compare loads from Option #1A and Option #IB to find estimated column sizes using column charts for timber

Connections can take various forms, such as exposed hangers or hidden embedded plates, depending on how the members are joined (co-planar, stacked, spliced, etc.). They rely on hardware such as bolts and/or lag screws, to hold large elements together. The choice of connection is an aesthetic and a structural consideration. (Figure 3.1.21)

Designing for Stability: At this point, we haven't learned about different strategies for providing sideways or lateral stability in frames. Chapter 6.0 will familiarize you with how frames can be stabilized. Some wooden pavilions use diagonal knee-braces to make Y-shaped connections between columns and girders to transfer vertical loads and brace the wood frame from moving side-to-side. This aesthetic is a conventional expression found in historic structures (like barns), or "do-it-yourself" pavilions—our challenge was to create a more refined expression so we'll look at using X-bracing or solid walls for bracing instead. Ideally, we will find that the bracing tactic can solve more problems, aesthetic and/or functional, by its inclusion.

Because we have a simple box form with a solid floor and roof (diaphragm), we'll add one type of stabilizer (a solid wall or a braced frame), somewhere along the frame on each floor level, in both x and y-axis orientation. Bracing one bay on each face will stabilize them all. As we develop the project, we can look to integrate these stabilizers with architectural functions, particularly the stairway and/or outer walls. By combining these ideas, we can get a more comprehensive framing plan that solves our structural and functional problems together. (Figure 3.1.22)

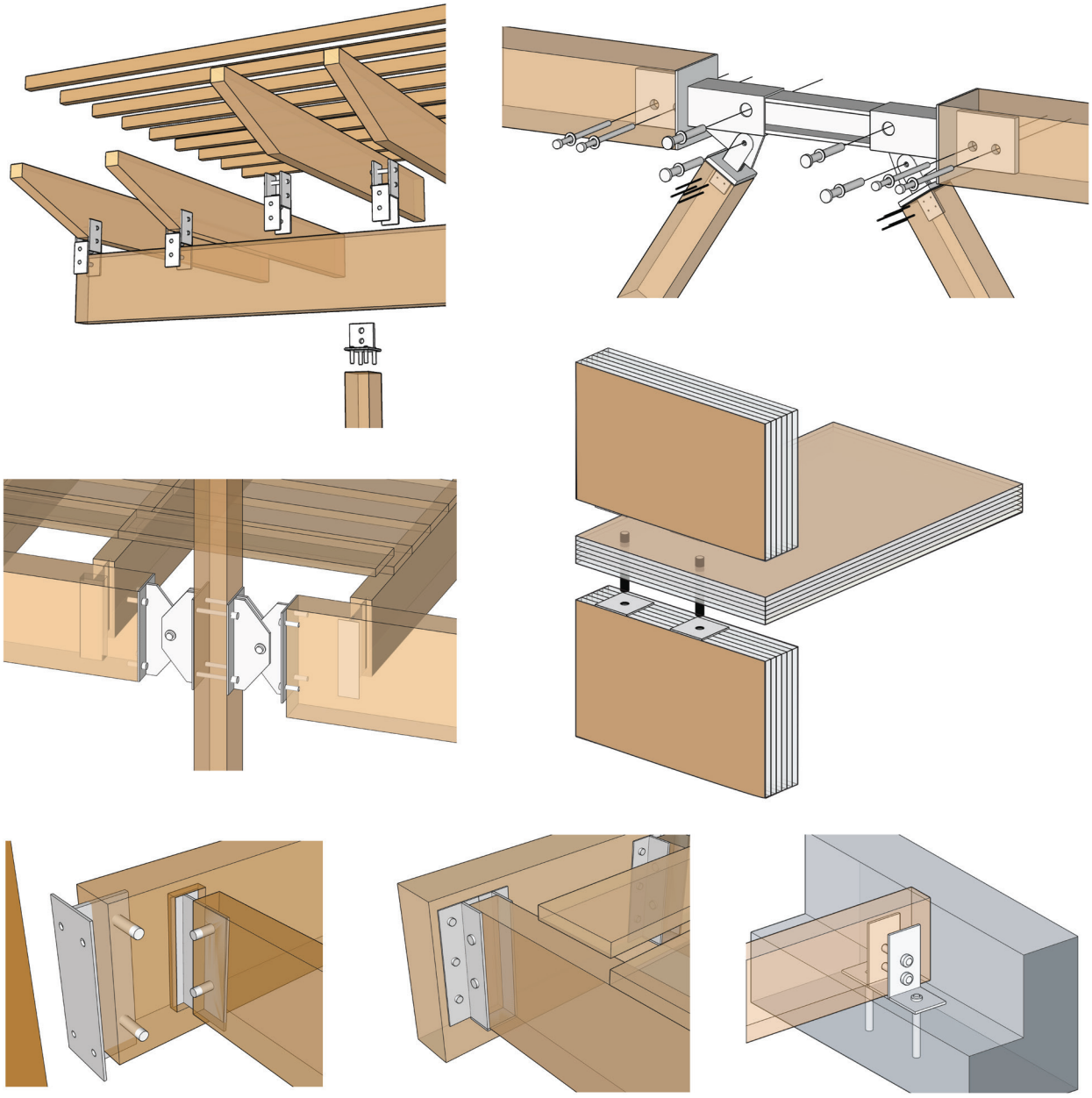


Figure 3.1.21 General details shown (left to right, top/down): Roof joist hangar, diagonal bracing bracket, pinned girder/column connection, CLT wall/floor (concealed), recessed (concealed) purlin/girder connection, exposed purlin/girder connection, and timber/concrete intersection

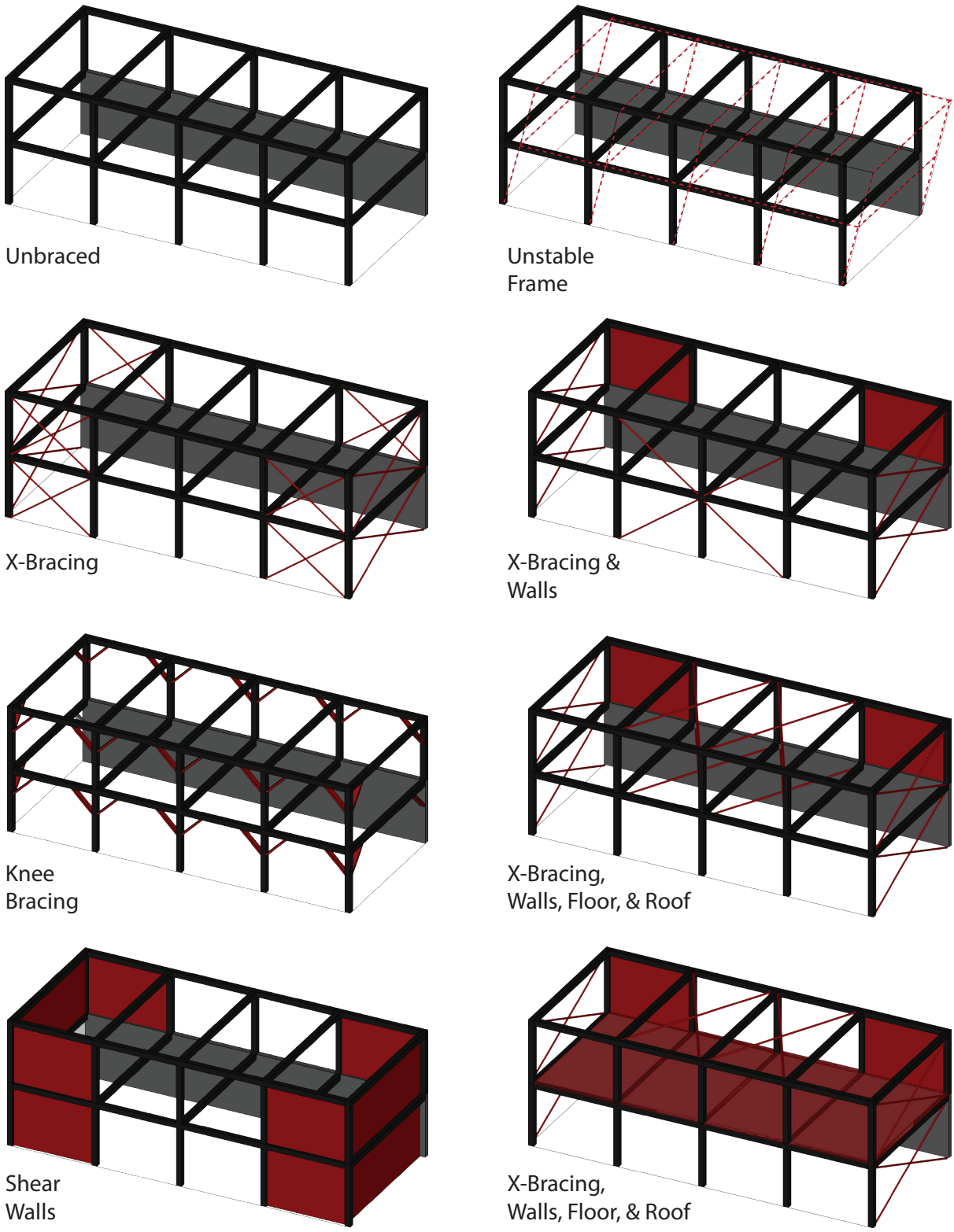


Figure 3.1.22a Bracing options shown with various consequences for aesthetics and functional correspondence

Figure 3.1.22b Mass timber bracing at Architecture and Environmental Design Building, Cal Poly San Luis Obispo

MAKING: DEVELOPMENT OF EXHIBIT TABLE DESIGN - PART II

Develop the exhibit tables to include more specific information about how the table top is framed, how it transfers its load to the supports, and the type of supports that will be used on the ends. How does the functional requirement for having supports only on the ends and a large variance in the weight of exhibits on the table affect your design? To keep the design of the table consistent, develop a "worst-case" design for the depth and supports.

To Discuss: Is the table top just a deck-like surface or is it a deck that spans to girders below? What are the pros/cons of each idea? What type of supports are used on the ends? How are the main pieces connected to facilitate force flow?

To Do: Determine how you'd actually size and test the structural elements. How would you keep the table from falling over side-to-side or front-to-back?

Design Development and Refined Schemes

The schemes we've drawn for Options #1A and Option #1B seem like they will work. We've met the one fundamental task of a schematic structure: *arrange load-carrying elements together in a way that facilitates force flow from the roof to the foundations*. But structural designs also need to correspond with architectural and environmental goals. We'll need to expand our range of assessment criteria: Do the schemes allow the best use of space? Does the structural expression enhance the articulation of interior spaces? Are environmental considerations addressed?

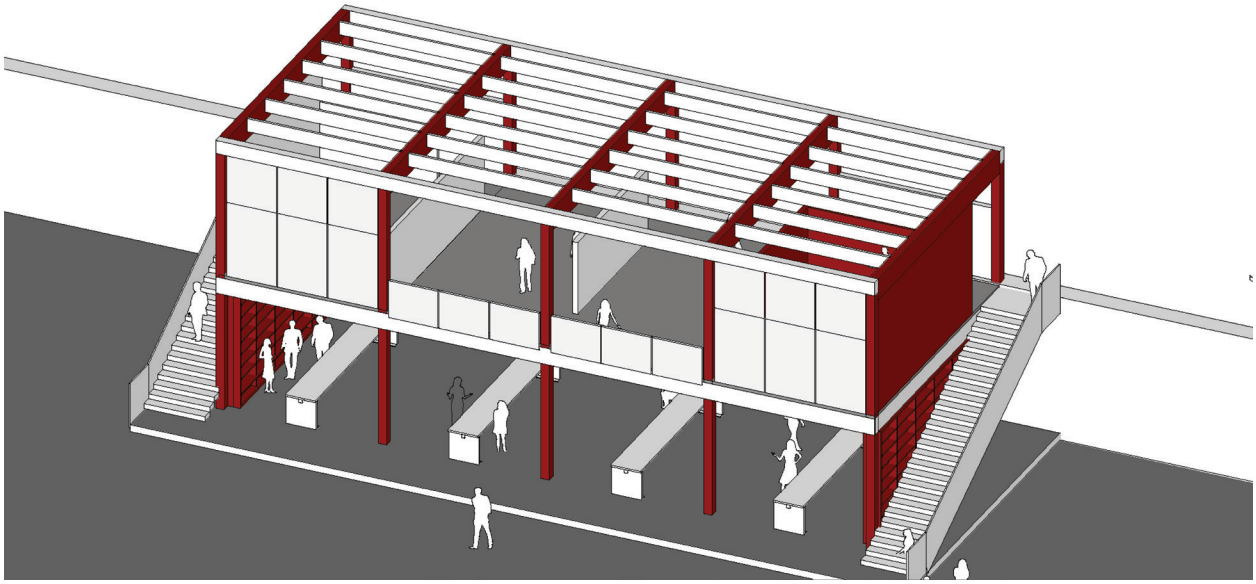


Figure 3.1.23 This scheme corresponds well with classroom spaces and uses moderately sized spanning members to allow for higher interior clearances

Most importantly, are there changes to the structural framing that would improve the proposals' structural and architectural effectiveness? (Figure 3.1.23)

Adjustments in the framing based on a combination of architectural and structural ideas can improve the building's effectiveness and responsiveness. We can reduce the spanning members' sizes, make the frame easier to construct, improve its responsiveness to environmental conditions, move the stair to improve its use, and explore other options for the building expression that result from these adjustments. All of these changes can happen if we introduce *cantilevers*.

Cantilevered Framing: Options #2A and #2B

The previous options assumed that the purlins were all supported on each end (*simply supported*), but

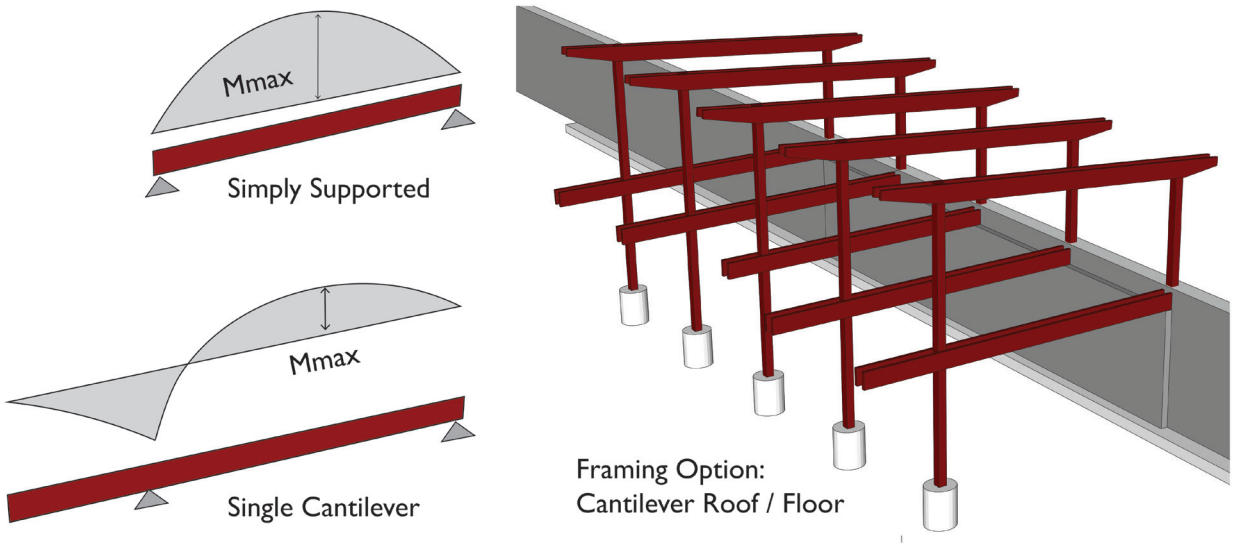


Figure 3.1.24 Adjusting the framing to include cantilevers and the lower maximum moments that can result

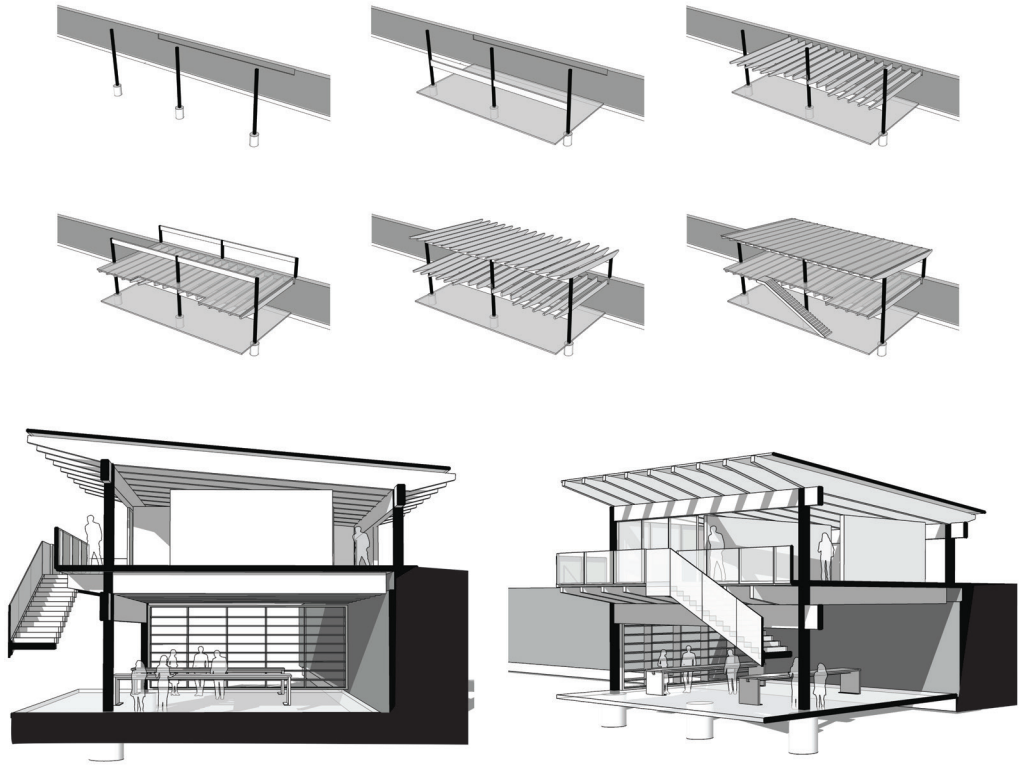


Figure 3.1.25 Option #2A, construction sequence framing diagrams and sectional perspectives showing stacked framing

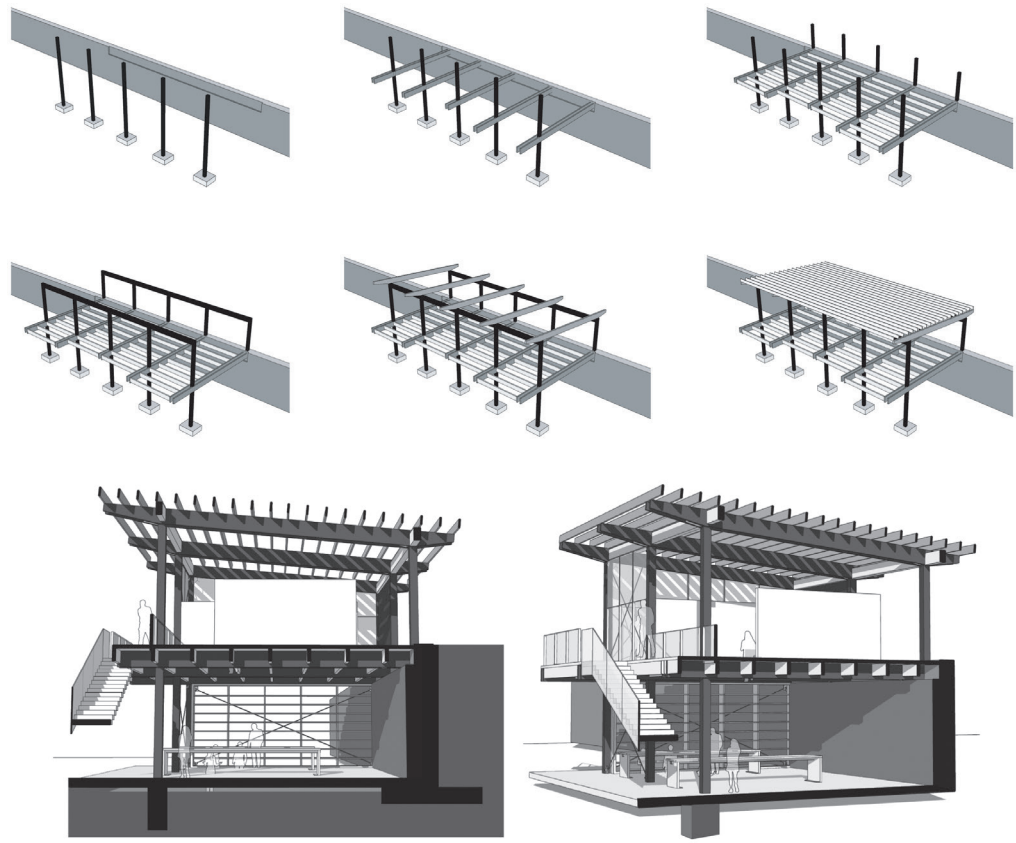


Figure 3.1.26 Option #2B, construction sequence framing diagram and sectional perspectives showing stacked framing on roof and planar floor assembly

what if they weren't? The moment diagrams for cantilevers shows us that extending a beam past the supports on one (or both sides) *reduces* the magnitude of the maximum moment by essentially reducing the span of the back-span beam. The resulting moment is split between the positive bending moment in the middle and negative moments on the cantilevered ends. Ideally the overhang length is extended so that the maximum positive and negative bending moments are equal, at ⅓rd the distance between supports. (Figure 3.1.24)

We can keep the two framing strategies for girder and purlin placement, but include cantilevers in both. To create a cantilever in Option #2A, the girders run east/west between columns and the purlins will need to run atop the girder to extend the floor out to the desired dimension (and roof to provide shelter). The connections between these elements will require a special steel connecting strap and perhaps some intermediate bracing between purlins along the bottom to prevent them from twisting or buckling. The stacked nature of these members reduces head height below, particularly on the ground floor. (Figure 3.1.25)

For Option #2B, because the column is at the end of the girder's span (running north/south), we can't just extend the girder outward without disrupting the verticality of the column. Instead we can move the girder to one side of the column or—better yet—sandwich the column between girders on either side so they can both extend outward. The purlins still span the same distance as they did before and can be easily framed. The roof doesn't have to have the same double girder articulation, but this is an aesthetically consistent option. (Figure 3.1.26)

Cantilevers are useful in both schemes. If we extend the roof outward so it is double-cantilevered, it will provide shading on the south and collect water on the north by tilting the roof girders. For the floor, we can extend the framing outward towards the open area of the site on the south, creating open balconies, shading the ground floor spaces, and allowing us to relocate the stair more centrally. Both framing options are improved with cantilevers.

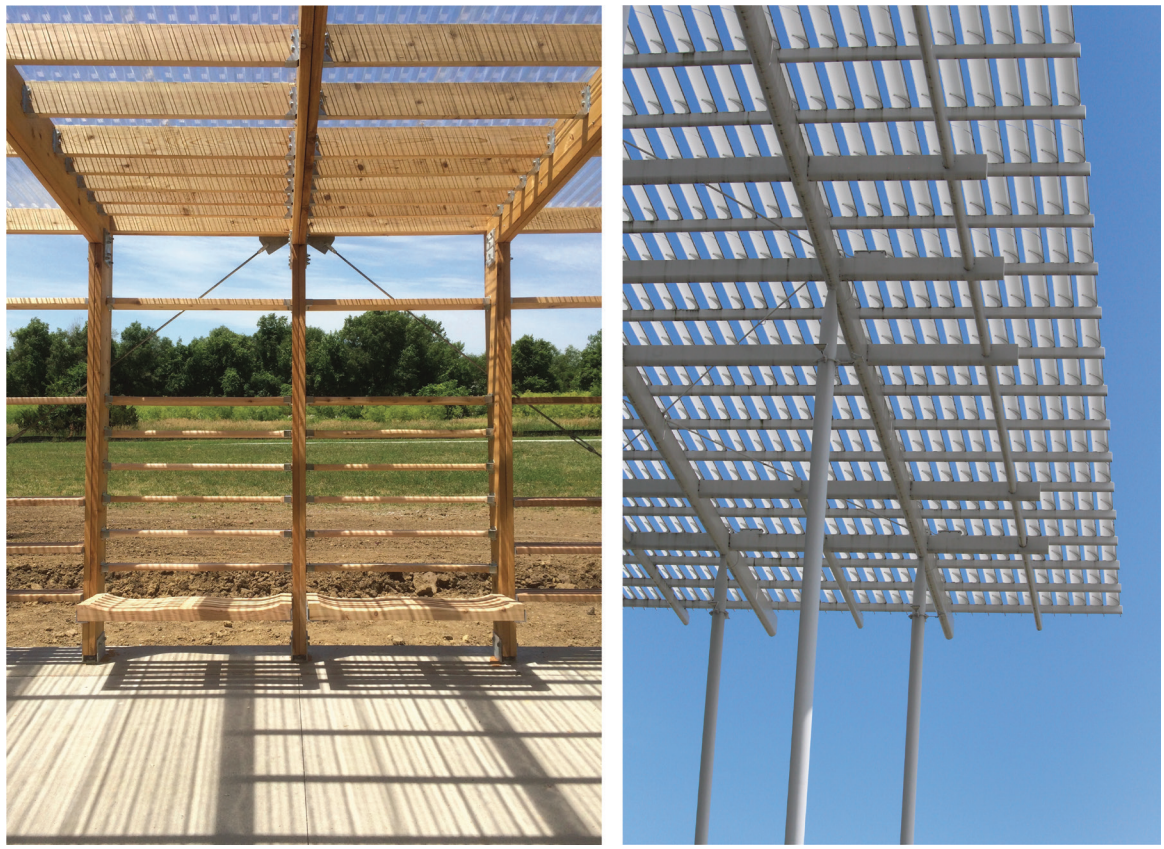


Figure 3.1.27 Filtering light is possible through layers of structure. Shown: Urbandale Shelter (Doyle and Whitehead, 2016) and Chicago Art Institute (Renzo Piano Building Workshop, 2010)

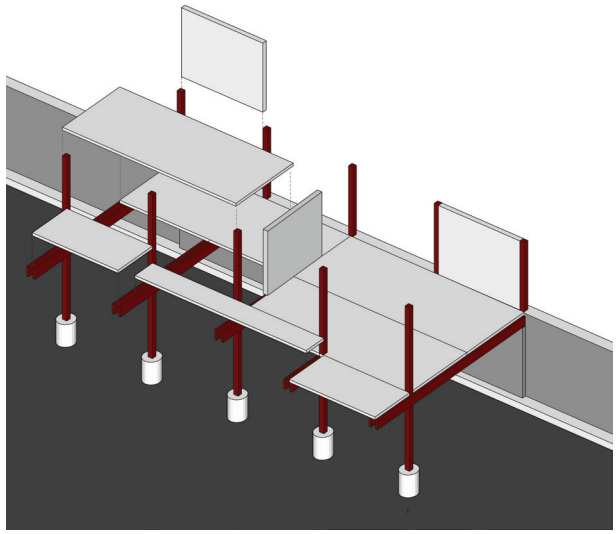
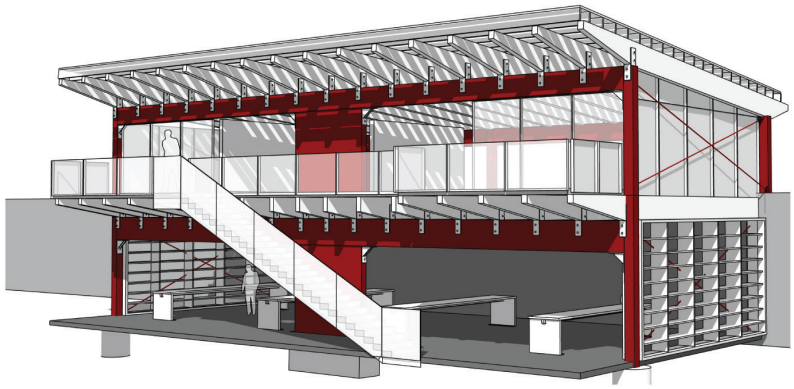
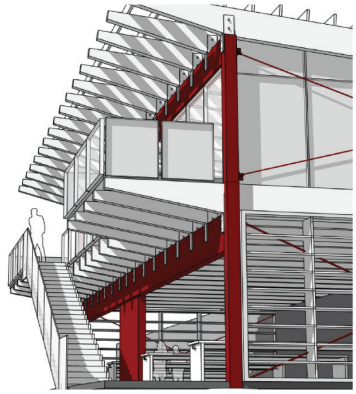
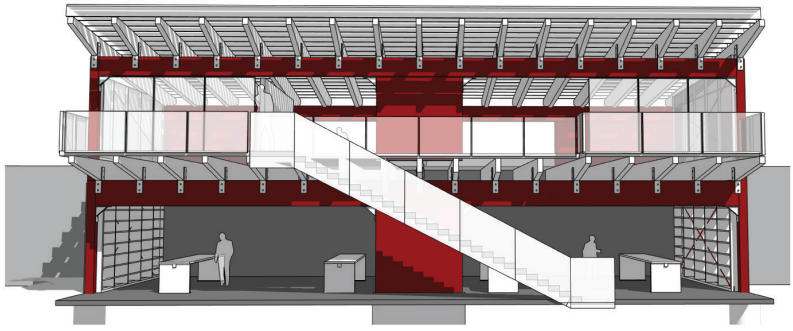


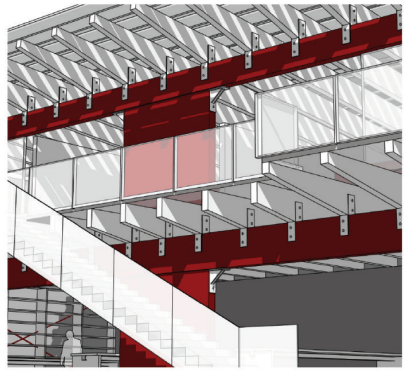
Figure 3.1.28a and 3.1.28b Framing diagram of CLT floor and wall panel installation—typically with wood-to-wood connections. The CLT can be topped with a thin slab of concrete if needed or finished at the edges with steel

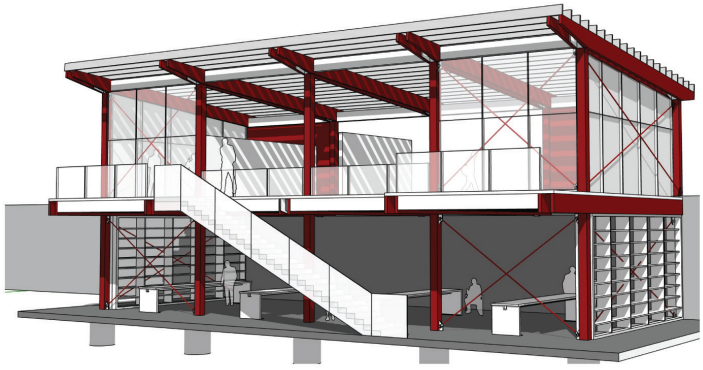


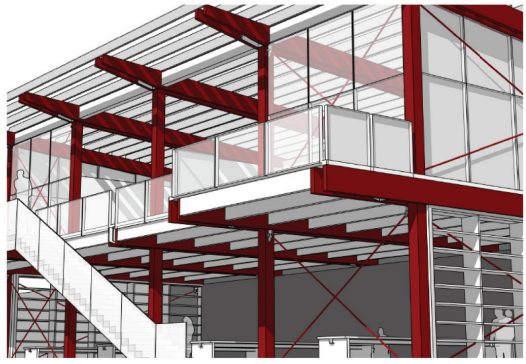


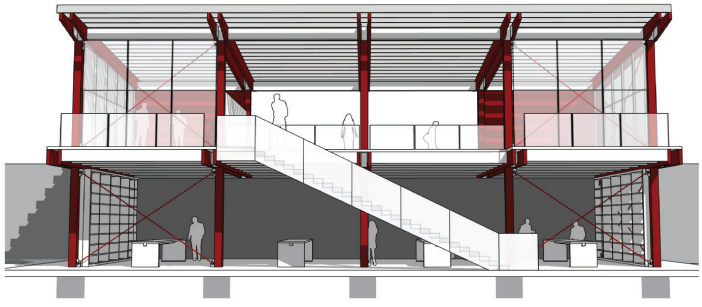


option #2A

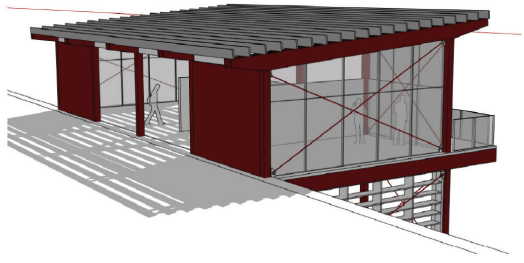








option #2B



Figures 3.1.29a, 3.1.29b, 3.1.29c, and 3.1.29d *(continued)*

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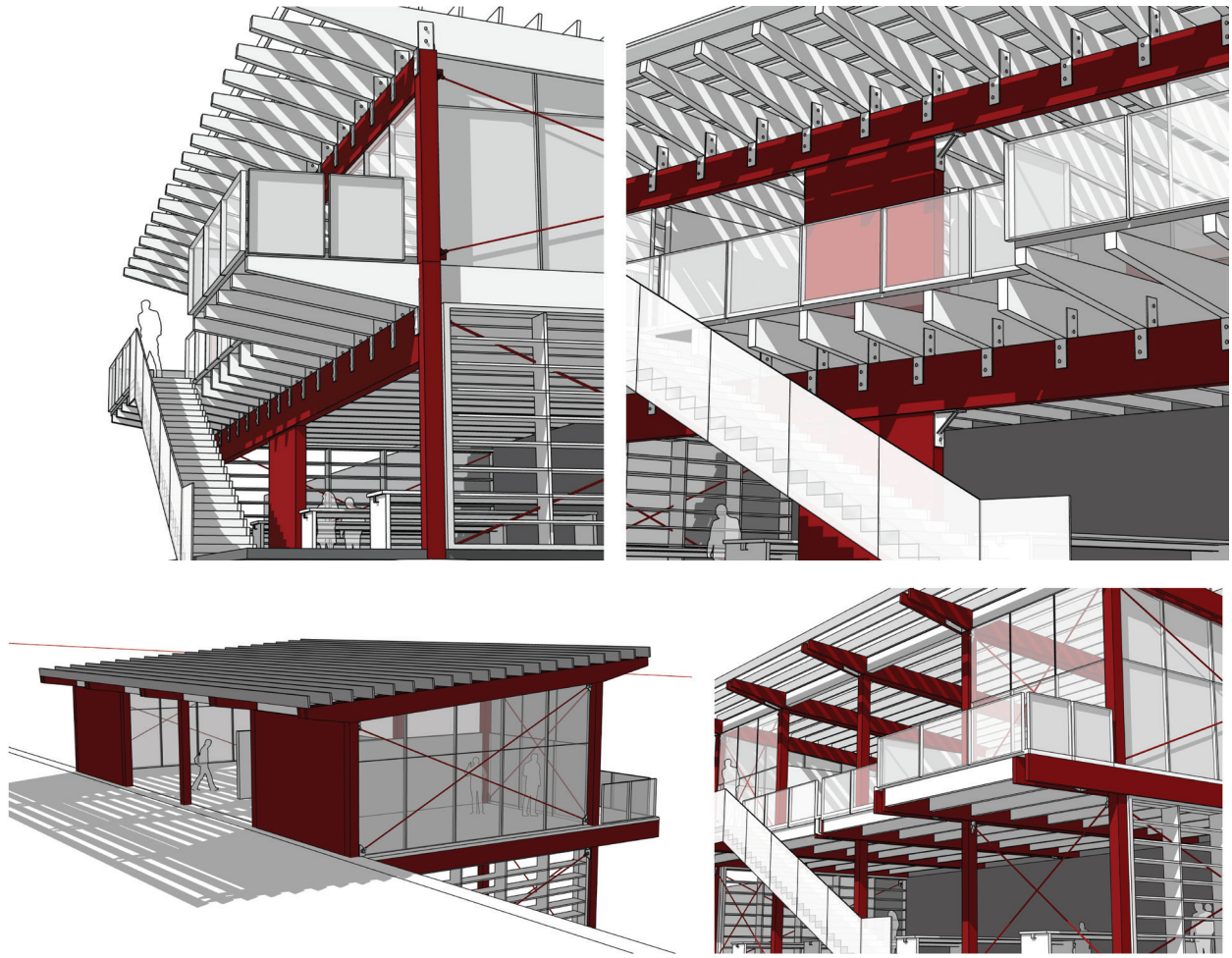

Figures 3.1.29a, 3.1.29b, 3.1.29c, and 3.1.29d Comparison of final schemes (top: Option #2A, bottom: Option #2B). Note the integration of program spaces, stairs, enclosure systems, and stabilization strategies. Schemes with cantilevers have functional, environmental, and aesthetic benefits

Additional Options and Refinements

Other framing options might be based on environmental or aesthetic reasons. If we wanted more mottled sunlight from above, we could double the frequency of the roof purlin framing and integrate a translucent corrugated fiberglass roofing deck. Doubling the number of purlins would refract this light as it hits the roof since the purlins have proper solar orientation. This wouldn't reduce their required depth, though, as length is the controlling factor in both bending and deflection, not load, and its length would be unchanged. (Figure 3.1.27)

We could also simplify the lower level ceiling (floor framing) by eliminating the floor purlins in favor of CLT decking. This would span in the same direction as the purlins, 16ft (4.9m) or 32ft (9.8m). We could improve the efficiency of this decking by having it run across multiple supports and acting like a continuous beam. Solid decking would have less overall depth than our previous framing, but it would be heavier, adding more load to our girders. The decking would be visible within the spaces, which could enhance the floor and ceiling aesthetics.

We might also use CLT panels for lateral bracing in vertical walls, which would keep the structure stable and help define interior spaces. If we added

shear walls in the classrooms on the north side (for bookshelves) and in the middle of the open classroom spaces on both floors (running north/south) we'd have walls that provide bi-axial lateral stability. These shear walls would resist lateral and vertical forces, stiffen the building, and improve the function of the spaces. (Figure 3.1.28)

Overall Assessment and Evaluative Conclusions

Throughout this process, we were able to take a general diagram for building spaces and add information about how we could enhance the structures. By limiting our designs to a simple structural frame made primarily from one material, we reduced the number of options and focused on layout and connections. We evaluated different ways of channeling forces through the framing system and assessed the functional and aesthetic consequences of doing so. A holistic evaluation of the options was possible because we included more detailed information about structural behavior, member sizing, construction sequencing, and connections. (Figure 3.1.29)

Frequently Asked Questions

Does anyone in practice actually use these calculations when they work on a project? Isn't that the engineer's job? Yes, architects are expected to understand these concepts and calculations. Not only does it make one a better designer, but in some states on certain types of projects registered architects can officially "stamp" a set of drawings with some structural calculations. Engineers are better qualified to do these calculations (and should be trusted to do so), but architectural calculations are useful in conversation with engineers to determine size, shape, connections, etc. Overall, think of calculations as one of many confirmation "models" you can use to clarify and confirm other design choices. And consider engineers as your collaborators in this process.

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Figure 3.2.0 Assembly of elements on John Deere World Headquarters (Saarinen, Moline, 1964)

CHAPTER 3.2

Better Beams and Slabs

Formal Experiments in Efficiency

Section-resistant structures, like beams and slabs, are so commonly used in our built environments that we might assume that they are inherently efficient. But pervasiveness isn't a reflection of optimal efficiency. Because their sectional shape determines their structural action, we can "hack" beams by manipulating their sectional shapes to make them more responsive and efficient. These hacks aren't a cheat, just clever logical enhancements of bending theory and beam behavior.

Making Better Beams: Concepts and Calculations

The most effective way of enhancing a structural element is to look at how structural resistance is generated. There are three factors to consider in section-resistant structures: *material capacity* (allowable bending, f'_b) and stiffness (modulus of elasticity, E), *loading conditions* (how loading (w) and *support conditions* (l) create a maximum bending moment (M_{max})), and cross-sectional shape (section modulus, S, and the related moment of inertia, I). To improve a beam's behavior or increase its efficiency, one (or several) of these factors needs to be addressed.

We will look at ways that these materials, connections, cross-sections, structural shapes, and even construction methods can be integrated to improve efficiency and performance. Instead of applying these lessons towards a single project, we'll complete smaller, faster demonstration and evaluation exercises to see and feel the difference these hacks make (i.e., more breaking than making as a way to help our thinking).

THINKING: STRUCTURES AROUND YOU

Find drawings and technical information about the structural framing used for a vehicle with high-performance standards (e.g., airplane, race car, etc.). How does the geometry of the frame relate to the function? What specific type of material is used (is it an enhanced version of the conventional material)? How is the material shaped (in profile and cross-section)? How is it fabricated? Speculate on the relationship of these factors and the relative efficiency of the framework. (Figure 3.2.1)

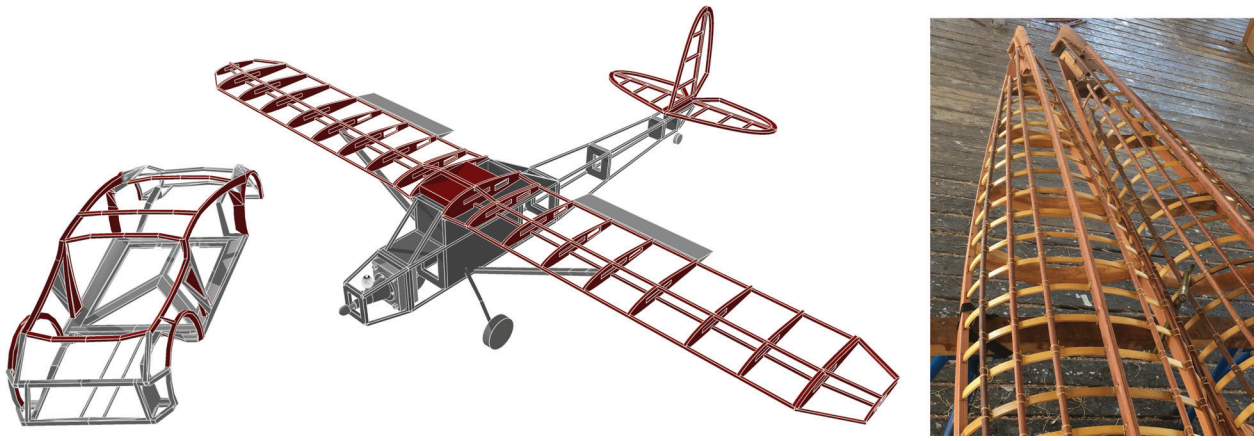


Figure 3.2.1 Examples of specialized framing for structures designed for optimal performance

Why Build Better?

This desire to make structures "better" is tied to a larger philosophical desire to integrate improvements. Our collective constructed history reveals a compulsion to improve upon the materials we use and the objects we construct—a desire to build better, stronger, or more effective objects. Richard Rogers suggests an ideological view of design that necessarily includes structures and construction: "If we lose control of the process of building, we become merely decorators of sheds. If we are simply decorators of sheds, we shouldn't be called architects." How does the cultural drive for innovation and efficiency affect our philosophical approach to design and construction? There are three viewpoints to consider, as to how and why we should build better:

In the late 19th century, Eugene Viollet-le-Duc suggested that some structural forms were better suited for some materials than others; he advocated designs that reflected rational means of construction and honest use of materials. This first "modern" theory linked materiality, form, and construction methods together as the means of being rational and efficient. Fifty years

Figure 3.2.2 The aesthetics and efficiencies of industrial practices have been consistent sources for architectural inspiration. Shown: Viollet-Le-Duc drawing for a 46m polyhedral hall (1865), car testing on Fiat factory workshop (Turin) from *Vers Une Architecture* (1923), and Fuller's patent drawing for Dymaxion Car (1937)

later, in *Vers Une Architecture* (1923), Le Corbusier reinforced this idea and suggested that modern design should be shaped by mass-produced, economically feasible, and technically-advanced industries. For him, the honesty of expression suggested by Viollet-le-Duc was about embracing the influence of industry. He equated modernity with efficiency and challenged designers to express these efficiencies through structures and construction (combining the "Engineer's Aesthetic and Architecture."). Conjoining innovations between industry, technology, and design was also Buckminster Fuller's primary consideration. He insisted that reducing an object's weight was an expression of efficiency and modernity; thus his query to architects: "How much does your building weigh?" (Figure 3.2.2)

All three viewpoints reinforce the idea that building structures are one part of a larger body of knowledge and that technology transfer between industries (material, technological, and construction) is a source for innovations. If we broaden our perspective to include other industries concerned with developing structural performance and fabrication, we'll see these "hacks" not just as a way to save time and money, but also as a way to improve how we design, build, and use our environments.

Designing elementally, so that each part is useful architecturally and structurally and not simply aesthetic, is an essential skill. The economics of structural assemblies are real and frequently the controlling decision-making factors. After all structural systems can account for 15–40% of the overall project budget. If a project is over budget, value-engineers will rarely find significant savings in the structural system if the proposal has been effectively and efficiently considered.

Inefficiencies of Beams

Why do we use beams? Compared to other types of structural systems, beams and slabs aren't materially efficient. Beams resist bending by a relatively inefficient mechanism that relies on sectional mass. Consider that some of the inefficiencies of beams may come from the process by which we design and manufacture them.

A beam's depth is based on the maximum shear and bending moment values. Although these stresses vary along the beam's length, the maximum values end up being applied to whole beam. Even if a large portion of the beam is subjected to lower bending moment forces, the beam's depth is based on the worst-case scenario. This is effective and safe, but it is a "one size fits all" approach that adds material and weight where it isn't needed.

Beams with a solid cross-section are also inefficient. Recall that beams resist bending by creating a force couple of compression forces along the top and tension forces along the bottom. These stresses are highest at the outer fibers of the beam and proportionately reduce until they reach the middle, *neutral axis*, of the beam. The stresses form a triangular distribution of force. Stresses are resisted by area and, therefore, a more effective beam cross-section would reduce materials around the neutral axis where there is little to no stress while maximizing material at its outer edges. (Figure 3.2.3)

Manufacturing: Problems and Opportunities

"Why do beams look the way they do?" Beams don't *have* to be solid with a consistent depth, of course, and in other industries they aren't. In architecture, the process of selecting beams has always been associated with a manufacturing system that favors repetitive, mass-produced, and extruded profiles with consistent depths. Efficiency is a relative term and mass production favors the status quo.

If a beam were redesigned to reduce its materials by 15% through a process requiring custom fabrication we'd need to assess if it was good exchange. Cost is roughly equal parts materials, fabrication, and construction, so it would have to be fabricated and installed with only a marginal penalty to justify any material saving. Most of the time there isn't a compelling building performance reason that requires these changes—unlike, say, an airplane wing that has performance goals for shape, strength, and weight. Buildings use planar surfaces for walls, floors, and walls, and rarely will a building's performance be tied to such stringent structural specifications. (Figure 3.2.4)

But manufacturing and construction fields are also inherently opportunistic. New materials and non-traditional methods for manufacturing and constructing

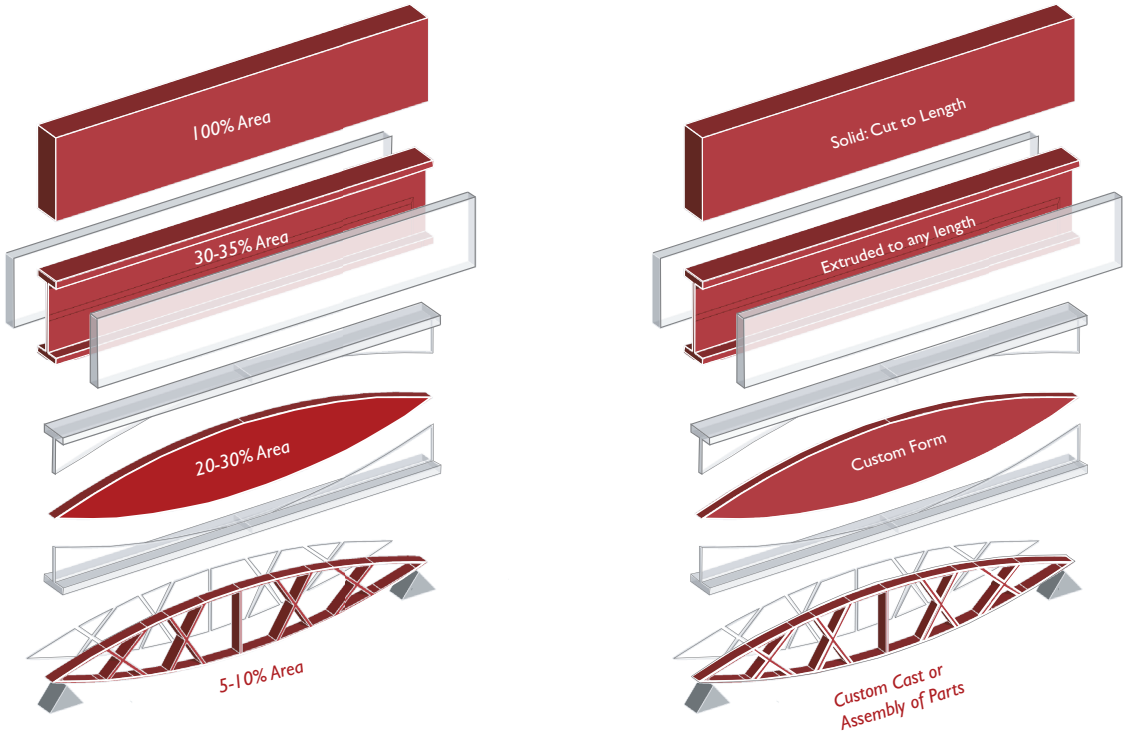


Figure 3.2.3 Potential adjustments to a typical beam section, profile, and mass to optimize the use of materials

buildings are constantly being developed and integrated from other industries. Architects need to translate these innovations into how we conceive and document the work. As the quantifiable standards for building performance increase (e.g., imagine a code that has a weight limit for buildings!) or as these experimental materials and methods prove themselves, we'll need to have ideas for what these innovations look like.

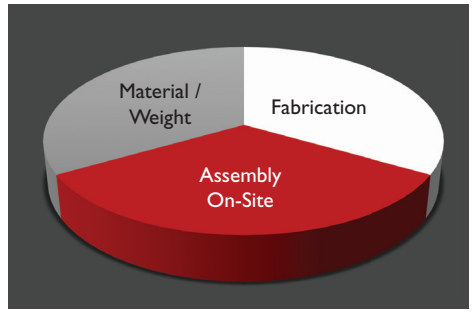


Figure 3.2.4a and 3.2.4b Although optimized beams are possible to envision, they are more difficult to mass-produce affordably by conventional means

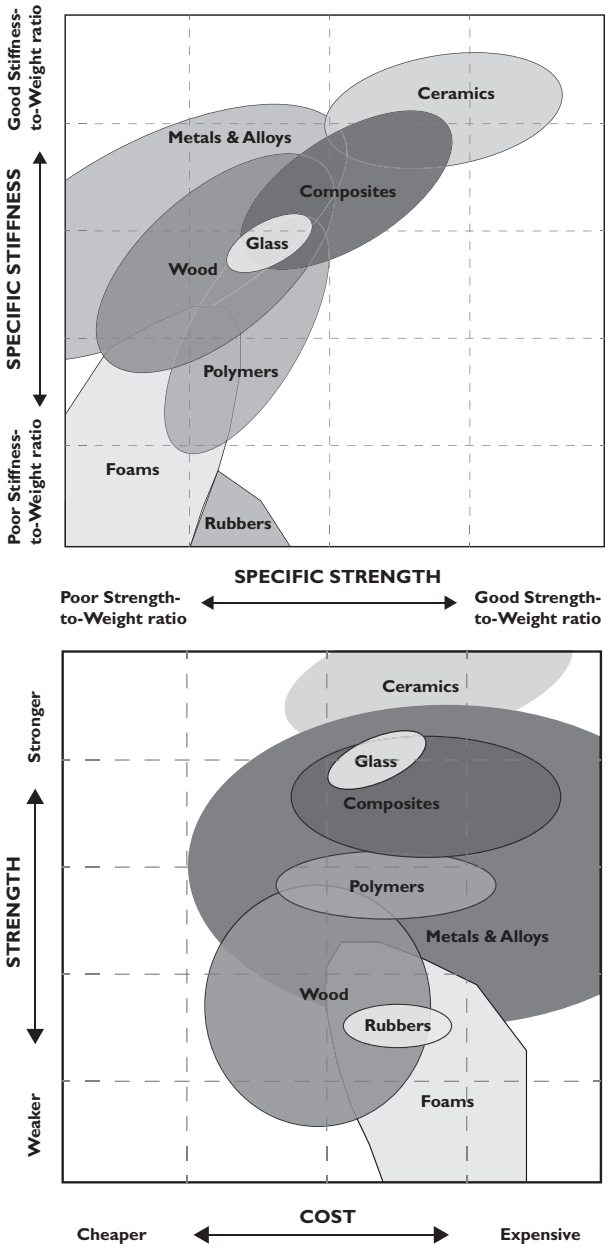
MAKING: MATERIAL MATTERS – HOW BEAMS ARE MADE

From a material utilization perspective, the mass timber beams we used in the previous chapter had a relatively inefficient profile and cross-section—they were just solid beams. But this solidity was necessary for efficient manufacturing, construction, and connections of mass timber. Solidity, and the associated inefficiencies in material use, reflects timber's inherent material qualities. But what about other materials that don't have the same constraints? Look at the ways steel beams are fabricated and how concrete beams are cast. What is the relationship between cross-sectional shapes, fabrication, and economy?

To Discuss: Is it "cheaper" overall to be inefficient in some criteria? Why aren't our manufacturing models based on material utilization and optimization?

What is involved in making a better beam? Let's begin with materials.

Material qualities are the most important indicators of structural behavior. Any product designed to perform a task needs to have some certainty about the way its materials will behave under stress. Sometimes it will need to be lightweight and flexible, like foams or rubbers, or heavy and rigid, like cast iron. Scientists test materials under compression, tension, bending, torsion, and shear and record how much they strain under stress, their elastic limits of deflection, and their ultimate stresses compared to their densities. These *material mechanics* (or strength of materials) are categorized by their behavior and plotted on charts (e.g., "specific stiffness," (stiffness-to-weight) ratio versus "specific strength" (strength-to-weight) or stiffness versus density). Selecting materials based on their performance standards is the first step towards increasing efficiency. See Appendix Table A.4 for a listing of material properties. (Figure 3.2.5)



Figures 3.2.5a and 3.2.5b Graphs showing strength/stiffness and cost/strength comparison of materials

We commonly use wood, steel, and concrete (and all the variations these entail) for building structures—all of these perform well. Industries that are concerned with lightweight, strong, and ductile material behavior also use aluminum, carbon fiber, high-strength polymer plastics, bamboo, and/or hybrid composites (e.g., aluminum sheeting over plastic honeycomb panel). These materials can be incorporated into building structures (as curtainwalls or panels), or other everyday objects (bike frames), but they are usually costlier than traditional materials, may not be available in the size and amounts needed to become a super-structure, and/or may be complicated to assemble. In other words, we *can* select lighter materials, but we may run into problems other than structural ones in doing so. Before doing so, we may want to look for "upgrades" that increase a material's inherent capacities to make it stronger in resisting bending (f'_b) or a greater stiffness (E) to resist deflection.

Strengthening Materials: Steel and Concrete

Steel and concrete can be altered at a chemical level to perform better structurally. Steel can be made stronger by increasing the amount of carbon (which increases hardness and strength) and admixtures can be added into concrete to increase its compressive strength and stiffness (known as "high-performance" concrete). Material strength can be increased so substantially that both materials are used in the world's largest structures.

Steel is particularly beneficial because it can be cast, fabricated, or otherwise amended to adapt to odd, difficult, or unforeseen conditions or shapes.

Figure 3.2.6 Changes in steel cross-sections and profiles allow building systems to run through the floors/ceilings and to accommodate irregular column grids and cantilevers. (Krause Center, Renzo Piano Building Workshop, Des Moines, 2018)

Standard steel products offer a variety of shapes, sizes, and thicknesses to meet many demands. These can be changed by cutting away pieces, adding new pieces on, or simply connecting one piece to another. Because of this flexibility and adaptability, steel is the material of choice for projects that push the limit on conventional framing. (Figure 3.2.6)

See Appendix Tables A.10, A.12, and A.13 for steel beam and column design charts.

Materials can be shaped in ways that make them more efficient at resisting bending stress and deflection. Steel can be bent into a metal stud, cast into a custom shape, or extruded into an I-beam. Concrete can be cast into nearly any efficient structural shape, from a double-tee beam to a thin shell. Remember, efficiencies in materials cannot simply be determined by strength-to-density measures; we need to assess how the material performs under specific structural conditions. A brick is strong—unless it's tensed like a rope. Steel is stiff—unless it's made into a spring. And paper is weak—unless it is folded. In order to make beams better, we need to consider the mechanics of the material and opportunities (and limits) for how we can shape it.

Beams can be made more resistant to bending and deflection by the way we shape them in cross-section. Because steel is equally good in tension and compression, an I-beam shape with thick outer flanges on both ends makes sense. But because concrete is good

Figure 3.2.7 Pre-casting concrete can create custom sectional profiles. In this T-shape, sleeves are cast into the section to allow for post-tensioning (Iowa State University, 2017)

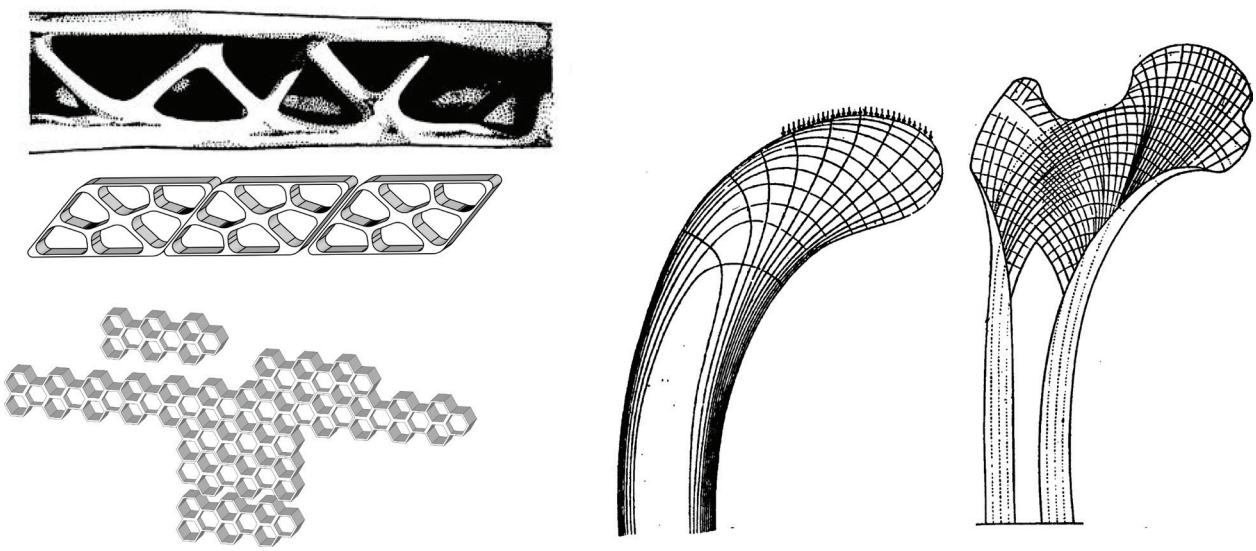
in resisting compression but has no real resistance to tension, we'd rightly expect to find steel reinforcing along the bottom of concrete beams and near their supports to resist shear. Steel has a higher allowable stress (f'a) rate (psi or kg/cm^2) so it can resist the same amount of stress as concrete with a significantly smaller amount of area. A concrete beam will need more material along its top edge, where compression occurs. At the bottom edge, concrete has to provide adequate cover around its steel reinforcing. Because concrete is heavy, we usually try to minimize its use wherever possible to avoid adding weight. But while concrete is affordable, its framework is often expensive, so it may be cheaper to accept structurally unnecessary material to make a solid concrete beam than to cast it as specialty shape. (Figure 3.2.7)

Shaping Sections for Efficiency: Modifications for Optimization

What would an optimized beam look like? There are two main inefficiencies in the conventional "solid" beams: longitudinal profile and cross-sectional distribution of material. If we wrote a performance specification for an optimal beam, we'd want it to be: strong, *ductile* (not brittle) but stiff against loading, lightweight (thanks to low density materials and efficient material utilization), resistant to buckling, and adaptable to connections with other structural elements. Basically, we'd only want to use materials where principal stresses occur within the beam.

To know where these stresses occur, we'll need to look inside a beam's cross-section and along its length. Specifically we'd be looking for locations to remove material. Robert Le Ricolais describes this challenge of lightening a structure with openings as paradoxical, but important: "the art of structure is how and where to put holes. It's a good concept for a building, to build with holes, to show things which are hollow . . . things that have strength but no weight."

Finding the right place for these holes isn't an exact science as forces can be transferred in many ways. Primarily we can use common sense to apply what we've learned about beam behavior. But there isn't one "right" answer as to how this can be done. These studies that suggest different potential solutions are called *topological optimizations*. They can be performed with genetic algorithms that favor certain factors over another. (Figure 3.2.8)



Figures 3.2.8a, 3.2.8b and 3.2.8c Shown (left to right, top/down): Metacarpal bone of a vulture (from *On Growth and Form*), biological brick prototypes (whitehead design workshop) and comparison of mechanical crane and human femur stress and strain distributions (from Wolff and Culman)

THINKING: CONCEPT > CALCULATIONS (OR CALCULATIONS > CONCEPTS?)

How would you evaluate different cross-sectional options to determine their effectiveness at resisting bending and deflection? How would we measure this? Consider the various options shown and try to determine which cross-section would have the highest resistance (and why). (Figure 3.2.9)

Cross-sectional Area and Resistance

A beam's resisting moment depends on its depth and the distribution of its sectional area. Stresses are resisted by cross-sectional area—the higher the force, the more area is needed (thus the psi or k/cm^2 rating of allowable bending stress resistance). It makes sense to put the most area where the highest internal forces occur in our beams namely, at their top and bottom edges. This is also why we reduce or eliminate materials near the center of the beam's cross-section, where stresses are lowest. An I-beam or tube looks the way it does because its mass has been moved to its edges and its middle has been hollowed out as much as possible. Because there is little or no stress acting around the neutral axis, this is a good location for small openings, if needed. (Figure 3.2.10)

We measure cross-sectional shapes two-ways: moment of inertia, I (in^4 or cm^4), and the section modulus, S, (in^3 or cm^3). Both measure the amount of area that a shape has and that shape's average distance from the neutral axis. *Higher values are more resistant to bending*. Two I-beams can have the same depth, but if one of the beam's flanges is thicker (i.e., more area at the outer edges) its S-value and I-value will be higher, and the beam will be able to resist more bending. Because I and S are essential measurements of structural performance, they are included in product literature and design manuals. (Figure 3.2.11)

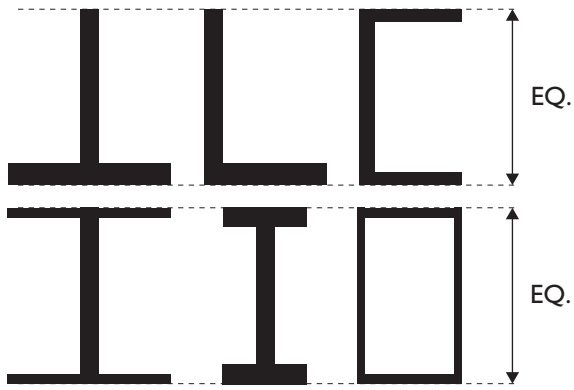


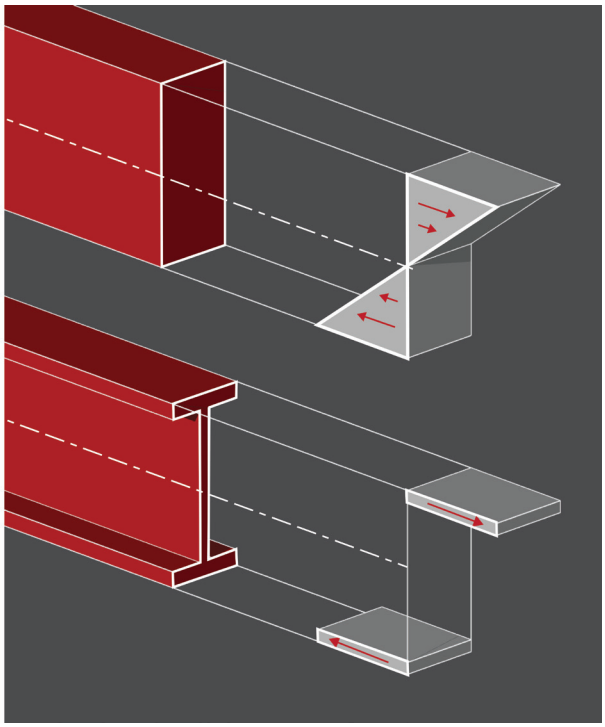
Figure 3.2.9 Which of these sections would be the most effective beam? And why?

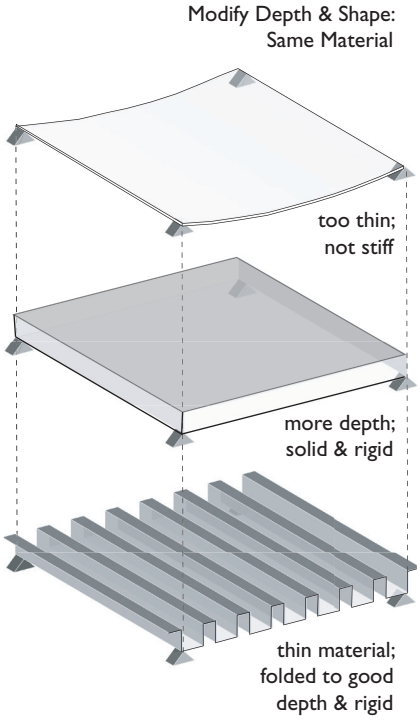
Longitudinal Stress Distribution and Shape

While optimizing cross-section is important, we can also reduce material by either refining a beam's profile to better match the bending moment, or by reducing materials in the beam's web.

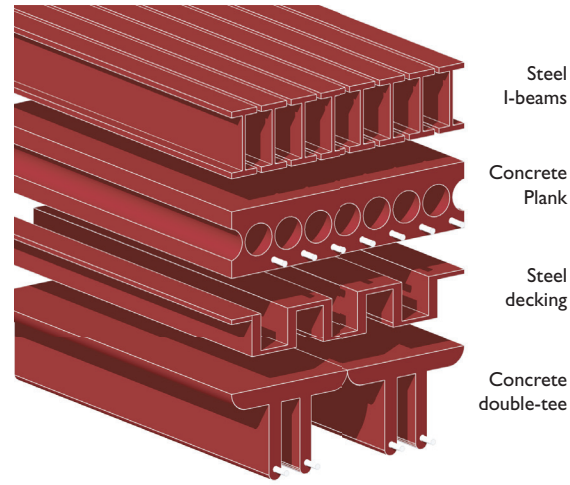
A beam's depth, remember, is related to bending resistance. If we shape our beam to be the deepest where the highest bending moment occurs, it won't have to be as deep elsewhere, where the moment force is smaller—thinking of a shelf bracket or a tree branch, which both get deeper toward their roots, where their internal moments are greatest. Because a moment diagram is drawn "to scale," it is actually a descriptive map of the changing magnitudes of stress—the highest point on a moment diagram is where the highest internal moment resistance is needed. Whenever possible, we match the form to the forces. (Figure 3.2.12)

We can refine the material distribution even further. A digital simulation of a beam under loading shows that beams resist bending longitudinally with a series of "arches and cables." The arches and the cables are densest at the point of maximum moment (in the middle). They are also farthest apart from each other here, forming a force couple with the maximum lever arm for the resisting moment. As moment forces





Figures 3.2.10a and 3.2.10b Effective beams locate cross-sectional area farthest away from the neutral axis (e.g., the top and bottom flanges) by manipulating the cross-section into a series of smaller "beams." 3.2.10b: Folding the plane stiffens the spanning element and increases its load-bearing capacity

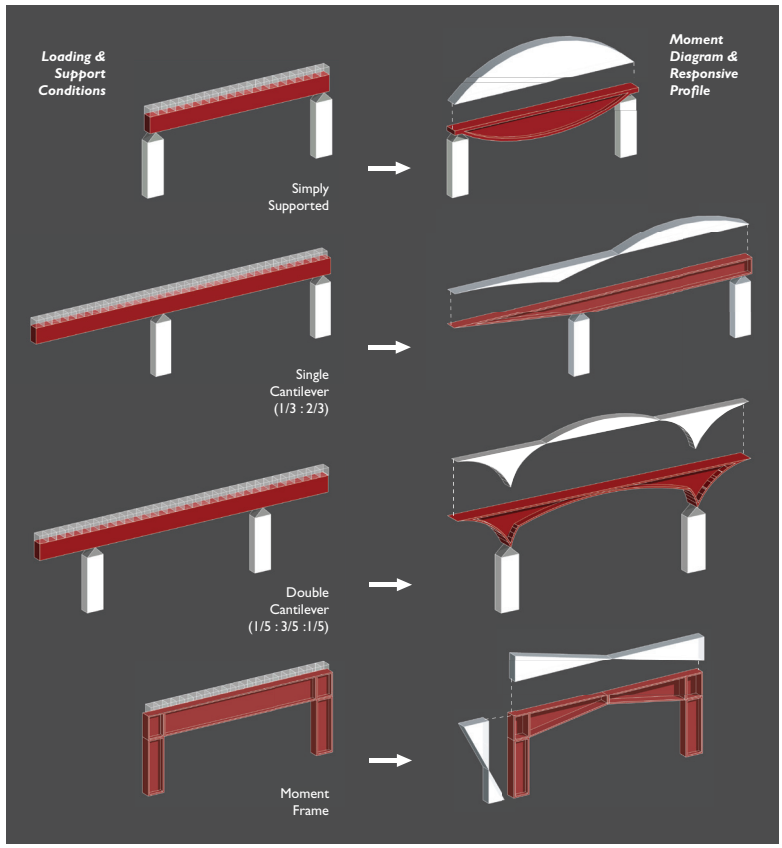


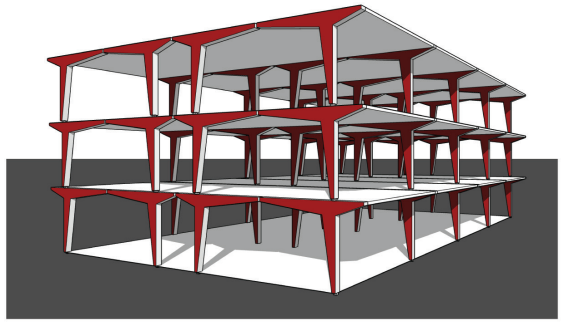
Similar distribution of sectional area

Figures 3.2.11a and 3.2.11b Common steel and concrete elements use a similar strategy for sectional distribution of area to increase efficiency of bending resistance and material utilization

diminish closer to the supports, their density dissipates and stress locations change profile. If we align material with these stress lines, the materials would be stressed axially, and they'd be able to resist against these stresses with an optimal amount of material.

To keep the beam stable from torsion or web buckling, these internal components also want to be triangulated. The end result looks a lot like a truss (a conclusion that we'll explore in Chapter 4.0). But again, we face manufacturing limits—digital fabrication





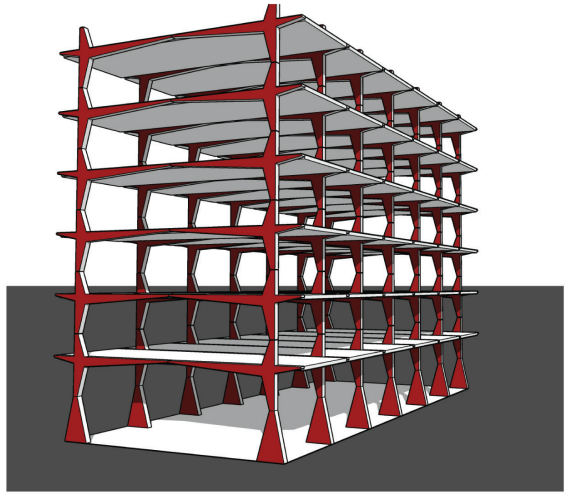


Figure 3.2.12a and 3.2.12b Changing spans and support connections can reduce maximum moments. Beam profiles can be shaped to match the moment diagrams and thus the building's expression

Figure 3.2.13a and 3.2.13b Flexible formwork, like fabric, can allow for beams to be shaped in structurally responsive profiles with minimal materials (Mark West)

has potential, but it would require us to translate these forms into steel and/or concrete to become structurally viable. Economic viability of all custom elements is also a question. Alternative forming methods, like fabric-cast concrete, also show great promise in providing efficient and expressive beams. (Figure 3.2.13)

Long Span Beams: Vierendeel Girders

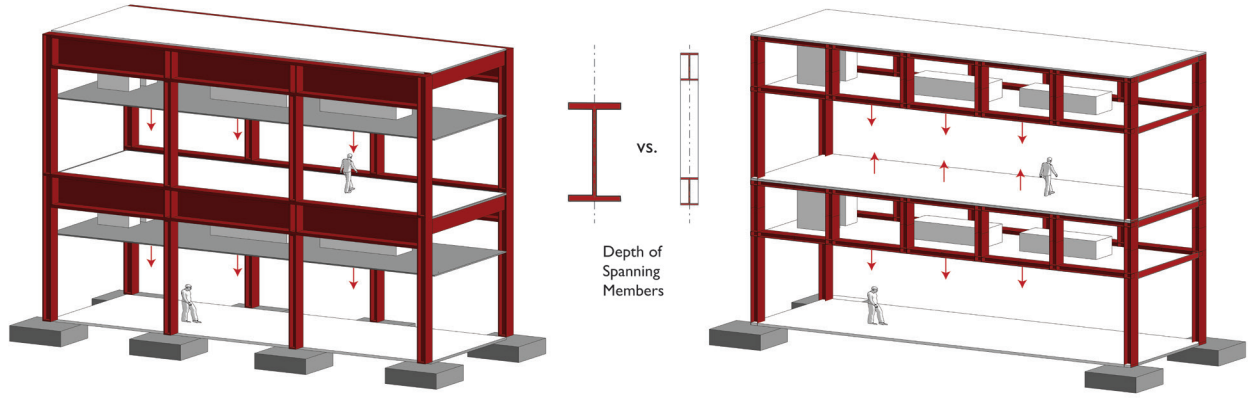
If a beam becomes deep enough and if we reduce enough area from its web, it becomes a *vierendeel girder*. These are giant moment resisting frames that can be the depth of an entire floor. They act as a hybrid of trusses and frames, but because they resist loads monolithically they also act like a beam and column system with fixed, connections. They can thus create efficient, long span column-free spaces. Vierendeel girders undergo high internal stresses, since every connection between their webs and chords is a moment connection that has to resist gravitational and lateral forces. (Figure 3.2.14)

Reducing Moment Force: Cantilevers, Continuous Beams, and Fixed Connections

Moment diagrams reflect loading and support conditions. Lower moments translate into smaller beams. We can rarely reduce the amount of load acting on a structure, since this is determined by code and material constraints, but we *can* vary location and/or frequency of supports to reduce maximum bending moments. Reducing beam spans by adding columns is the most direct way to reduce moments but this isn't always an economical or functional possibility. Two other strategies that designers employ to reduce moment forces in beams without increasing the number of columns include: moving supports inward from a beam's outer edges, and/or making the beam "continuous" across multiple supports.

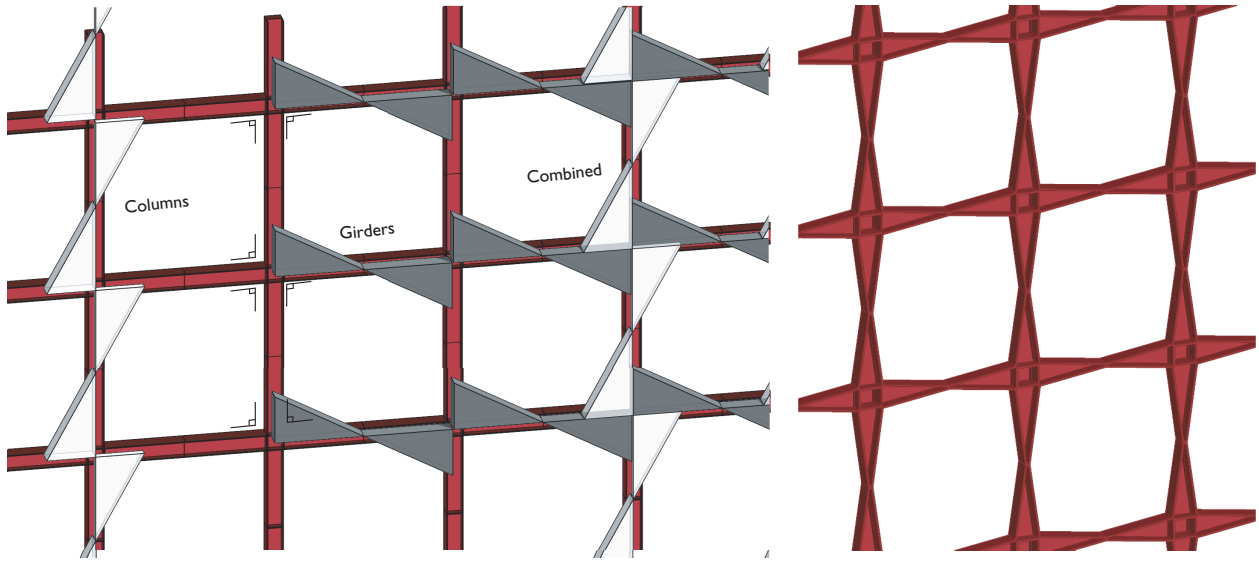
Cantilevers: If we move column supports inward from the ends of a beam, creating a double-cantilever, we can spread the moment out between positive and negative values across the span, reducing the overall maximum moment (Chapter 3.1). The overhang to back-span ratio is ideally a 1/3 : 2/3 ratio. This isn't always a possibility with the building program, but we can also use this strategy to create a continuous beam. Unlike a conventional beam that spans between two supports, a continuous beam spans across several supports.

Gerber Beams: The term "continuous" is a misnomer, because the continuity is typically achieved by linking several smaller overhanging beams that are either singly or doubly cantilevered. These beams are spliced end-to-end at a point along the span known as the *inflection point*, where the bending moment is zero. This inflection point is also the location of maximum shear, so the beams have to be connected with a shear plate to resist these stresses. This continuity reduces maximum moment because the cantilever action reduces the effective length of the beam's span. This approach, sometimes known as a Gerber beam system, is frequently used for long or



Traditional Framing:
Deep girders, additional columns, & lower ceiling height.

Vierendeel Truss Girder:
Additional structural depth, fewer columns, dedicated service space



Moment Diagrams in Fixed Frame

Optimize Shape: Increase Area at Maximum Moment Locations

Figures 3.2.14a and 3.2.14b Comparison of framing layouts between traditional framing with deep girders and vierendeel girders 3.2.14b: Vierendeel girders can be shaped to be responsive to the internal moment stresses at the connection points between the girders and columns

MAKING MATTERS: CREATIVE INTEGRATION OF STRUCTURAL PRINCIPLES

The Beinecke Rare Book & Manuscript Library at Yale University (1963) features an expressive example of vierendeels. Its outer walls are large vierendeel girders that span 131ft (40m) to columns in its four corners. These walls support the interior floors and the roof. Although the outside wall looks like cast-concrete boxes with marble panels, it's actually a series of structural modules: 8ft 8in (2.6m) prefabricated steel crosses that are welded together end-to-end and top-to-bottom to get the square shapes. The entire perimeter wall acts like an enormous vierendeel truss beam (with ample holes throughout) spanning from one corner to another.

To Discuss: What are the structural principles that would explain why the modules are formed and connected together in the way that they are? (Figure 3.2.15)

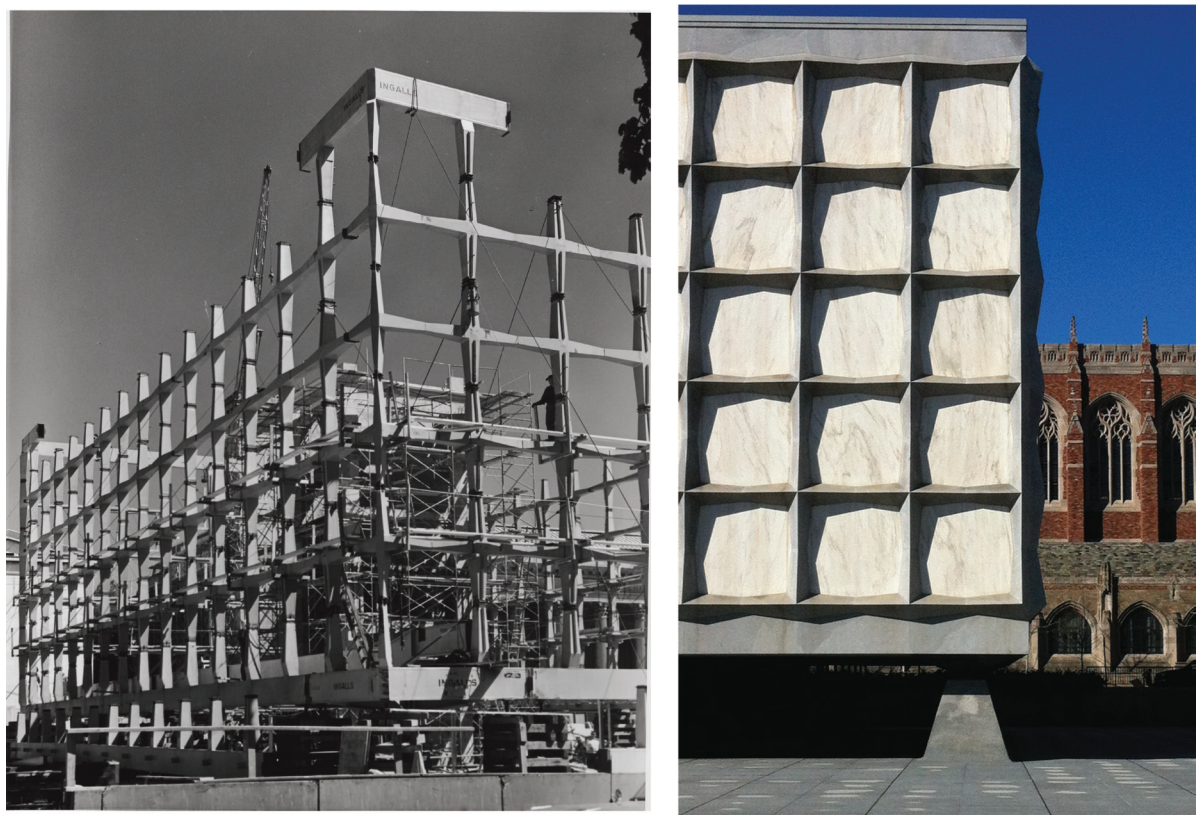


Figure 3.2.15 Beinecke Library under construction (1962) and exterior view of one support column (SOM, 1962)

intermediate spans in buildings and infrastructural projects. Because it spans across several supports, the beam is *indeterminate,* and cannot be analyzed by the statical analysis we've used. It can, however, be modeled digitally. (Figure 3.2.16)

Fixed Connections: Up to this point we've assumed that beams have been connected to supports using pinned (or hinged) connections. These have allowed the beam to freely rotate under bending, which means that all bending resistance must come *internally* from the beam without any assistance from the connections. But what if the connection could help resist bending moments too? In other words, what if we *fixed* the beam connection to the column in a way that didn't allow the beam to freely rotate (as we saw in vierendeel girders)? Bending moments in the beams would be reduced because the connections and thus the columns would share burden of resisting bending. Moment connections are difficult to analyze and construct, which makes them expensive and particularly challenging in timber or steel construction. However, if we cast girders and columns in concrete, we automatically get this effect.

Let's put all these ideas together: To maximize efficiency of a spanning element, we can combine the structural benefits from continuous beams, shaped beams, and fixed concrete connections. Imagine the benefits of reducing the overall bending moment if we use a mesh of continuous "beams" fixed together in order to span across multiple supports. In principle, we'd be taking linear beams and making them planar. *That is a structural slab.* (Figure 3.2.17)

Structural Slab Basics

Section-resistant systems collect and transmit loads as either composite assemblies or as a monolithic slab. Whereas composites rely on discrete elements to span horizontally in one direction and transmit their load

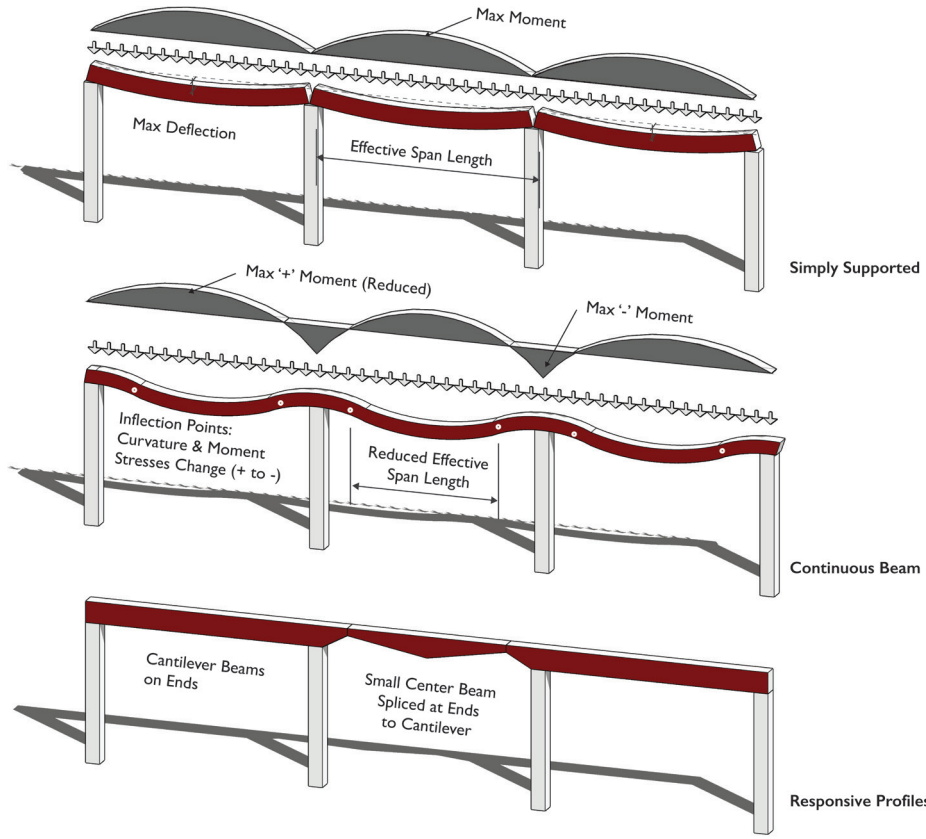


Figure 3.2.16 Diagrams of continuous beams. Maximum moment is reduced across multiple spans. Internal compression and tension stresses are inverted at connections where the moment is negative

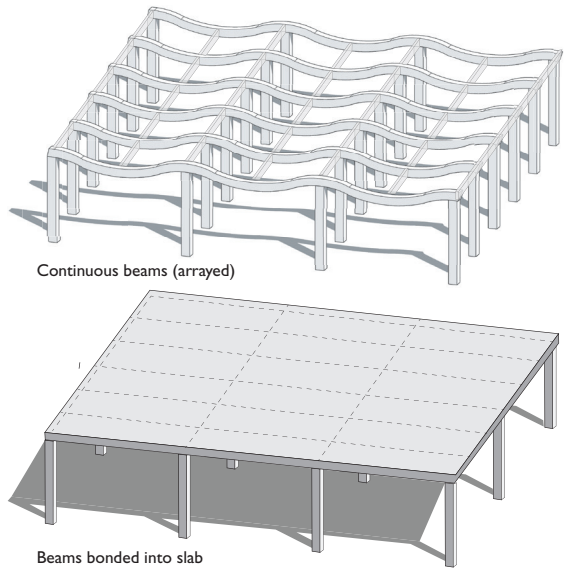


Figure 3.2.17 Connecting arrayed beams into a solid surface—like a slab—retains the structural benefits of reduced moments as continuous beams with fixed connections

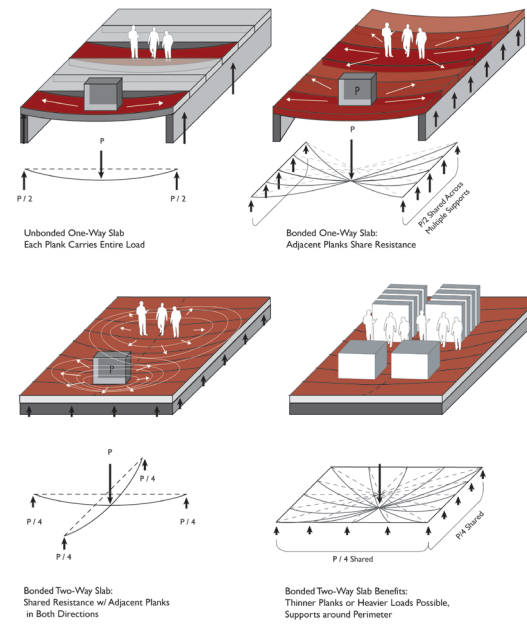


Figure 3.2.18 Principles of slab behavior: Shared resistance and reduced deflection is a function of slab proportions and location of supports

to another bearing element, slabs push all of their load-collecting elements together into one plane—providing resistance in multiple directions.

Picture a slab as a series of beams laid side-by-side. If these beams were unbound to each other they'd each transmit their loads along their axis. But if we bind them together, they start to resist loads differently. Each piece bends less because it is recruiting the structural capacity of the adjacent 'slices' next to it. The bending load is now being transferred in two directions, along its length and side-to-side. In other words, the load is distributed as a *two-way* system. Because they are connected, the slices act like continuous beams with fixed connections. This gives them a reduced "effective length." This two-way transfer of loads and the added stiffness of monolithic behavior makes slabs much more efficient than unbound beams. *Resistance is always more effective when the burden is shared.* (Figure 3.2.18)

Appendix Figure A.8 shows the depth-to-span ratios of various slab assemblies—sorted by materiality.

LEARNING BY MAKING

Part I: Create a structural slab and column supports made entirely of paper products to span 12in × 12in (30.5cm × 30.5cm). The slab needs to support 2.5 pounds (1.1kg) across the span to the columns in the four corners (fix the columns to a base). The material is the constraint—manipulate the cross-sections to support (and transfer) the loads.

Part II: Build a 14in × 18in (35cm × 46cm) slab using chipboard only. The columns remain spaced at 12in × 12in (30.5cm × 30.5 cm) centered below the slab, which requires the slab to resist cantilevered forces. Design the slab to resist 12.5lbs (11kg) of weight uniformly distributed across the surface.

To Discuss: How did the different material qualities affect your design choices? How did the different span and support locations change your configurations? How did you create cross-sectional characteristics in the slabs to transmitted and resisted loads? (Figure 3.2.19)

Figure 3.2.19 Examples of "slab lab" experiments (SxD, Iowa State University)

Shaping Slabs

Concrete is the most common material for structural slabs because of its monolithic behavior and durability. Slabs that are solid flat plates are common solutions for carrying heavy loads between supports *without* additional purlins and girders. Non-orthogonal plans are possible if we provide a regular grid of supports. With enough room for rebar, nearly any slab shape is possible. (Figure 3.2.20)

Slabs expressed as uniform, planar extrusions of a beam miss a further opportunity to optimize, since we can also manipulate a slab's cross-sectional properties. Slabs, after all, are section-resistant structural members and subject to the same opportunities for modification as beams, which we've seen above. But how and where we choose to reduce, or increase, the depth of the slab is dependent on which direction we intend the forces to flow.

Resistance in slabs is shared in multi-directions, but loads always seek the shortest distance to a support or the pathway with the stiffest elements. Therefore, the distinction between a one- or two-way slab is determined by the proportions of the framing system and, of course, column placement.

One-way slabs have a rectangular bay shape in which the loads are being primarily transferred in the slab's short direction and collected by a concrete girder that spans the long direction between columns. Its structural depth can be articulated by creating "ribs" cast-in-place using either formwork "pans" or with precast Tee beams or planks. (Figure 3.2.21)

Two-way slabs don't have a short-side. They have square proportions with a network of beams fixed together and columns equally spaced around their perimeter. Some two-way slabs take the form of a waffle slab (deeper beams with hollowed out voids between). These are created by pouring the slab and beams together atop pans that are secured to the formwork below. Two-way slabs are efficient, but they offer less integrative possibilities with building elements like conduits, lights, ducts, and sprinklers—*unless* the pan system is modified to provide voids. (Figure 3.2.22)

A slab's internal stresses aren't really distributed in a grid, though. One-way and two-way classifications suggest an orthogonal grid layout and expression but static forces don't automatically follow the same geometry. Forces follow patterns of structural

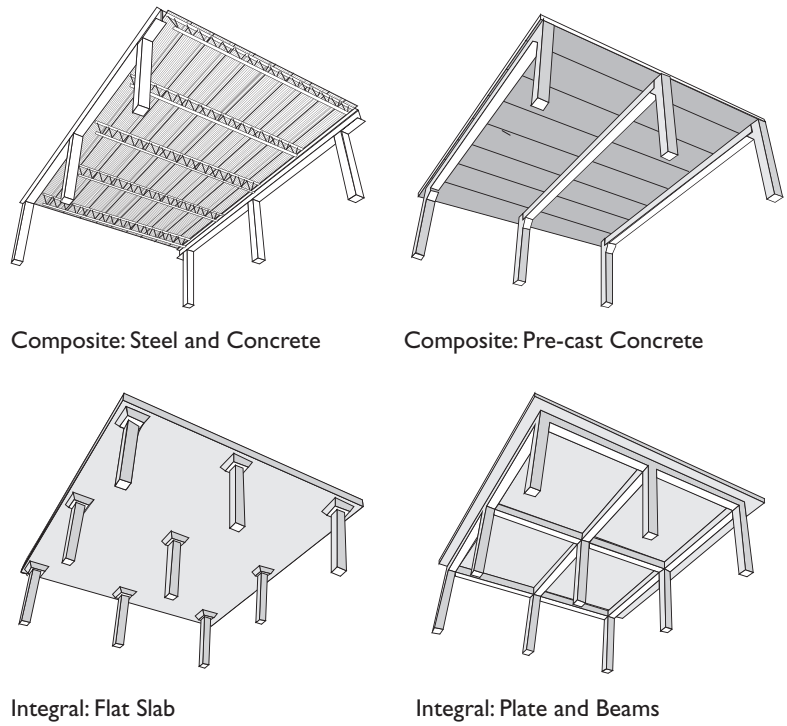


Figure 3.2.20 Slabs can be composites (wood, steel, or precast) or solid bearing surfaces (cast concrete)

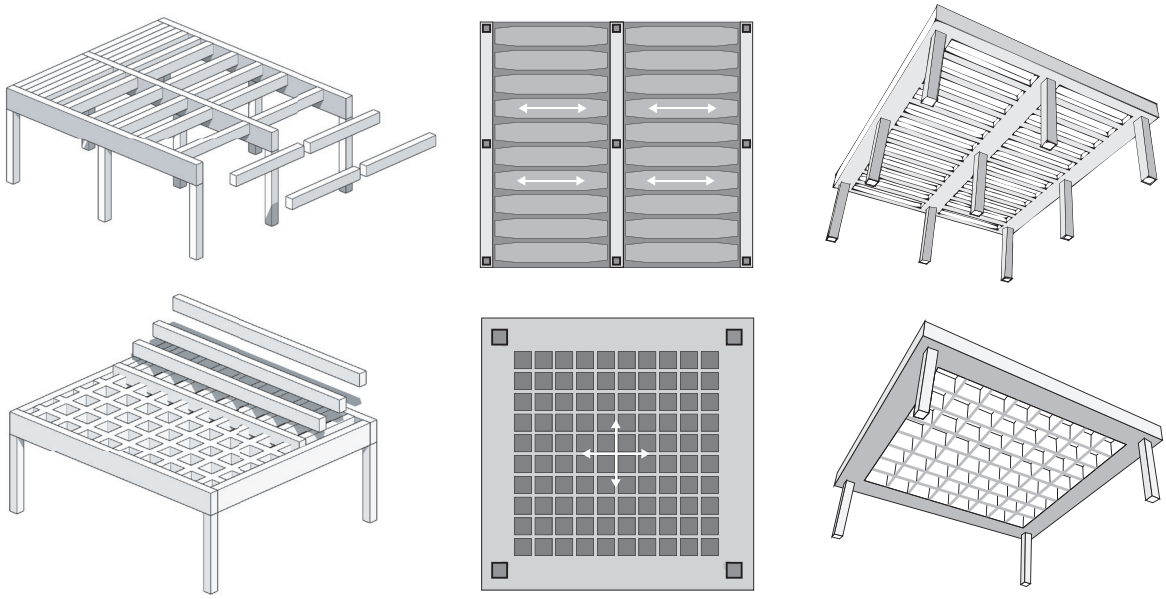


Figure 3.2.21 One-way framing (top) and two-way slab framing (bottom). Beam and girder directions reveal how forces are transferred

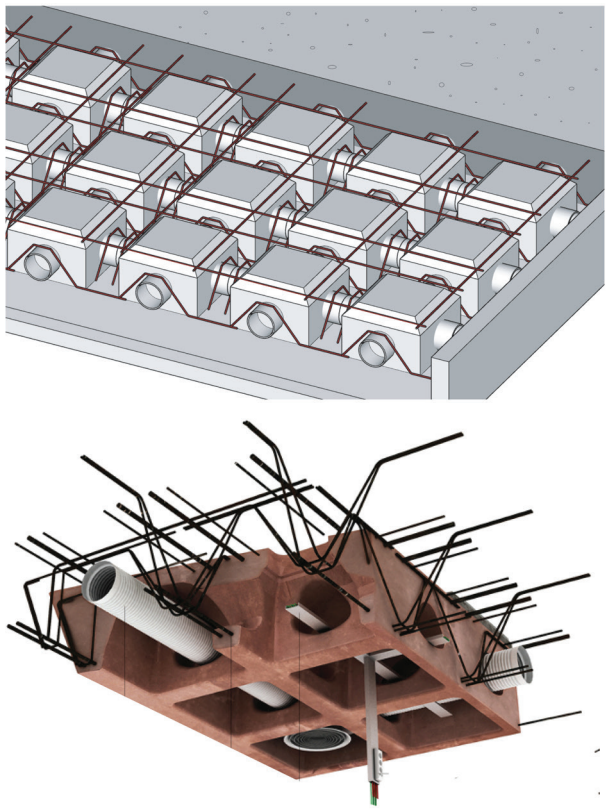


Figure 3.2.22 Integrated voids can reduce weight and simplify coordination with mechanical and electrical items. Shown: Holedeck system

actions—patterns we can draw with *isostatic lines*. If we can create the fiberglass pans with a similar geometry, we can cast the slab deeper in these locations to articulate the forces in the slab. (Figure 3.2.23)

Improvements through Construction: Pre-stressing and Post-tensioning

Designing better beams *isn't* a matter of waiting for viable materials—they already exist—or knowing what the beams or slabs should look like—we already do. The challenge is finding ways to translate these innovations into a viable design and construction process. For that we'll need to look for ways to build better too.

One solution for slabs is to manipulate how they are reinforced by considering how they are built, specifically by *pre-stressing* them. To do this, we modify the beam's form to overcome concrete's natural weakness to tension, compressing the ends together by pulling strands in tension until the beam becomes subtly arched, or *cambered*. Once the concrete is cast and sets to its specified strength, the extra tension in the strands is released and the camber disappears when the beam is finally loaded. The compression/tension

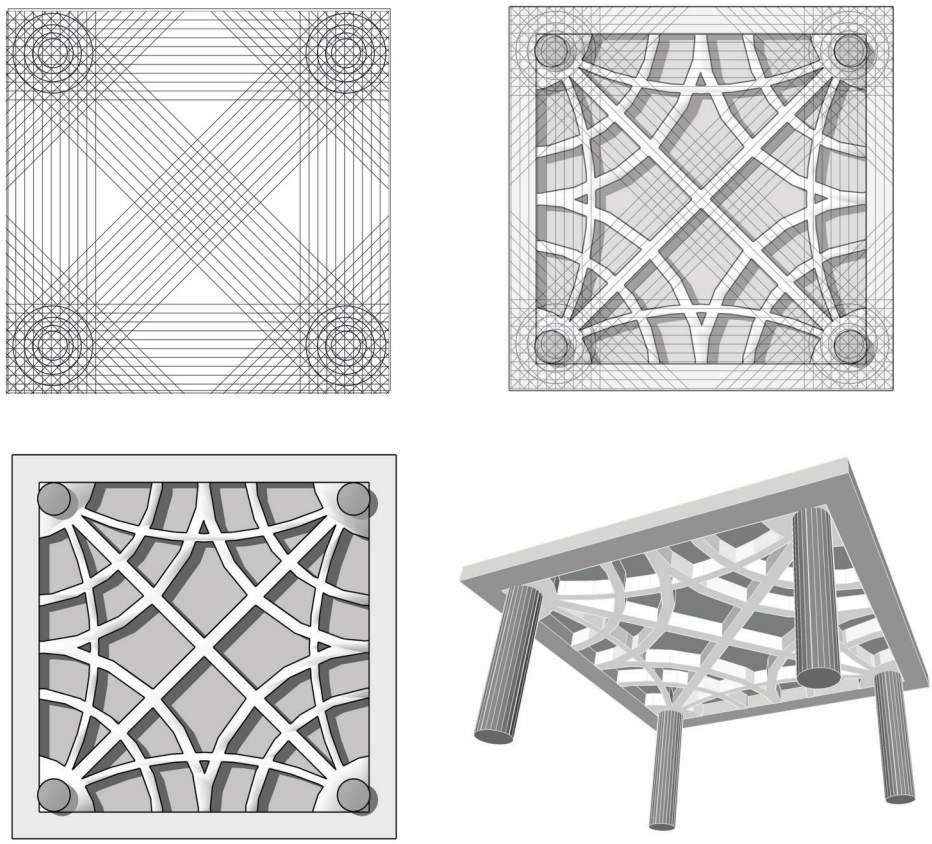


Figure 3.2.23 Isostatic lines can articulate the intensity of stress locations as deeper beams within more shallow slab surfaces

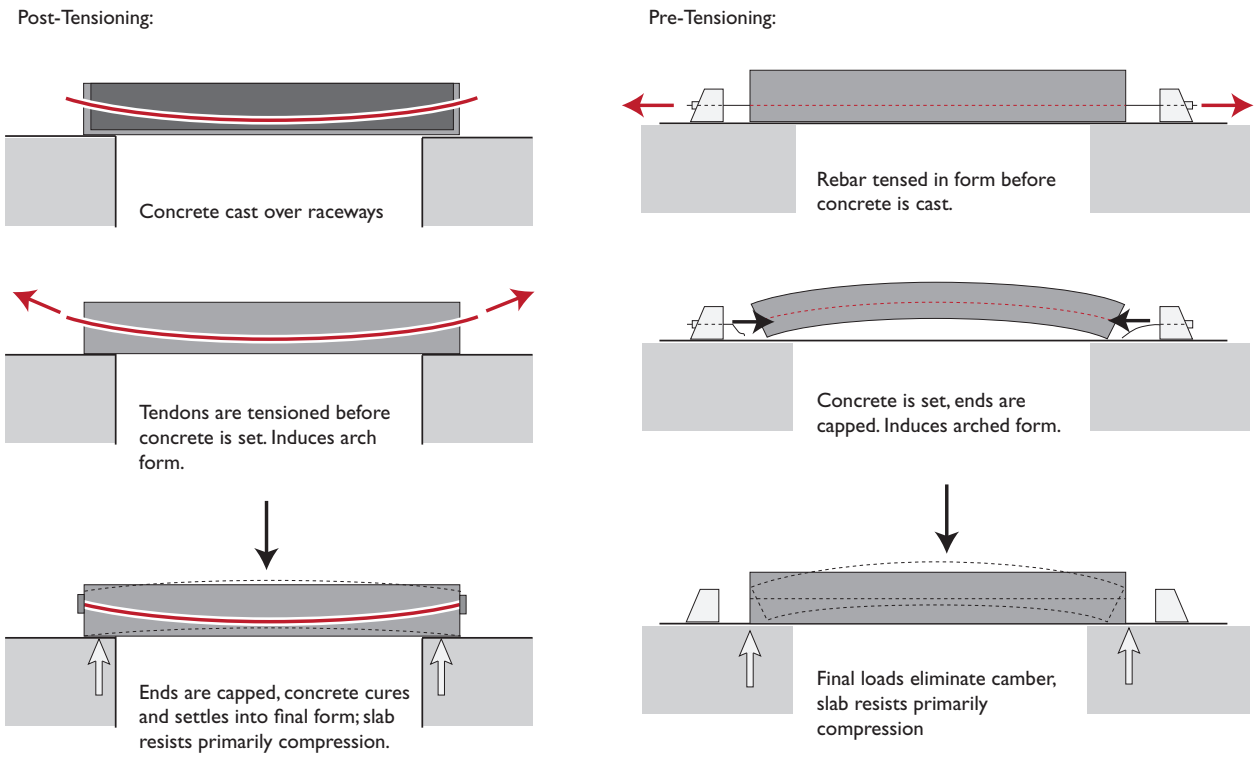
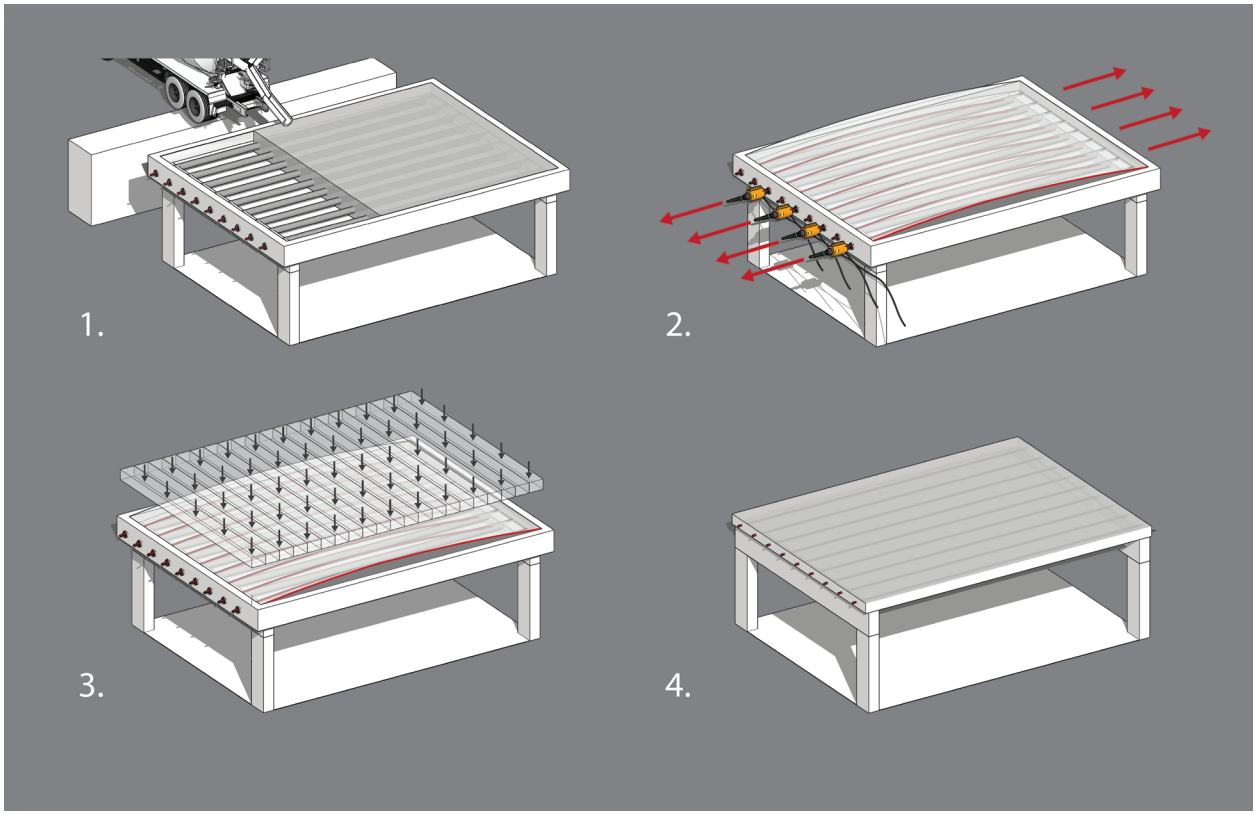


Figure 3.2.24 Diagrams illustrating the pre- and post-tensioning processes



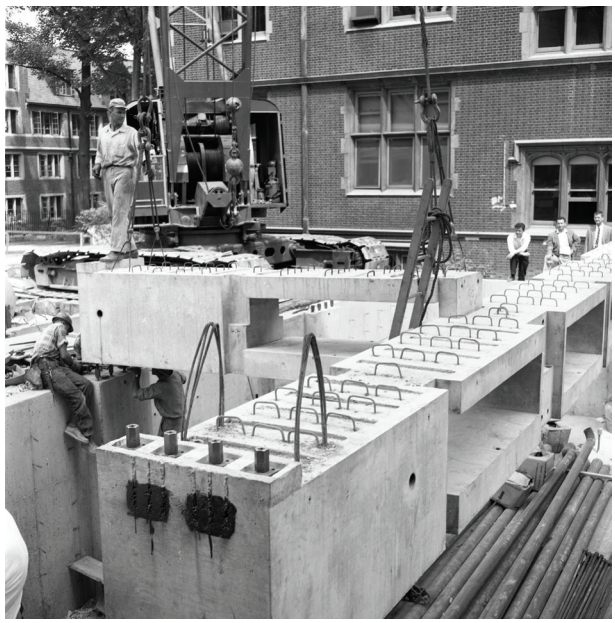


Figure 3.2.25a and 3.2.25b Construction process for post-tensioning elevated slab (casting, tensioning, applied additional loading, and final in-situ condition). Shown: Richards Medical Research Lab building (Kahn, Philadelphia, 1957)

force couple of the bending moment is relieved. In its final form, the beam resists compression across its entire depth, instead of simply its upper half, which allows it to be shallower. (Figure 3.2.24)

If pre-stressing isn't possible, another option with similar benefits called *post-tensioning* may be. In this process, before the concrete beam or slab is cast, reinforcing tendons are grouped within flat hollow steel raceways that extend to the slab edges. The raceways are spaced several feet apart, running either one-way or two-way as intended by the designer. After the concrete is poured over the raceways, but before it is completely set, a hydraulic jack pulls the tension wires tightly, compressing the slab. The ends of the tendons are fixed in place and concrete is patched over the ends, leaving a distinctive evidence of the process. Post-tensioned slabs, particularly if they are continuous across multiple support points, can span very long distances with minimal depth. (Figure 3.2.25)

LEARNING BY MAKING: PRE-STRESSING

Part I: Design and cast your own simply supported pre-stressed beam that spans 36in (91.5cm). This will require a carefully constructed formwork and a strategy for tightening and fixing the beam's reinforcing on the ends. What materials would be a good match for pre-stressing? How much camber would be necessary? Build a second beam with the same cross-sectional profile that *isn't* pre-stressed. Once both beams are cured to full strength, test them side-by-side. Carefully observe changes in deflection, cracking, and overall bearing capacity of the beams.

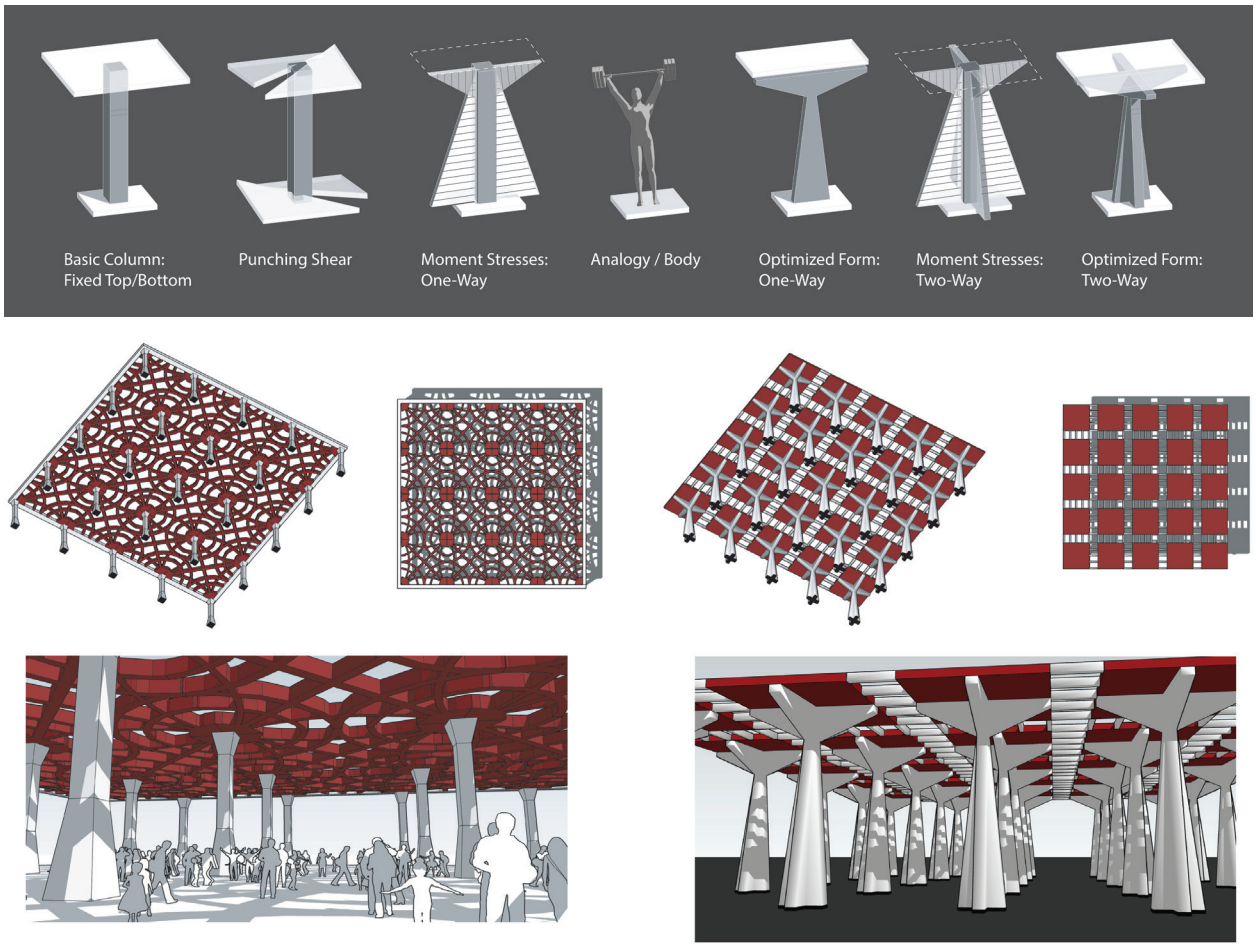
Part II: Using drawings and three-dimensional models (physical or computer), generate a proposal for how this beam could be translated into a building framing system, including a slab, using a post-tensioned process. How would your considerations change? Where would you connect the pieces together?

Strength and Shape in Column Design

Columns can also be made more efficient by distributing extra materials specifically in locations that prevent failures of buckling and punching shear. The threat of buckling can be resisted by reducing a column's length—a shorter column is less likely to buckle—or by adjusting its shape in plan—a square, circle, or a cruciform because bi-axial symmetry equalizes the resistance to bending in all directions. Finding other improvements depends on the column's connections. See Appendix Figure A.14 for column sizing estimates.

If it is pinned on both ends, buckling is most likely to occur at its center, like the bending moment in a beam turned on its end. Accordingly, columns can be shaped like a vertical beam—a pylon with more material toward the middle. If the column is fixed to the foundation, like a cantilever, its highest moment will be at its base, so the column should be wider there and taper up. If it is also fixed at the top, the column will gather load from its tributary area like a double-cantilever—thin on the ends and deep at the connection. This shape should be intuitively easy to understand, as we used this pose in our body structures (Chapter 1.0), and since the moment diagram suggests this shape. (Figure 3.2.26)

Similar shapes are also effective against punching shear. Columns risk shearing through a slab surface on account of their large concentrated loads where the two elements meet, like a pencil through paper. To avoid this, columns can be shaped with a large broad cap at their tops; capitals that spread loads across a large area. This is the same strategy columns use to release their load to the foundations at the ground, but inverted. These modifications can be quite expressive; column caps become tributary load diagrams. (Figures 3.2.27 and 3.2.28)



Figures 3.2.26a and 3.2.26b Potential evolution and optimized forms for columns showing wider areas on the top, bottom, and plan. Structural principles can be expressed in columns and slabs

Figure 3.2.27 Expressions of load transfer between load collector and columns. Shown (left to right, top/down): Madrid-Barajas International Airport (Rogers, 2006), Clemson University, Lee Hall III (Phifer, 2012), Miller House (Saarinen, 1957), Atocha Train Station addition (Moneo, 1993), Denver International Airport (Fentress, 1995), Itamaraty Palace (Niemeyer, 1970), Sagrada Familia Cathedral (Gaudi, 1883), St. John's University Library (Breuer, 1966), Clemson University, Lee Hall III (Phifer, 2012)

Figure 3.2.28 Frank Lloyd Wright's dendriform columns in the SC Johnson Wax Administration Building and the full-scale prototype testing process (60tons of sand) required for code approval. (Wright, Racine, 1939)

THINKING: STRUCTURES AROUND YOU

Find examples of "columns" in everyday objects and compare these to the column design principles discussed above. Are the proportions in plan bi-axially symmetrical? Is the cross-sectional area distributed to the outer edges of the element? Is the connection between the column and the load collector pinned or fixed? Next, find an example of an architectural column that seems more "expressive" in its form than a conventional solution (e.g., cast concrete columns, lattice grid columns, etc.)

To Discuss: What qualities does the archite ctural column have that helps it to resist against buckling?

Building Better: Off-site Construction

Another solution for improving our structures' performance is to integrate digital fabrication techniques—frequently in coordination with off-site construction methods. As Kieran Timberlake suggest in *Refabricating Architecture*, architects should conceive of buildings not just as mass-produced elements but as larger pre-assembled parts, akin to an automobile. Doing so allows us to provide more quality and scope for less cost and time. This is common for many phases of a project's construction (e.g., doors, equipment, etc.) but not the structure. (Figure 3.2.29)

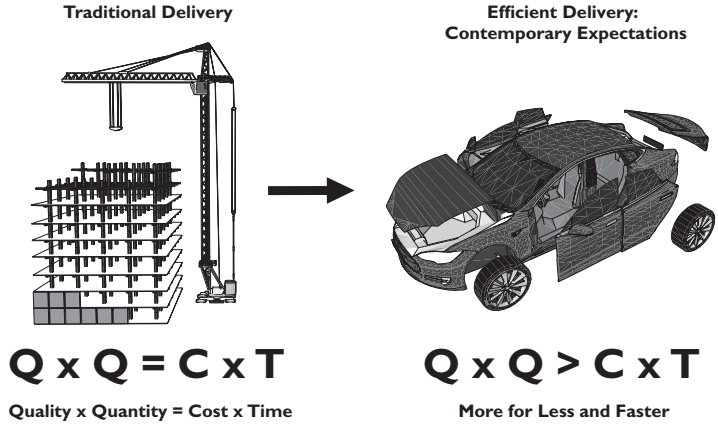


Figure 3.2.29 Economic expectations of assembly and aesthetics have changed how buildings should be designed and delivered. Quality and quantity must equal cost and time constraints. Shown: Illustration of principles from *Refabricating Architecture*

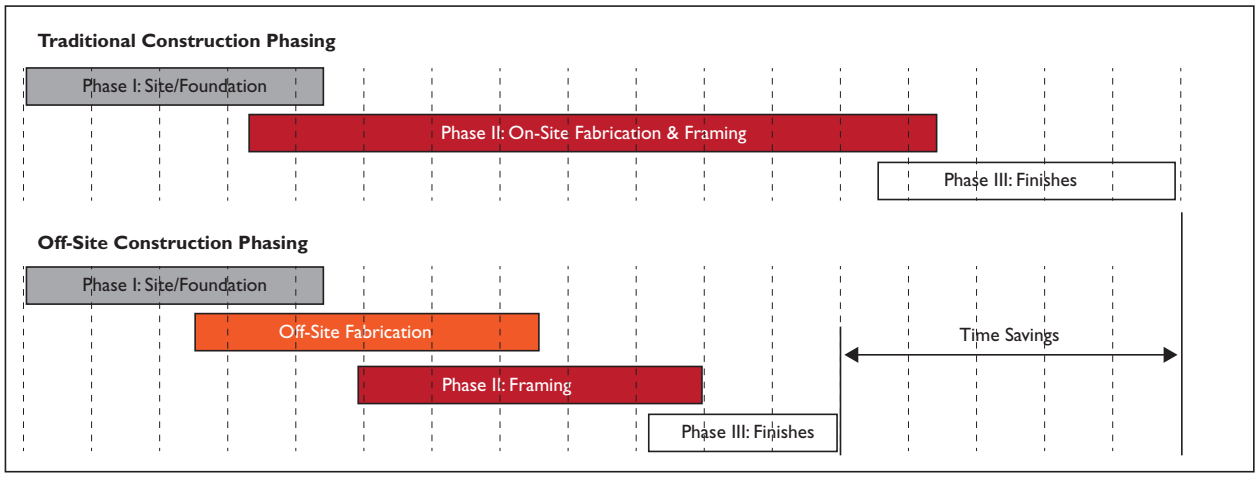


Figure 3.2.30 Typical project construction time line comparisons between traditional and off-site

Pre-fabricated structural elements have been integrated into buildings ever since the Crystal Palace (Chapter 1.1), but typically as smaller parts (beams and columns) that are assembled in the field, not as full modules. Previous chapters have extolled the efficiency associated with modularity in structures and systems. Off-site construction takes this idea to a next level of economic and construction efficiency. One idea is to have portions of a building that can be built off-site in larger volumetric modules (e.g., a box with the floor, walls, and roof), so they might ideally be assembled faster with more exact tolerances. (Figure 3.2.30)

As of this writing several "stacked cube" construction projects at moderate scales have been successfully completed and a growing set of economic data demonstrates the benefits of off-site construction. Although it is understandable volumetrically, experiments in off-site fabrication shouldn't be limited only to boxes. Off-site fabrication's promise is compelling. Factory controlled settings for assembly have greater quality control and enable custom shapes, with specialized digital fabrication tools, at lower cost. Each element can be designed and fabricated with minimal materials and shaped to their most effective forms. (Figure 3.2.31)

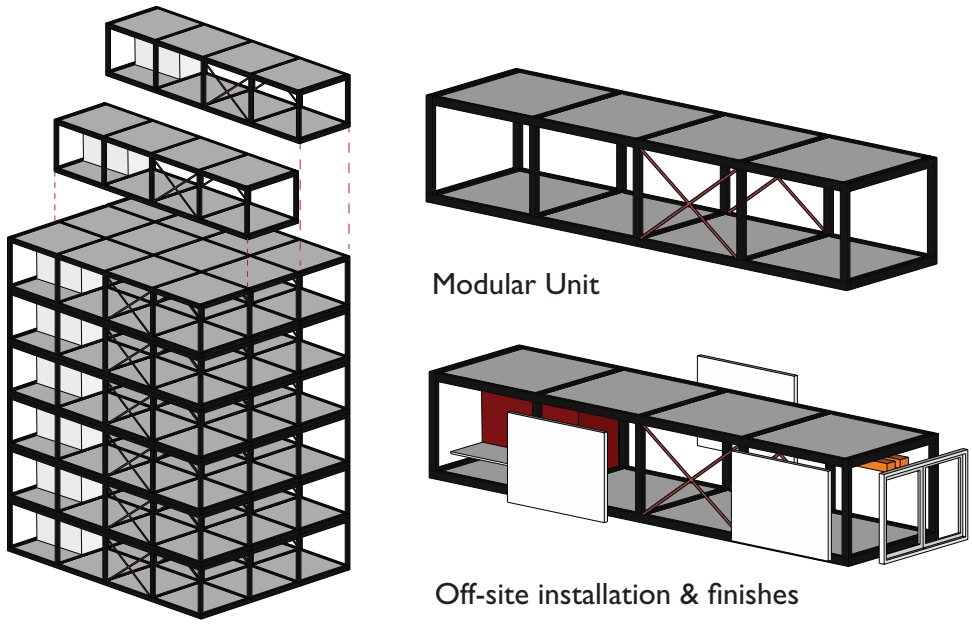


Figure 3.2.31 Critical contemporary practices rely on an ability to optimize structural and construction principles as an integrated architectural solution

FOLLOW THE FAILURES: PRE-FAB TOWER

Early experiments in off-site construction have usually started with the structural idea of stacked boxes. SHoP Architects and Arup used this modular construction approach for the 32-story Atlantic Yards/B2 BKLYN (now named "Pacific Park") which was intended to be the largest prefab tower in the country. Their design included a super-structure framing system for the stairs, elevator, and building lateral supports connected to module boxes that were to be stacked up and connected together to complete the living spaces. But the project was beset with many delays related to political, economic, and technical issues.

To Do: What were the structural issues the project faced? What caused the misalignments to occur?

To Discuss: What other formative lessons about the relationship between design and construction does this project offer?

Collective Considerations for Building Better Beams

History has shown the importance of developing design, construction capacities, and material technologies together to advance structural design—and beams are no exception. There are different ways to shape the section and profile of spanning elements, including relatively radical departures from the norm. We *could* make beams like this, but we'd need to develop new ways to manufacture them as they couldn't be "rolled" like steel, and casting them would require complicated formwork. Speculative beam shapes can be printed or fabricated using new materials and innovative methods. In industry, these shapes are often produced through a process called Additive

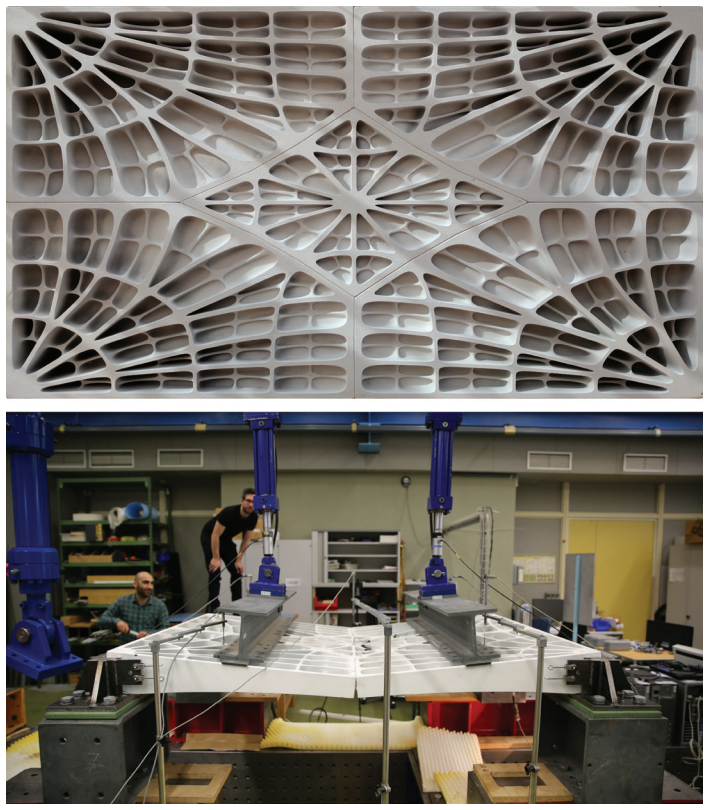


Figure 3.2.32 3D-printed floor system testing and prototypes (Block Research Group, ETH, 2017)

Manufacturing (AM) that forms complex shapes by affixing layers (like 3D printing). Implementing this for an entire structural system is currently limited by the lack of viable and affordable structural materials.

Successfully integrating digital fabrication opportunities isn't solely dependent on new materials, new industries, or even 3D printing. Existing materials and methods can be hacked and improved if we understand the intent of our efforts. By connecting digital design and analysis with digital fabrication processes, we can create structural prototypes that are efficient, effective, and expressive.

The Block Research Group at ETH Zurich consistently examines this fundamental relationship between design optimization and fabrication. Their "3D-printed floor system" combines the naturally resistant forms of a double-curved Guastavino vault with isostatic lines to create a materially efficient slab. Its expression comes from its structurally efficient form and its manner of assembly. These holistic efforts to combine high-performing materials with structurally responsive forms and advanced methods of making follow challenges set forth by Viollet-le-Duc, Corbusier, Fuller and others. (Figure 3.2.32)

Frequently Asked Questions

If construction and economics plays such a big part in building design, wouldn't we be better off only learning the cheapest way to build something? Don't confuse conventions with cost-effectiveness. Think of economics as an intellectual principle that can be guided by ethics and professional judgment. As a first step, don't cede your ability to be a creative problem-solver simply because you are told something is just "done that way." You should be able understand what makes one type of solution seemingly more "affordable" than others before you propose changes. The strategies we've discussed above may suggest modest or radical changes from certain practice standards, but most of these solutions are quite commonplace in the industry already. Our profession shouldn't settle for convention if new solutions are a better match for the architectural goals of the project and if they are customized for the purpose of enhancing efficiency. How else does innovation occur?

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PART 4

Resistance and Vectors

Figure 4.0.0 Construction of Blimp Hangar One (NASA, Sunnyville, c. 1934)

CHAPTER 4.0

VECTORS

Trusses and Triangulated Planar Assemblies

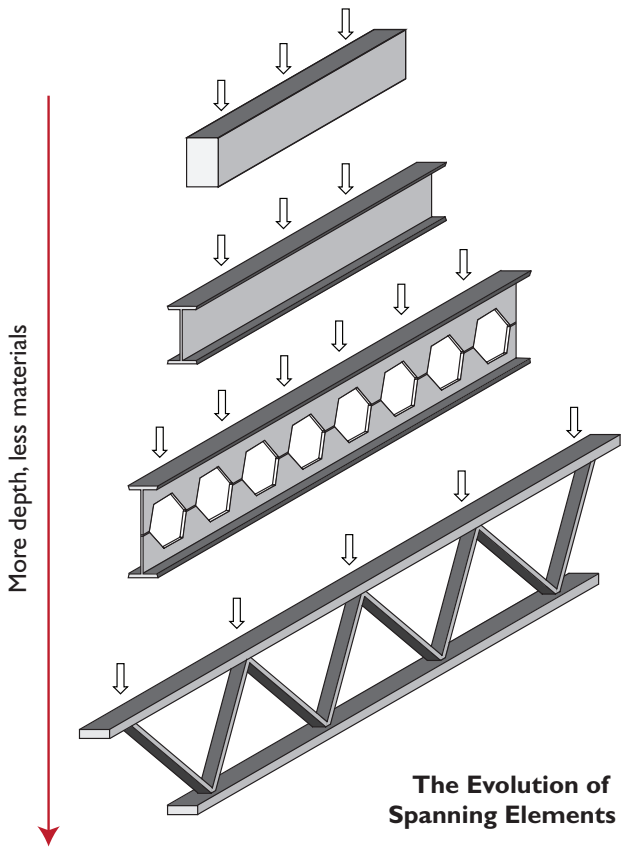
Trusses, or vector-resistant structures, are the most materially efficient and functionally effective structures ever designed. These qualities result from their distinctive features—triangular panels. Truss design is, in part, an exercise in how to creatively embrace their efficiencies.

What Makes Trusses Efficient?

To understand how trusses work, it's helpful to think of them as being like "hacked" beams—deep, but lightweight elements with materials strategically placed in the most effective cross-sectional locations.

But trusses don't behave like beams. Instead of relying on only a mass of material to resist bending, trusses transfer stresses through short, straight-lined elements (*webs or struts*). Because trusses are lightweight and materially efficient, they can span farther with less material than beams. Cecil Balmond describes their relative effectiveness by stating, "Measure for measure, on weight and strictly engineering parameters, the truss is hard to beat." (Figure 4.0.1)

These triangulated panels can be assembled to make larger geometric configurations that resist stresses collectively. These are called *vector-active* systems because they split downward forces into vectors, creating only tension and compression stresses. Because these stresses are axial, trussed web members can resist them with their entire cross-sectional area. Their use in the world's longest spanning structures and in modest, cost-effective, utilitarian buildings alike, attests to their usefulness. (Figure 4.0.2)



Figures 4.0.1 Truss as an evolved beam: Lighter, deeper, and stiffer with an effective distribution of cross-sectional materials

Design Challenge: When Beams and Arches Won't Work

It's difficult to intuit why triangular panels make effective systems of resistance and conveyance—we can't make our bodies into multi-panel trusses to feel how they work. But if we compare trusses to the most structurally effective qualities of arches, cables, and beams we'll see why trusses are a responsive alternative.

Consider options for a roof structure across a hypothetical building with a span that exceeds a beam's capacity. How would your spanning elements evolve if the supporting walls weren't able to resist thrust from arches or vaults? What solutions might you propose? Because span is the controlling design factor in bending and deflection, it is of paramount importance for the solutions to be light, stiff, and self-stabilizing without thrust.

These conditions aren't all that unusual—they are the same programmatic, economic, and physical conditions that compelled many early experiments in truss design. As new programs demanded larger open volumes, structural solutions evolved to span them. Many of the following options mimic the historical attempts created by designers and builders to solve this problem. (Figure 4.0.3)

Option #1: ***Bent Beams***. *Matching structural forms with moment diagram profiles is an effective means of load transfer. Unfortunately, bending beams into an arch form or tilting them into a triangular peak creates an outward thrust at the base. We'd have to resist the thrust somehow.*

Option #2: ***Bigger Beams***. *The span is nearly twice as far across as the longest available beam. Splicing two flat beams together won't work, as bending stress at this splice (the point of maximum moment in the middle) would be untenable. If we staggered*

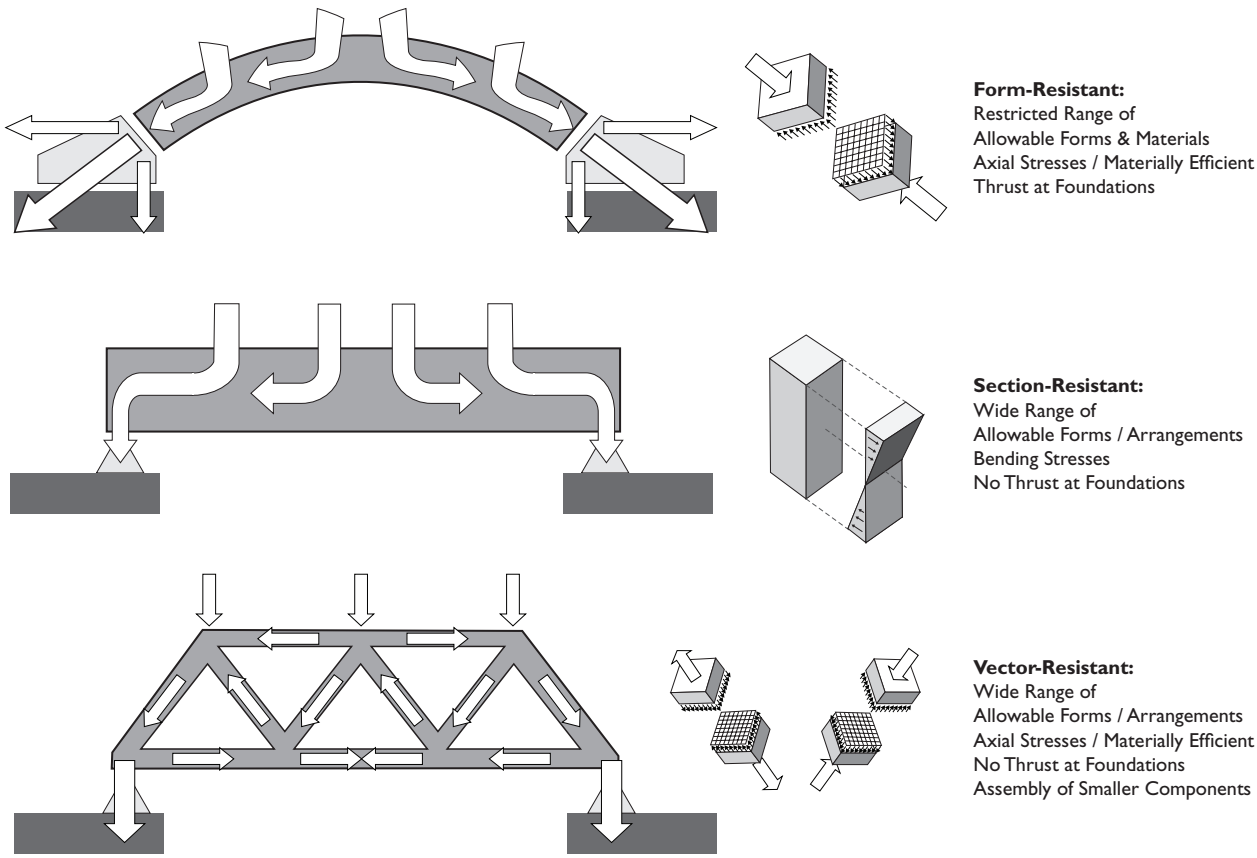


Figure 4.0.2 Comparing form, section, and vector-resistant force distribution

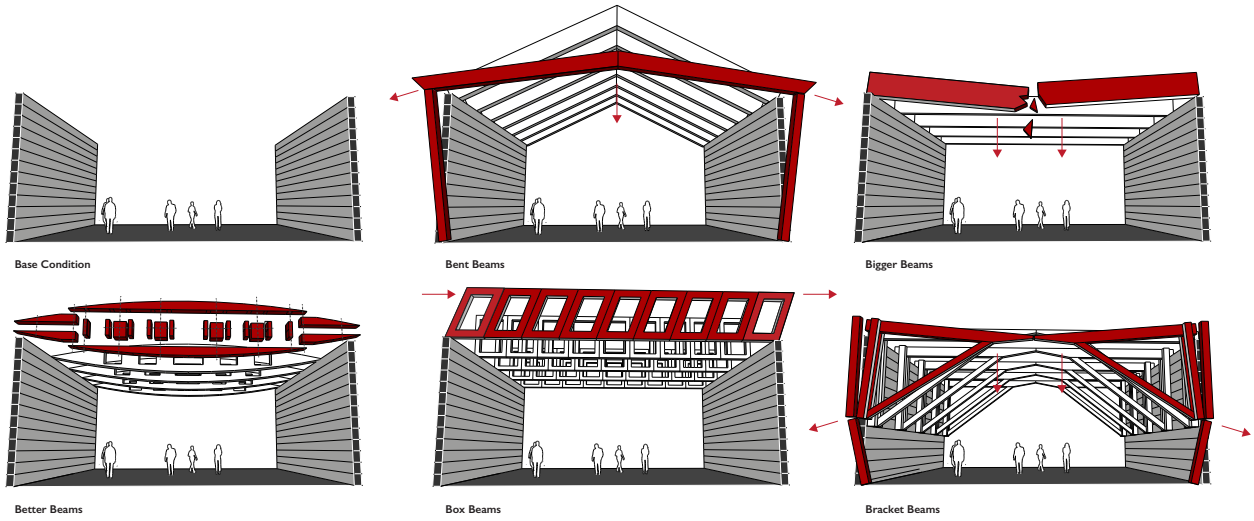


Figure 4.0.3 Potential failures of traditional spanning solutions

and spliced several beams together, as was done in the 15th to 18th centuries, we'd add too much dead load weight—a fundamental problem and limitation for long span structures (e.g., Galileo's "square-cube paradox")—and we'd still have high bending stresses at each splice point.

Option #3: ***Better Beams***. *We could try to create one large "optimized" beam. We'd have to design the beam's top and bottom edges with more cross-section to resist higher tension and compression stresses, create an "arch + cable" profile that was deepest in the middle, and brace the web from buckling by adding diagonals. Constructing it would be difficult. It wouldn't panelize easily into regular parts and the bending resistance would depend on monolithic behavior that common splices couldn't provide.*

Option #4: ***Box Beams***. *We could solve many of these construction concerns if we created rectangular frames that were connected end-to-end. Unfortunately, unless we connected them with moment connections (like a vierendeel truss), the panels would be geometrically unstable and they would collapse into a hanging cable form of parallelograms. To solve this, we'd have to brace the panels laterally.*

Option #5: ***Cantilevered Brackets***. *Instead of spanning across the roof with one element, we could think of the structure as two triangular "brackets" that cantilever off opposite walls and meet in the middle. These brackets would be geometrically stabilized and their form would reflect a cantilever's moment diagram which would be helpful. But the cantilever still inflects a horizontal thrust as part of its bending moment connection at its root. Again, we'd have to resist this outward force with another structural component.*

Design Solutions: The Utility of Trusses

For most of these solutions, we could resolve their problems by adding components to triangulate the configurations. (Figure 4.0.4)

Option #1: We could avoid the thrust if we make an A-frame, with a new member along the bottom to tie it together. The top part would resist compression like an arch—but without the thrust—while the new member along the bottom would resist compression.

Option #2: We could still splice the beams together but instead of relying on the splice to resist bending we could add a new element that would triangulate and stabilize the form. A vertical strut in the middle of the span, below the splice, would then hold the bottom of the strut in place by triangulating it with the supports. This inverted V-shape is helpful because it adds depth in the center of the span

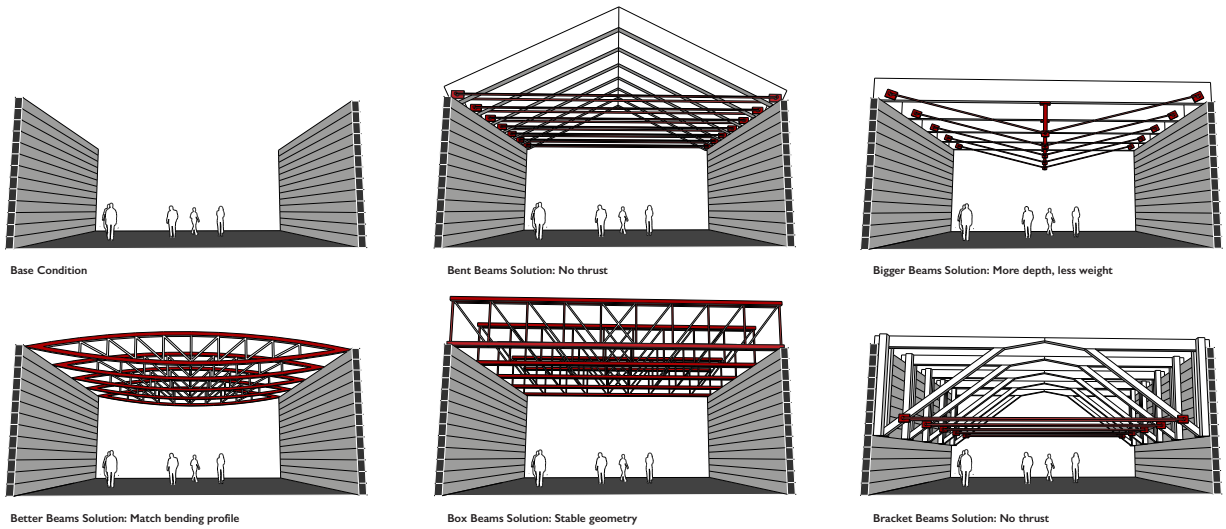


Figure 4.0.4 There are specific ways that triangulating the structural members can resolve the spanning problems. The solutions create an internal equilibrium between compression and tension internal stresses

(matching the moment diagram like the A-frame). The top part and the strut are in compression, the bottom diagonals are in tension, and—because it is triangulated—there is no outward thrust. This is commonly called a *trussed beam*.

Option #3 and Option #4: The solution to both of these schemes is similar. If we redistribute the cross-sectional area of the spanning member to its top and bottom edges and stabilize the interior web, we could create a lightweight but stable version of a beam. If we add a triangulating element between the rectangular frames, it would no longer deform under loading.

Option #5: We can use the same approach as Option #1 to resolve the thrust in the cantilevered bracket. If we add another member across the bottom as a tie-back, we'd resist the outward force of the bracket. Instead of two triangles, now we'd have three. The efficiency of the form and load resistance would remain.

Compared to arches, cables, and beams, the benefits of trusses under these conditions are clear. Because trusses don't have outward thrusts, they can be used in framing systems like beams; but, because they resist stresses axially like arches and cables, they are more lightweight and efficient than beams. Because of their triangulated panels, trusses have a high strength/stiffness ratio, they maintain an effective depth-to-span ratio, and they can be effectively fabricated and erected into form-resisting shapes. As long as their triangular panel forms are connected together at their end *nodes*, any number of solutions are possible.

Internal Equilibrium in Trusses

The critical concept to understand about truss behavior is this: The forces within a truss are in equilibrium—across the entire truss and panel-by-panel or joint-by-joint. The components within a truss are either pushed in compression or pulled in tension. We can see this equilibrium if we look "inside" a truss. (Figure 4.0.5)

Consider one refinement to one of our initial solutions. Combining smaller elements in self-stabilizing triangles allows us to span farther than the individual members could have spanned by themselves (an apt benefit for Option #4 and Option #5). We could even amend the shape of the individual panels to create a profile that matches the moment diagram. We could combine the solutions for Option #1 and Option #2 by creating two smaller trussed beams, tilting them up into an A-frame shape, and linking them together

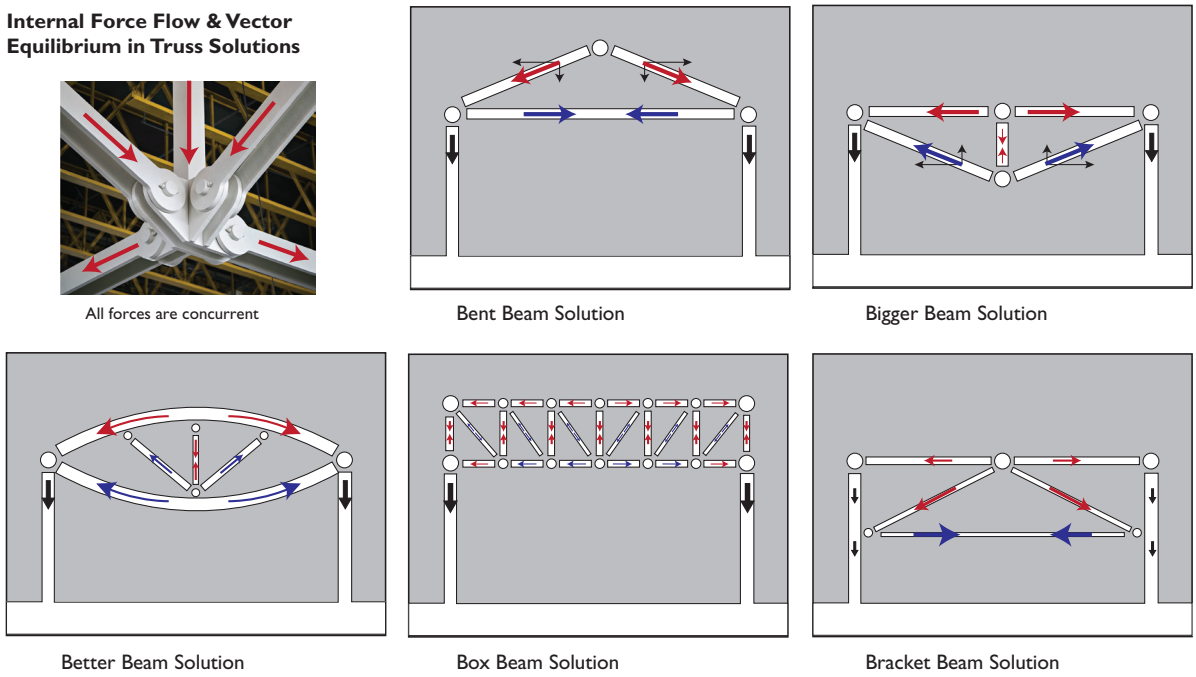


Figure 4.0.5 Force distribution diagrams of the truss solutions (red = compression and blue = tension)

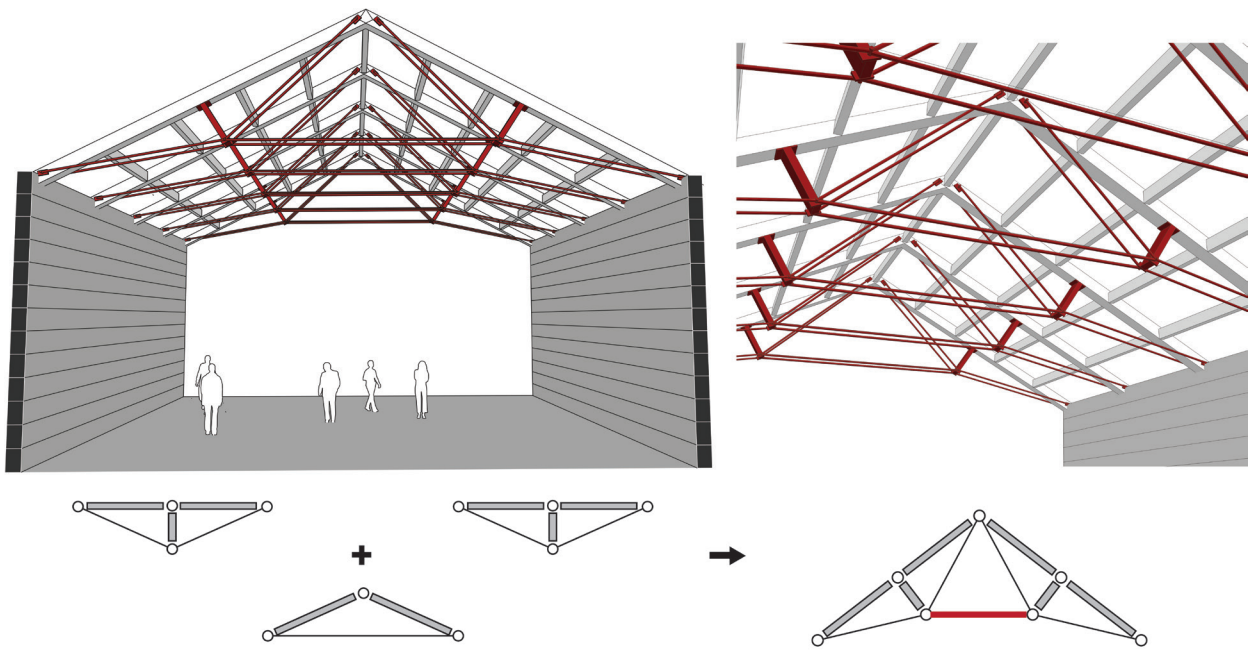


Figure 4.0.6 Example of the first "engineered" iron truss design by Camille Polonceau, a French railway engineer (1839)

Figure 4.0.7 Contemporary examples of the strut and cable truss system. Shown: Chicago Art Institute (2006), Harvard Art Museum Expansion (2014), both by Renzo Piano Building Workshop, and Louvre Museum entrance pyramid (Pei, Paris, 1989)

horizontally. Their outward thrust would be relieved if we connect the base of the posts together with a tension wire. (Figures 4.0.6 and 4.0.7)

Panels and Parts: Design Options for Trusses

Trusses' efficiencies are easy to recognize, but to apply these lessons to our designs we'll have to understand how variations in their configurations affect structural behavior. Consider the options: Trusses can be panelized (one-way) or spatial (two-way), their chords can be parallel, curved, or sloped, they can have many panels or just a few, they can be tall or short, their diagonal web members can be angled in many ways, and they can be made of wood, steel, iron, etc.

These options allow us to optimize trusses using the same strategies we've seen in other structural systems: Flat panel trusses can be curved into arched trusses (to resist bending stress), they can match the funicular shapes of arches or cables, trusses can become moment resisting portal frames (or vierendeels), multiple panel trusses can be combined into two-way resisting systems like slabs (known as space frames), and trusses can be formed into efficient doubly-curved three-dimensional forms. (Figure 4.0.8)

How do we assess the consequences of each of these factors (steel versus wood, taller with more panels versus shorter with fewer, etc.)? Because design is about problem-solving and comparing options to find responsive solutions, we have to understand how this resistance is provided. We need to move beyond broad principles of truss behavior into the truss' inner-workings. Not surprisingly, we'll begin by looking at their basic module: the triangular panel.

Material Matters: Engineered trusses were a product of the mid-19th century advancements in mathematics and material science, but for centuries prior to this, wooden trusses were integrated into a variety of building types throughout the world.

To Discuss: What were the material limitations of creating trusses that were exclusively made of timber? In other words, how does the behavior of a truss suggest (or challenge) the use of certain building materials? Why did many trusses use different materials for the connections between truss members?

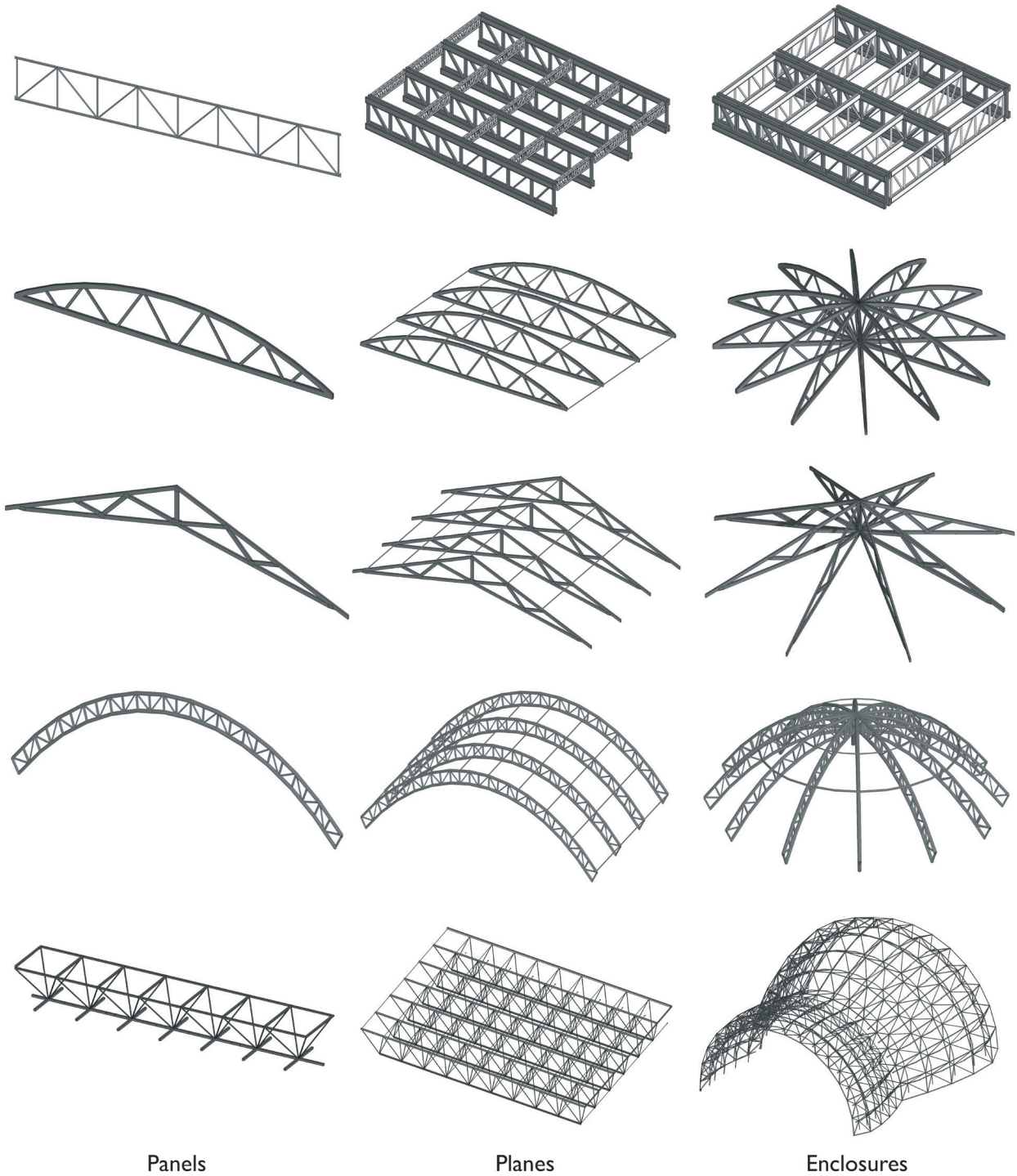


Figure 4.0.8 Truss members are comprised of smaller triangulated panels that can be arrayed, gridded, rotated, or otherwise combined into spatial volumes

Truss Behavior: Panel by Panel

To design with trusses, we need to understand how their triangular panels resist stresses and how these panels' geometries affect the amount of stress within them. We'll use demonstration models and gather certain "rules" for how trusses work from each demonstration and an understanding of basic terminologies of trusses: Chords (top and bottom), diagonals (web members), panels (the module), and panel points (connections of panels). (Figure 4.0.9)

Model #1: Plane and Basic Configurations

We are trying to determine how forces move through a triangulated panel, and how the truss' basic parts work. A single point doesn't provide geometric resistance because it is unbound. Joining two points makes a line that can resist stress axially (pulled or pushed), but can rotate. If three non-linear points are connected together with lines, they'll form a plane that is naturally resistant to movement. *Try it*. Make a plane by joining three sticks together at their ends with a pinned joint and push down at one of the node points. You'll see and feel it slightly move and stabilize by compressing or pulling it. The pin joint directs the line of action along the axis of the resisting member. Pushing elsewhere on the truss, like in the middle of a chord member, will bend it and negate the efficiency of structural action. Unlike cable or arch systems, these triangulated panels ensure that the trusses are self-stabilizing and that they don't exert outward thrusts. (Figure 4.0.10)

In order for truss panels to work, the following must occur:

Truss rule #1: *Panels must be triangular.*

Truss rule #2: *All members must be connected with pinned joints at the ends (the "nodes").*

Truss rule #3: *To avoid bending, loads must be applied at the pinned nodes.*

Model #2: Panel Modifications and Stress

How does triangular geometry affect the amount of internal stress? What happens if the truss becomes taller but the applied force remains the same? We

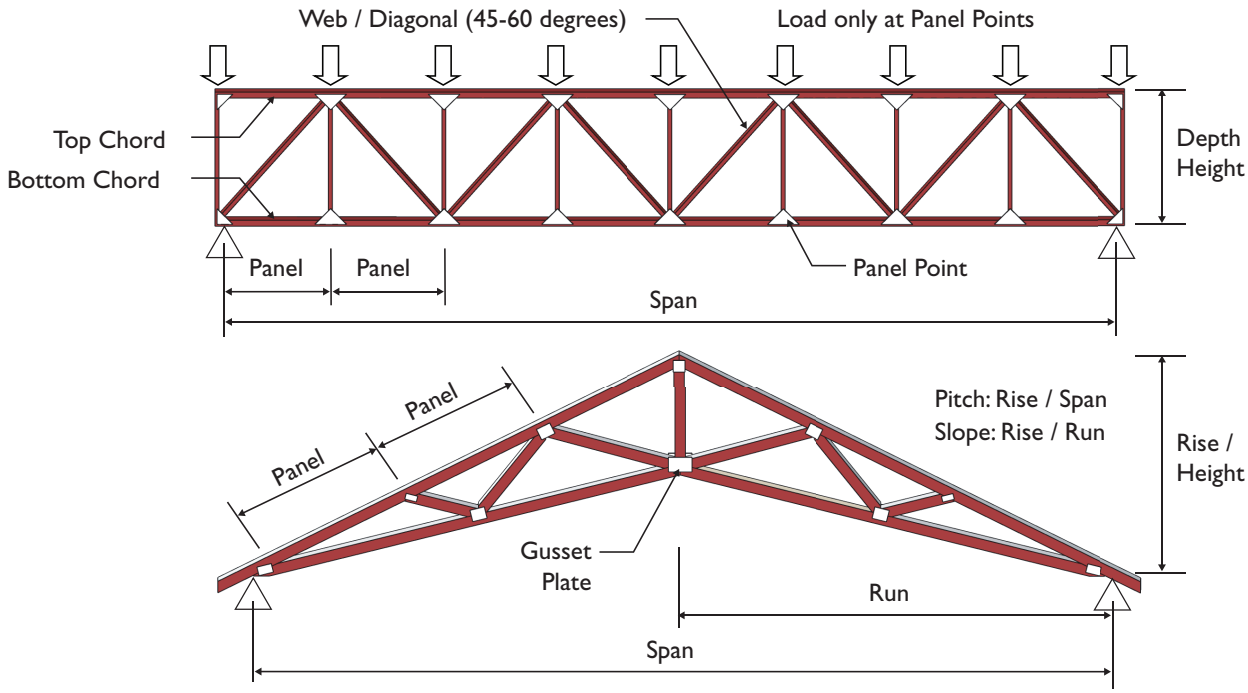
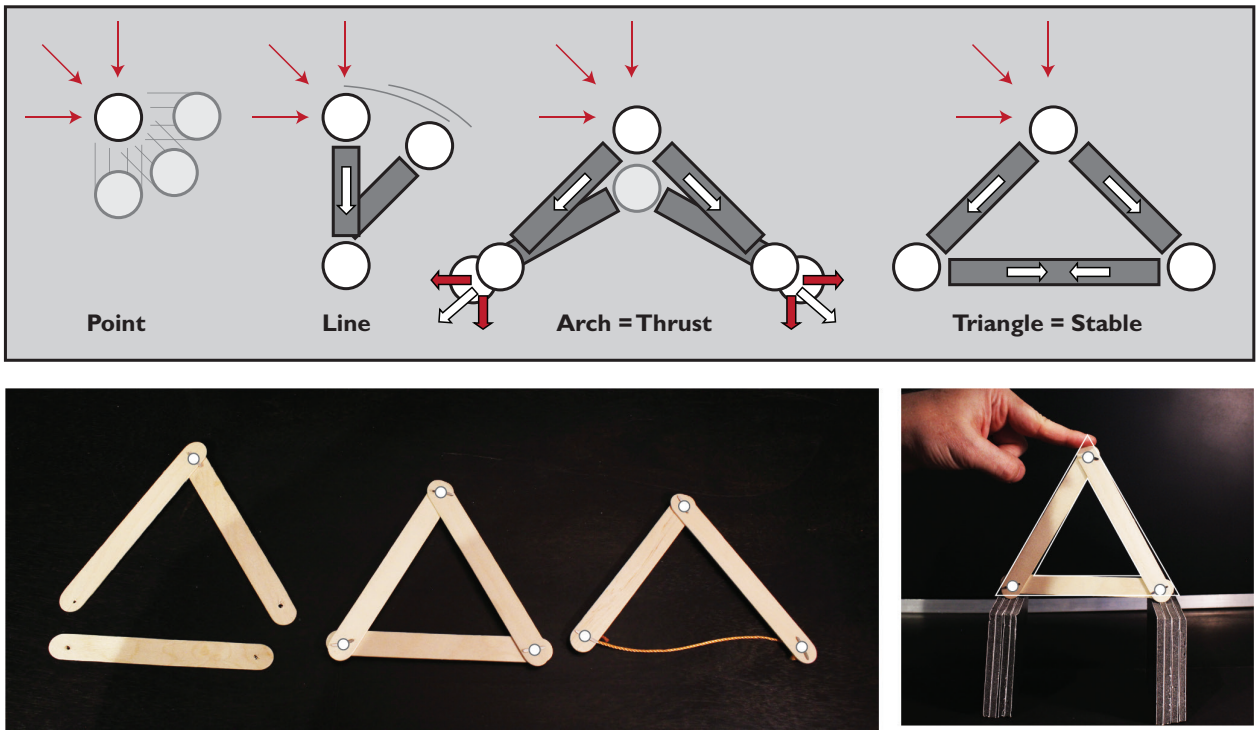


Figure 4.0.9 Basic terminology of trusses



Figures 4.0.10a and 4.0.10b Photos of model experiments. Diagrams show how points that are fixed to each other through straight-lined members are eventually stabilized through triangulation

don't need to run calculations to answer these questions. We could apply what we learned from basic body structures (Chapter 1.0)—the wide versus narrow stances of the weight lifter are analogous to the width of truss diagonals. If we want a more specific value, we could confirm these stress levels without having to run calculations using graphic statics to draw the internal stresses.

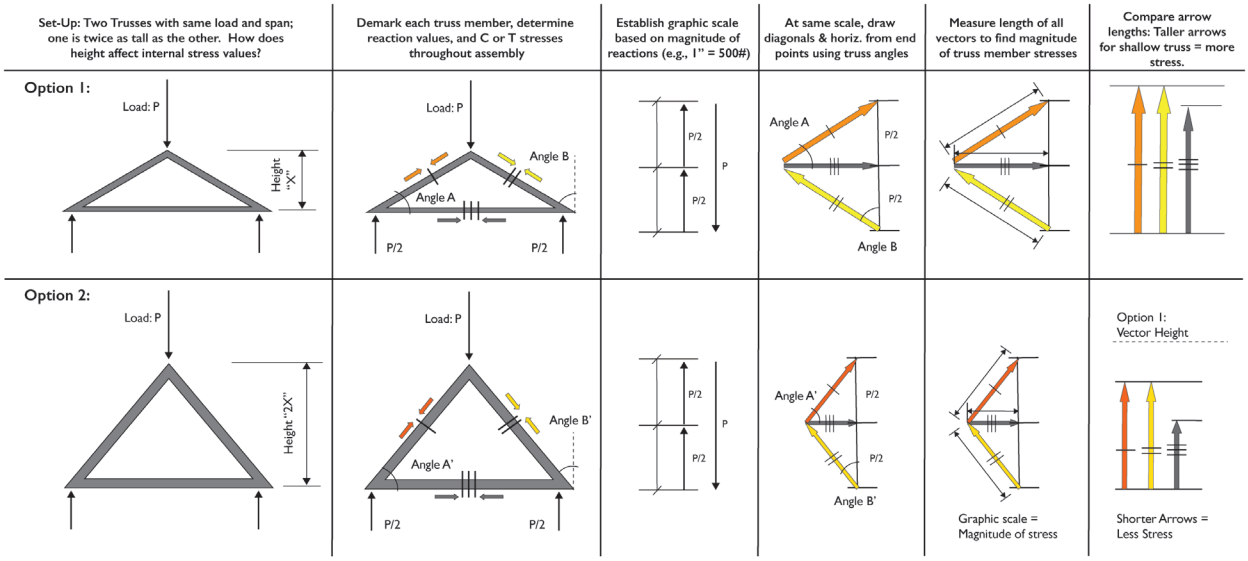


Figure 4.0.11 Comparison of internal stresses between a taller and shorter truss using free-body diagrams and vector lengths

From basic vector mechanics, we know that resultant forces have horizontal and vertical force components that are geometrically related. Raising the angle of the diagonal reduces the horizontal force component (as the vertical magnitude is fixed based on loading) and therefore lowers the resultant force as well. We could measure the components' lengths to find the forces' magnitudes and based on the direction of arrows, we can see which components are in compression and tension within the truss. (Figure 4.0.11)

BREAKING: MATHEMATICAL AND GRAPHIC MODELS

There are two common tools used to calculate internal stresses in trusses: one method is visual (graphic statics) and the other is mathematical. By determining internal stress inside each truss component, these tools anticipate deflection and determine how large the resisting members need to be.

Graphic Statics is still the tool of choice for many contemporary practitioners because of the direct way that it translates physical form into vector diagrams. This evaluation compares applied forces with the geometry of the truss diagonals and chords to find the value of internal forces in each element. These comparisons can happen roughly with sketches, or precisely with minimal drafting; either method allows immediate feedback on the design and performance of different designs. Digital versions of graphic statics programs can be used for educational and demonstration purposes. (Figure 4.0.12)

The two mathematically based systems of evaluation, *Method of Joints* or *Method of Sections*, rely on an assumption of translational and rotational equilibrium throughout the truss. Because truss components are

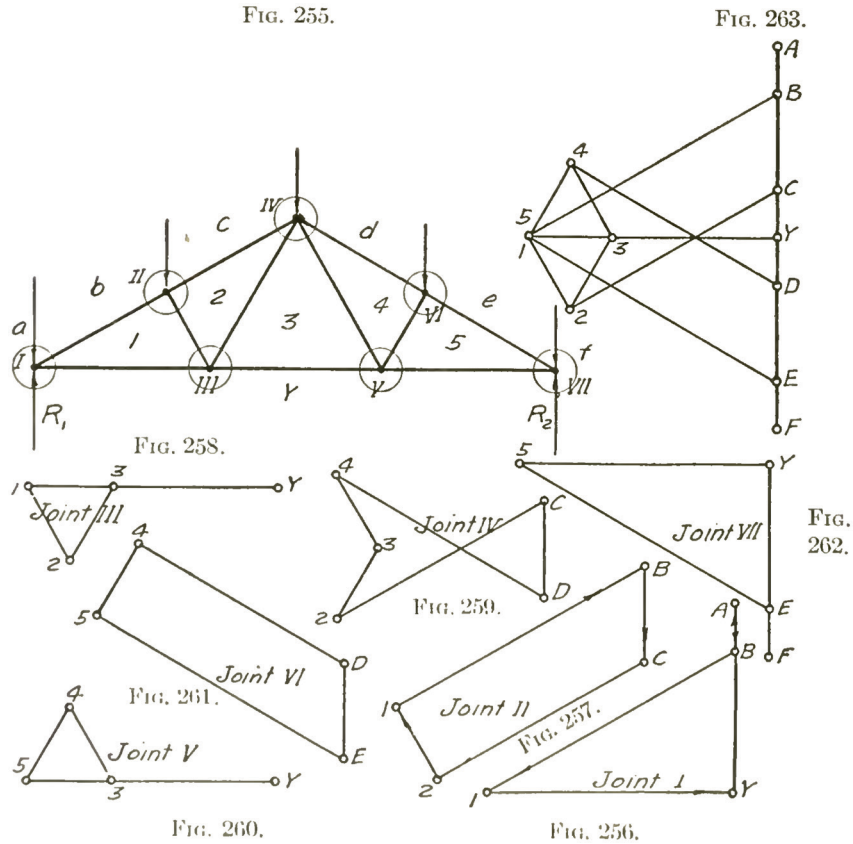


Figure 4.0.12 Example of graphic static truss analysis (Wolfe, Graphical Analysis, 1921)

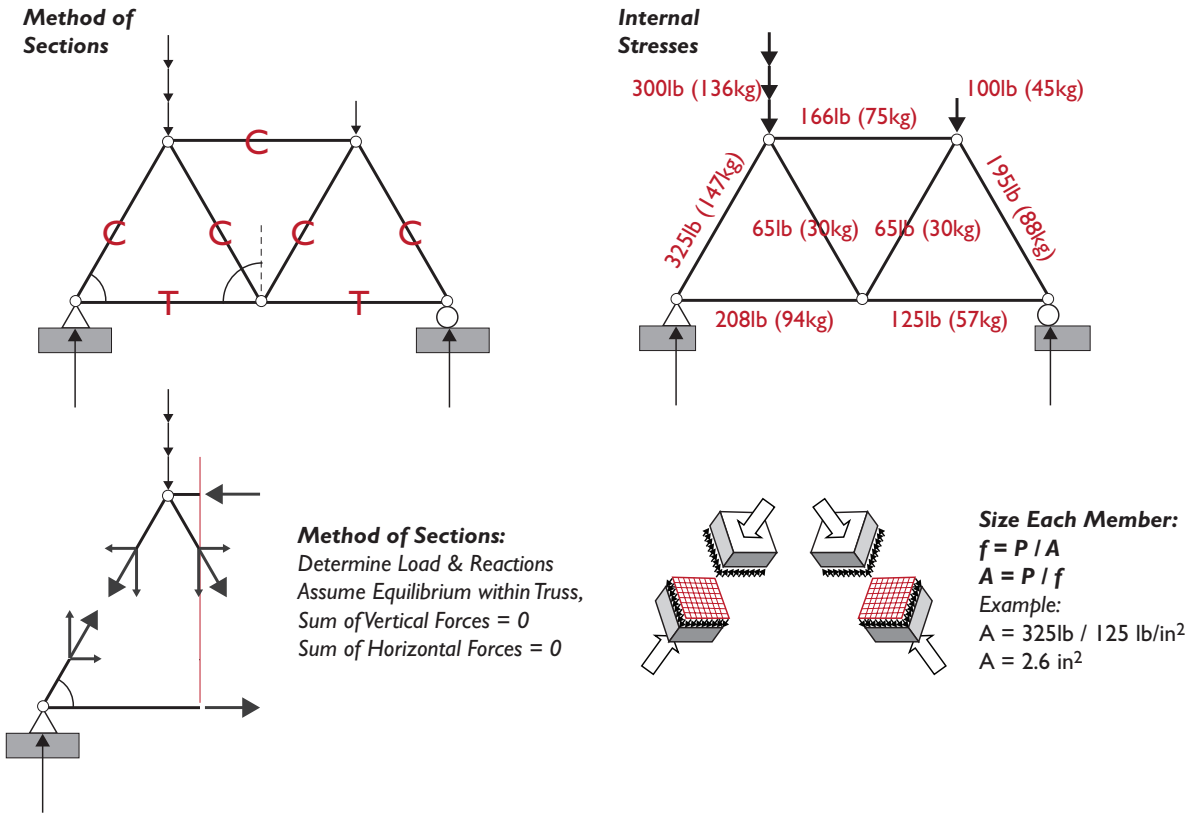


Figure 4.0.13 Method of Sections diagram of equilibrium and resulting internal forces

triangulated, these methods also reveal how much stress is in each individual part of the truss and whether it is being compressed or tensed. Because the mathematics involved in this type of analysis involves only basic trigonometry there are many free online truss "design" tools. (Figure 4.0.13)

Knowing that there is less stress in the more vertically-inclined diagonals may lead to a conclusion that the trusses should always be as tall as possible, but this may not be reasonable or advisable. Taller trusses take up more ceiling clearance space and longer diagonals under compression are at more risk of buckling. But what if we kept the truss height the same and simply reduced the width of the panels, thus inclining the verticals more steeply? Although we would gain efficiency in the amount of material needed to resist these stresses within each panel, we'd have to add more panels to the truss and thus more materials and connections overall. (Figure 4.0.14)

Truss rule #4: *The number of truss panels and the angle of the diagonals affects the magnitude of internal stresses.*

Economic Considerations for Trusses

Consider the constraints of construction as well. Truss economy is based on materials, fabrication, and erection. Eventually the collected loads transmitted to the end supports will require larger diagonals. Having different sized members across the span is structurally logical, but economically specious. Fabricating trusses gets more complicated

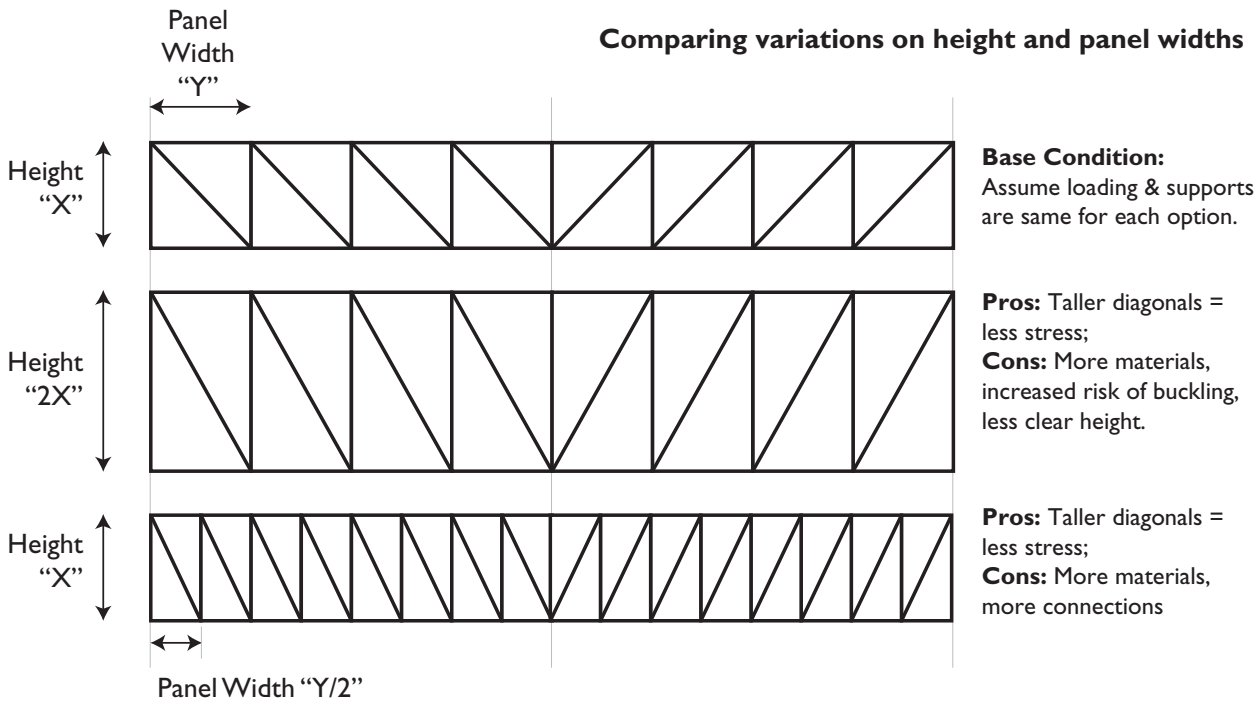


Figure 4.0.14 Pros and cons of varying height and panel width

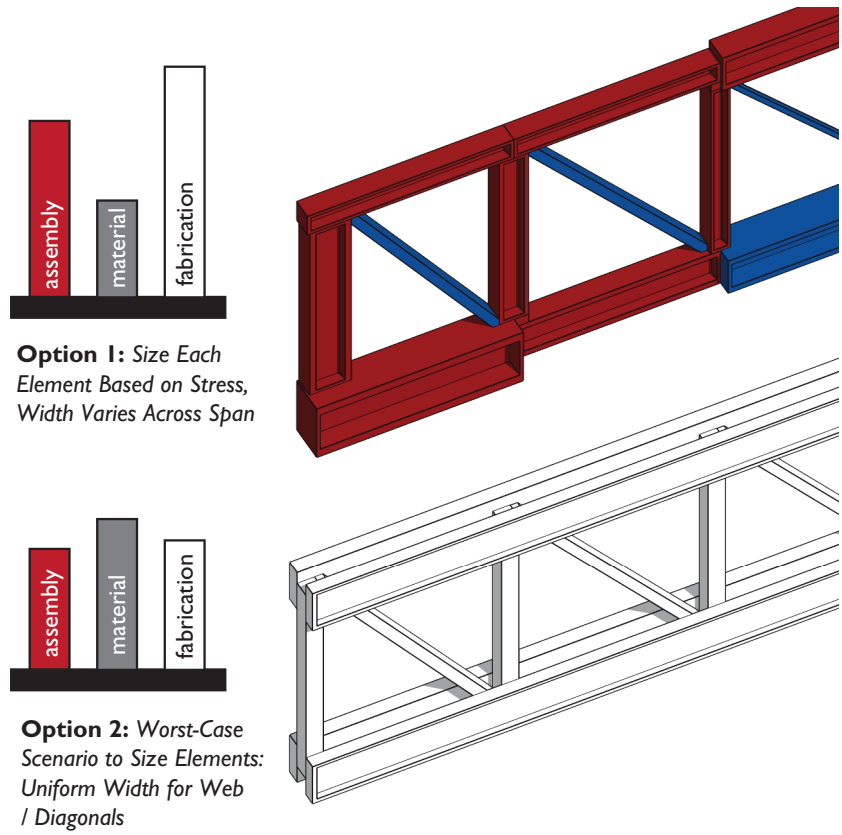


Figure 4.0.15 Balancing the economic considerations of sizing trusses for specific loads versus increased fabrication cost. Cost is also a measure of efficiency

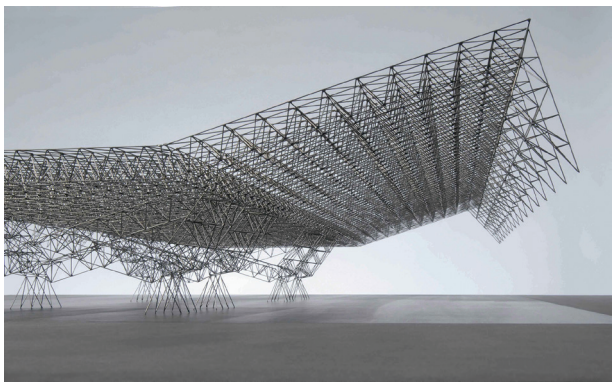
with variously-sized members and additional material cost is often off-set by faster fabrication. Finding ways to join several members together at one connection point of the truss panel is difficult as the line of action for all members must be concurrent and the connections must resist concentrated loads. The worst-case scenario for load resistance may dictate other members' size, or the profile size may remain consistent but its thickness may vary. (Figure 4.0.15)

Truss rule #5: *Trusses are useful in part due to their relative economy—but not all truss configurations are economical.*

THINKING: UTILITY AND ECONOMY AS A STRUCTURAL PROBLEM?

Because trusses can be used to construct very large structures from small, easily portable, straight-lined members, they are of great interest for military applications. In 1955, Konrad Wachsmann completed a proposal for the U.S. Air Force for a construction system using trusses that could be used to build and demount hangars faster than conventional construction techniques. To prove his idea's viability, he relied on models of the structure, sequential construction drawings, and a full-scale prototype of the universal joint that would join all elements together.

To Discuss: What are the possible roles for physical models in conveying complex formal and structural ideas? To what degree do experimental ideas need to be prototyped? (Figure 4.0.16)



Figures 4.0.16a and 4.0.16b Wachsmann's model for U.S. Air Force Hangars and student analysis model for the universal joint (SxD, Iowa State University)

Model #3: Push or Pull?

A truss's resolution of loads into only axially stresses is beneficial—its components can be relatively small because axial loads can be resisted by each triangular component's entire cross-sectional area. Even if you know that the triangulated web members are resisting only tension and compression it is difficult to understand which elements are resisting which types of stress. A physical model can show you if you use two sticks and a string. Place the string along the bottom and press down at the top point (watch the string tense). Now rotate it and press down again (watch the string sag under compression). Unsurprisingly, the A-framed components were resisting compression, like an arch, while the bottom component tied them together to resist thrust. (Figure 4.0.17)

What happens to the stresses if we change the number of panels and their orientation? Reorient the panel so the string is at the bottom (to resist tension) and link this panel with another similar panel at the base (share a node point). Stabilize the

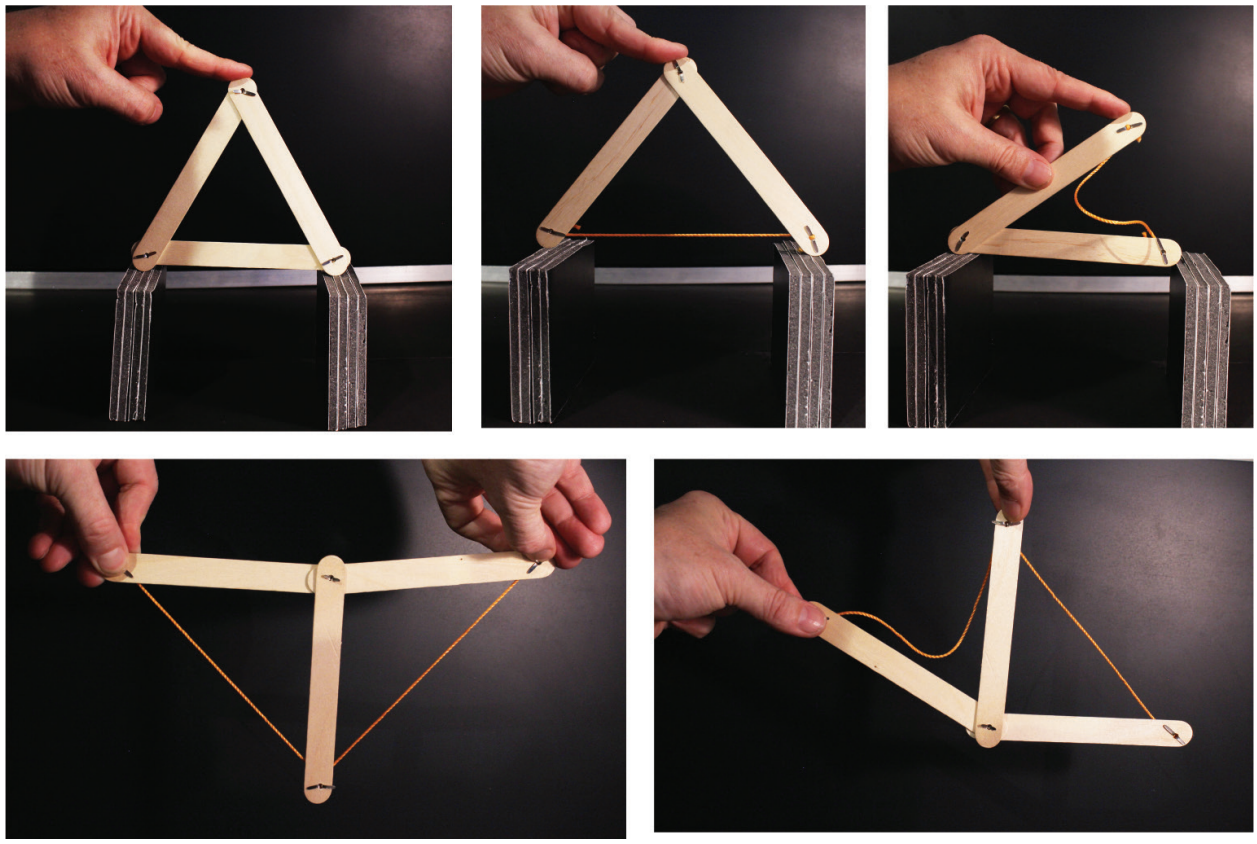


Figure 4.0.17 The inclusion of a string in the model demonstrates which members are in compression or tension

form by adding a member across the top. Notice the economy of materials that you get by linking three separate triangles together—two fewer diagonals and three fewer nodes. When you apply a load to the top, will the same members compress and tense now that they are combined together? What happens if the truss becomes four panels, or changes its overall form? (Figure 4.0.18)

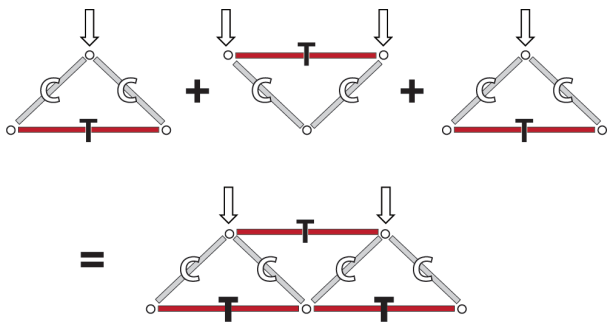


Figure 4.0.18 To understand which members are in compression versus tension, imagine the truss (in the internal stress) first as separate panels

The analogy of a truss as an "evolved beam" can help us visualize internal stresses across its length. Beams and trusses, when simply supported on the ends and uniformly loaded, have the highest stresses at mid-span; compression stresses at the top and tension at the bottom. A more refined force flow diagram of the beam revealed that the compression forces looked like arches and tension stresses like cables. This is useful as an observation to help visualize the stresses in multi-panel trusses. For example, when truss diagonals "hang" as downward diagonals towards the center span, they'll be in tension. (Figure 4.0.19)

> **Truss rule #6**: *The direction of the diagonals within a truss, in respect to the support location, will determine the type of internal stress (compression or tension).*

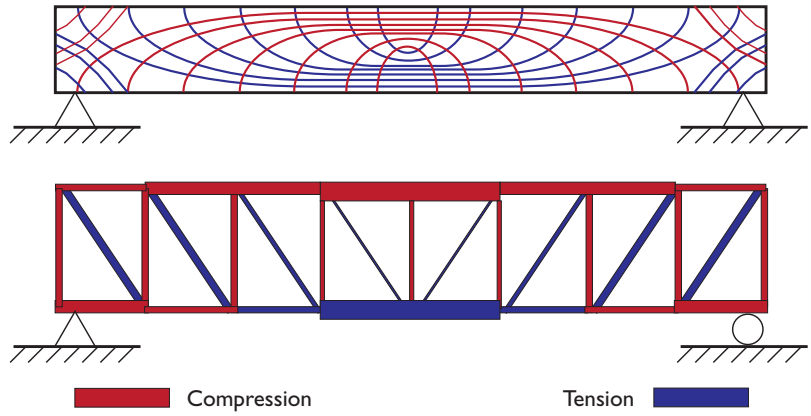


Figure 4.0.19 Internal force flow of a beam and stress distribution for a truss of same length. Location of compression and tension stresses and patterns of distribution across their lengths are similar

Internal Stress in Diagonals

Knowing which portion of the truss is in compression and which is in tension is important because it helps predict potential modes of failure. Most materials used in trusses (wood, steel, etc.) are equally good in tension and compression, however wood elements with tensioned connections risk shear failure. Members in compression tend to buckle like columns

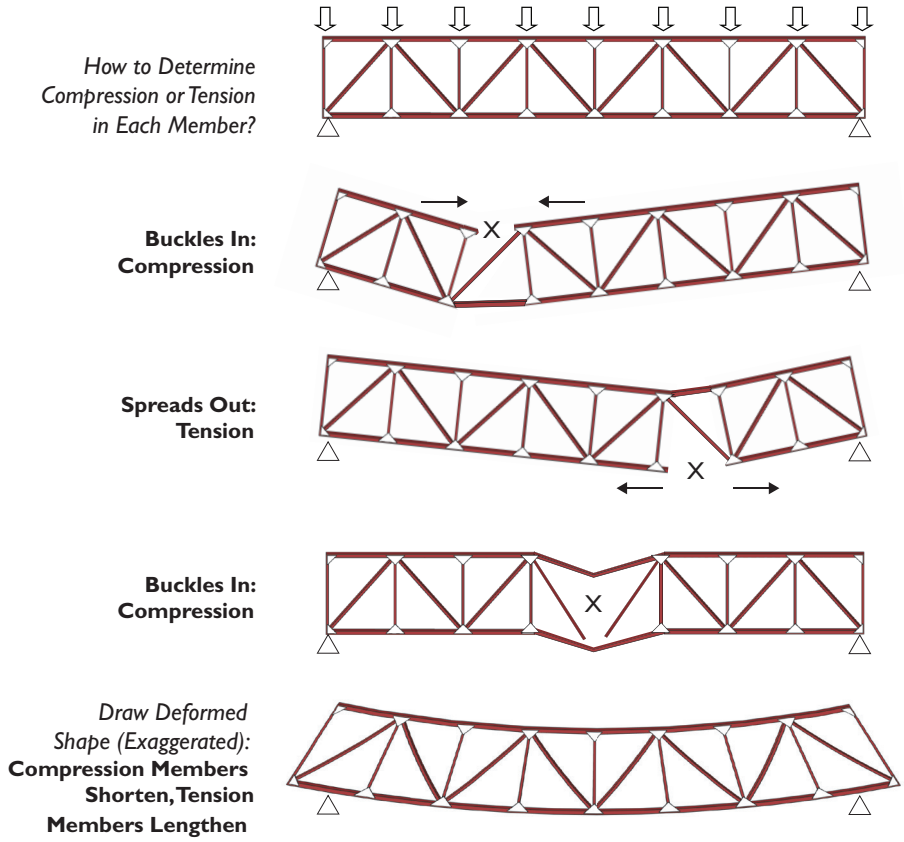


Figure 4.0.20 Estimating internal stress by visualizing failures when a component is removed

if their loads are too high or if they are too long proportionally. We are more concerned with failure at the connections, however, in tension members.

You can select materials that behave like sticks or string to respond to these stresses. Trusses, of course, are multi-component assemblies, so when we change the types of components used in each panel, we must adjust the overall design of the truss accordingly. In the king-post truss option, in which a single compressive vertical strut is placed between a compressive top chord and a V-shaped tension chord at the bottom, we could select a material that was good in compression for the struts, along with a buckling-resistant cross-sectional shape, while we could select a different material and potentially a smaller profile for the tension member.

How do we know which members are in tension or compression in multi-paneled trusses? We can visualize a member's internal force by thinking about what would happen to the truss under loading if that particular chord or diagonal was severed (or use a physical model to help). If the truss collapses inward, it was in compression, if it opens up outward, it was in tension. We can perform a similar exercise by looking at the length of each truss member when deformed under loading: compression members shorten, tension members lengthen. (Figure 4.0.20)

> **Truss rule #7**: *Compression elements in trusses are sized to resist buckling, tension elements are most at risk at the node connections.*

Historical Bridge Types

Timber trusses have been the subject of some experimentation and prototyping throughout the centuries, it was Andrea Palladio's "inventione" sketches of timber bridges in 1570 that lent a more

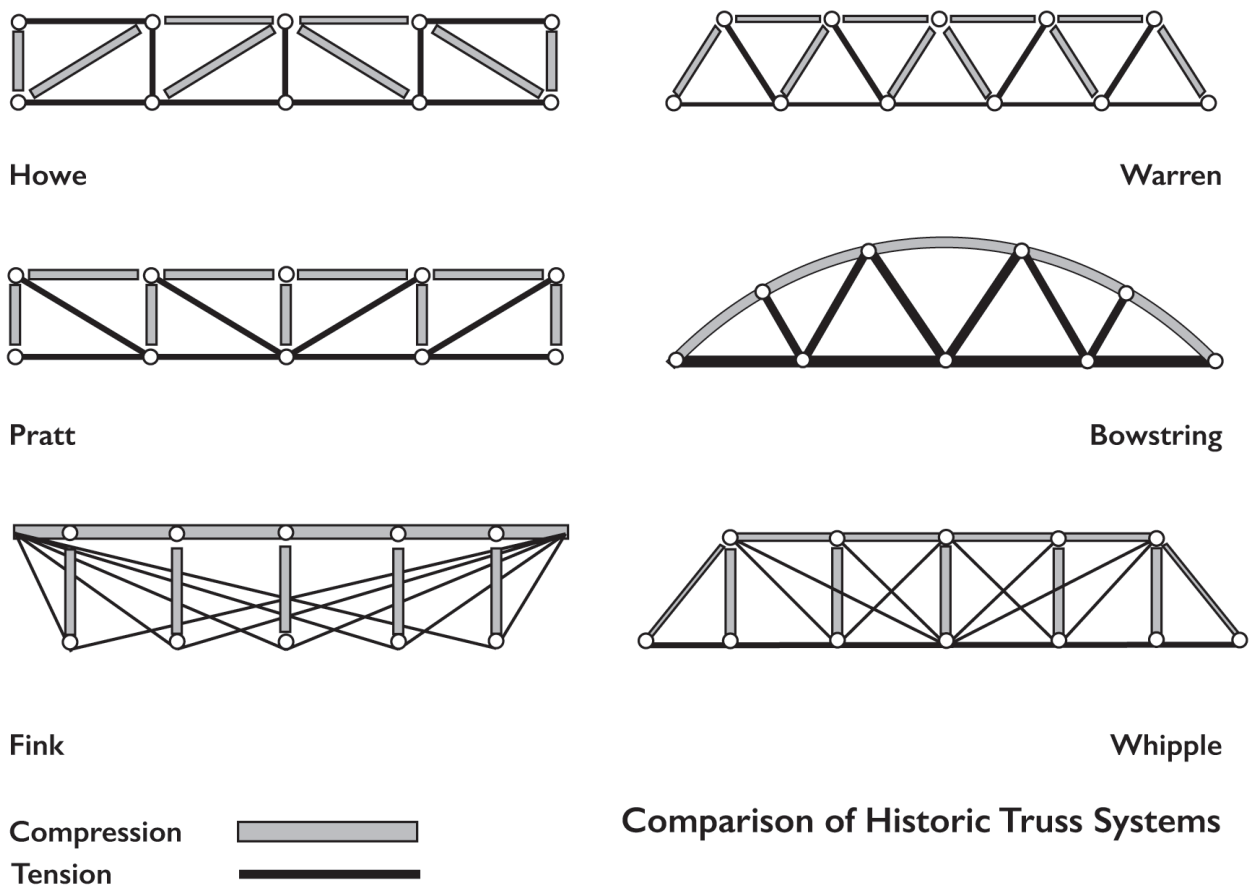


Figure 4.0.21 Historic truss examples with type of stress indicated for internal members

scientific basis to the diagonals' orientations. As arched masonry bridges spanned farther over the next two centuries, they needed correspondingly long spanning timber formwork below. Many of these scaffolding solutions were pre-cursors to later truss designs. In the mid-19th century, as wooden trusses evolved into iron, reliable tension-connectors for trusses became available, and part of the new profession of engineering specialized in designing and prototyping trussed bridges.

Diagonal members' orientation became a distinguishing feature for different patents. The Howe truss (1840) used wooden X-bracing compression members in the panels and vertical iron rods in tension—an appropriate match between materials and stresses. The Howe design was later modified to have only single diagonal members in each panel, sloping upwards towards the center of the span so that they would be in compression. The Pratt truss (1844) also included vertical members and diagonals, but it flipped the diagonal direction to slope downwards towards the middle of the span, which put the vertical members in compression and the diagonals in tension. The compression members were larger to resist buckling. You can "feel" this solution if you simply hold a heavy shelf out from your waist—your arms are tensed and your body is compressed. The Warren truss (1848) was made exclusively with equally spaced diagonals across its length; its main benefit was the economy of equally sized diagonals, since the compressive member was the worst-case scenario. Two types of trusses altered their profile to make a more funicular shape: The Bowstring truss (1841) had a curved top chord and the lens-shaped Lenticular truss (1869) used compressed verticals and tensioned diagonals between an arch and a corresponding cable. These were highly efficient materially but difficult to construct. Two other bridge types that integrated tension members in notable ways were the Whipple and Fink trusses (1861); both hold vertical members in compression with a series of tension wires that cross over multiple panels—a similar structural scheme as those used later for aircraft wings. Most truss design over the ensuing decades looked for ways to optimize an already efficient system. (Figure 4.0.21)

Trusses in Building Design

Finding an optimal balance between material, overall form, panel geometry, and individual member sizes makes truss design challenging. While much of the detailed work is completed by engineers and fabricators, there are basic design parameters that can guide our initial efforts. Because trusses are multi-panel assemblies of smaller triangulated shapes, they can be adapted into many different forms and profiles that cannot be achieved by solid beams.

Trusses are often framed like beams in one-way or two-way arrangements such as decking, purlin (or joist), and girder trusses. The farther apart they are spaced, the more load they'll each carry and the deeper the trusses will become. (Figure 4.0.22)

Trusses are economical in spans between 30ft–200ft (9.1m–61m). Their allowable span and depth/span ratios vary by material and loading/support conditions, their geometry (parallel chord, sloped,

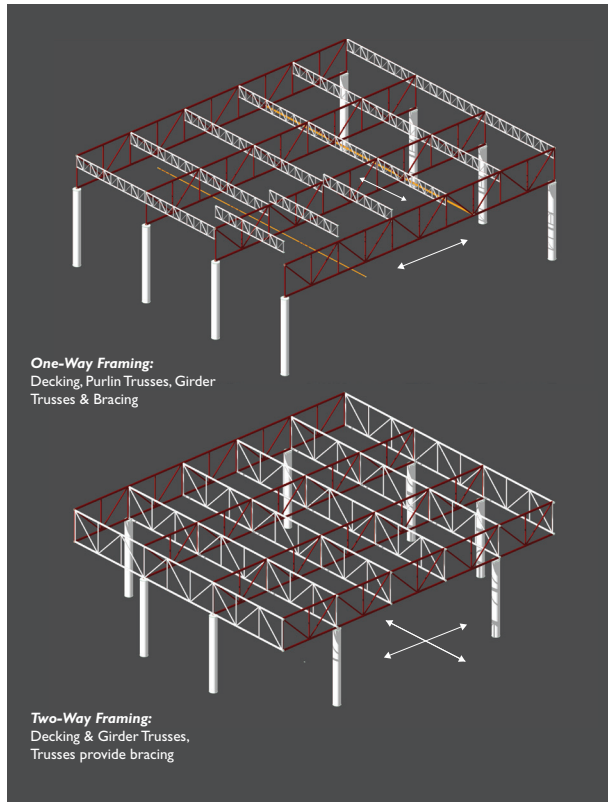


Figure 4.0.22 One-way or two-way framing options for truss systems. Load distribution changes depth of girder trusses (red) and purlin trusses (white)

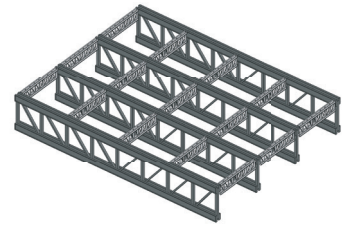
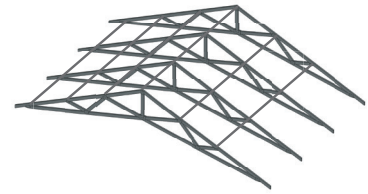
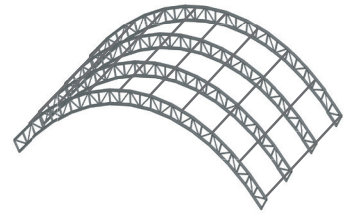
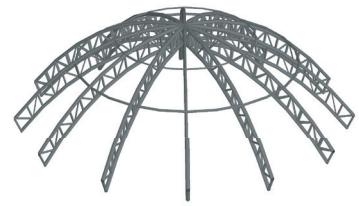
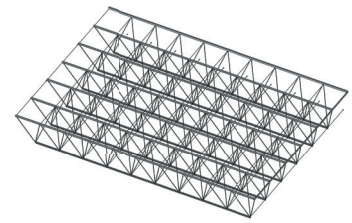
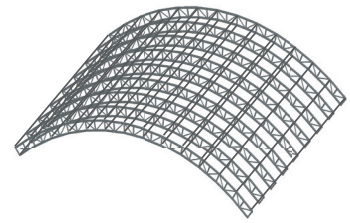
Descrip.		Diagram	Mtl.	Typical Span	Depth-to-Span
Flat Panel	Parallel		Wood	40-120 ft. (12 - 36.5 m)	1:10
			Steel	40-300 ft. (12 - 91.4 m)	1:10 -1:12
	Pitched		Wood	40-120 ft. (12 - 36.5 m)	1:10
			Steel	40-150 ft. (12 - 41.7 m)	1:6 - 1:8
Curved Panel	Arched		Wood	50-250 ft. (15.2 - 76.2 m)	1:40
			Steel	50-350 ft. (15.2 - 107 m)	1:100
	Domed		Wood	100-300 ft. (30.5 - 91.4 m)	1:6 - 1:8
			Steel	100-500 ft. (30.5 - 152.4 m)	1:10 - 1:20
Space Frame	Flat		Wood	40-120 ft. (12 - 36.5 m)	1:6 - 1:10
			Steel	80-400 ft. (24.3 - 122 m)	1:15 - 1:25
	Curved		Wood	40-200 ft. (12 - 61 m)	1:6 - 1:10
			Steel	80-600 ft. (24.3 - 183 m)	1:20 - 1:25

Figure: 4.0.23 Depth-to-span rules of thumb for truss sizes based on panel form and materials

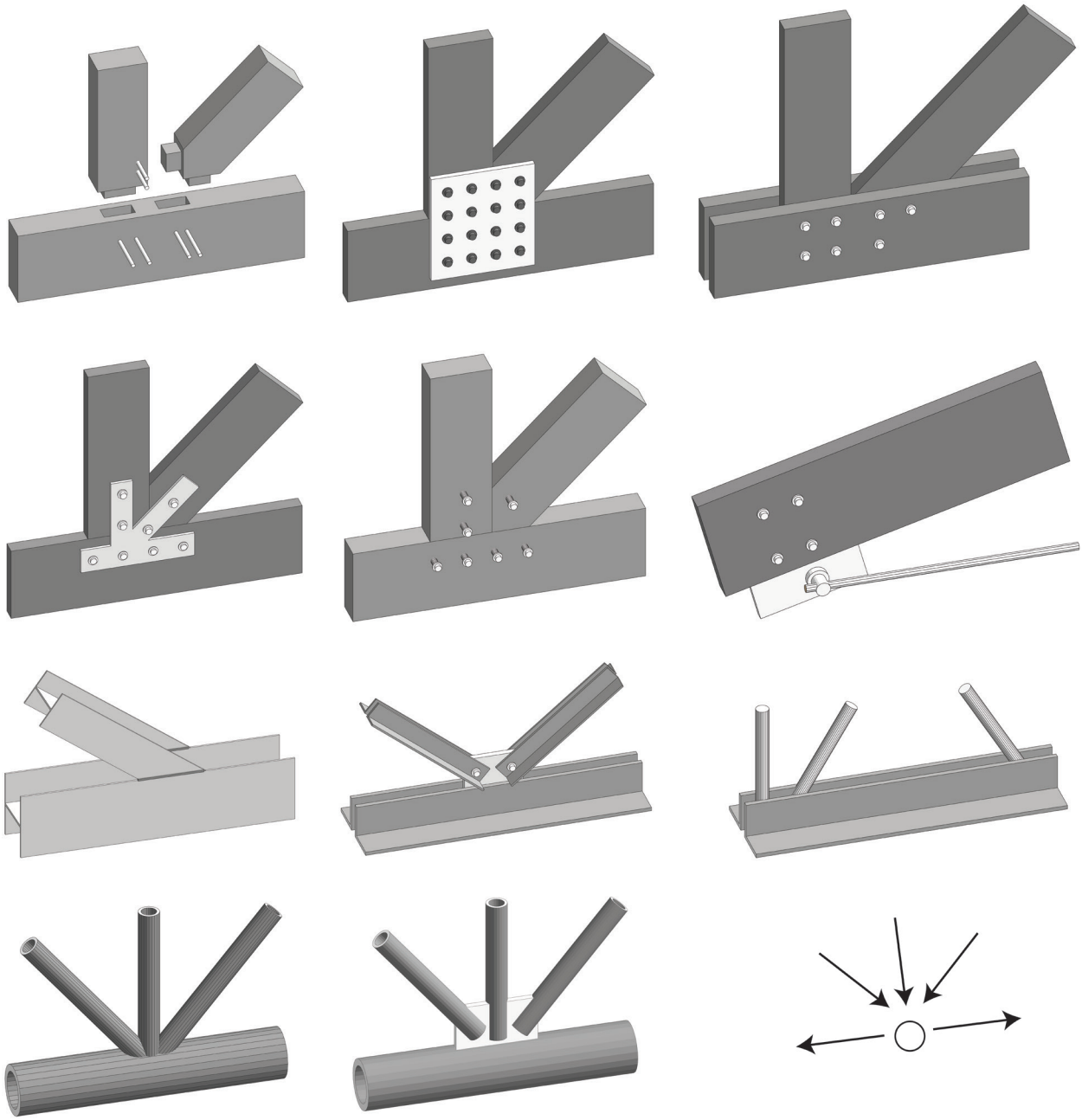


Figure 4.0.24 Truss connections must be concurrent. Connections can be exposed or hidden, doweled, welded, or bolted

and/or bowstring), the framing system (one-way versus two-way), and the role of the truss in transmitting loads (joist truss or girder truss). Rules of thumb include common ratios of depth/span that can vary from 1/10 for a flat wood truss (with a maximum span of 100ft (30.5m)) to 1/50 for a curved steel space frame (with a maximum span of 500ft (151.5m)). Deviations need to be off-set with other adjustments. (Figure 4.0.23)

Truss connections vary by material, intended structural performance, cost, and aesthetics. Trusses are typically made of steel or wood. Wood trusses require steel connectors, since tension connections in wood risk shear failure. Steel has considerably higher allowable stress capacities (f_a) and stiffness (E), and because it can be spliced, welded, and connected together without loss of strength, it is the preferred choice for long spans. (Figures 4.0.24 and 4.0.25)

Figure 4.0.25 Heavy timber trusses with steel connectors and tension ties at UMass Design Building (Leers Weinzapfel, 2017)

BREAKING: KEMPER ARENA, 1979 COLLAPSE

After a moderate storm, the roof above Kansas City's Kemper Arena ponded with water, which caused the roof structure to deflect further downward, allowing more ponding. This led to the roof's partial collapse. This structure's framing was unique: Three large prismatic trusses (spanning greater than 300ft (91.4m) were exposed above the roof enclosure while the secondary (and tertiary) trusses were suspended below.

To Do: Why the building collapse? What role did the layout or framing of the system play in this failure (or elevated risk)?

To Discuss: How did this failure affect the practices of structural and architectural design of long span systems?

Two-Way Spanning and Space Frames

Trusses are materially efficient and lightweight, but this litheness can become a liability with heavy point loads or dynamic loads. To avoid buckling or dynamic movement, trusses in a one-way framing system need to have their bottom chords braced at regular intervals. This can be accomplished with horizontal members connecting several trusses together along the bottom, X-bracing diagonals between bottom chords and top chords, or by changing the truss from a panel to a prismatic truss. (Figure 4.0.26)

If we space out the purlins and girders into a two-way configuration the purlin would eventually carry the same weight (and have the same depth) as

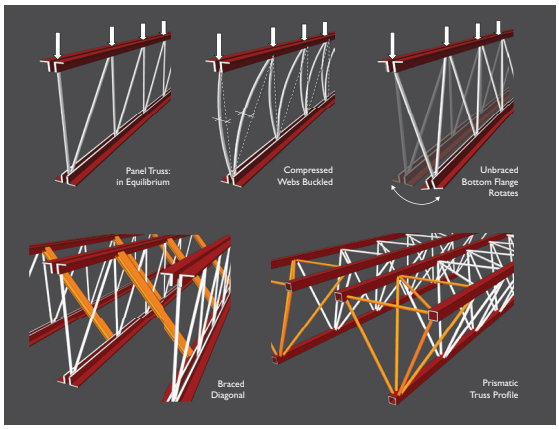


Figure 4.0.26 Bracing (orange members) strategies against buckling and rotation—an option not shown is bracing running flat along bottom chord across multiple trusses

the girder. Combined, they'd provide bracing and transmit loads like a "waffle" pattern of trussed diagonals. Forces follow the stiffest path of resistance, so when we stiffen the project equally in two directions, we create a two-way system of shared resistance—or a *space frame* truss system. Whereas planar trusses distribute loads one-way, space frame modules are a three-dimensional truss system that distributes loads in multiple directions. Two-way systems are most effective in square shaped bays (up to 1–1.25 plan aspect ratio).

The first space frame experiments, intended for towers and flying machines, were prefabricated steel tetrahedrons patented by Alexander Graham Bell in 1904 (published in *Tetrahedron Principle in Kite Formation*). Other designers and mathematicians experimented with these systems, noting their similarities to organic surface structures. The triangulated panel was particularly useful as a building block for larger structures. These tetrahedral structures weren't built frequently at the time, but it wasn't lack of creativity or knowledge that impeded their integration into buildings; it was the inability to construct connections that were reliably strong, stiff, and affordable. (Figure 4.0.27)

Unlike a waffle slab, which uses a square grid of beams, space frames rely on trusses' triangular forms to provide resistance. When the individual pieces are assembled, they form a mesh of triangulated supports. Unlike a flat, single-layered lattice mesh, space frames have depth between the top and bottom layers that allows them to remain self-stabilizing, efficient, and rigid. This depth provides increased resistance to deflection, which increases the span's potential length. Because there are so many modules, their resistance is redundant—the collapse of one part won't collapse the entire system. Despite the added geometric complications, bending loads within the trusses are still resisted within their members axially. Overall, space frames have the most efficient depth-to-span ratios of any long span system (depth is 4%–7% of span) and can provide the largest available column free spaces, upwards of 500ft (152.4m). (Figure 4.0.28)

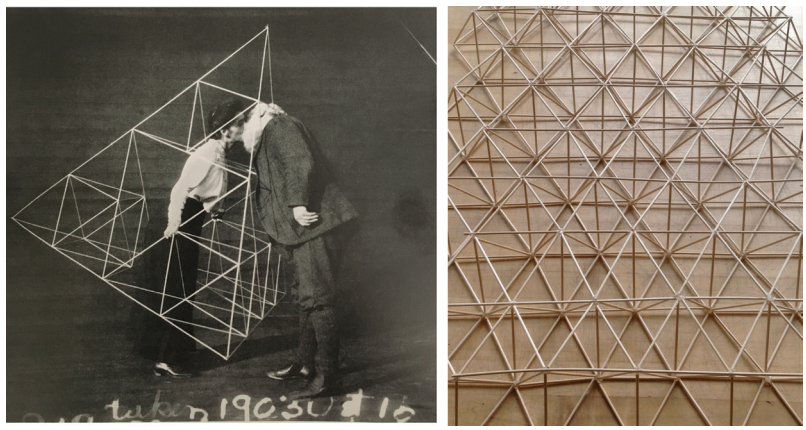


Figure 4.0.27 Graham Bell, his wife, and an early prototype of space frame kite. Right: Close-up view of student model of space frame geometry

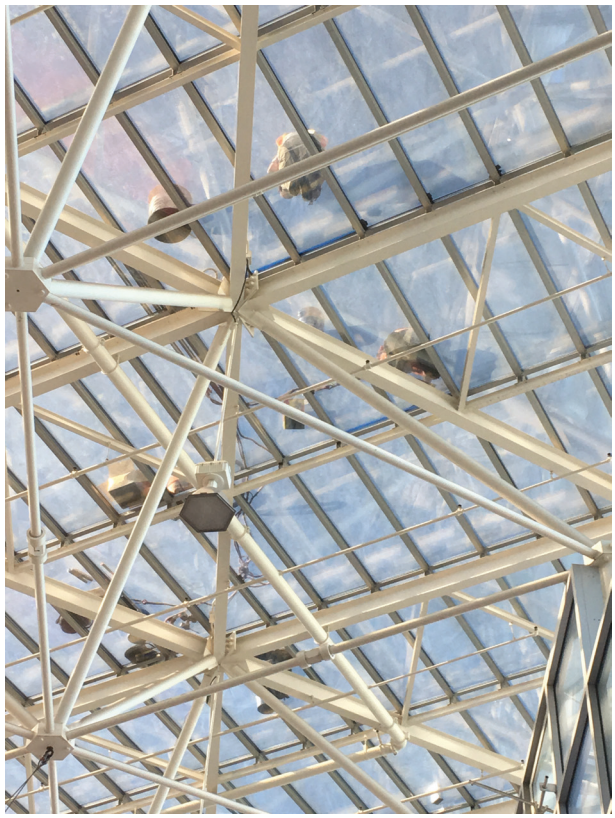


Figure 4.0.28 Steel space frame roof structure, Museum of Flight Great Gallery, Seattle (Jack Christiansen, Engineer, 1965)

Space Frame Connections

All the structural benefits of space frames depend on their connections, which comprise between 20% and 30% of their overall weight. Connections must be designed to resist the concentration of all accumulated stresses in the frame. Depending on the number of grids and the type of geometric solid shapes, more than a dozen members could come together at one connection point, each with a specific geometry. Properly designed connections have to ensure these forces meet concurrently and resolve themselves. A space frame's strength, stiffness, material efficiency, and constructed economy (and accuracy) is only as good as its connections.

The earliest spatial trusses were a compilation of straight-lined members with custom connections at each end—typically labor-intensive welded joints. This construction system compounded even the smallest inaccuracies in the length or angle of diagonal. Imagine the difficulty of attaching more than a dozen diagonals to a single point—over and over again across a long span while maintaining the overall geometry of the structure accurately. The type of connectors are the top determinant of potential construction efficiencies or liabilities in this system.

Designers worked to develop a "universal" joint. Connections can be welded (gusset plates), bolted (Uni-strut), or made of cast ball joints to which diagonals can be attached (MERO Ball). Many of these connections are proprietary systems which need to be selected during the early planning stages planning to understand their limits. (Figure 4.0.29)

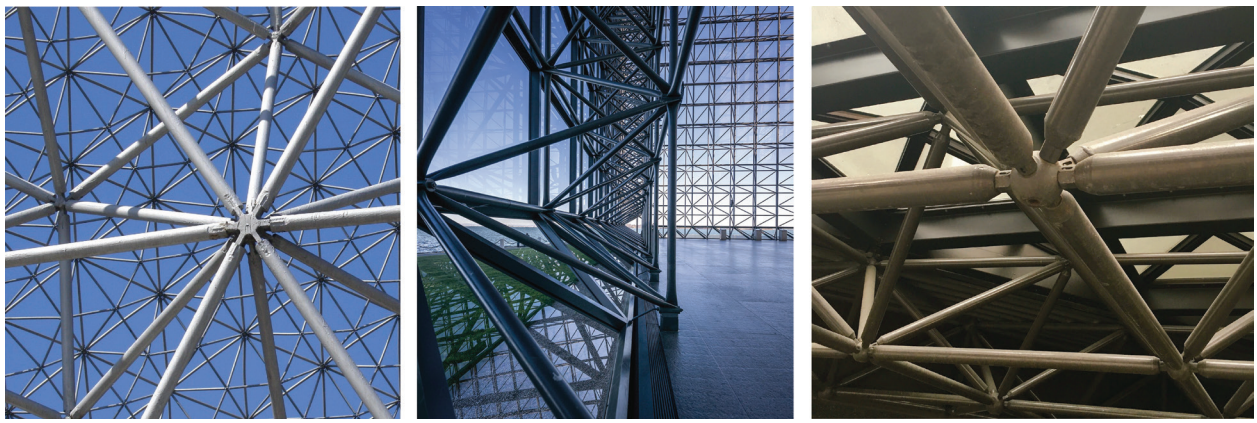


Figure 4.0.29 Three space frame connections: Montreal Biosphere (Fuller, 1967), JFK Library (Pei, 1979), canopy with MERO Ball (Seattle, 2017)

Designing with Trusses: Load Grounders

Because trusses transmit loads without thrust, they don't require modifications to receive these loads, like columns. Only the top chord needs to rest on the supports, as the diagonal member below will pull tension stresses upwards to the connection).

There are standard types of column/truss connections, but these don't preclude opportunities for modifications. Because columns are concerned with buckling, it makes sense to change the traditional "stick" form, particularly if it works with the remainder of the truss framing or if the structure's scale is so large that a traditional column won't suffice. When trusses are connected to their vertical

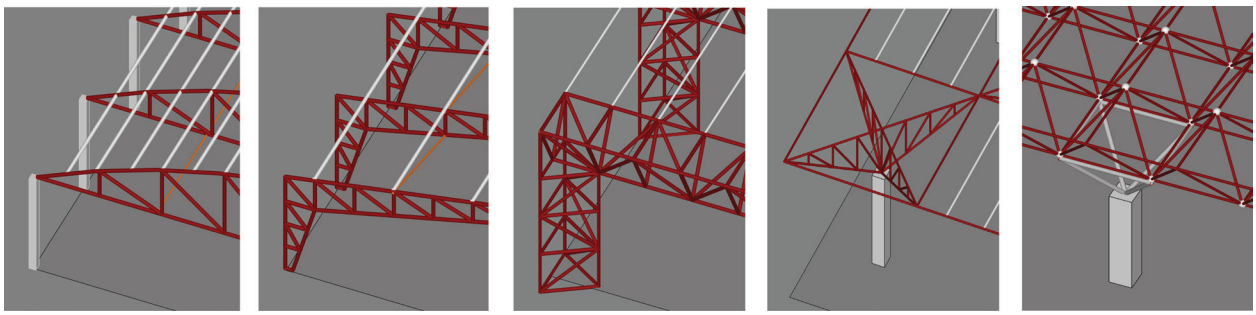


Figure 4.0.30 Options for column supports: Point load, portal frame, prismatic frame, umbrella, and space frame cap

Figure 4.0.31 Examples of trussed supports: Eiffel Tower (Eiffel, Paris, 1889) and King's Cross Station (McAslan + Partners, London, 2012)

supports like portal frames the columns will also become trusses. If the supporting truss is prismatic, it often makes sense for the supporting column to also become prismatic. Supports for a two-way framing system should broaden the area of support to avoid concentrated loads and punching shear—this can take the form of a larger umbrella frame or a smaller prismatic truss module that matches the space framing. (Figures 4.0.30 and 4.0.31)

Geodesic Domes

Not all trusses are configured into planar surfaces and vertical columns. There are times when the span dictates a greater level of efficiency or the design/function requires more volume. Even though space frames are already materially efficient and able to span long distances, their efficiency can be improved by curving their planar form. By having two layers of lightweight trusses (top and bottom chords) with stiffening diagonals between, the added rigidity of a single (or double) curved surface and the inherent structural advantage of curved "arch-action" these curved space frames exceed the span possibilities of any other structural type. These surfaces can provide three-dimensional volumetric enclosure if the truss extends to the ground. (Figure 4.0.32)

Figure 4.0.32 The Montreal Biosphere, created for the Expo '67, spans 249ft (76m) and is 203ft (62m) high

In the late 1940s, Buckminster Fuller realized that the structural and assembly problems of a double-curved space frame could be solved by applying basic mathematics and geometric principles to make the

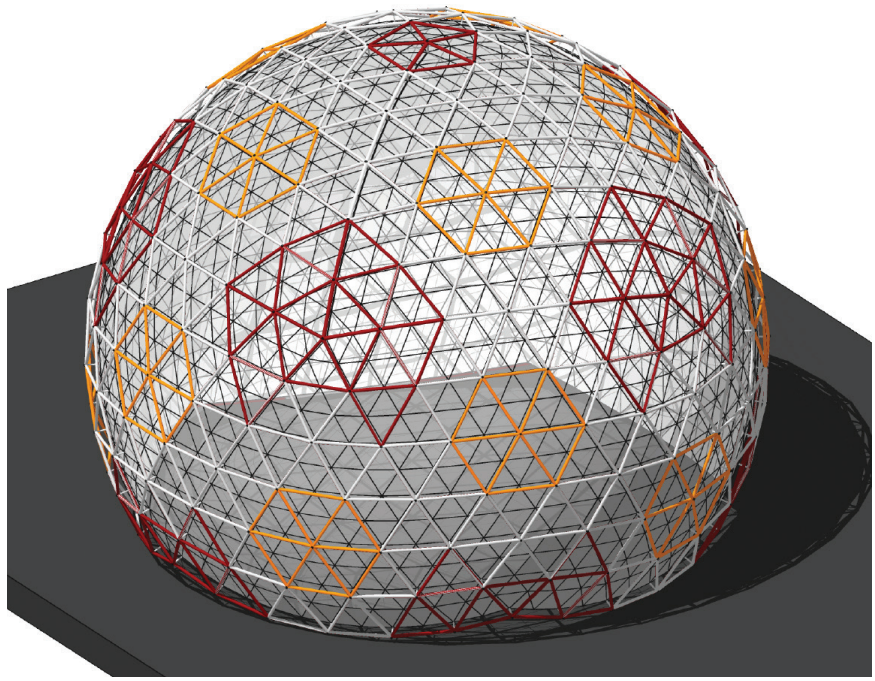


Figure 4.0.33 Geodesic panel layouts are based on a frequency of particular three-dimensional geometries (e.g., Frequency 16 Icosahedron) and the mathematical models upon which these are based

Figure 4.0.34 Full-scale geodesic prototypes were developed by the U.S. Navy to serve as lightweight portable hangars for aircraft. Shown: Marine HRS-1, Quantico, 1954

surface from regularized three-dimensional truss modules. After years of developing drawings and building prototypes at various scales, he created a spherical shape comprised of small interlocking platonic solids—a geodesic dome. The form of a geodesic is based on a sphere (typically a hemisphere) that is subdivided with trussed modules (triangles, hexagons, and pentagons), similar to a soccer ball's pattern. The more frequent the modules, the more accurately they can represent the double-curved surface of a dome. Of course, not all functional programs fit in a dome—the benefit of the form was also its major liability. Today there are thousands of geodesic domes constructed worldwide. (Figures 4.0.33 and 4.0.34)

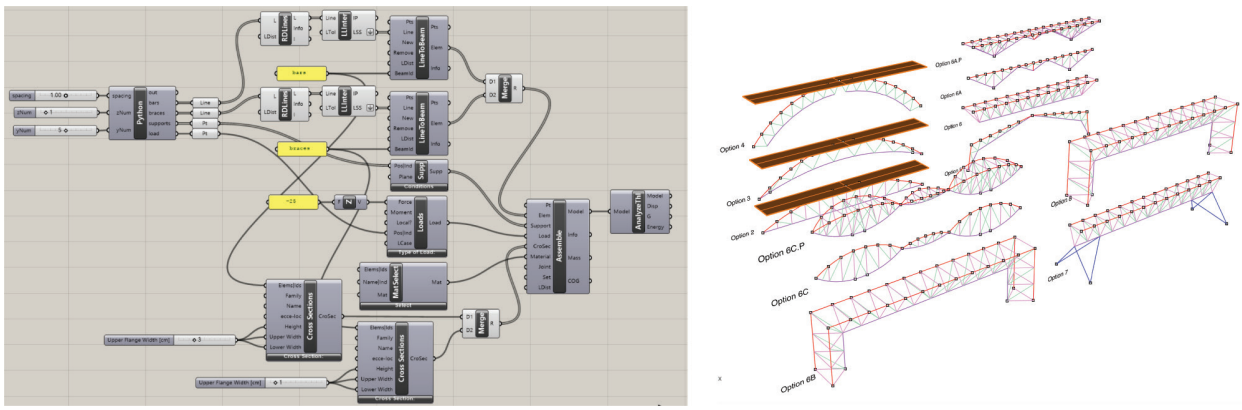
Contemporary Tools for Truss Design

Most of the tools we've seen in this chapter have been confirmation tools that are used to understand the amount of stress (vectors) within individual members. We've seen examples of instructional models, graphic analysis, mathematical calculations, and testable prototypes. Each of these tools advances the designer's ability to create (and improve) trusses. But every tool has limits. Depending on the method, variables can be too open-ended or too prescriptive to be a fluid design tool for rapid evaluation and redesign. Contemporary digital tools can bridge this gap between design and evaluation.

Because truss design's factors (profile, height, panels, etc.) are interrelated, it is useful to be able to alter these parameters. Some web-based programs offer digital versions of analysis akin to the method of joints or sections—you input a truss diagram, assign loads, and it will show the resulting stress quantities and type (tension, compression, or no stress). Other, more advanced, programs use algorithms that equate all of these three-dimensional factors together and give instantaneous feedback to a three-dimensional model when a parameter is changed (e.g., by moving a "slider" to change the number of panels, you can see the effect on the total weight, stress distribution, and elongation in the proposed design). (Figure 4.0.35)

Generative Design: Interactive and evolutionary algorithms are available even in web-based platforms to explore how changing variables can influence truss designs. Generative designs create high-performing alternatives based on a single idea and a set of performance standards—there is no single solution but a range of many to consider.

These are helpful ways to explore the ramifications of their proposals in quantitative terms without detailed structural information. Designs are instantly modeled and analyzed (giving each a numerical score based on material volume). Advanced versions sync with other parametric models, which allows designers to include more qualitative aspects too. (Figure 4.0.36)



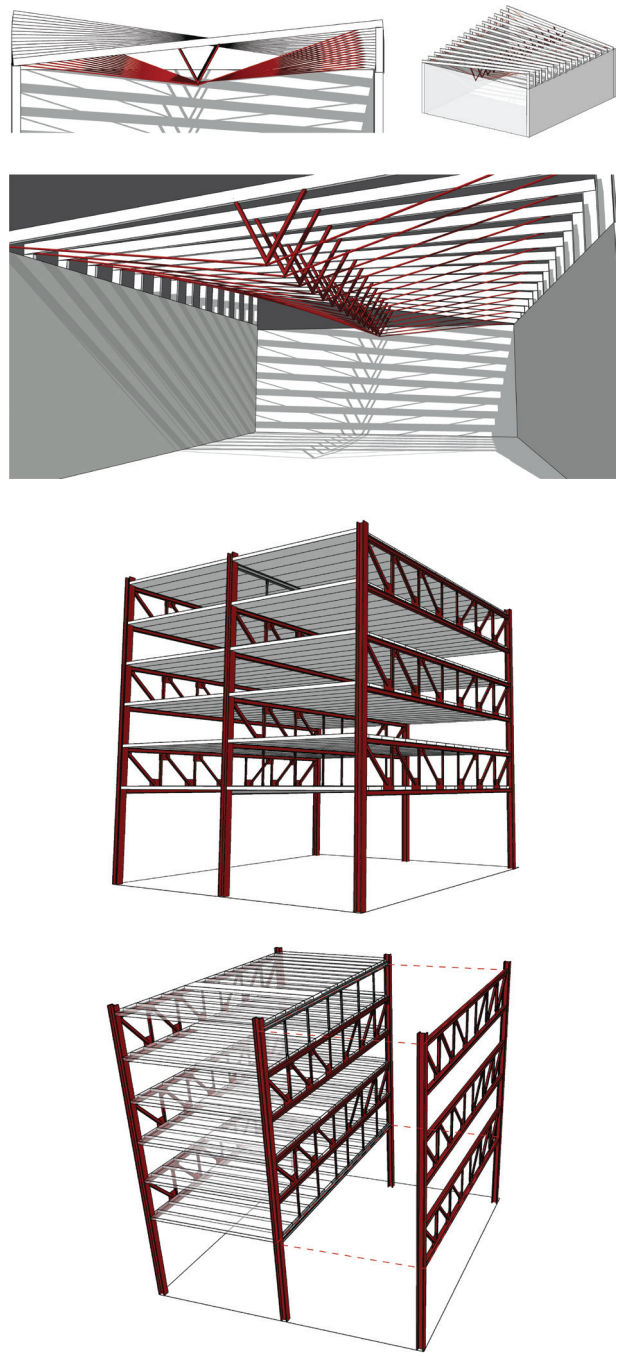
Figures 4.0.35 A digital workflow diagram that parametrically combines information about the truss elements, loads, and supports for rapid analysis and design testing. It can be translated into a set of algorithm scripts for rapid design and analysis

Figure 4.0.36 Generative software allows complex forms to be envisioned and fabricated such as the Interplanetary Lander prototype created by NASA and Autodesk in 2018

These programs are useful for large-scale truss designs with indeterminate conditions. The computer helps visualize complex behaviors, but designers are still in control in terms of how they manipulate the parameters. They are often written by the engineers that work on these types of projects to improve their tools. As these tools have evolved, one central effort has been to close the feedback loop between design and analysis more quickly—allowing trusses to be less intimidating as a structural option with a linked combination of visual and computational data.

Frequently Asked Questions

How much responsibility do architects usually have in "shaping" the trusses used in their designs? This depends on the project and the designer. There are some cases where the most economical "off-the-shelf" trusses recommended by the engineer would be the best choice (perhaps for a ceiling in a warehouses). But in other spaces where the trusses are defining features of the building's structural expression (e.g., arenas, convention-centers, airports, etc.) architects should be deeply involved. Many newer design and analysis tools facilitate designers' involvement (architects and engineers) regarding both aesthetics and performance. Fundamentally, trusses should be part of the spatial and aesthetic conditions of the design; this may require the truss to become expressive or volumetric. (Figure 4.0.37)



Figures 4.0.37a and 4.0.37b Two examples of ways trusses can shape volume and expression of space. Shown: Representation of University of Minnesota Rowing House, VJAA, and "Skip-Stop" truss framing scheme

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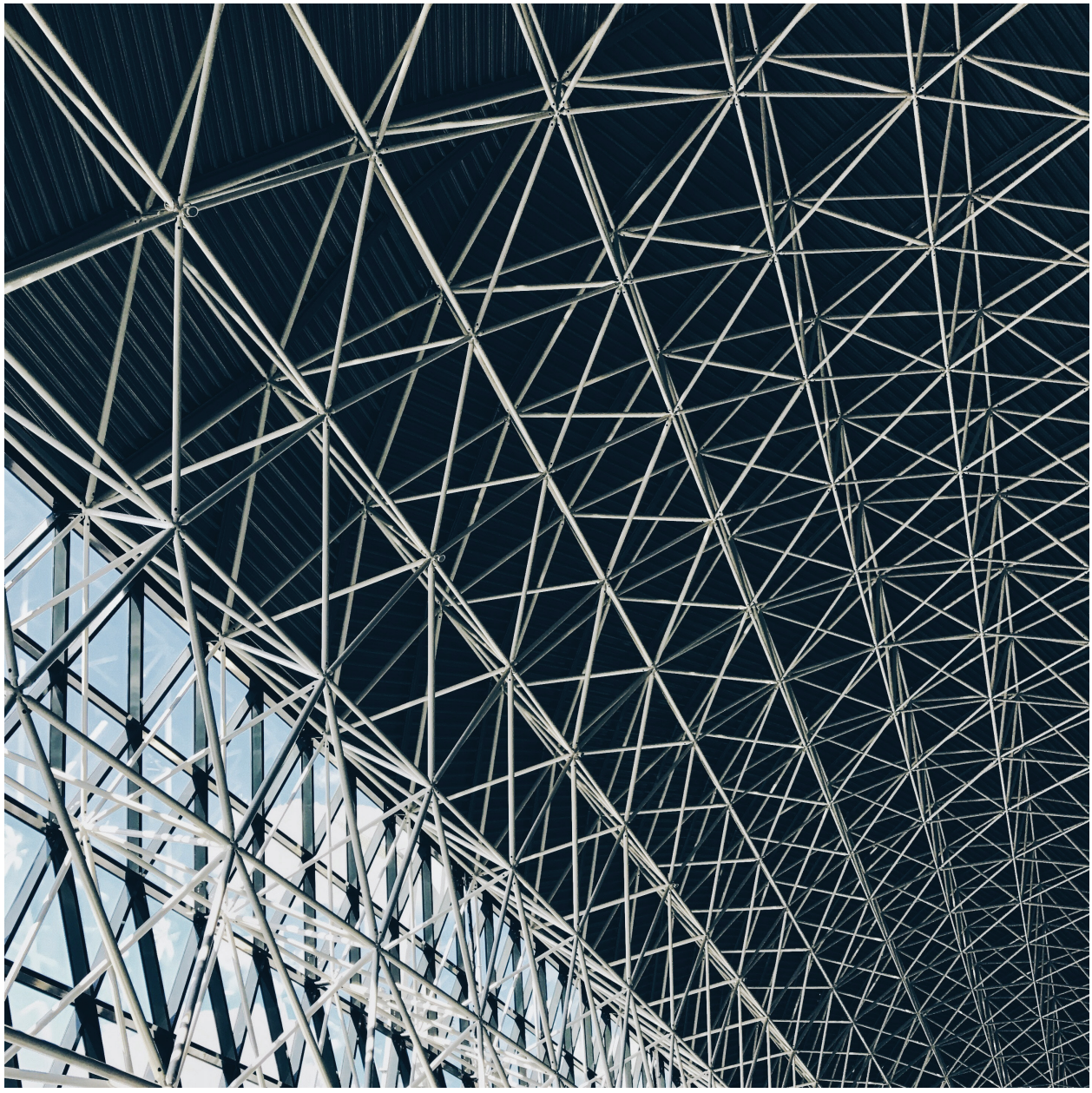


Figure 4.1.0 Curved space frame creates layers of light and space

CHAPTER 4.1

Planar and Spatial Trusses

Long Span Solutions

To span long distances, structures need to be lightweight and stiff with a viable strategy for assembly. Both planar and spatial trusses meet these criteria. Trusses can be framed like beams, arches, or slabs, but they are inherently more efficient than these types. The ability to composite an assembly from smaller trusses increases overall effectiveness and expands the range of potential solutions.

Trusses and Long Span Challenges

Because of trusses' inherent ability to span great distances with minimal material, they are frequently the choice of structural systems for enclosing large, open spaces.

"Long-span" buildings, with spans approximately 60ft (18.2m) or farther, exceed the limits of standard structural elements and have distinct architectural and structural challenges. Most long span buildings (exhibition halls, airport hangars, sporting arenas, train stations, etc.) can be enclosed in a number of ways, so designers need to go beyond formal and functional evaluative standards to include more influential structural standards of cost and performance. For example, deflection is the controlling design factor, so long span elements need to be both *lightweight and stiff*. The scale of the building amplifies the impact of *every* decision, so designers need to balance structural performance with aesthetics, materiality, constructability, and economic viability for each potential choice.

Accordingly, the tools used to design, develop, and assess long span systems need to include a broader range of considerations. We need to create representational and analytical models—ideally with a fast feedback loop. Models help determine if our structural domain is appropriate (e.g., type of truss arrangement) or how specific conditions can be amended and improved (e.g., truss profile, panel sizes, etc.). These factors compel a more detailed and specific assessment matrix. (Figure 4.1.1)

See Appendix Figures A.16 and A.17 for estimations on allowable spans for long span systems based on the type of structural system and depth.

Long Span Design Challenge: High-Speed Rail Station

Our project is a new train station for the I-35 High-Speed Rail Authority, located outside Minneapolis-St. Paul, Minnesota. The proposed station is the endpoint of the north–south high-speed line that follows Interstate I-35 south to San Antonio, Texas. The building has a large footprint that includes four program areas: an entry/drop-off area, a station house with offices and support programs, a large open lobby/waiting area, and the open-air platform for the trains. Because of the inclement upper-midwestern weather, the entire

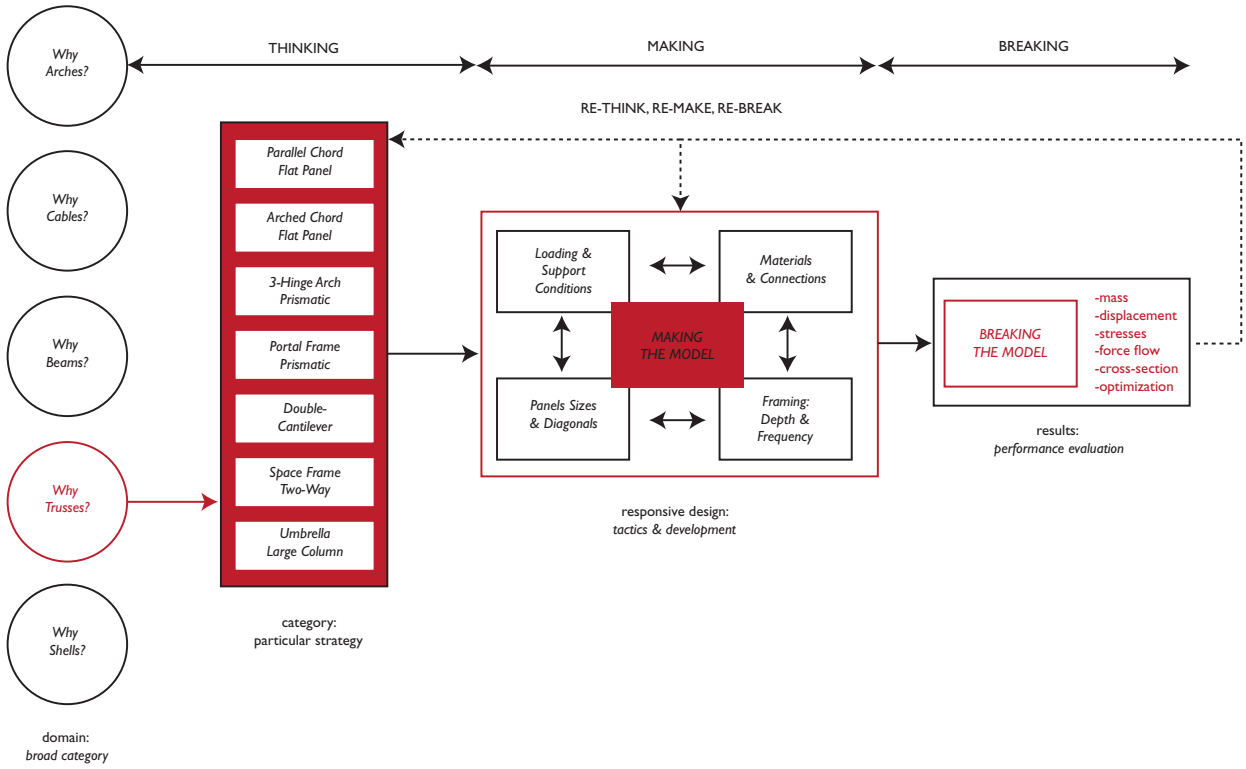


Figure 4.1.1 General "Think, Make, and Break" workflow diagram. Each category will become more detailed depending on tools used and phase of design

building is to be covered by a large roof with skylights for passive winter heating and daylighting. Train stations were the initial testing grounds for the first long span trusses and many contemporary stations continue to use trusses as their primary structural systems.

A dimensioned plan with a structural grid spacing, a massing model, and building section were established at the initial project meeting. Your goal is to develop these strategies to a more specific set of proposals that can be analyzed and evaluated qualitatively and quantitatively. The most significant challenge is the 208ft (63.4m) clear span because it demands a long span structure. We'll explore different options to understand structural and spatial implications of different layouts, roof forms, and cross-sections of trusses. (Figure 4.1.2)

Truss Framing Options

Trusses provide distinct advantages: Their small triangulated components can be easily re-configured in cross-section and/or profile to improve their structural advantage (e.g., an arched truss). Triangulated panels provide efficient means of force distribution within a self-stabilizing geometry; they make trusses light and stiff (two of the key requirements for long spans). Trusses are the obvious, and perhaps the only choice. As a result of their configuration, trusses are inherently lighter, and as Buckminster Fuller explains, "The sophistication of a building varies inversely with its weight."

Because they are assembled from smaller modules, trusses can either be planar (like a beam) or more spatial (like a prism), depending on the desired structural action, aesthetics, and stability requirements. Significant synergistic advances in material technology, fabrication methods, and analytical tools have improved the ways trusses can be integrated into long span structures; under certain conditions, trusses can span over 500ft (152m). (Figure 4.1.3)

How do we determine and develop truss arrangements? We can return back to what we've learned about how other spanning systems (e.g., arches and beams) are integrated into framing layouts. This includes the primary determination of framing as either a *one-way* or as a *two-way* system.

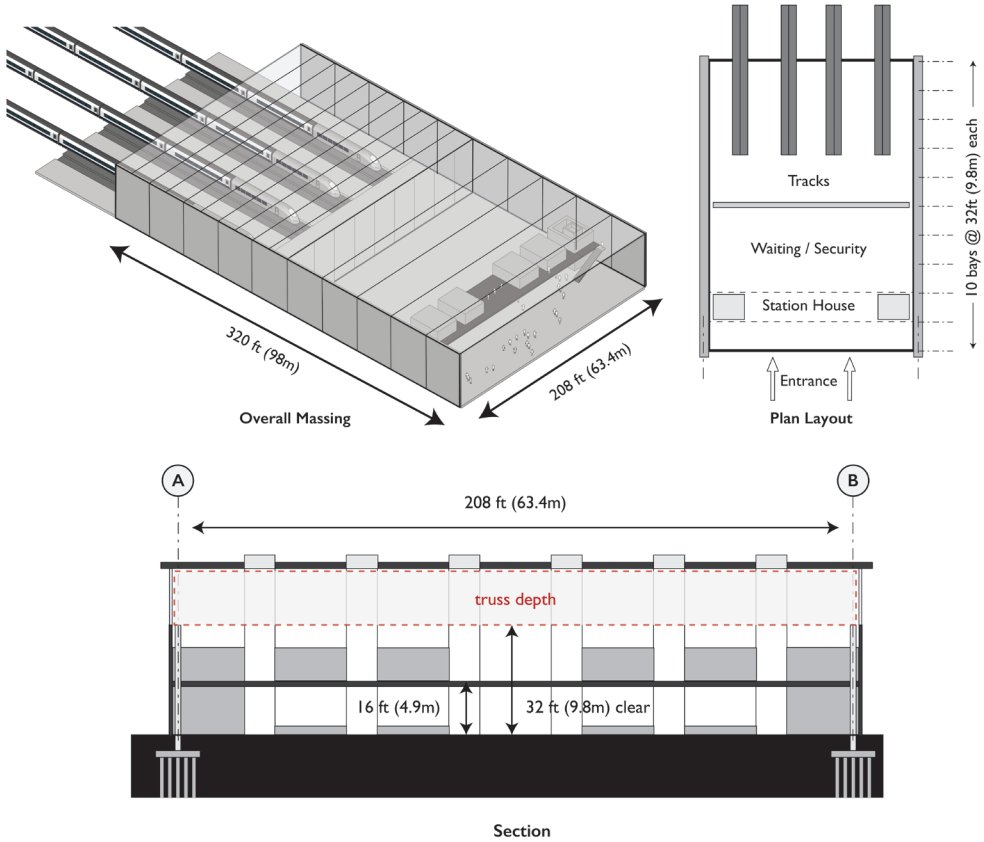


Figure 4.1.2 Train station initial images and constraints: Massing, plan, and sectional diagrams

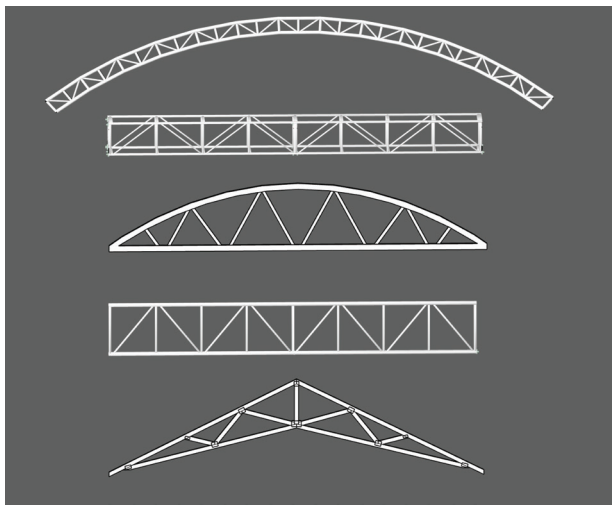


Figure 4.1.3 Triangles are the building blocks that allow for different profiles and forms

One-way systems are distinguished by their framing elements' linear nature with a composite assembly of stacked load-collecting elements that are connected perpendicularly. Each element transmits stresses along its length to supporting elements below. The spans and depths of these elements range from the small decking to large girder trusses as the load collects. Two-way systems, as the name suggests, transmit loads in multiple directions. Because these structures are complicated, with many parts, they are only economical for spans of 100ft (30.5m) or more. These arrangements will also need load stabilizers, in the form of bracing, to keep the trusses from warping out of plane and buckling under loading. (Figure 4.1.4)

In our schemes, we'll use the same "hacks" we learned from beams. These include three options for "better beams" (e.g., efficient cross-section, matching moment diagram profiles, double-cantilevers), two options for curving or stiffening the span (arches and portal frames) and two options for two-way spans (umbrella framing and space frames). Each broader option will have possible modifications that may improve the structural performance (e.g., varying the truss spacing, panel sizing, height, etc.) that we'll test,

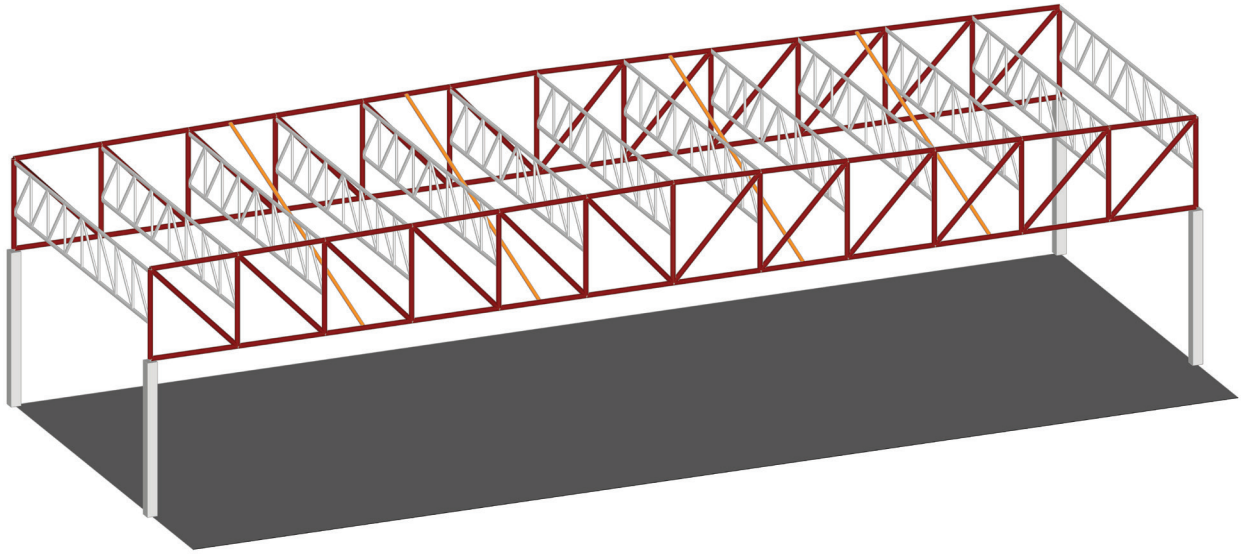


Figure 4.1.4 A single bay of one-way truss framing. Purlin trusses (grey) transmit to girder trusses (red) which span between columns. Trusses are stabilized from buckling (orange)

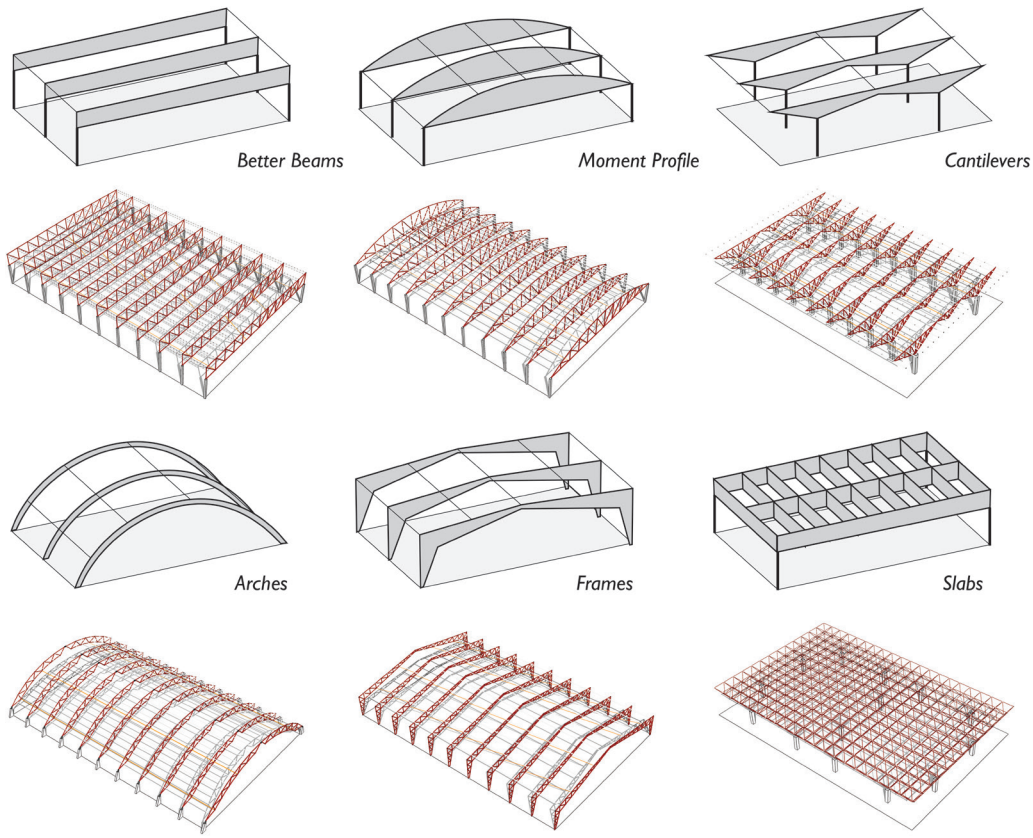


Figure 4.1.5a Comparison of traditional beam framing solutions expressed as trusses

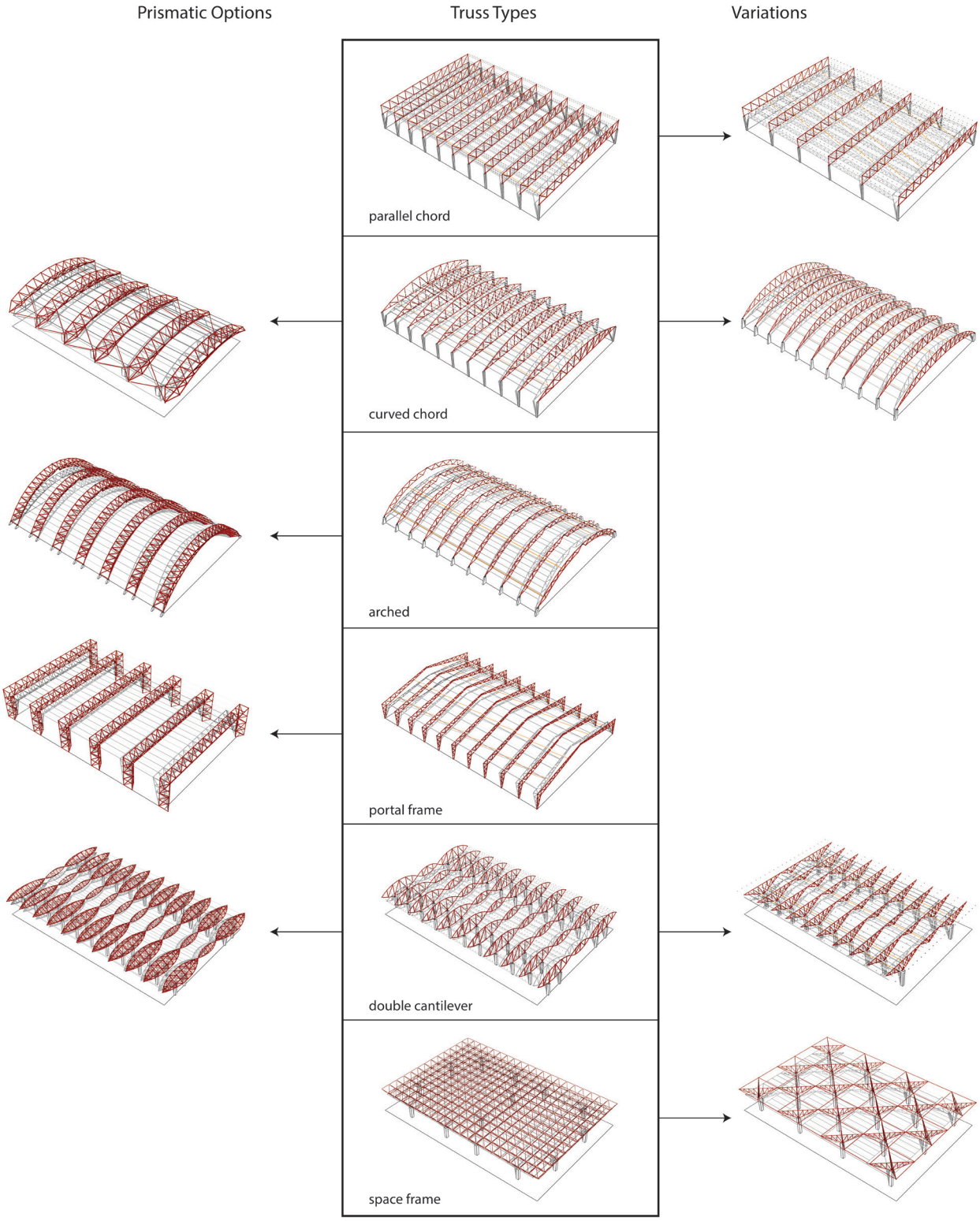


Figure 4.1.5b Graphic matrix of selected truss options with variable conditions

too. Finally, we'll find that each scheme will also have an option to change the main framing members from flat panel trusses to spatial prismatic trusses. Note that these may look like specific solutions, but we are merely concerned about the overall profile of the form and location of supports at this point. (Figure 4.1.5)

Establishing Criteria for Long Span Truss Evaluation

In order to assess these options, we are looking for more than just compliance. Critical evaluation depends on the shared priorities of the architectural and structural design criteria. We want to create testable models that allow us to evaluate quantitative and qualitative aspects of the design.

Our goal is to develop a set of pros/cons for each option—but we'll need specific data to do so. For each option we will look for existing examples to better assess aesthetic and spatial consequences and we will model the schemes three-dimensionally to visualize the number of elements and how they are assembled. We'll use digital assessment tools to determine internal stresses, the resulting sizes of truss members (reflected in overall mass), how much the truss will elongate/deform under stress, and how much interior

TOOLS FOR MAKING AND BREAKING: PART ONE – PHYSICAL MODELS

Developing physical models is a good way to visualize buckling and deflection in trusses. These fundamentals are necessary to understand before the digital modeling process begins—particularly for designers who are new to framing layouts and truss designs.

To Do: Design and build your initial idea for this design problem using wooden dowels and hot glue. Establish a scale for the model, how much weight it should carry, and set a performance goal for how far it should span and how much deflection is allowable under loading.

To Discuss: What tools did you use to generate a truss design? How did you assess the height/depth of the trusses, the panel sizing, and direction of diagonals? How did it perform under testing (specifically discuss the consequences that resulted from the limitations of flexible materials like wood and glue). How productive was the feedback from a physical model in refining your design? What other information would you want to get from a testable model? (Figure 4.1.6)

Figure 4.1.6 Form-finding and remedial truss testing with physical models (SxD, Iowa State University)

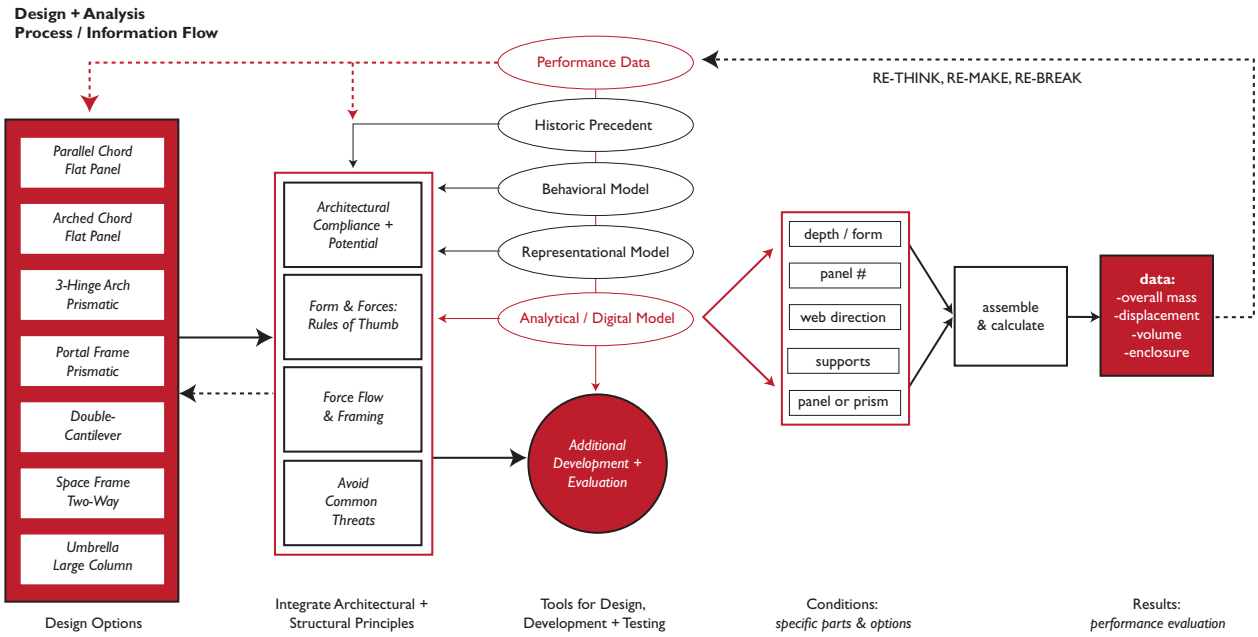


Figure 4.1.7 Detailed workflow diagram with expanded criteria as an outline for the digital modeling process

volume and exterior surface area will be required, reflecting economics and energy performance. For clarity, we won't analyze the prismatic trusses in the same way. These options all share similar advantages of added stiffness and stability but require additional materials and more complicated fabrication methods. (Figure 4.1.7)

We don't have a clear method to assess construction costs, but we'll note whether certain systems present unique (and potentially expensive) construction challenges. Finally, we'll need to understand that certain assumptions about materials (steel) and member sizing (worst-case) need to be applied uniformly to each scheme at the beginning of this design process and do NOT represent final truss sizes. These results are simply a road-map to help us refine our designs using data-based information, not endorsements of a final scheme.

Station Roof Truss Option #1: One-Way, Parallel Chord, Flat Panel

Most truss framing schemes are one-way systems. This simple arrangement of repeated and arrayed units adds a predictability (and economy) to design and construction—one that matches our rectangular plan arrangement. *This is where we'll begin*. Our first option (Option #1) uses parallel chord, flat panel girder trusses to span the space with columns on each end. Purlin trusses will span between these girders, and decking (and/or skylights) will span between the purlins. We'll estimate the depth of each element in this arrangement to evaluate this approach's relative effectiveness.

In one-way systems, each element's spacing affects the others. The decking's allowable span determines the purlin trusses' (or *joist trusses*') spacing. The girder trusses should collect loads from purlin trusses only at their panel points/nodes (Chapter 4.0), so the purlin spacing dictates the girder truss's panel widths. The girder truss's panel proportion is based on the distance between purlins and the depth of the girder, which is based in turn on its span. Ideally the diagonals within each truss panel will be angled between 45 and 60 degrees to effectively resist loads. But the girder truss' depth and its member sizes are also based on the load it has to carry, which is dependent on the span of the purlins! To avoid this circular logic, we can design in the opposite order, starting with the spacing of the columns on a grid. (Figure 4.1.8)

Spacing the columns 32ft (9.75m) on-center corresponds with the program and seems to be a visually

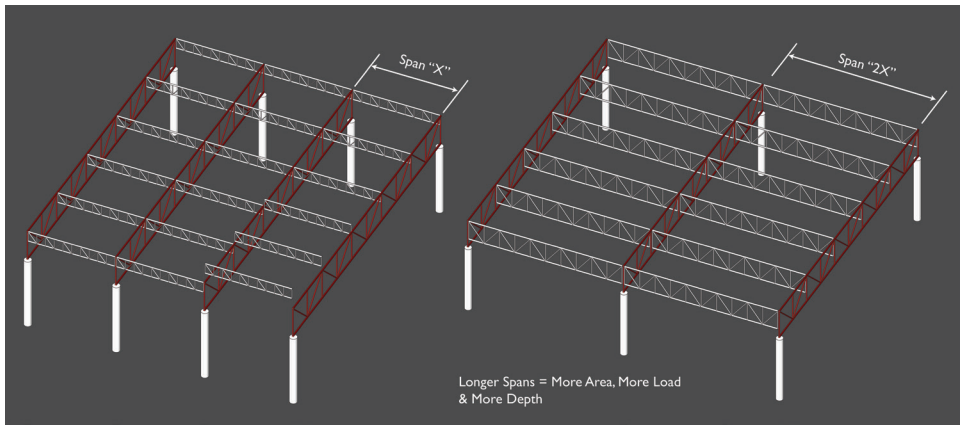


Figure 4.1.8 The fundamental relationship between span and resulting depth of the trusses. More area = more load = greater depth + more materials

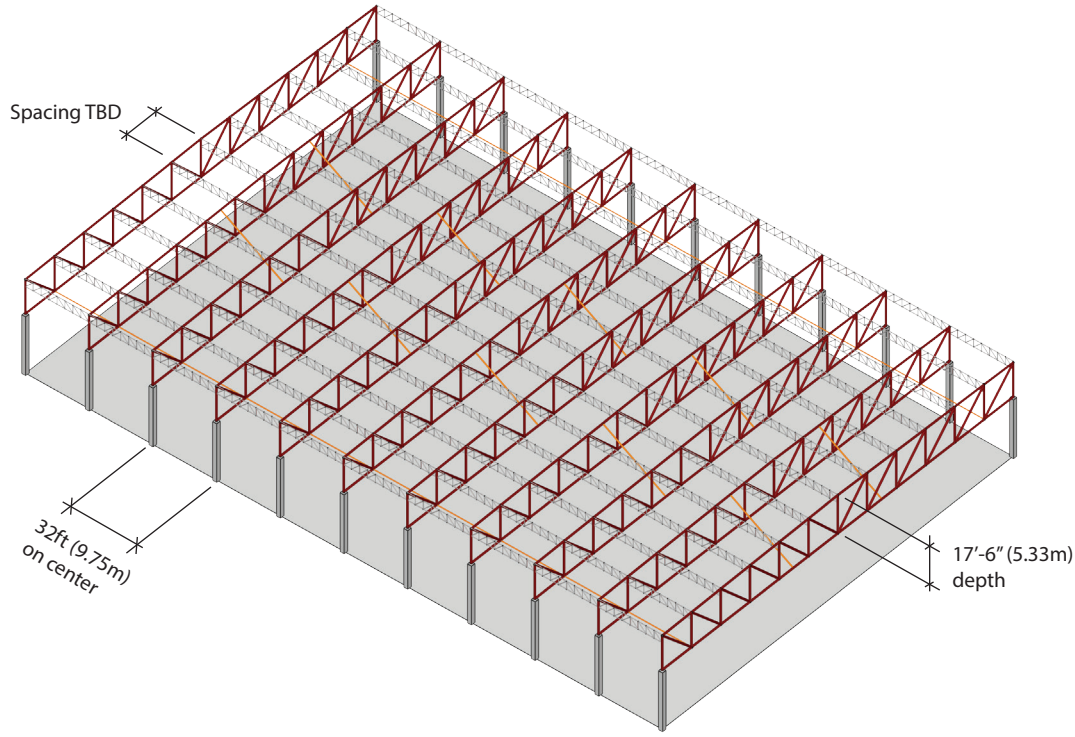


Figure 4.1.9 Option #1 proposal massing and framing layout

reasonable distance. Using our rules of thumb for parallel chord steel trusses (Chapter 4.0) we find the depth-to-span ratio is 1:10–1:12. The truss spans 208ft (63.4m) so our most optimistic depth will be 17ft 6in (5.3m). (Figure 4.1.9)

After depth is established, we can estimate the number of panels in the truss across the span. Because loads should be transferred to trusses on their panel points/nodes, the spacing of panels within a truss is dependent on the spacing of the load collectors above it (e.g., if decking spans 6ft (1.8m) this sets the purlin spacing, and therefore the panel size). This is balanced with the overall truss depth because the diagonals within the panels are typically set between 45 and 60 degrees. We can divide our truss into twelve equal panels (at 17ft 6in (5.3m) each), which allows our diagonals to be

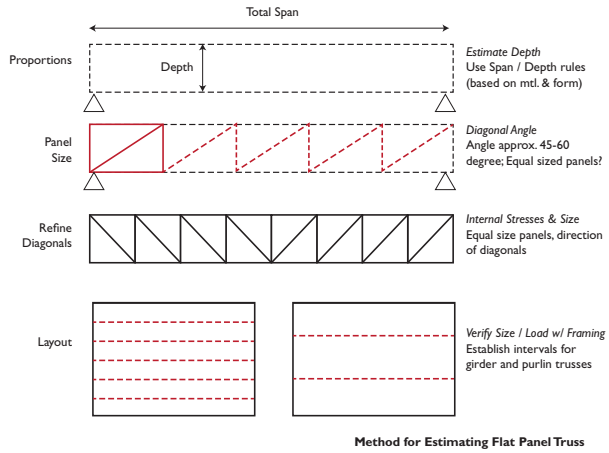


Figure 4.1.10 Rapid (and rough) method for estimating height, depth, and panel sizes for a flat panel truss using rules of thumb

at 45 degrees. If we assume that our diagonals should only be in tension, like a Pratt truss, they would slope downward towards the center of the span from each end, meeting in the middle. (Figure 4.1.10)

Our purlin/joist trusses would span 32ft (12.2m) between the girders, less the thickness of the trusses. If we space them at 17ft 6in (5.3m) on-center to align with the girder truss' panel points truss we could conservatively assume that the purlin trusses would be 4ft (1.2m) deep based on the depth-to-span ratio for purlin trusses of 1:8–1:10. Atop the purlin trusses we'd use steel roof decking with a depth that is sized to span 17ft 6in (5.3m); this is approximately 4–6in (10–15cm) deep based on span tables. These girder trusses would be too long to fabricate in one piece, so we'd need to pick splice points at approximately ⅓rd of the span. Finally, these are deep girders, so we need to provide regularly spaced bracing at their bottom to keep them from buckling, perhaps at every other purlin, or 35ft (10.6m)). Drawing this in three-dimensions helps to visualize the composite assembly, particularly if each type of element is assigned a separate color (e.g., girders (red), purlins (white), and bracing (orange)). (Figure 4.1.11)

Station Roof Truss Option #1: Sizing and Evaluation

Truss members resist only axial (normal) stresses, so there is a mathematical relationship between the amount of stress (P) and the cross-section area size

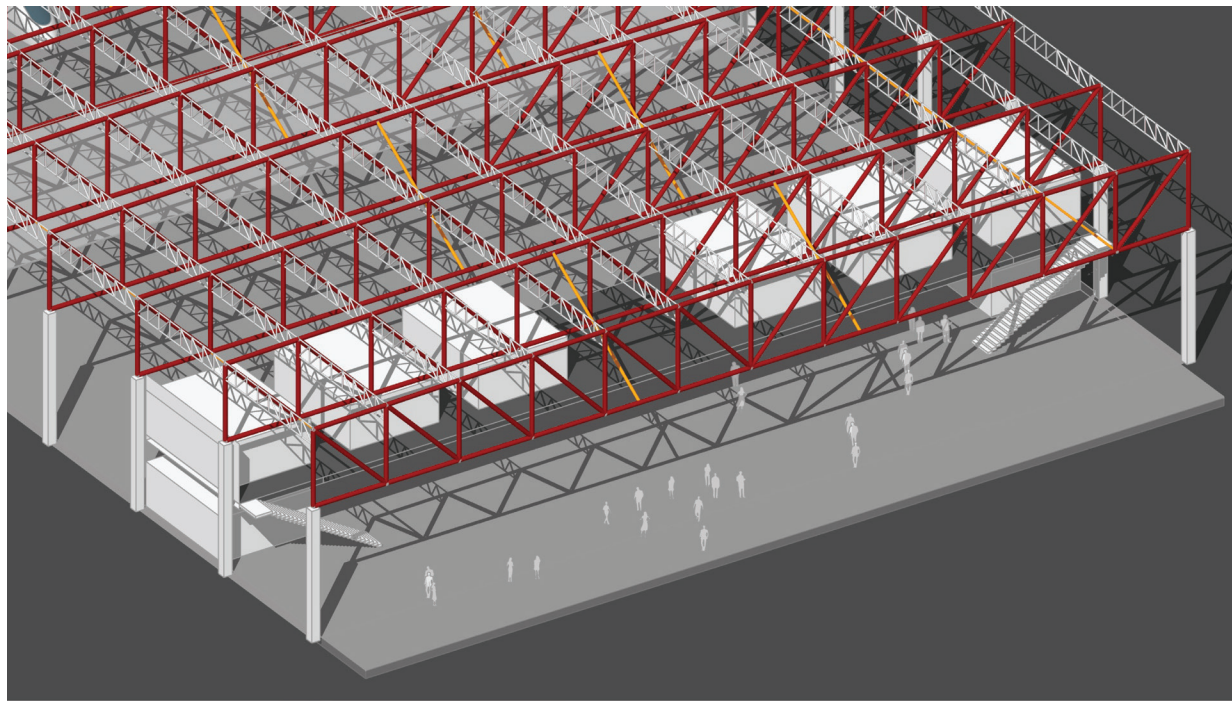


Figure 4.1.11 Up-close view of Option #1

(A) of the resisting element (f = P / A). We'd probably size the chords and diagonals differently across the length of the truss based on the anticipated stress at each point (e.g., higher stresses in the middle and the ends will require bigger elements with thinner and smaller members elsewhere). But consider why this might not work in terms of fabrication.

Would we really have different sized diagonals, some much larger than others *and* a variance in the size of the top and bottom chords along its length? This variety of member sizes would become costly—planar trusses favor some level of uniformity in member sizes—and would increase the connections and potential points of failure. Instead, truss designers look to optimize member sizes to keep sizes consistent across the length. For example, using W10 × 20 steel members for lightly-stressed areas and heavier but similarly sized W10 × 90s for higher stressed areas. For this reason, we'll assume a uniform sizing for our truss members based on "worst-case" loading. (Figure 4.1.12)

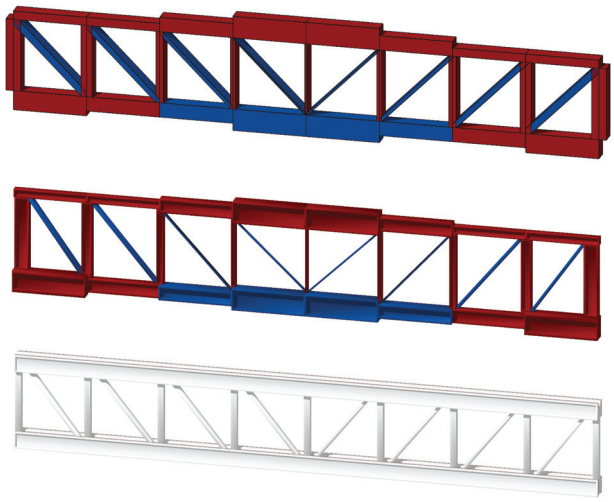


Figure 4.1.12 Visualize the inefficiencies inherent in creating a truss with each portion of the chord and/or diagonals sized specifically for the anticipated stresses (top: Stress intensity, middle: Match element size with stress, bottom: Worst-case sizing for uniformity)

To assess this scheme, we initially look at a girder truss' stresses and its deflections. Our digital model tells us how much stress each element of the truss needs to resist and we find that stress in each element varies across the span of the truss. A model shows how stresses vary by changing color and line weight—darker saturation equals higher stress. Deflection is typically the controlling design factor in long spans. In this case, we will limit the allowable deflection to L/360 (approximately 7in (17.6cm)).

Some qualifiers about the digital model and the results: This digital analysis model was initially set up with an assumption that all member sizes would be the same, but since its deflection was excessive, we redesigned ("optimized") it to vary the member sizes to reduce deflection. We've assumed a "worst-case" scenario sizing where all of the girder truss members would be a 6 × 6 × ½ in hollow steel tube and columns are 36 × 36 × 2 in hollow steel tubes—regardless of their stresses. The mass includes only the girder trusses and columns (as purlins and decking would be a uniform load). The dead load only includes the weight of the truss (no additional elements) and a live load factor is included in compliance with local snow loading code conditions. (Figure 4.1.13)

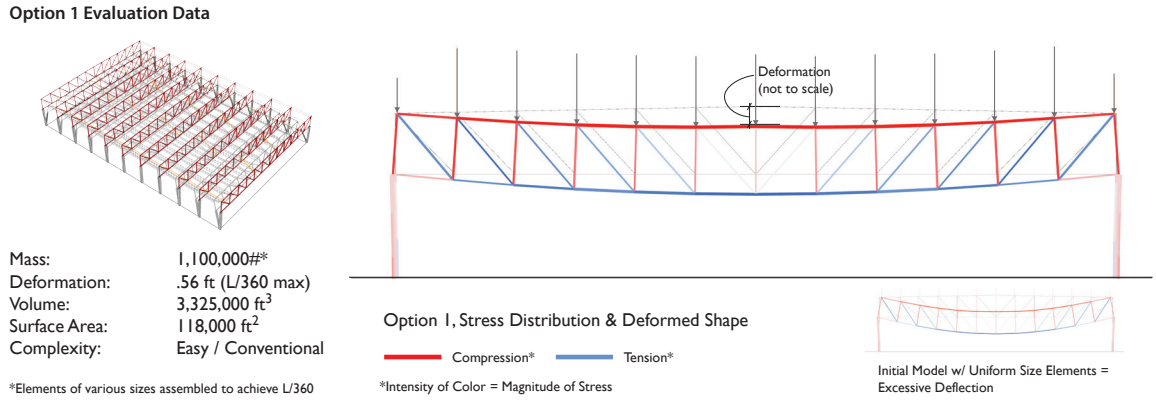


Figure 4.1.13 Option #1 evaluation data and stress distribution

TOOLS FOR MAKING AND BREAKING: PART TWO – DIGITAL MODELS

We'd like to rapidly assess the effects of truss height, panel spacing, etc. on performance, so we'll use a digital design tool. To more easily study our options, we'll select one particular option: A parametric structural engineering tool, in this case, a plug-in for the 3D modeling tool, Rhino, called Karamba which gives a great deal of latitude to the designer.

Analytical software relies upon us to understand the structural element's basic conditions (components, layout, support, materials, etc.) and to create an algorithm that assembles and analyzes aspects of the design to create helpful feedback. There is no single "right way" to set-up a parametric analysis model but there are steps to define the parameters of the model, identify the parts/components, assemble them, define the material, loading, support, and framing conditions together to get an accurate assessment. The "garbage in/garbage out" axiom applies—if we don't understand the basic conditions of trusses in setting up the model, we won't get reliable data.

Digital modeling can create representative, instructive, and analytical information in *one model*. These aspects shouldn't preclude or exclude other modeling techniques—particularly if other methods are useful for representation or instruction—however, a digital model provides a holistic assessment of architectural and structural information. Setting up the algorithm to include broad categories (the domain) and specific conditions for arrangement and loading is critical. Once assembled and calculated, proper digital models should be looped back into the conditions to quickly assess variations. (Figure 4.1.14)

To Do: Find examples of tutorials for setting up parametric models for truss analysis. Diagram how information is collected, assembled, and sorted and compare this with other modeling and assessment techniques. Collect information about the factors you'll need to model: chords, panels, diagonals, loading, supports, materials, etc. and determine a range of options (or sliders) that you'd want to establish for each (if any).

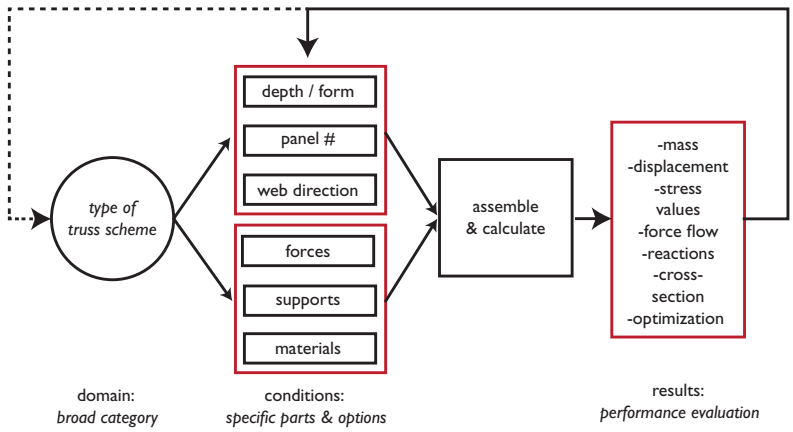


Figure 4.1.14 The workflow created for analyzing trusses for this design problem

Option #1: Variations – Panel Sizes and Purlins

By digitally modeling the initial Pratt truss scheme (Option #1) and defining the design parameters as variables we can change the truss' factors or layout in order to understand their consequences. How would changing one element affect the others? For example, what if we changed the girder spacing or size of panels?

We could explore the consequences of doubling or halving the spacing of girders (64ft (19.5m) or 16ft (4.9m)). Without running calculations, we could

rule out any benefits of halving the spacing because we know that span length is the controlling factor in deflection—length is cubed in the equation. Deflection is the controlling design factor for long spans, so we can't solve it with more trusses. Doubling the girder spacing (Option #1A) would greatly increase the size of the girders and purlins (girders would be carrying twice the load and purlins would span twice as far); we'd halve the number of columns and foundations but they'd have to be significantly larger.

What effect would changing the number of panels have on the amount of materials or stresses (Option #1B)? Diagonals with more vertical orientation will have less stress than more horizontal members (Chapter 4.0), but will this make a difference across a long spanning multi-panel truss? Doubling the panels would allow us to double the number of purlin trusses as well, as they always rest on the panel points. This would reduce the decking's depth and span, so we'd have some material savings there, but would this off-set the additional structural members in panels? (Figure 4.1.15)

By altering our digital model and applying our pre-set conditions we can get a visual and data-based assessment. For Option #1A, the stress levels have doubled in the girder truss as a result of the new spacing, so the member sizes have had to increase in size, mass, and depth, all of which increase volume and surface area. Despite reducing the number of girder trusses by half, the girder and purlin trusses' increased depth, coupled with increased member sizing, only marginally decreases the overall mass. Most importantly, the purlin trusses are now "long span" trusses too and subject to deflection concerns. There are few potential benefits to this option. In Option #1B, the smaller panel sizes have added more mass to the truss (which isn't helpful economically or structurally) and they haven't stiffened the truss against deflection any better than Option #1. (Figure 4.1.16)

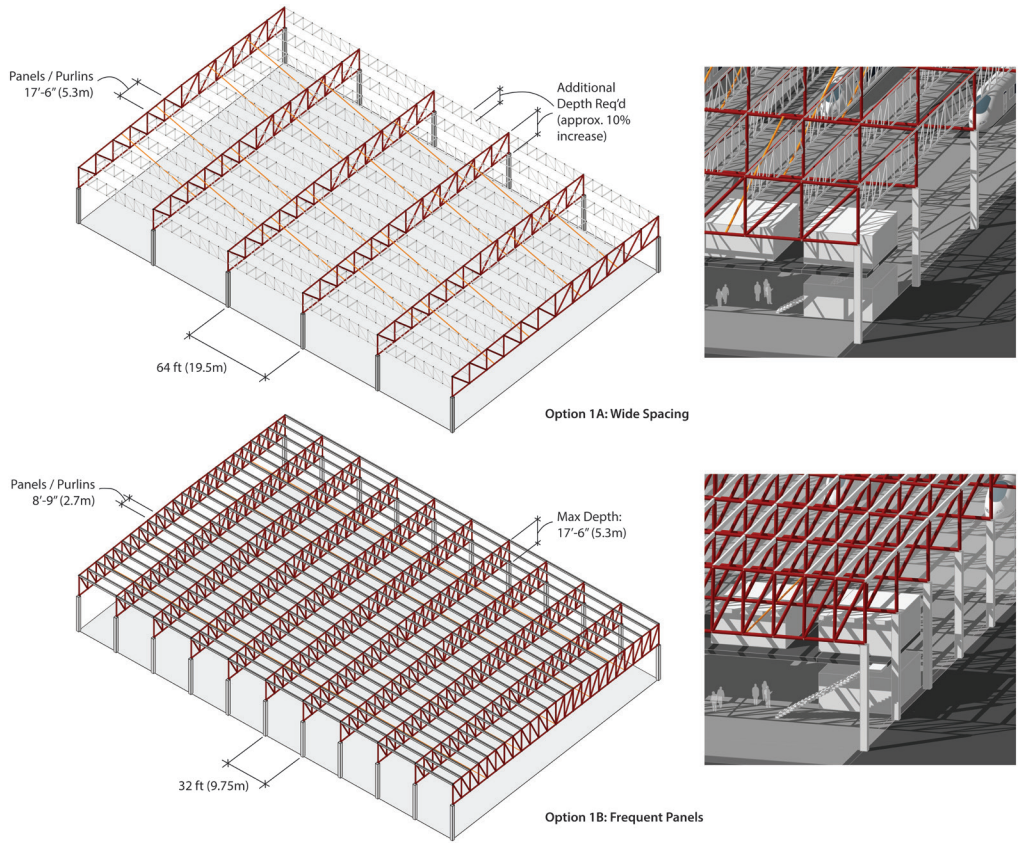


Figure 4.1.15 Option #1A and Option #1B variations

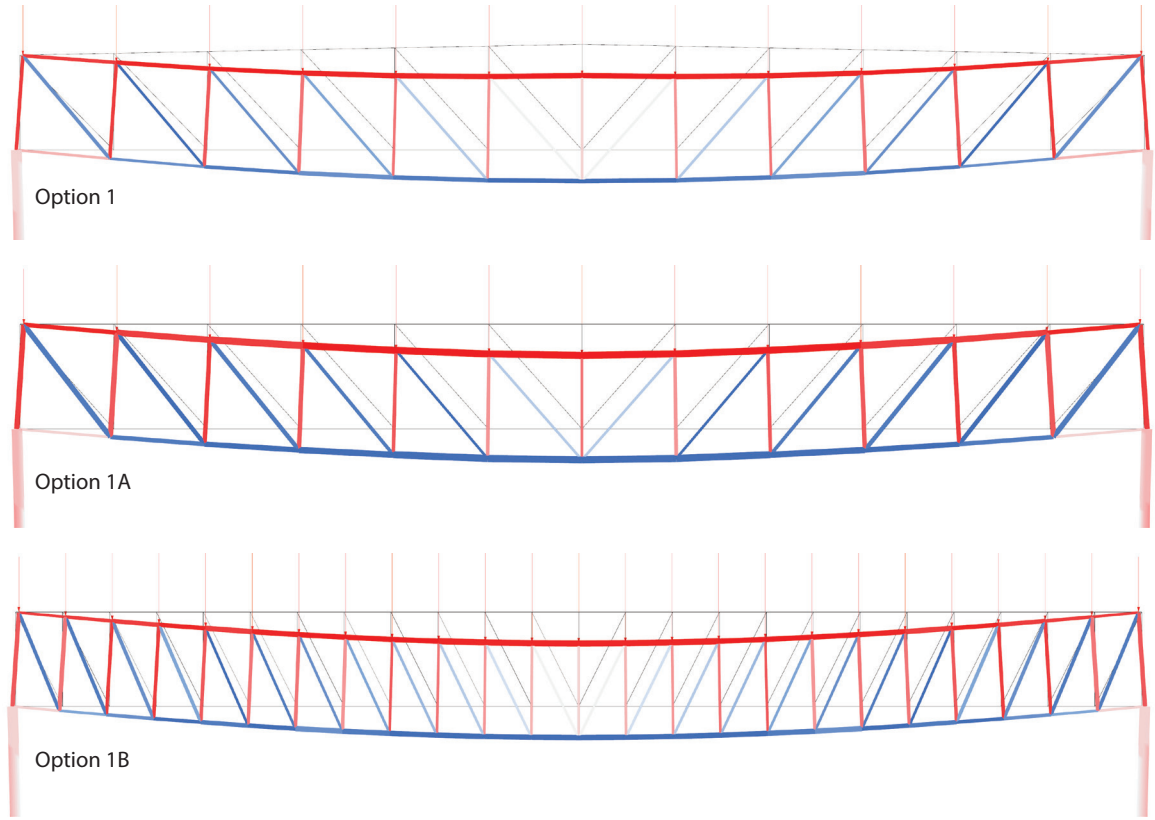


Figure 4.1.16 Comparison of stress levels and deformation in Options #1, #1A, and #1B. Member sizes would be based on limiting deflection and internal stress levels

Overall Option #1 is conventional and unremarkable structurally and architecturally—it gives us a base line for data regarding mass, deflection, interior volume and exterior surface area. Perhaps we could increase the responsiveness (or reduce the volume) by changing its form. The ability to shape an element's form is highly advantageous in long span applications—especially if these adjustments could be made without harming the structure's effectiveness, affordability or fabrication.

MAKING AND BREAKING: EVALUATION BY TESTING

In order to better understand the relationship between the amounts of materials, parts, weight, efficiency and performance of different trusses, your task is to build scale models (and digital simulations) of three types of trusses that span 96ft (29.2m) designed to support uniform load across the span. Using modestly sized wooden dowels and string (or wire), build three types of trusses: *Panel, prismatic, and king-post* at ⅛th scale (24in (61cm) between supports). Build two panel and king-post trusses and space them far apart so each carries an equal portion of the load. When completed each truss should hold 10lbs (4.5kg) of load. Use a 50psf gravity load plus 20psf for live loading for your digital simulation. Your schemes will need to include any elements required to keep the trusses stable side-to-side and/or to resist any

(continued)

(continued)

buckling under loading. You are comparing feedback from hand-made wooden models and simulated steel trusses—the results will clearly vary.

To Discuss: Discuss the relationship between truss depth, length of truss members, and behavior under loading. Which scheme used the most materials if you include all sticks and string? What are the liabilities of using fewer elements in the truss? How does the different feedback from the model help your ability to assess truss design from a schematic level? (Figure 4.1.17)

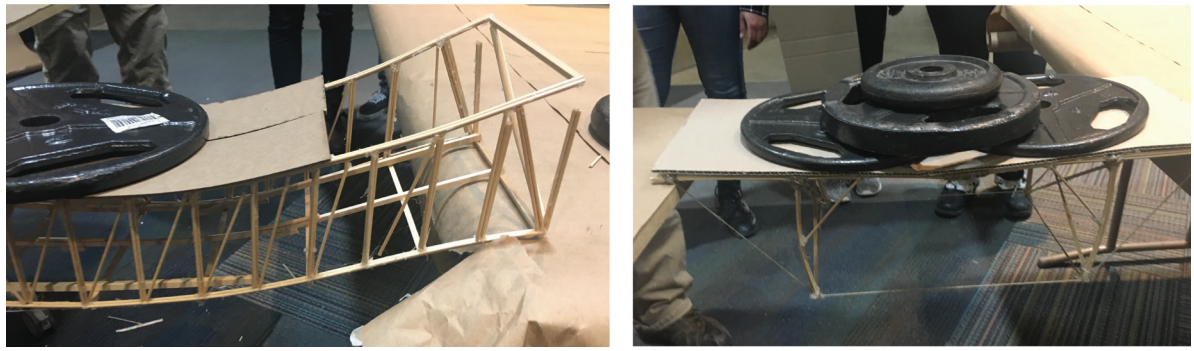


Figure 4.1.17 Testing models reveals the different deformations and locations of concentrated stress in various truss arrangements—tension members pull apart and compression members buckle (SxD, Iowa State University)

Station Roof Truss Option #2: Curved Profile Panel Trusses

Flat trusses, like flat beams, suffer structural disadvantages by their geometry—particularly when compared to truss forms that curve or arch. Because trusses are made from smaller, lightweight elements, they can easily accommodate different configurations. Trusses' top and bottom chords can be configured in different ways (parallel, pitched, curved, etc.) and trusses can be shaped to match architectural desires and spatial requirements.

For our first variant (Option #2) we will use a bowstring truss (curved upper chord, flat lower chord) with a profile that matches the moment diagram (i.e., deepest in the middle, shortest on the ends). The truss rests on the columns at the ends and rises up with a 1:8 depth-to-span ratio (approximately 26.5 ft (8m) at the highest point) with a regular curve. The top chord is in compression, like an arch, and the bottom chord is in tension, like a tie-back cable. This prevents any thrust at the supports. The diagonals resist stresses based on their orientation (in this case, compression). We could modify the bowstring by also curving the bottom chord to create a crescent-shaped truss (Option #2A). The profile would still match the bending moment (maximum depth of 20ft (6.1m) in the middle of the span), but because the bottom chord also arches up (20ft (6.1m) from the bearing point), we could lower the bearing height on the ends to reduce the overall volume and still achieve the required clearance. (Figure 4.1.18)

These revisions perform better than Option #1 in terms of mass and displacement—proof of arch-action's advantages. The stress model shows how the forces are concentrated in the arched top and tensed bottom chords with marginal compressive stress in the diagonals. As a result of the arched roof, Option #2A encloses less volume than Option #1 (500,000 cubic feet less) and it has reduced the exterior enclosure area by more than 23,000 square feet—both significant improvements. (Figure 4.1.19)

This curved roof form isn't without consequences. An arched truss has some of the benefits and liabilities that come with an arch. Unlike parallel chord trusses that rely on bottom chords to resist thrust through tension, crescent trusses will generate some thrust at

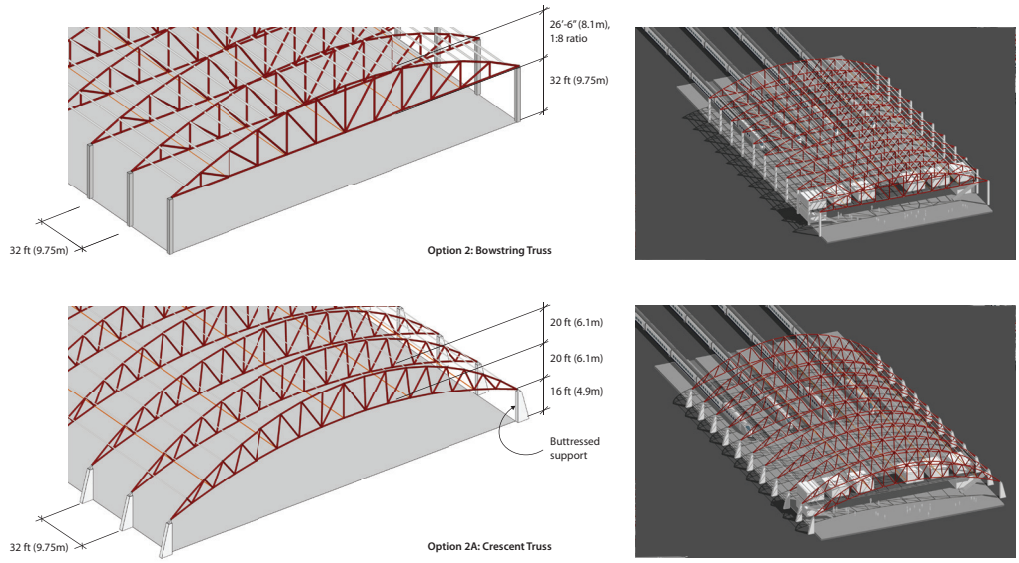


Figure 4.1.18 Option #2: Bowstring Truss and Option #2A: Crescent Truss comparison

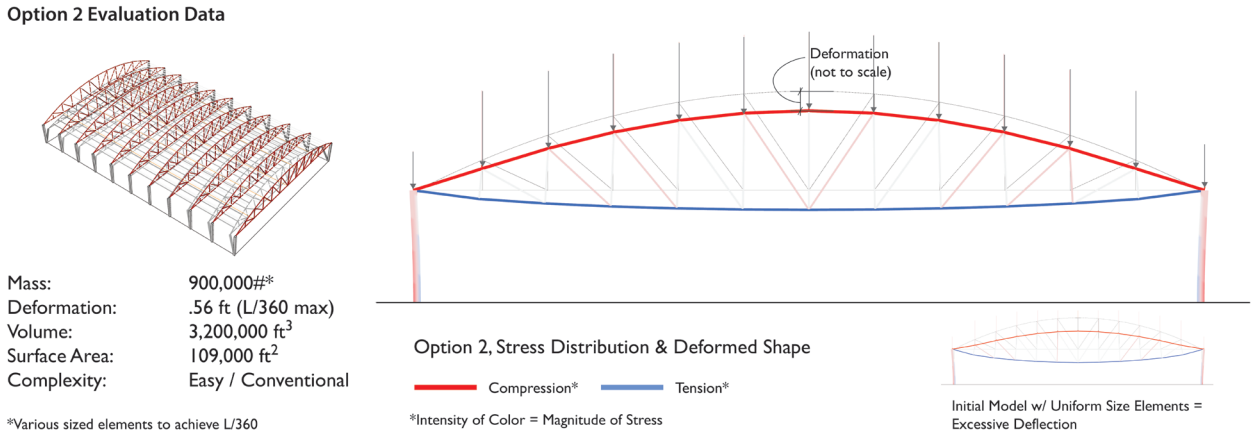


Figure 4.1.19 Option #2 and #2A evaluation information. Note the tension on the inside face of columns that results from thrust from #2A

their ends. It would be problematic to introduce an outward horizontal force atop a column as it would create a large overturning force. We could reduce this by adding buttress supports or by blending the bottom chord of the truss into the columns.

Station Roof Truss Option #2 Variations: Prismatic Truss and Branch Columns

Consider the benefits of keeping the crescent form, but changing the planar trusses into a V-shaped *prismatic* cross-section. In this arrangement, the top of the truss would split into two chords with a single bottom chord. If we assume the prismatic truss is equally as deep as a panel truss, the V-shape sides would angle up between 45 and 60 degrees each, creating a prism with a wide top. This truss has three important benefits: it creates a self-stabilizing cross-section, because it is triangulated, it reduces the purlins' spans between girders, and it can reduce the girder truss' effective depth because it behaves like two angled panels trusses to resist bending. (Figure 4.1.20)

Making our crescent trusses prismatic would allow us to space the column locations farther apart, because our girder trusses are wider. We could spread out the

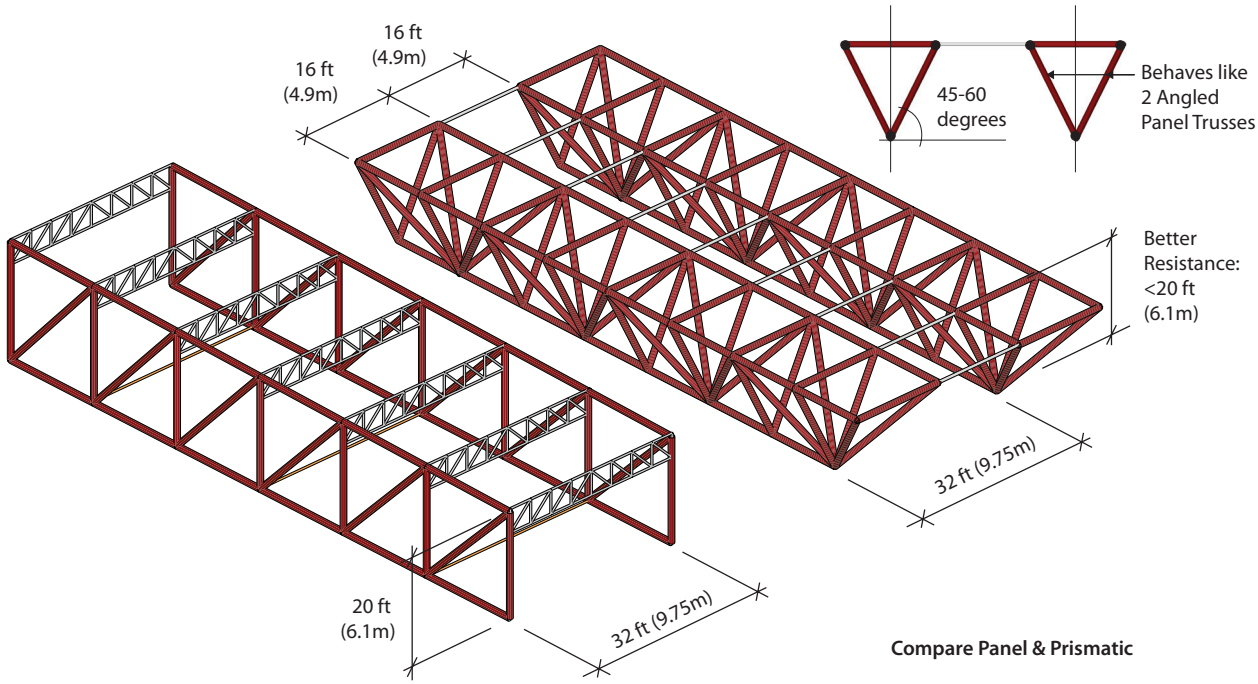


Figure 4.1.20 Comparison of panel and prismatic trusses with same center-line spacing between

columns under the center of each girder, or we could explore a more radical approach in which the columns weren't under the center of each prismatic truss. Offsetting the columns between two girders (64ft (19.5m)) reduces the number of columns and creates a more interesting composition. Instead of a single large vertical element, we could create branch-like supports that extend up at angles and connect to the prismatic girders at multiple points. This option increases geometrical complexity but it could reduce the amount of material in the girder trusses and column supports—a hypothesis worth testing. (Figure 4.1.21)

A similar scheme can be found in the Hamburg Airport Terminal where curved prismatic trusses

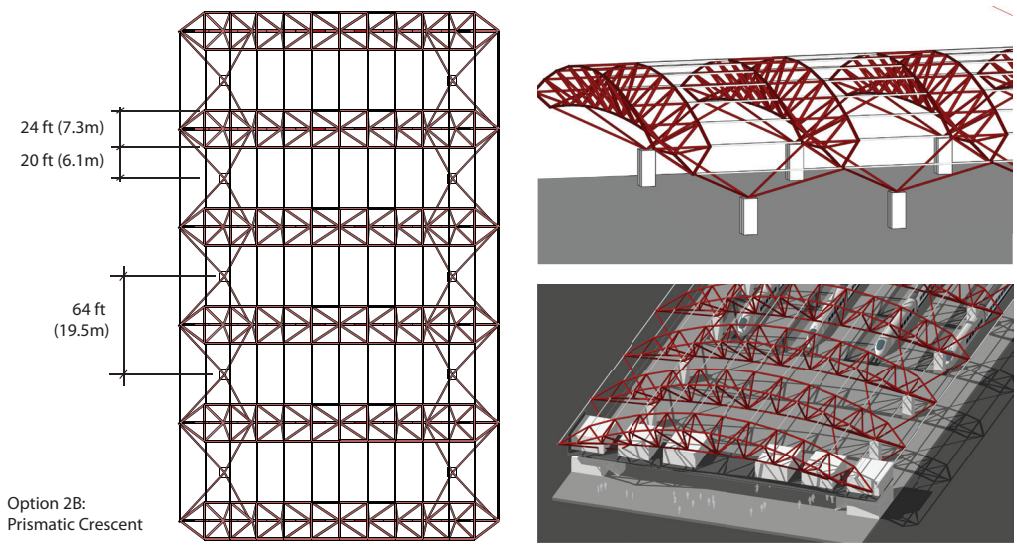


Figure 4.1.21 Option #2B prismatic crescent truss: Framing plan, axon, and atmospheric view

Figure 4.1.22 Hamburg Airport Terminal 2. Skylights above arched prismatic trusses with off-set columns (Architekten von Gerkan, Marg und Partner, 1993)

with skylights connect to four steel tube supports that branch out from the columns' middles. Angled compression members at the column supports are at risk for buckling so each support is a cigar-shaped pylon that is thickest in the middle where buckling is most likely. (Figure 4.1.22)

Station Roof Option #3: Arches

The logical architectural and structural extension of the bowstring and crescent shapes would be a complete arched truss (Option #3). A vaulted roof form makes sense; it matches form with forces (compression) which lightens the structure, the curvature stiffens it against any abhorrent loading (lateral loads) that would cause bending stresses, its interior air volume is reduced, and the building enclosure of walls and roof is consolidated as one surface.

Architecturally, we want to make sure the functions aren't compromised by the reduced height at the two ends, so we'll establish an arch geometry that works with our conditions—particularly the Station house rooms and the train clearance on the sides. Ideally, we'll use the most effective funicular profile for the arch and repeat them regularly using a one-way framing layout (approximately 64ft (19.5m) at its highest point). Arched trusses can have very thin depth-to-span ratio of 1:50–1:100 but we'll err on the conservative side of this estimate. (Figure 4.1.23)

To control movement within the truss and at its supports, we could use either a two-hinged or a three-hinged arch (Chapter 2.1)—a three-hinged approach is shown. There is more bending stress in the middle of the two arch segments, so we could thicken up the trusses there to be more responsive to loading. The arches rest directly on the foundation supports (with a pinned connection) which need to be buttressed to resist the thrust forces at the base.

Our analysis confirms the significant benefits of the arched form: It requires less material, is

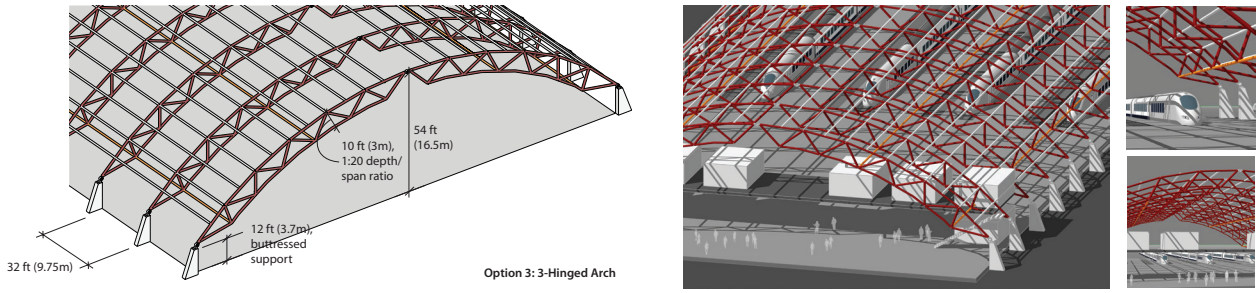


Figure: 4.1.23 Option #3, three-hinged arched truss: Framing, axon, and interior views

significantly stiffer, and even with a relatively steep radius it has 10% less interior air volume and building skin than the other two options. The stress distribution analysis shows that all truss components are in compression, a factor unique to the arch truss. The supports at the perimeter do need to be connected directly into the ground (Figure 4.1.24), however, or buttressed to resist horizontal thrust.

Station Roof Option #3A: Arch Variations

One of the largest liabilities with trusses under compression is buckling—a threat that is exacerbated when the truss is planar *and* arched. A potential solution is to create prismatic arched trusses (Option #3A). As we've seen, changing the cross-section to a 16ft wide (4.9m) prism with 45 degree angled diagonals stabilizes the trusses and allows us to change the supports' spacing. We could increase our span between girders and columns to 48ft (14.6m) on-center and maintain a 32ft (9.8m) clearance for the purlin beams. This allows the Station house to fit between the trusses and is well within the allowable span for purlin beam tubes. If we consolidate the girder trusses' supports we will reduce the foundation supports by ⅓rd. (Figure 4.1.25)

The Waterloo International Station in London (1993) by Nicholas Grimshaw and Partners is an instructive example for how an arched roof can be improved with trusses. This project had unique site and section constraints that required one of the tracks to be at the edge of the site, which meant that a funicular arched form wasn't possible. To meet required clearances, the trussed roof form had to become a flattened three-hinged bow arch (spanning 160ft (48.5m)) with an asymmetrical profile and a non-planar cross-section.

Figure 4.1.24 Option #3, arched truss stress and deflection evaluation

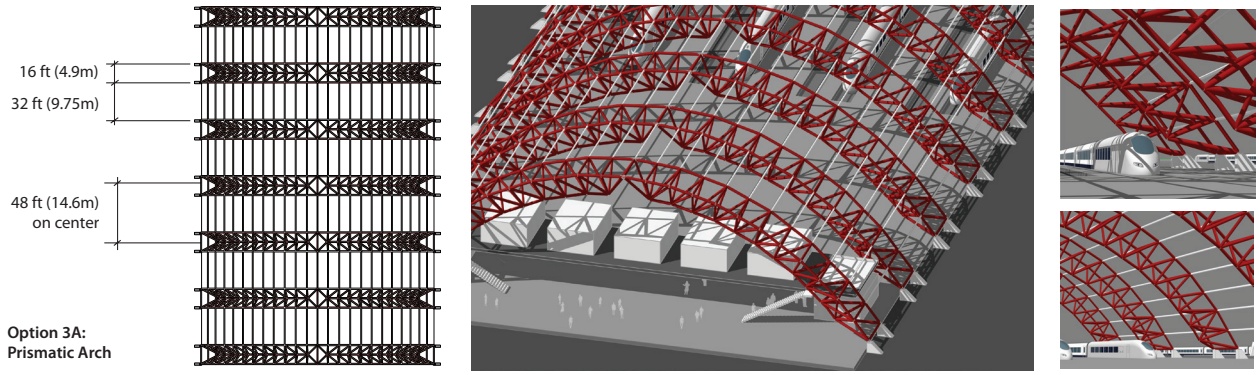


Figure 4.1.25 Option #3A, prismatic arch: Framing plan, axon, and interior views

To gain the necessary interior clearance, a minor truss of each arch was moved to the outside of the enclosure, expressing its compression and tension elements. At the hinge point, the truss switches to the interior of the enclosure, with the same strut and tension rod expression. Although these modifications were structurally and functionally logical, the unconventional nature of the scheme required additional modification during design and fabrication. Three-dimensional CAD models (made with early, less accurate programs), sketches, models, and prototypes all helped design the cladding details. Eventually, a full-scale structural bay was built off-site to test the erection procedure and coordinate structural and cladding details between trades; this led to significant and important changes in the connection details. (Figure 4.1.26)

Figure 4.1.26 Waterloo International Station's three-hinged prismatic truss (Grimshaw, 1993)

MAKING AND BREAKING: VARIANTS AND OPTIMIZATION

The most impactful benefit of correct digital model algorithm is the ability to study the impact of changing certain variants without having to rebuild the entire model. Once you have developed a general consensus that a certain framing arrangement or truss type is correct, you can optimize the efficiency of truss. Most parametric modeling software has an option called a "slider" that allows you to manipulate a factor within a set range (e.g., number of panels, height of truss, etc.) and instantly see the results. These manipulations are helpful tools in decision-making and in communicating the consequences of certain actions to collaborators and users.

To Do: Vary the truss height and panel sizing factors separately using your digital model. Record the impact of the changes.

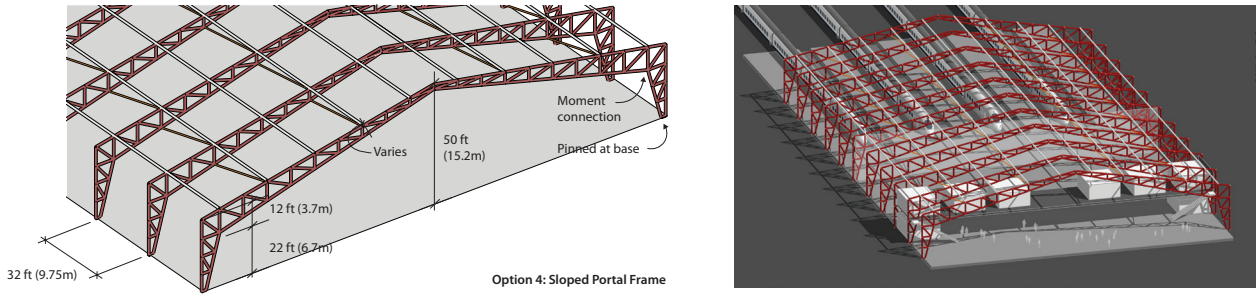


Figure 4.1.27 Option #4, portal frame scheme

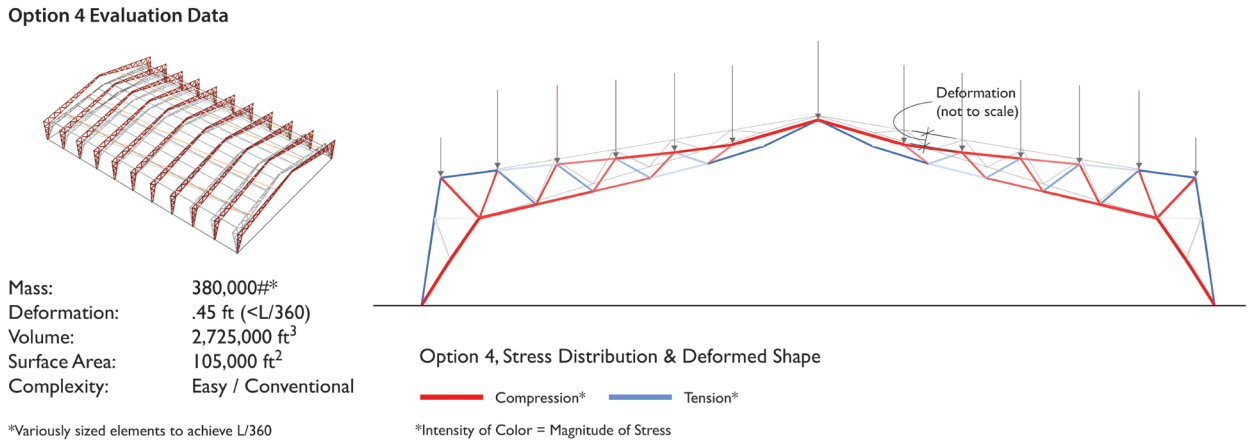


Figure 4.1.28 Option #4, portal frame evaluation

Station Roof Option #4: Portal Frame

A three-hinged arch form also resembles the types of compressive forces and overall profile found in a *portal frame*. Portal frames, or rigid frames, are useful in long spans because they reduce the amount of bending load in the middle of a long span by sharing moment stresses between the column and the beam (Chapter 6.0). The maximum load is accumulated and resisted at the point where the column and beam meet, thus the deeper profile and most materials at this point.

In our application, we could space portal frames at 32ft (9.75m) on-center. To gain a structural advantage, the profile of the frame should be sloped, and the roof could thus be gabled up towards the center, reducing the bearing height on the ends and reducing interior volume as we did in the arched scheme. Because of the high moment forces and torsion that comes from portal frame behavior, we'd want to switch from I-beam or angles to steel tubes throughout. The triangulated nature of the system automatically provides a depth that is useful in resisting moment forces *and* trusses can be easily shaped into the columns as well. (Figure 4.1.27)

Figure 4.1.29 Galerie des Machines, three-hinged portal frame with clear span of 377ft (115m) and a height of 159ft (48.5m)

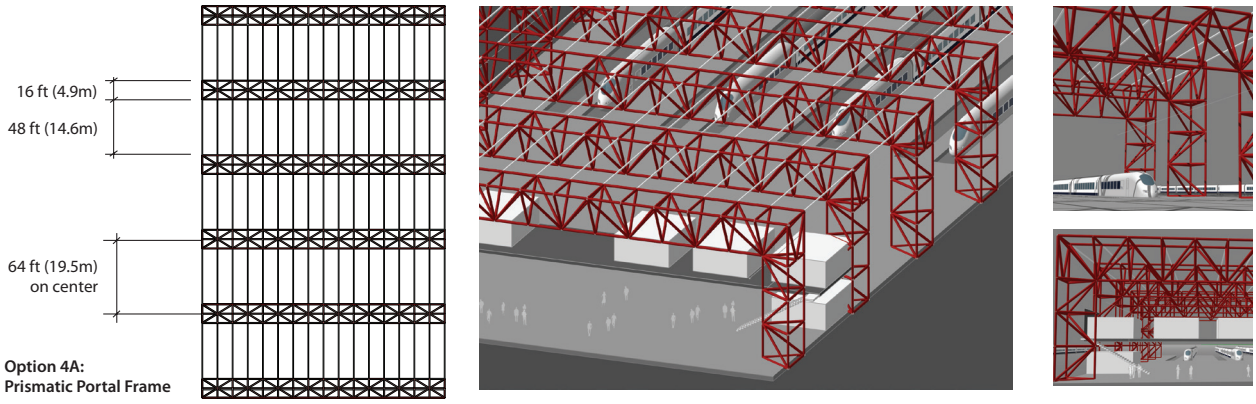


Figure 4.1.30a Option #4A, prismatic portal frame: Framing plan, axon, and interior views

Analysis shows how compression and tension forces are distributed across the span—a pushing and pulling on opposite sides of the member that generates large internal moment forces. This scheme's benefits become clear. The deflection, mass, interior volume, and building skin are all reduced compared to our flat and bowstring trusses. Additional variations would need to reduce deflection at the peak. The effectiveness of this scheme demonstrates why three-hinged portal frames have been popular choices for train stations, exhibit halls, and industrial warehouses, including the 1889 Galerie des Machines at the Exposition Universelle in Paris. (Figures 4.1.28 and 4.1.29)

Station Roof Option #4A: Prismatic Portal Frame

The gabled roof gives a distinctive aesthetic that may or may not be desired. As an alternative, we can create a moment frame without a pitched roof, but using a prismatic truss. The size and depth of this truss would be similar to the prismatic arch but without the arched form and thrust. There is efficiency in making a moment connection between the column and truss—a prismatic column can help make this connection. These columns become volumetric, which suggests locating building services and programs within these side walls.

A more detailed analysis would show whether these proportions would affect performance but we know that flattening the roof will lose the structural advantages of force transfer and stiffness that come with a curved or sloped roof. Because this scheme has additional interior volume and surface area, there would need to be a significant functional or aesthetic reason to use flat prismatic trusses instead of the arched option. (Figure 4.1.30)

Figure 4.1.30b The prismatic girder trusses at the Calgary International Airport are supported by tree-like column caps to allow the light to penetrate the space and spread out the structural load

Station Roof Truss Option #5: Double-Cantilever Trusses

To make the trusses perform like a "better beam" we will try a more radical configuration. As we've seen in beam designs, a double-cantilever system reduces maximum bending moments. To enclose the same amount of space, we'd have to reduce the clear span by

THINKING: STRUCTURAL STRATEGIES AND SPACE PLANNING

The Sainsbury Center (1978) by Norman Foster is a simple rectangular museum with trusses that span 113ft (34.4m) across the open space and glazed ends. Prismatic trusses form the roof and support columns on the two sides. The shape of the prismatic trusses (8.2ft deep (2.5m) × 6ft wide across the top (1.8m)) allows building systems to run through the trusses across the roof. But what is the intent of using these trusses for the wall/column supports? Why space these all so closely together (11.5ft (3.5m)) across the length?

To Do: Look at the plan of the building and exterior elevations (consider all the functional and mechanical needs of a museum), in order to determine the connection between the structural and architectural planning principles.

moving the columns in from the two sides and nearly halving our clear span. We could accommodate these columns by coordinating their placement between the tracks and leaving clearance around them.

We'd double-cantilever two trusses out from these supports and place a third truss in the middle supported by the ends of the other two (similar to the Forth Bridge diagram in Chapter 1.0). These should be shaped to reflect the depth of a double-cantilever—wide in the middle, narrow on the ends—with straight or curved chords. The simplest version looks like a series of V-shaped trusses reaching out from the column and capped with a flat roof. (Figure 4.1.31)

Our analysis model shows bigger problems than we've seen in the other schemes. This isn't perfectly analogous to the Forth Bridge (Chapter 1.0, Body Structures) because it isn't tied down on the two ends. Our analytical model shows that without these tie-downs the two larger trusses would rotate inward and deform drastically. Even switching the connections to moment connections won't resolve these issues, but they would add the complication of bending stress. We *can* add tie-downs on the ends but we would still have excessive bending stresses. (Figure 4.1.32)

We could resolve this "tipping inward" imbalance by amending the number of supports to create a more balanced double-cantilever (Option #5A). The optimal profile of the trusses would become lens-like, or *lenticular*. These are the most materially efficient truss shapes because they reflect optimal arch *and* cable forms and require minimal diagonal and vertical web framing. Unfortunately, by making them lenticular we've added depth, which exacerbates the risk of web buckling, and we've increased the amount of material. We'd have to add frequent bracing along the bottom chord, add X-bracing from the bottom to the top chords, or change the truss' profile to something more volumetric. (Figure 4.1.33)

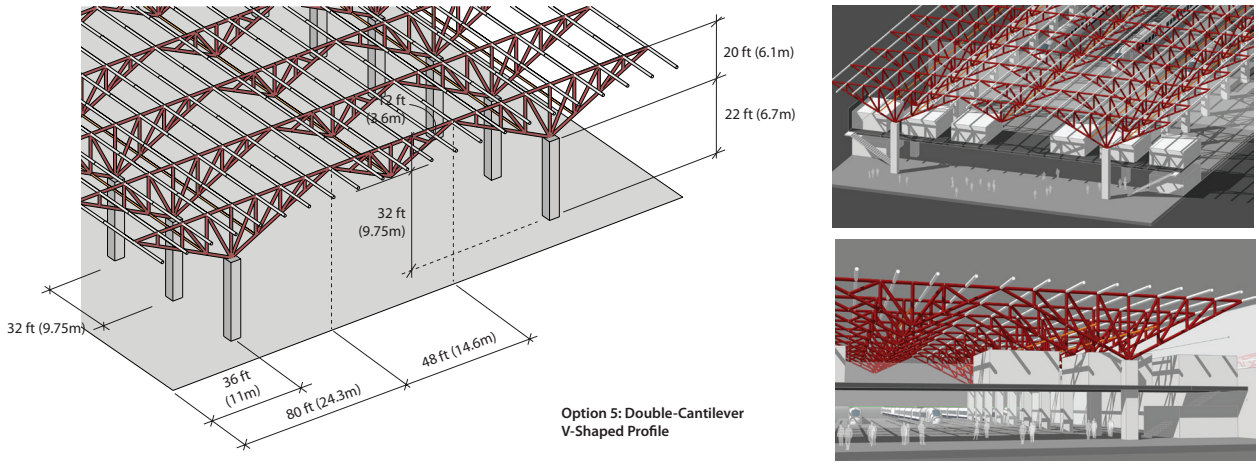


Figure 4.1.31 Option #5, double-cantilever V-shaped truss scheme

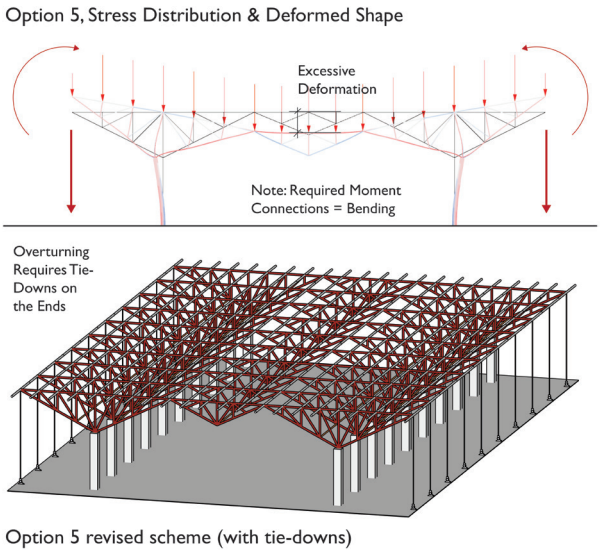


Figure 4.1.32 Option #5, double-cantilever evaluation and revised configuration with tie-downs

There are no significant savings in material, building volume, or enclosure area, but this scheme has *significantly* reduced the amount of deflection due to the reduced span of the double-cantilever. If we had a very strict design standard of deflection (L/480, L/960), this might be our best option. Overall it also performs well structurally, it provides a unique expression of structural logic, and offers a heightened experiential impact. (Figure 4.1.34)

As with other schemes, we can change the panel trusses to more volumetric or prismatic elements. The double-cantilevered lenticular and prismatic truss schemes combine strategies of structural efficiencies. If we made the lenticular truss three-dimensional it would be more expensive to fabricate and erect but would not provide significant quantitative savings, except for rigidity against deflection. Not all evaluations are quantitative; from a qualitative point of

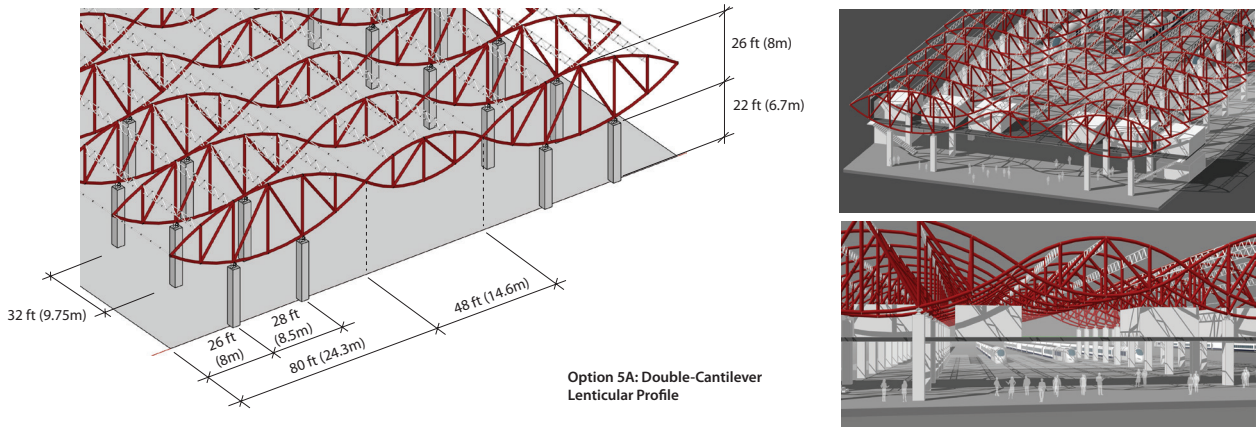


Figure 4.1.33 Option #5A, double-cantilever with lenticular profile

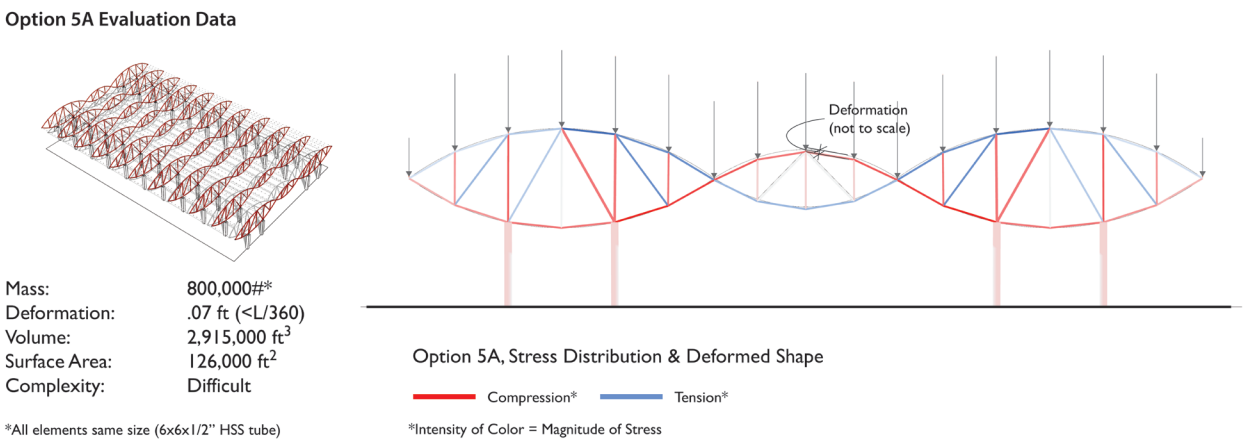


Figure 4.1.34 Option #5A, double-cantilever evaluation

view, this option creates a stunning interior expression of structural dexterity. (Figure 4.1.35)

A similar scheme was used at the San Francisco International Airport (2000) by SOM (architecture and engineering). Their design placed skylights above the middle trusses to guide visitors into and through the building—while exposing one grid line of the structure on the building exterior. The trusses are lens-like in plan and elevation, which gives them a rich formal quality and a highly efficient design (it spans 380ft (115.8m) between columns with 160ft (48.7m) overhangs on each end). (Figure 4.1.36)

Station Roof Option #6: Two-Way Umbrella Supports System

If we wanted to explore the most efficient possibilities for a truss design, we'd also want to test a two-way framing system. One possibility would be to convert the double-cantilever V-shaped truss of Option

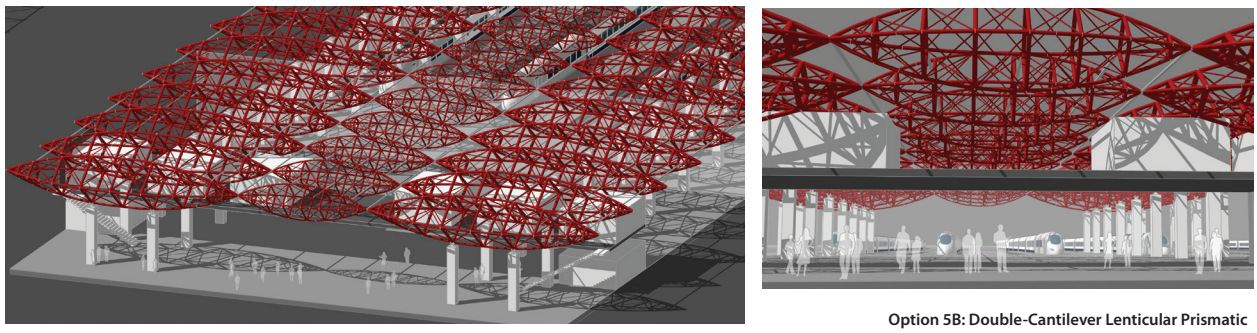


Figure 4.1.35 Option #5B, Double-cantilever lenticular prismatic scheme

Figure 4.1.36 Interior view of San Francisco International Airport terminal (SOM, 2000)

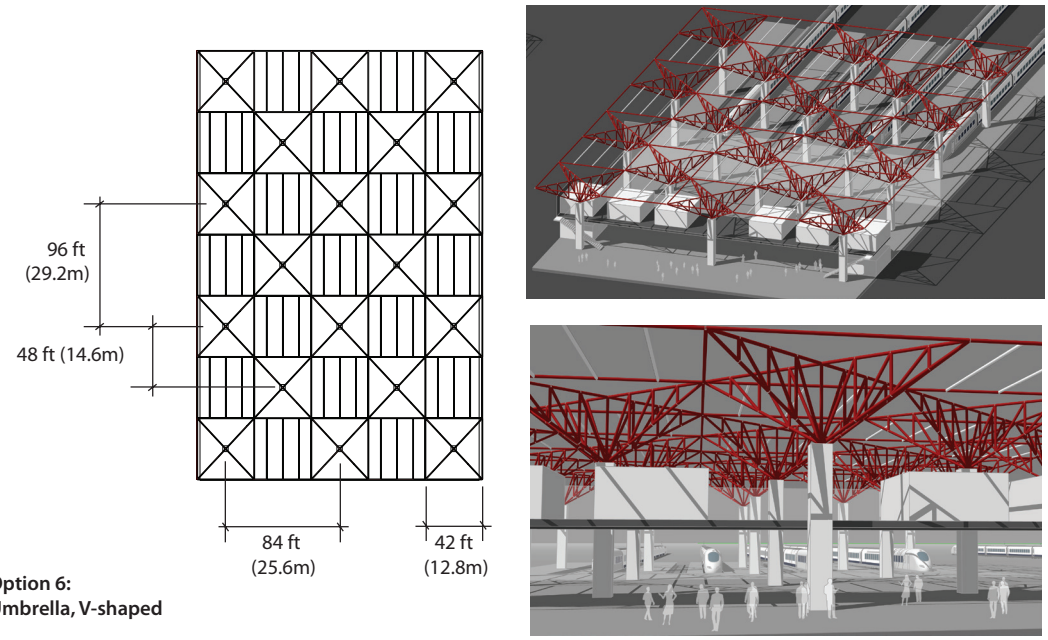
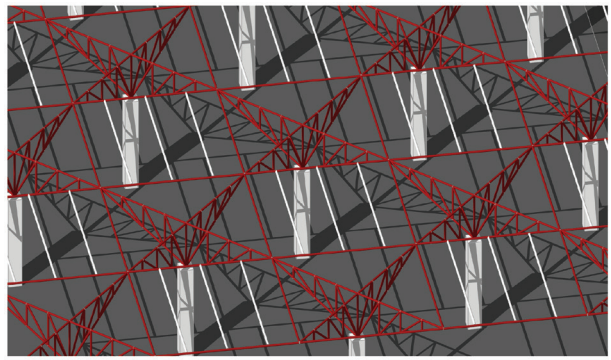


Figure 4.1.37a Option #6, two-way umbrella

#5 into a more spatial framing strategy, like a series of umbrellas. The logic is sound: if it can effectively span in one direction, why not add another double-cantilever truss perpendicularly and span two-ways? If trusses can be made more efficient by thinking about better beams and arches, then why not mimic the form of efficient columns too?

The umbrella structure can be arrayed in a slightly staggered configuration and still provide the same support (Option #6). Of course, this moves the columns from the outer edges into the space, but as long as we can accommodate the functional constraints of the train station, this shouldn't be a problem. We could bring light in at regular intervals between the columns, and we already know the double-cantilever system has minimal deflections. We've seen similar schemes in the previous chapters, including the addition to the Madrid Atocha Train Station (Moneo, 1993). (Figure 4.1.37)



Mass:	850,000#*
Deformation:	.3 (<L/360)
Volume:	2,975,000 ft^3
Surface Area:	114,000 ft^2
Complexity:	Moderate

*Variously sized elements to achieve L/360

Figure 4.1.37b Evaluation data for Option #6, umbrella scheme

Station Roof Option #7: Two-Way Space Frame

Our final option for a two-way space frame is analogous to an efficient slab (Option #7). Like slabs, it has distinct advantages (long spans with minimal depth) but unique complications.

Because space frames are two-way systems, they function best in a square bay (1:1–1:1.25 ratio), so the first step would be to find locations for columns in the middle of our span that don't disrupt the program of the train platforms. Once we have established

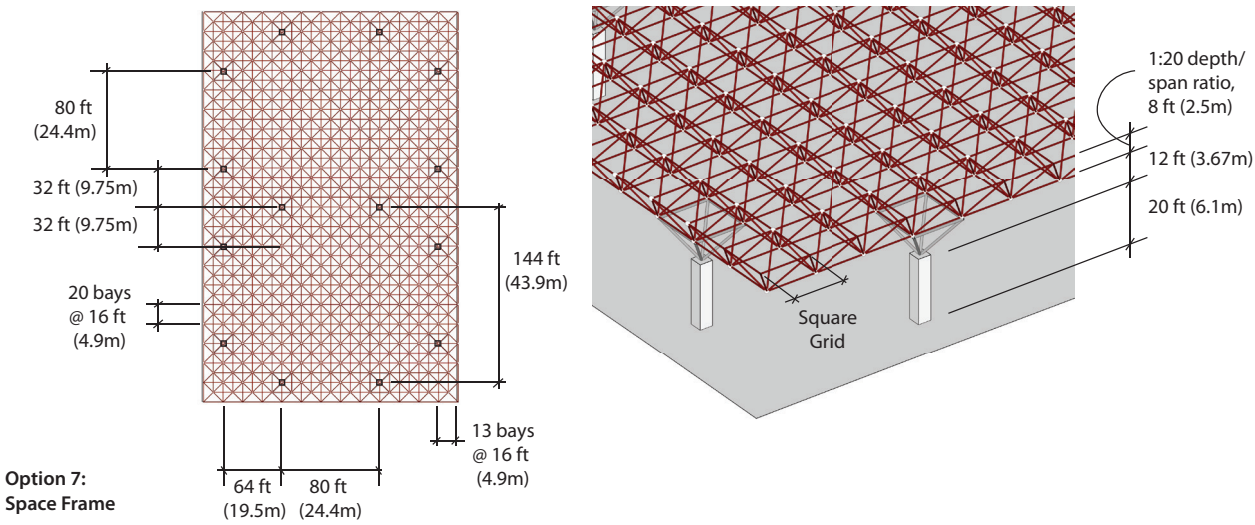
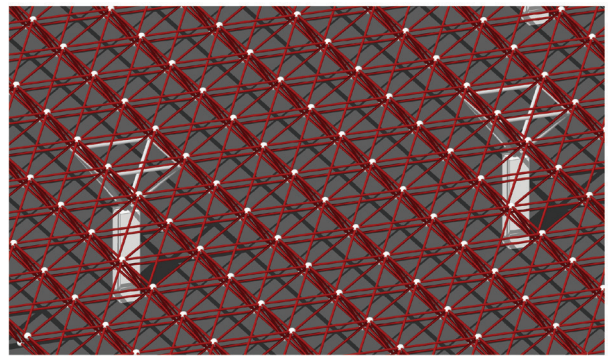


Figure 4.1.38a Option #7, two-way space frame truss framing plan and axon

nearly-square bays (48ft × 42ft (14.6m × 12.8m)), the next step is to determine the depth between the top and bottom layers of the grid and the module size. These are mutually dependent factors. Module proportions are determined by the geometry of the top and bottom grids and the diagonals that connect them—typical steel space frames have a depth-to-span ratio between 1:15–1:25. Standard modules make for more economical construction and fabrication. (Figure 4.1.38)

Space frames are classified by the plan geometry of the top and bottom grids (e.g., two-way, diagonal, three- and four-way grid). These grids are interlocked by diagonals, so the space frame is also defined by the geometry created by the diagonals (e.g., half octahedrons (pyramids) or tetrahedrons). The diagonals within the modules should be angled between 30 and 60 degrees. For spans more than 300ft clear (91.4m) space frames are constructed as triple-layers in order to avoid buckling in the diagonals. Diagonal square pyramids with off-set square grids generally use the least steel. If we locate columns along two sides and assume a depth-to-span ratio of 5%, we would have modules 10ft (3m) deep. To find the size of the module in plan, we'd compare this depth with an acceptable angle for the diagonals—let's assume 60 degrees—and look for a dimension that would be evenly spaced across the plan in two directions; 16ft (4.9m) × 16ft (4.9m) seems to work for both factors. (Figure 4.1.39)

Because the load is transmitted in two directions the columns should distributed in two directions as well. The number of columns determines the length of span and thus the depth of the truss. If the columns



Mass:	810,000#*
Deformation:	.56 (L/360)
Volume:	2,975,000 ft^3
Surface Area:	114,000 ft^2
Complexity:	Difficult

*Uniformly sized elements, (3x3x1/2 HSS tubes)

Figure 4.1.38b Evaluation data for Option #7, space frame evaluative data estimates

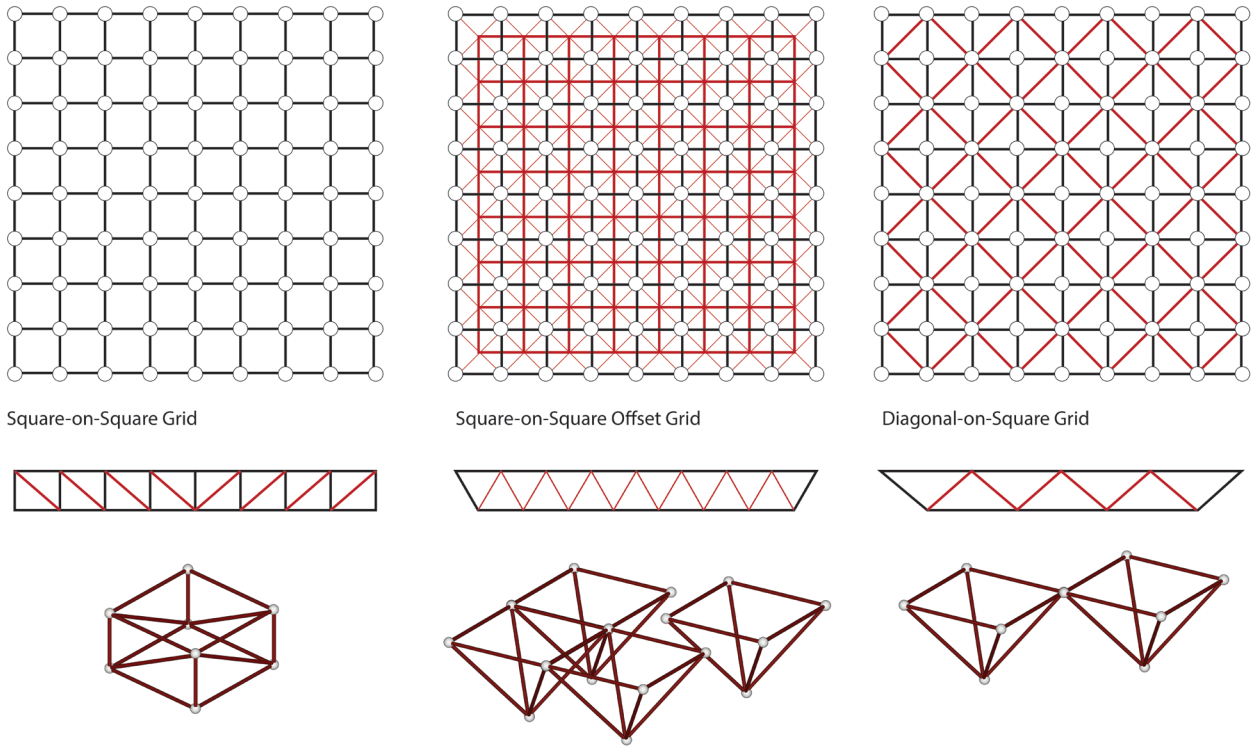


Figure 4.1.39 Three possibilities for double-layer space frame grids and modules

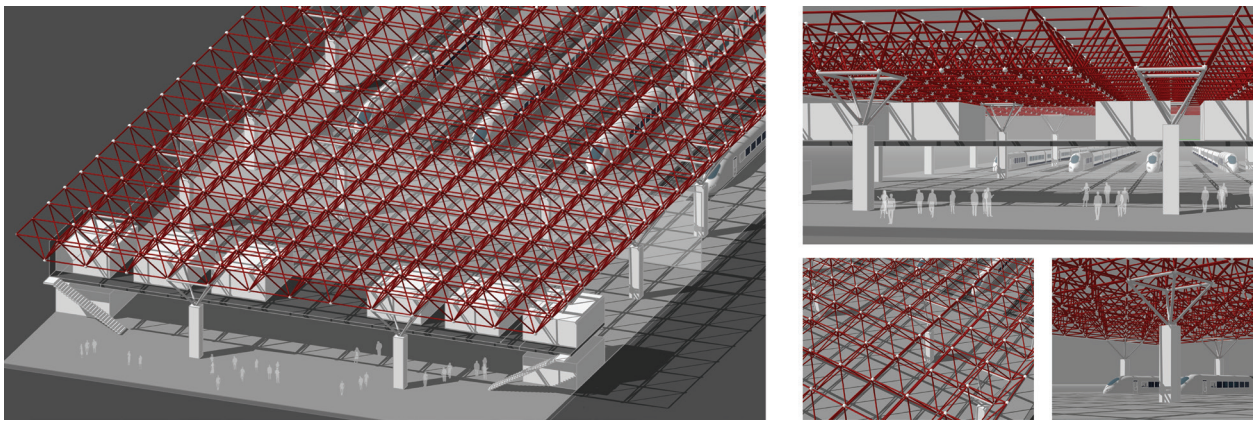


Figure 4.1.40 Option #7, two-way space frame axon and interior views

are spread out evenly across the span, the minimum and maximum stresses will even out, which will allow a more uniform size in the modules. A column in a two-way system collects loads like an umbrella centered in a square grid of trusses above. Like all load-collecting elements, forces consolidate at the columns. Accumulated loads being transferred through a single point support into the columns creates a large concentration of forces. As a consequence, modules around a column need to have larger components and we'd be concerned about a punching shear failure. To avoid this, column connections with a space frame need to have larger bearing perimeters that attach to multiple points across the trusses. This reduces spans and minimizes the differences between localized stresses. (Figure 4.1.40)

BREAKING: FOLLOWING FAILURES – HARTFORD CIVIC CENTER COLISEUM

On January 18, 1978, just hours after hosting thousands of people within the arena, the space frame roof structure over the Hartford Civic Center collapsed. The largest snow storm in five years had loaded the roof asymmetrically and caused it to deflect so much that a portion of the roof collapsed—creating a progressive failure that eventually brought down the entire structure. Roofs aren't supposed to fail because of snow loads, but because the space frame details were built differently than intended (observations that were noted as early as 1972), the space frame module failed.

To Do: Research this project to find out why it failed.

To Discuss: Did design flaws cause the failure, construction flaws, or both? How was the pyramid intended to be built? How was it actually built and why did this matter? How could it have been prevented?

Additional Options: Curved and Free-Formed Surfaces

Considering trusses and columns not just as a series of triangles but as a planar mesh of geometric volumes unlocks another evolutionary way of thinking about the design. If we change the form of the individual pieces, we can curve the overall form in one or two directions. This introduces new spatial and aesthetic options and could improve the efficiency of a system that is already efficient. A curved surface is stiffer and more effective in transmitting loads and resisting deflection. Curved profiles can be extruded into a vault, revolved around an axis to

Figure 4.1.41 Construction framing of double-curved surface using space frame trusses at Heydar Aliyev Cultural Centre (Zaha Hadid Architects, Baku, 2012)

make a dome, or expanded into other less-regular "free-formed" surfaces.

If we had enough depth and computational power to document and analyze them, nearly *any* forms made from curved space frames would be possible. Digital tools allow designers to define spaces based on complex geometries of doubly-curved surfaces—not all of which have been based in structural logic. To document and construct these projects, digital tools from other industries (aerospace, shipbuilding, etc.) have been integrated into engineering and fabrication processes. These changes, along with digital optimization tools, led to building projects that challenged the planar nature of walls and roofs.

Of course, the term "free" is misleading; gravity and load transmission are inescapable. The term indicates the possibility of not following conventional forms informed by load transmission alone. These new forms have consequences. When a space frame is integrated into an appropriately-designed roof system it resists loads efficiently. When this surface is combined with other elements (like the façade) or takes a form for the roof that isn't structurally driven, the structure's elements will all have to carry more load and different types of bending stresses. In these forms, the components' triangulation isn't useful in the way it is in a truss—it is more of a formal convenience to create smaller modules to form curved surfaces from straight-lined members.

We can suggest and analyze almost any form with digital modeling, but this doesn't mean we necessarily should. Being space framed and double-curved forms doesn't make them efficient—it only makes them expressive. These forms are possible, but even with advanced digital fabrication techniques, they are still complicated, expensive, and may be based on specious reasoning. (Figure 4.1.41)

Constructing Long Span Trusses

For every proposal we've explored, we also need to consider the consequences of construction. Because these

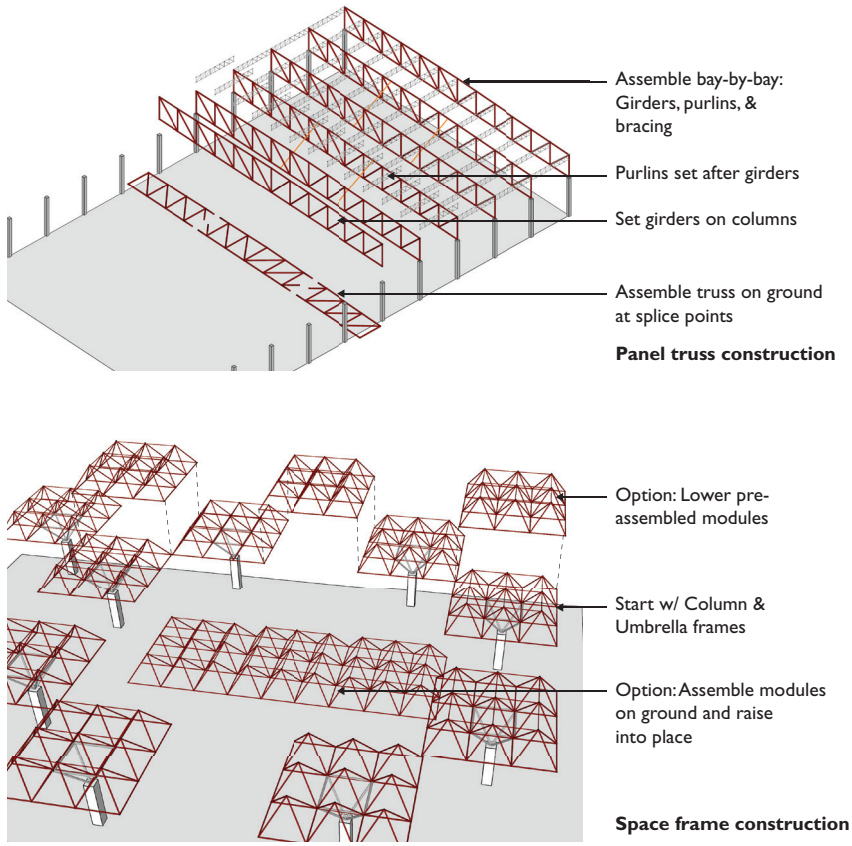


Figure 4.1.42 Diagram of construction sequencing for truss panel or space frame modules

are long spanning structures the load-bearing elements will have to be spliced together from other, smaller parts. There are limits to how big an item can be fabricated, shipped, and lifted into place. Remember, construction costs for steel structures are divided equally between the cost of materials, fabrication, and erection, so economic gains in one regard must be compared with others. More parts generally means more construction cost. We can reduce the number of parts by using larger sized modules or less frequent splices.

One-way trusses are generally erected like beams (e.g., foundation, column, girder, purlins, bracing, and finally decking). They can be constructed in a bay-by-bay order along their lengths. For prismatic and space frame trusses, the challenge is finding a composition of cells/modules that works structurally and allows them to be prefabricated into larger modular elements that are easily transported and erected. Space frames can be erected in three ways: by assembling modules in the air (atop scaffolding), by first installing strips or blocks (between or around columns) and infilling from there, or by assembling the frame on the ground and hoisting it into place with derrick masts, hydraulic jacks, or cranes. (Figure 4.1.42)

Consequences and Conclusions

These are all wide-ranging solutions with divergent ideas about structure and space. We can't comprehensively assess them against one another at a schematic

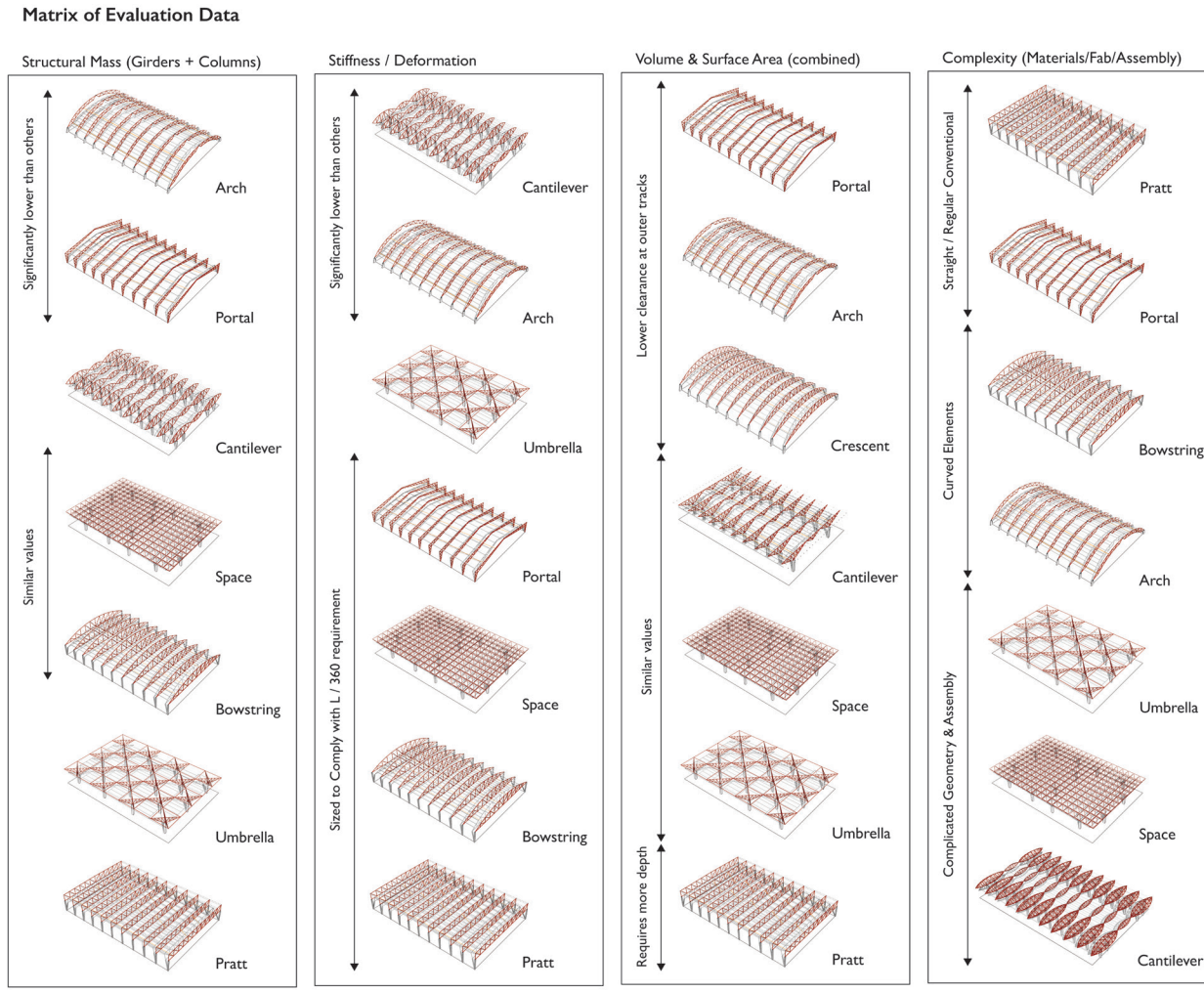


Figure 4.1.43 Comparison of all truss options based on evaluative data

level of development. But we can look at them all in terms of mass, displacement, volume, surface area, and complexity. Each factor has a structural, environmental, and/or economic impact. Consider this: Conventional building enclosures cost $75–150/sf, so a reduction of $10,000/sf becomes a significant one-time economic and environmental saving, with lifetime energy savings associated with less air volume to heat/cool. Many times, a valid case can be made for a more complex architectural form than a simple orthogonal box.

We can use our data points to compare the pros and cons of all of our schemes. Our data confirms several core structural principles (Figure 4.1.43):

- *Conclusion #1*: By manipulating the form to better match with the transmission of forces (arches, portal frames) we can reduce the amount of material needed and stiffen the structure against excessive deflection.
- *Conclusion #2*: Roof curvature reduces air volume and surface area for building enclosure—both of which are significant factors in cost and operation.
- *Conclusion #3*: The most traditional/ conventional flat panel trusses aren't very efficient when comparing their material and structural behavior. This inefficiency makes deeper members, which adds volume and surface area—a double penalty. But the relative ease by which they are fabricated and erected make them viable options.
- *Conclusion #4*: Non-conventional schemes (double-cantilevers, umbrellas, etc.) have structural, aesthetic and experiential advantages; they require a well-coordinated fabrication and assembly processes to remain viable.

By converting panel trusses to prisms, new benefits and consequences also emerged. We changed the supports' spacing to stabilize the truss, and created a more complex spatial expression of support, integrating skylights with prismatic trusses. Of course, without supporting data on fabrication costs and the difficulty of construction of the curved or prismatic trusses we can't make a complete assessment.

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PART 5

Resistance and Surfaces

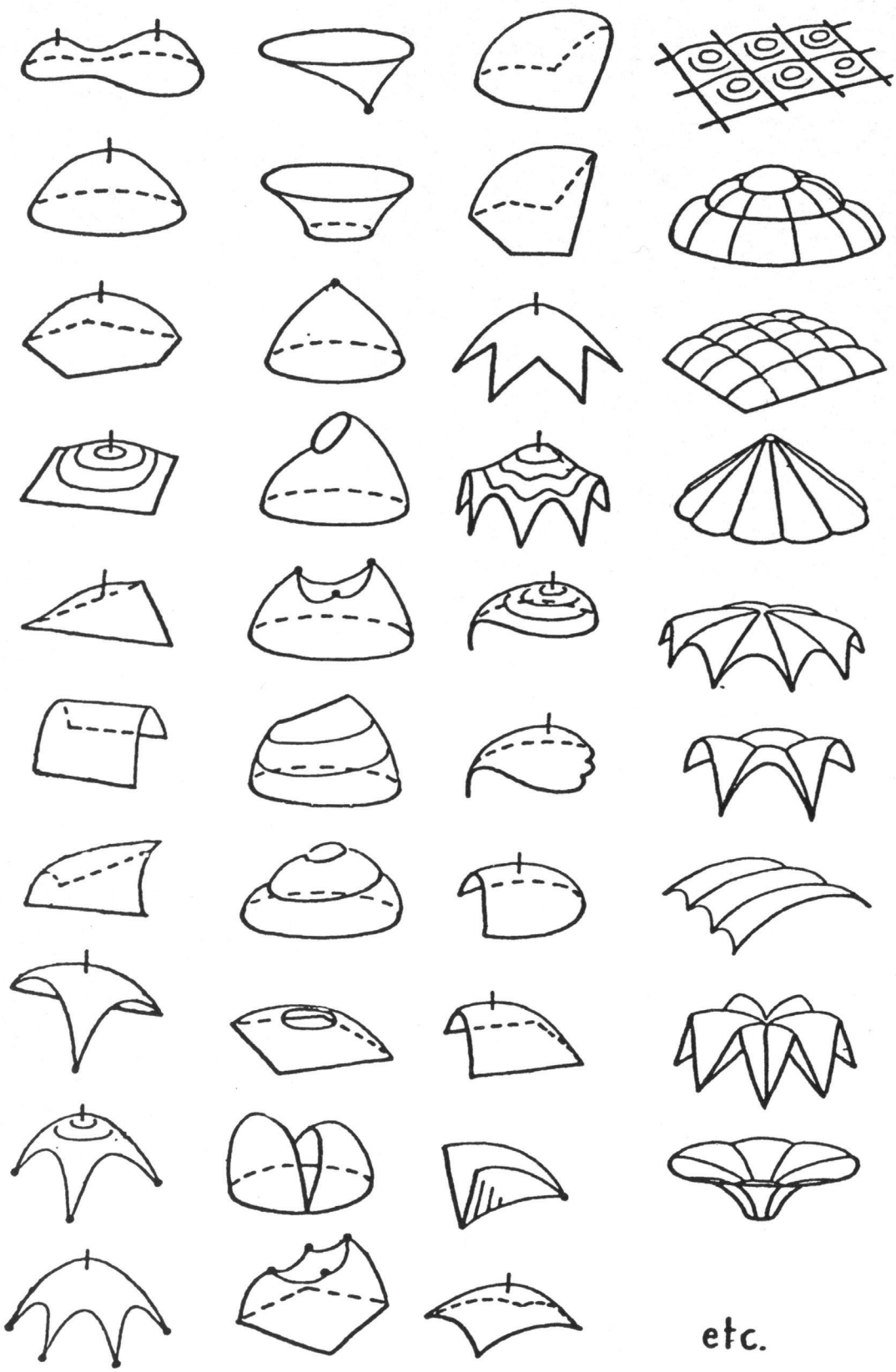


Figure 5.0.0 Heinz Isler's + 39 sketch, 1959

CHAPTER 5.0

SURFACES

Structural Art, Utility, and Assemblies of Shells

Learning how to create structures that are simultaneously effective, efficient, and expressive is this book's central purpose. Well-designed surface-resistant shells are the paragon of this endeavor. Shells unite structural and architectural considerations by creating surfaces that define spaces and provide support—with an unmatched material efficiency.

Shell Fundamentals

Structural shells, or *surface-resistant structures*, are three-dimensional load-bearing surfaces made from thin and stiff materials designed to resist external loads through their membrane. Because they are efficient, they can enclose large amounts of space using minimal material and few supports. Unlike other systems that require additional elements to enclose a volume, shells provide resistance and stability through one element—their surface.

Curving or folding surfaces is an effective functional, structural, and economical strategy found in everyday objects (e.g., molded plastic furniture, lampshades), industrial applications (e.g., automobiles, construction equipment), and other structural components (e.g., corrugated metal decking). Because there are profound limits in translating these forms into buildings, shells are less pervasive in architecture than product design. But when efficient strategies combine form, material, and construction methods together, shell structures create remarkable buildings. (Figure 5.0.1)

In 1956, Felix Candela contemplated the question of why shells existed as structural forms when other alternatives might suffice. "Why should we trouble to look for new forms or worry about design when it is so much easier to demand just a little more resistance of a certain material?" He argued that shell design was an intellectual and creative evolution of design that could combat the "tendency towards mental slothfulness." Shells are an important, yet often enigmatic, type of structure.

In this chapter, we'll look at different ways that shells can be defined: Regular geometry, curved surfaces generated from ruled surfaces, mathematical models of double-curvature, physical models of hanging fabric, or even digital modeling techniques. We'll explore how each method evolved and how the relationship between structural form and construction shaped the ingenuity, economy, and evolution of shell structures.

Figure 5.0.1 One of the earliest experimental shell forms, the Zarzuela Hippodrome by Eduardo Torroja (Madrid, 1935)

Membrane Action Theory

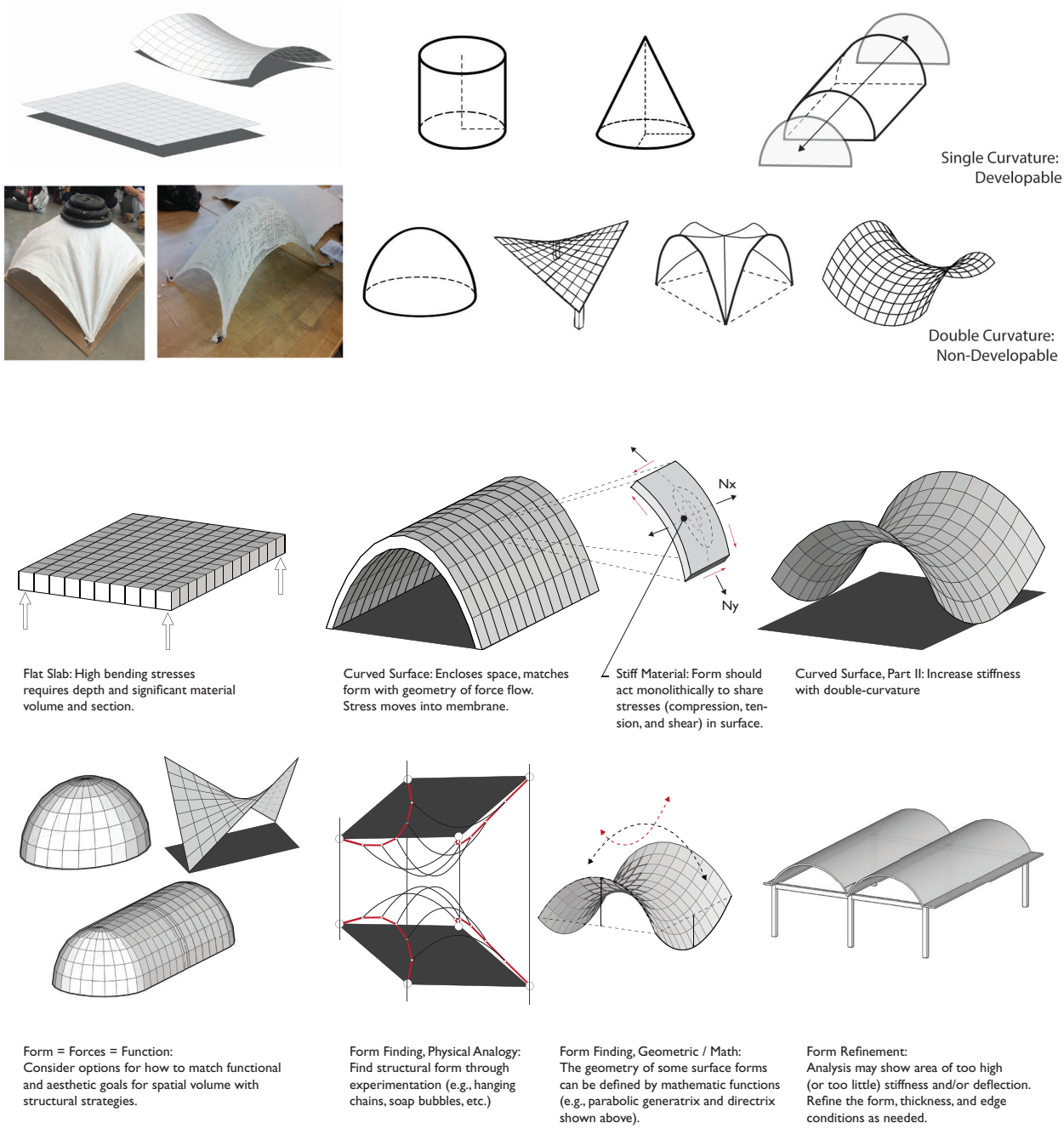
Every structural system we've looked at up to this point has relied, in part, on the structural element's depth to provide resistance against applied loads. How can shells be so thin and span so far? The answer lies in shape and surface. *A single structurally-resistant shape is always more beneficial than an awkward accumulation of materials.*

Shells work like form-resistant arched systems that match their forms to the forces applied to them. But shells improve upon this resistance by using their curved, stiff surface to attract, resist, and transmit the loads all in the plane of the membrane. Our primary task is to find forms that resist and transmit loads effectively. As long as the surface material is stiff (like a tortoise shell, not a rubber ball) it will attract the vertical and horizontal forces into the shell's surface. Stiffness also helps to resist against concentrated point loads (e.g., pushing into a rubber ball deforms it so it's less effective as a shell). (Figure 5.0.2)

This creates "membrane action" by which the structural surface contains all the three types of forces (vertical "meridinal" forces, horizontal "hoop" forces, and shear stress). We don't want to introduce bending forces into the membrane; the curvature, not the thickness, keeps it from bending. Concrete is the traditional material of choice for shells because it can be cast into different forms, it's inherently stiff, and it behaves monolithically as a compressed bearing surface.

Simply put: *Form follows forces. Forces follow stiffness. Curvature creates stiffness.*

Shells must be stiff. Look at shells in everyday objects and you'll find different curvatures—curves with a broad radius in the body and tight folds that bind the edges. We must develop strategies



Figures 5.0.2a and 5.0.2b Shells get strength from curvature and stiffness of membrane. Common shell form classification and their structural principles of behavior

that help these thin structures maintain their stiffness. If the curved form isn't fixed geometrically, the shell won't work—it will buckle, spread out, etc.

Shell shapes must be restrained. For a shell to hold its shape firmly, its boundaries/edges often have to be folded or double-curved to increase its resistance to deformation. As long as the shape is restrained in

a form-resistant configuration, shells can resist and transmit loads long distances. Discontinuities in shell surfaces or openings can disrupt the form's rigidity. (Figure 5.0.3)

Ultimately, shells are efficient spatial enclosures with thinner surface thickness-to-span ratios than an egg, which requires a high level of technical acumen. Shell's primary loads are their own self-weight, so reducing this weight by creating efficient load-resisting forms is essential. It may seem obvious, but *thin shells should be thin*.

Basics of Shell Forms and Tools for Design/Analysis

Although shell forms are ubiquitous, designing shells isn't very intuitive for most architects, perhaps because they seem difficult to design. Shell structures provide support, enclosure, and interior volume for building functions all within the same system, so they require the most comprehensive range of integrated "architectural" considerations. Of course, these considerations can't be made independently of structural behavior.

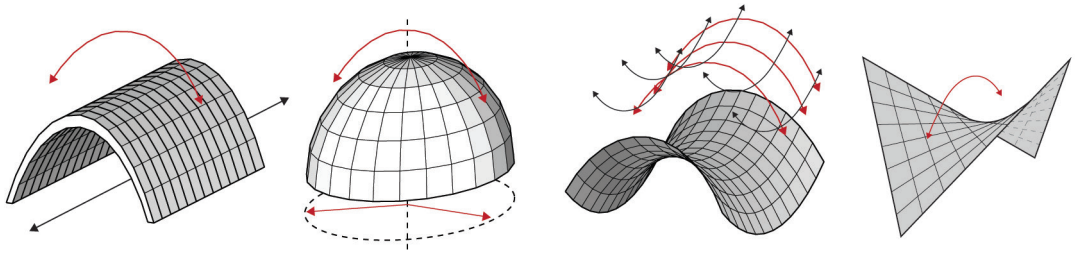
Many types of shell forms will work structurally; some are defined by mathematical functions, natural principles, or other physical analogies. Shell surfaces can be single-curved or double-curved, based on lines that are translated along an axis (vault), rotated around an axis (dome), or combined into a saddle shape with two different, perpendicularly curved lines (known as director curves and translating curves). Double-curved surfaces can also be derived from straight lines (called ruled surfaces) when the perpendicular director and generating lines are both tilted along axis points (e.g., hypars, umbrellas, etc.). Since their curvature determines their performance, shells shouldn't become too flat. In the most basic terms, the folded or curved surface is able to carry a load across a span better than an un-curved surface because the curvature makes it taller and stiffer. (Figure 5.0.4)

Figure 5.0.3 Eames Molded Plastic Armchair for Herman Miller (1950)

Having design options for shells shouldn't be confused with having design freedom. Designers that haven't understood the relationships between function, expression, and economy of shells have struggled. There are five common mistakes shell designers make: improper form and curvature, lack of stiffness in the membrane material, excess material (too heavy), discontinuities in the surface (unresolved edges and support conditions), and the complications of construction, including technical and economic consequences.

Matching form to forces with a curved and stiff surface material are shells' mandatory conditions. The dead load weight of the shells is their primary design load, so reducing this weight by making the material thinner follows as a condition of proper form and material selection. Because shells rely upon continuity of load transmittance within their surfaces, they are sensitive to discontinuities (like openings) in their surfaces or differences in stiffness at their edges or boundaries. Loads are attracted to the stiffest elements so any differences in stiffness will attract loads to areas where they may not be expected. There are always exceptions to these five common problems—with enough resources nearly any shell could be built. The interaction between architects and engineers to evaluate these choices is a critical part of the design development. (Figure 5.0.5)

Architects and engineers look at shell design differently. Engineers analyze shells by looking for how the form corresponds with anticipated force transfer,



Developable:
Single-curvature:
Curve translated along straight line.
Spans: 65'-200' (20m-60m)
Least expensive (60% max)

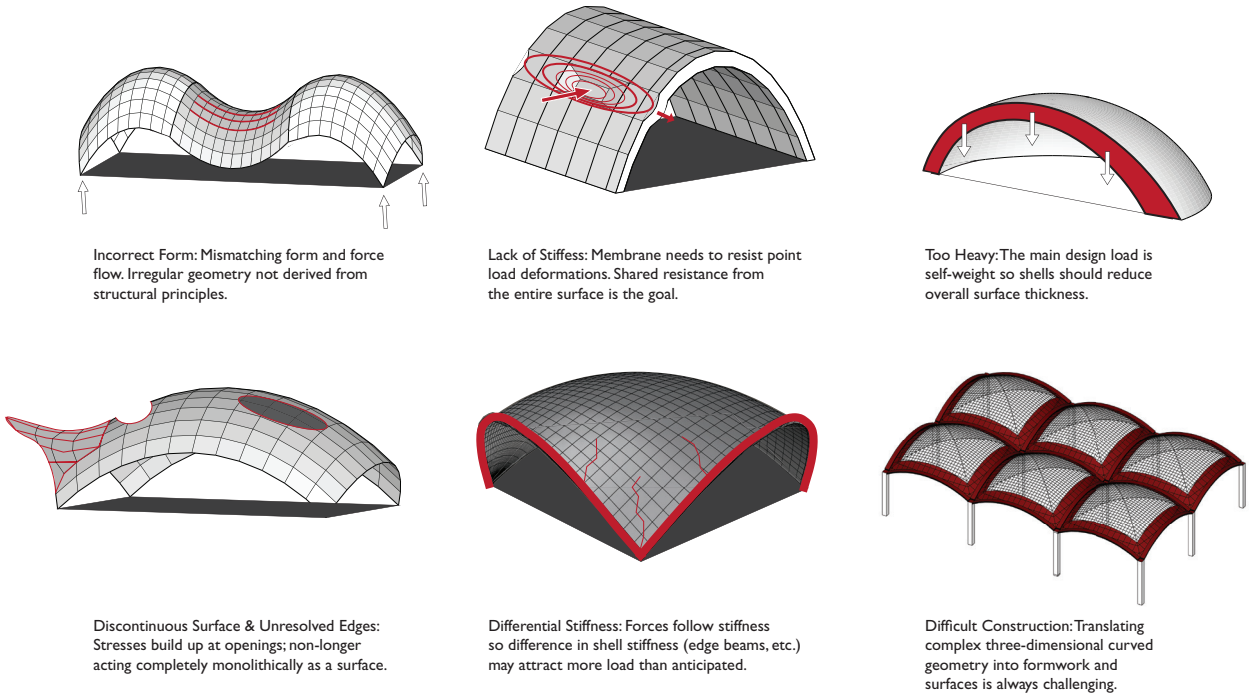
Synclastic:
Double-curvature:
Curve rotated around central axis.
Spans: 135'-500' (40m-150m)
Expensive (90% max)

Anticlastic Curve:
Double-curvature:
Curve translated along a directing curved line.
Spans: 65'-225' (20m-70m)
Most Expensive (100%)

Hyperbolic Paraboloid:
Double-curvature:
Straight lines translated and rotated.
Spans: 65'-200' (20m-60m)
Less Expensive (70% max)

Figure 5.0.4 Shells are grouped by their formal properties because their behavior is based primarily on form

and then determine if the membrane surface meets ultimate load capacity and stiffness requirements. Engineers use two analytical techniques. The *finite element analysis* (FEA) method (and/or digital simulations of this method) determines areas of concentrated stress, while the *displacement method* determines overall elasticity. Analysis work depends on engineering principles found in arches, beams, columns, and basic material mechanics. It involves complicated mathematics that are beyond this book's scope and aren't included in most structural engineering curricula. Because form and construction of the shell are



Incorrect Form: Mismatching form and force flow. Irregular geometry not derived from structural principles.

Lack of Stiffess: Membrane needs to resist point load deformations. Shared resistance from the entire surface is the goal.

Too Heavy: The main design load is self-weight so shells should reduce overall surface thickness.

Discontinuous Surface & Unresolved Edges: Stresses build up at openings; non-longer acting completely monolithically as a surface.

Differential Stiffness: Forces follow stiffness so difference in shell stiffness (edge beams, etc.) may attract more load than anticipated.

Difficult Construction: Translating complex three-dimensional curved geometry into formwork and surfaces is always challenging.

Figure 5.0.5 Five common failures of shells

the most important factors, experienced designers need very few calculations.

Learning about Shells

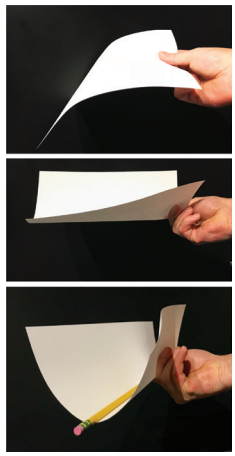
Shells are difficult to envision, document, and construct because they are complex curved three-dimensional forms. How would you document the proportions of an egg in drawings? Or build it accurately? We will need to use tools to create, develop, and assess their performance including: drawings, physical models, and digital simulations. In the following chapter, we'll develop schemes in correspondence with established rules of thumb for span, height, thickness, and geometric proportion based on nearly a century of rigorous data.

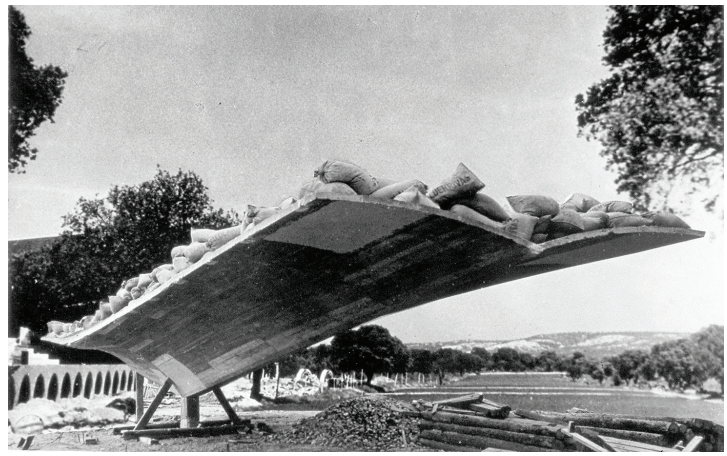
LEARNING BY MAKING

The most decisive influence on a shell's effectiveness is the combination of material behavior and the form of the curved surfaces. Manipulating a material by bending or folding it offers structural advantages. Forces follow stiffness and curved (or folded) surfaces provide stiffness *and* a means to transmit the forces.

Try it. Modify a piece of paper to carry the weight of a pencil. Now manipulate it again so it can carry the weight of this book across a small span. When the paper's surface is curved in your hand or folded into a fan-like cross-section, it becomes capable of carrying loads hundreds of times larger than the weight of the paper itself! Different forms allow the same material to hold different weights. Notice the importance of restraining the shape under loading so it doesn't spread out. Throughout the chapter, examples will be shown of different physical prototypes created to better understand the structural behavior and behavior of different options.

To Discuss: Why is it advantageous to add stiffeners or diaphragms on the sides and ends? (Figure 5.0.6)





Figures 5.0.6a and 5.0.6b Although the form of a folded plane is intuitively strong, the calculations of force flow are complicated. Torroja testing a full-scale model for the Zarzuela Hippodrome (Madrid, 1935)

Shells have always served as a microcosm of larger issues in design and construction. We'll look at issues of utility, aesthetics/expression, construction history, economics, and the tools used to analyze the shells to see how these all have influenced the development of shells and contemporary advancements.

Early Shells: Domes, Vaults, and the Arch and Beam Analogies

Domes and vaults have been used for centuries to span long distances and enclose large spaces. They're generated from a familiar element: the compressive arch. A dome is an arch that is revolved around a central axis. Vaults are arches that have been extruded (or "translated") along an axis. Because a regularly curved line defined these forms' geometry, they could be documented and constructed using simple tools and methods.

Domes are the oldest, most well-established structural forms. They enclose the most volume with the least surface. The dome's double-curved (or *synclastic*) surface creates a predictable and repeated geometry. Dome profiles can be spherical, parabolic, or elliptical. Most of the stresses in the dome's upper surface are compressive, while those in the lower portion are where it tends to bulge out in tension. Because domes primarily act in compression, like arches, their proportions follow many of the same rules of thumb as arches. (Figure 5.0.7)

Domes can be solid, but voids can be cast into them to save weight. This strategy was used in two of the world's largest and most enduring shells: Pantheon and Hagia Sophia. As long as the surface remains stiff and the consolidated stresses around the openings are able to be resisted, some discontinuity in the surface is acceptable. Creating openings or changes in the dome's thickness can save significant weight. The dome surface can also be articulated by gridded ribs, a Whipple configuration of lattices, or even geodesic patterns.

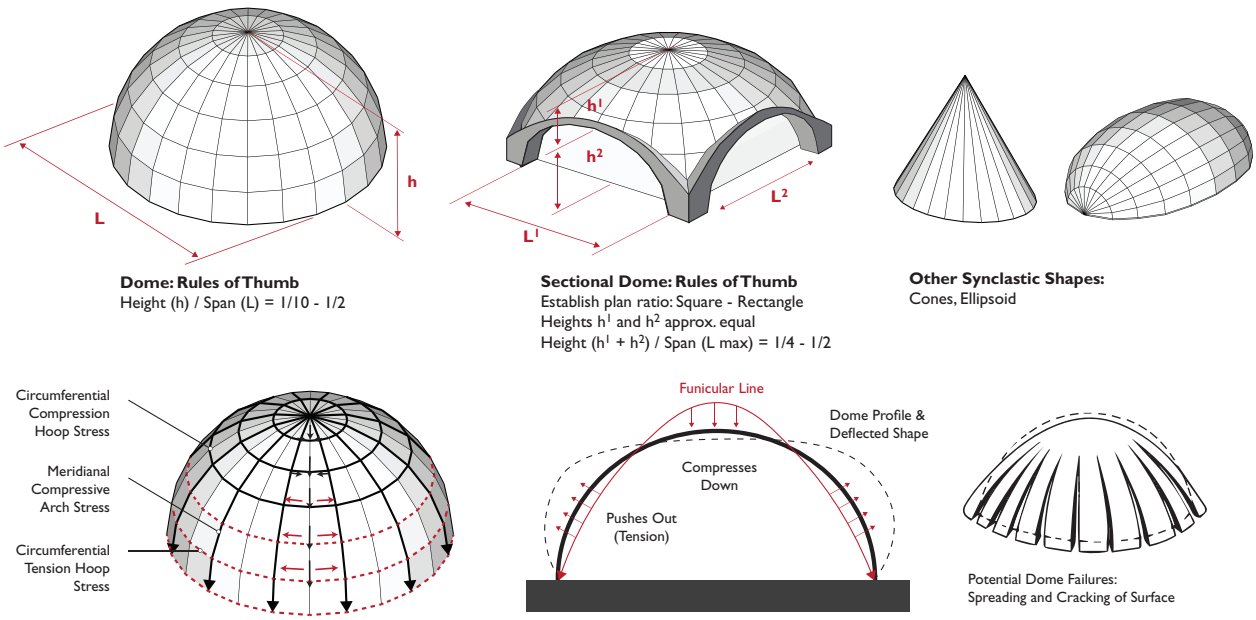


Figure 5.0.7 Dome guidelines, stress, and other synclastic forms

THINKING: FORM FOLLOWS FORCES

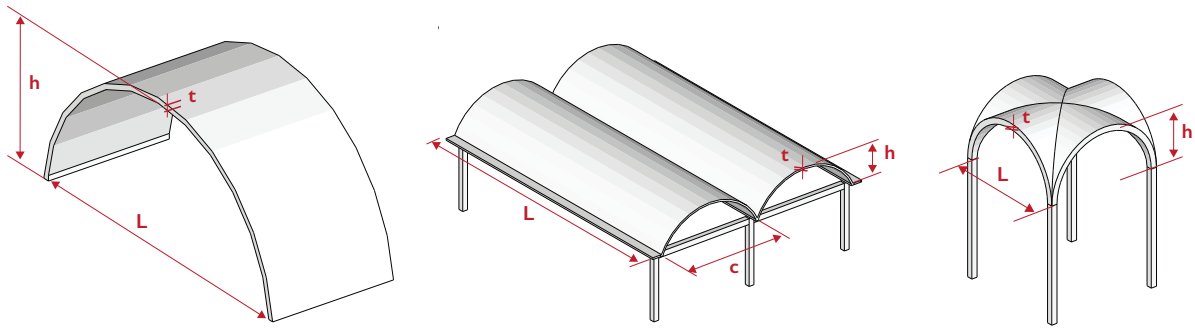
For nearly 2,000 years, the Pantheon rotunda in Rome (AD 126) has served as a reminder about the enduring importance of matching form, material, and articulation. The height of its dome and the diameter of the interior plan were designed to allow a 142ft (43.3m or 150 Roman feet) sphere to fit inside—making it the largest free-spanning dome in the world for 1,300 years (it's still the largest unreinforced dome). Although the rotunda's interior is spherical, its section varies in thickness, like a compressive arch, with a thickened base and a narrowed apex (including the oculus at the top). The rotunda's gridded waffle pattern is a reflection of the two stresses domes endure: vertical meridional forces (resisted by the arches), and circumferential hoop stress that runs latitudinally. The Pantheon was constructed from what was then a highly-advanced mixture of concrete-like material (placed in horizontal layers and tuned to anticipated stresses).

To Discuss: Look at the building section and discuss how the materials are distributed in section (including the oculus). (Figure 5.0.8)

Figure 5.0.8 Interior view of Pantheon roof. The gridded articulation is a reflection of arched and hooped internal stresses found in a dome

Vaults are surfaces of translation generated by extruding an arch with a consistently curved profile line along an axis. For centuries, barrel vaults and their variants (groin vaults, ribbed vaults, two-centered vaults, etc.) provided structurally predictable behavior with easily defined and constructed geometries. Vaults look like arches but they don't completely behave like arches—it depends on the orientation of their curve in relation to their supports. Short vaults, with curves parallel to their spans, act more like arches while long vaults, with curves perpendicular to their spans, behave more like curved beams. Like all surface-resistant structures, vaults need to have their curved forms restrained on the sides and ends to ensure proper load transfer. (Figure 5.0.9)

If curved surfaces are too complicated to design or construct, similar advantages can be gained by folding surfaces into planes that will then act as flat plate girders. Because folds stiffen surfaces, they collect loads at each folds' peak and valley. Whereas a slab transfers loads to supporting girders on its ends, a folded structure creates additional supports with each successive fold. Because the folded slab is now inclined



Short Vault: Rules of Thumb
Height (h) / Span (L) = +/- 1/3
L = 100' (30.5m) h = 35' (10.7m) t = 4" (10cm)
L = 200' (61m) h = 70' (21.4m) t = 5" (13cm)

Long Vault
Depth (c) / Span (L) = +/- 1/3 - 1/4
Height (h) / Span (L) = 1/12 - 1/8
L = 100' (30.5m) c = 25' (7.6m) h = 5' (1.5m) t = 4" (10cm)
L = 200' (61m) c = 66' (20m) h = 25' (7.6m) t = 5" (13cm)

Groin Vault: Square Plan
Height (h) / Span (L) = +/- 1/3
L = 50' (15.3m) h = 15' (4.5m)
t = 4" (10cm)

Figure 5.0.9 Three types of vaults and general proportional guidelines

vertically, it has a higher effective depth, which resists bending like a beam. The plate structure's rigidity and strength relies upon the pitch and height of the plates—if the pitch is too shallow the structure loses its effectiveness. Although plates are monolithic and joined together, they need to be restrained at their ends where loads transfer to vertical supports and on their edges, usually with horizontal extensions along their sides and face plates on the ends. We'll explore the design and evaluation of vaults and plates in the ensuing chapter, but paper and weights will provide a fast introduction to the pros and cons of folded plate structures. (Figure 5.0.10)

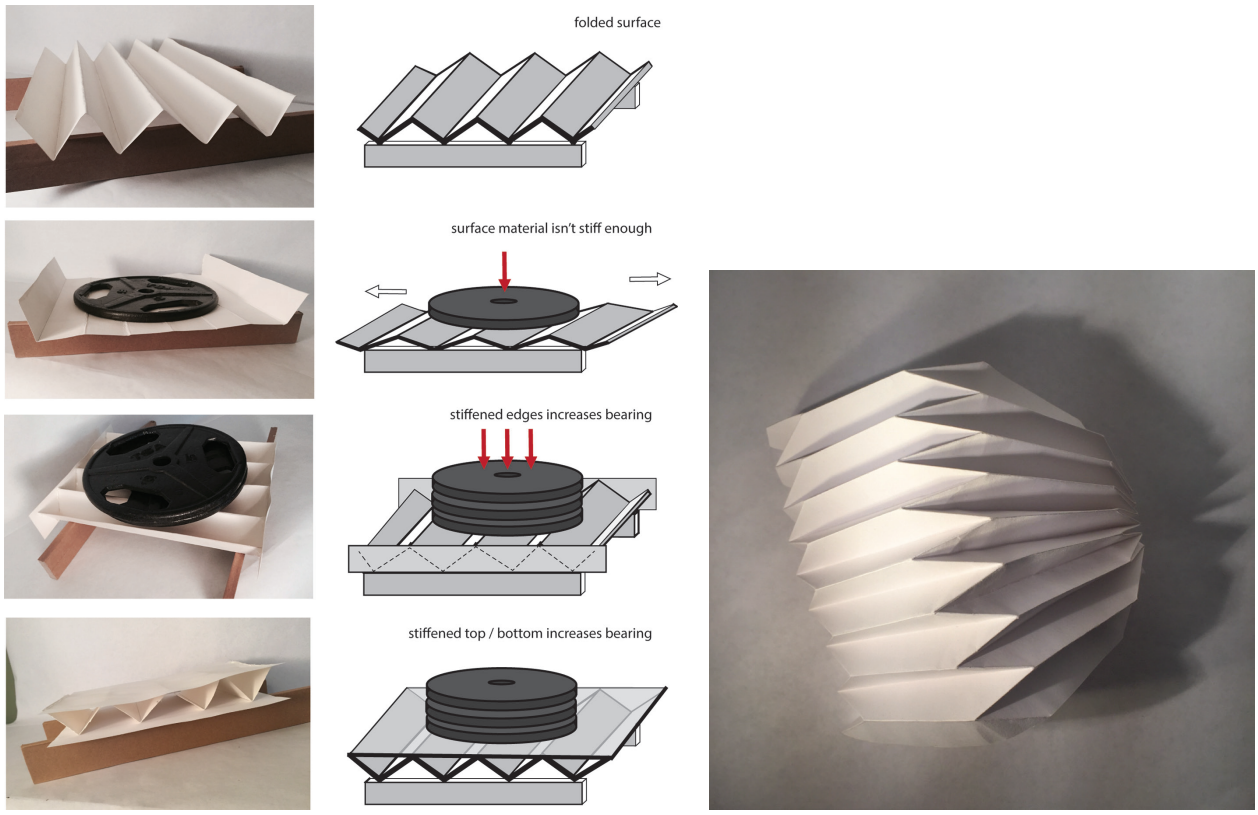
Thin Shells in the Early 20th Century: Grid Shells and Concrete

Domes and vaults were used throughout our history, in part because they behaved predictably and their geometries could be easily documented and constructed. Masonry shells were used because there wasn't any viable alternative system or material to enclose similar volumes. By the beginning of the 20th century this was no longer the case. Curved dome and vault forms could be lighter, more expressive, and easier to make using beams and trusses.

These new structures enjoyed of the benefits of stiffness and load resistance that curved forms provide, but instead of relying upon a monolithic surface to resist loads, modern grid-shells (or lattice structures) combined smaller elements made of iron and steel. Lattice domes and braced barrel vaults followed natural patterns (e.g., Whipple truss dome framing), mathematics (e.g., geodesics), or construction convenience (e.g., diamond-shaped lamella grids). These shells offered expanded formal and material options that weren't possible before. But other complications occurred as these structures became lighter.

In grid shells, the supporting material is concentrated in some areas and absent in others, so the thinness and flexibility of the single layer grid are vulnerable to asymmetrical or heavy point loads, which can cause deformation and instability. If the grid is too open, it will lose its membrane behavior. The differential stiffness in the surface is a defining variant between lattice shells and thin shells of concrete. Because lattice shells aren't monolithic, they aren't as stable as solid shells. The joints between each of their elements need to be rigidly connected, often as expensive fixed connections. Construction of double-curved surfaces from straight-lined elements—particularly with rigid connections—was quite complicated. These structures were also incomplete without an additional enclosure system, all of which suggested a return to solid shells. (Figure 5.0.11)

This return was spurred by a planetarium structured from a hemispheric geodesic dome built of iron framing in Jena, Germany, in 1922. The need



Figures 5.0.10a and 5.0.10b The bearing capacity of folded plates increases when the form is held in place by edge stiffeners or changes to the geometry

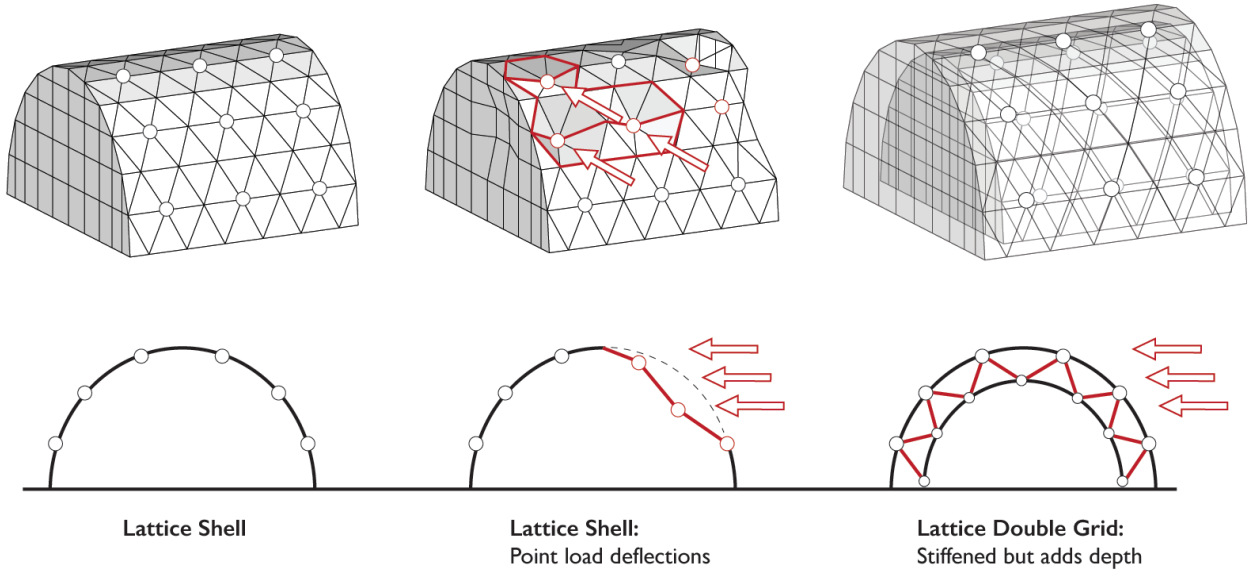


Figure 5.0.11 Discontinuous surfaces, like lattice shells, are limited by point load deflections

for a smooth, solid projection surface led its builders to cover the spherical framework with wire-mesh and spray-on concrete, creating a hybrid system that had the thinness of a braced dome but with the solidity of a shell. The solution was so successful that it was patented as the Zeiss-Dywidag (or ZD) system, which was eventually expanded to include engineering and construction methods for domes and vaults. ZD systems in steel and concrete were built throughout Europe. The patent was sold to only one engineering firm in the U.S., Roberts & Schaefer from Chicago, who recognized the opportunity to expand the construction market in the U.S. to include concrete thin shells. (Figure 5.0.12)

Figure 5.0.12 Zeiss-Dywidag dome under construction, 16m span at 3cm thickness (Jena, 1923)

By the late 19th and early 20th century, concrete shells in the U.S. were limited to utilitarian vessel-like structures, like grain silos, and infrastructural projects. Shells weren't economically viable for most buildings, because of the concrete's limitations. Concrete was generally used for projects that needed to be fireproofed and durable, like warehouses and hangars. Concrete beam-and-slab configurations had limited spans, which adversely affected functional utility, and their formwork was expensive. Thin shells, however, offered significant improvement, since they could provide significant structural, architectural, and economic benefits. Shells increased material utilization, increased spans, and provided integral enclosure systems, all while maintaining concrete's fire-resistance and durability.

MATERIAL MATTERS: SHELL SURFACES

A successful shell requires rigidity in its form and a stiffness in its surface—ideally both factors are combined in a monolithic form. As such, the material choices for shell are somewhat limited and usually involve cast concrete. Concrete has consequences for construction as it requires continuous reinforcing. To cast a traditional shell, one builds the building twice: once out of formwork and the other out of concrete. The formwork is often more expensive then the final shell. Because of these drawbacks, for more than a century, designers have looked for ways to improve the economics of shell building—including the incorporation of fabric formwork.

One of the first examples was the "Ctesiphon" method developed by James Waller (1884–1968) after World War II. In this system, Waller constructed funicular arches (pure compression shapes with minimal centering below) and draped heavy fabric coated with concrete between them—the fabric was the formwork, the reinforcing, and the interior surface. These experiments were continued by others through the decades. Recently, research and production methods have come under more focused scrutiny by Mark West and the Centre for Architectural Structures and Technology (CAST) at the University of Manitoba. CAST has developed methods for casting highly expressive and efficient beams, columns, panels, and portions of shells—including methods initiated by Waller. (Figure 5.0.13)

(continued)

(continued)

Figures 5.0.13a and 5.0.13b Ctesiphon shell construction process from Waller's scrapbook (project unidentified, approximately 1950). 5.0.13b: Fabric cast precast panel for Hanil Construction Company Visitor Centre (Byougsoo Cho Architects, Chungbuk, 2009)

Standardizing Shells

Despite these benefits, few shells were built before the mid-20th century perhaps because generating and analyzing monolithic curved bearing surfaces proved too complex. Such topics weren't included in engineering or architectural education. The situation was complicated by their reliance on a thin surface to span long distances and the need to be as lightweight and stiff as possible. Further, such thin surfaces didn't leave room for redundancies, which made them high-risk structures to design and complicated surfaces to build.

By the 1930s, only a few pioneering engineers had created long spanning concrete shells, including Eduardo Torroja (1891–1961) and Pier Luigi Nervi (1891–1979). Although the forms produced by Torroja and Nervi were both viable, analyzing them mathematically was too complex—there wasn't any accurate means of testing monolithic double-curved concrete forms for stiffness. Both made physical models of their projects and subjected them to testing, looking for confirmation of behaviors that their calculations had predicted. (Figure 5.0.14)

In the U.S., the knowledge of how to create and build shells was proprietary. The hundreds of thin shell terracotta tile "Catalan" vaults spread throughout prestigious projects in America's largest cities from the late 19th and early 20th centuries were nearly all created by Rafael Guastavino's Fireproof Construction Company. His tile arch system (discussed in Chapter 1.1) was patented in 1885, providing a cost-efficient and reliable method for enclosing large amounts of space with thin, shallow vaults.

Like Guastavino, Roberts & Schaefer saw the proprietary value of combining design and construction services. They served as the sole provider of design services for concrete ZD thin shells in the U.S. for decades. Thanks to the efforts of the firm's main engineer, the godfather of American shell design, Anton Tedesko (1903–1994), and high-profile projects such as the Hershey Arena in Pennsylvania (1936), concrete shells' utility became evident and their popularity spread.

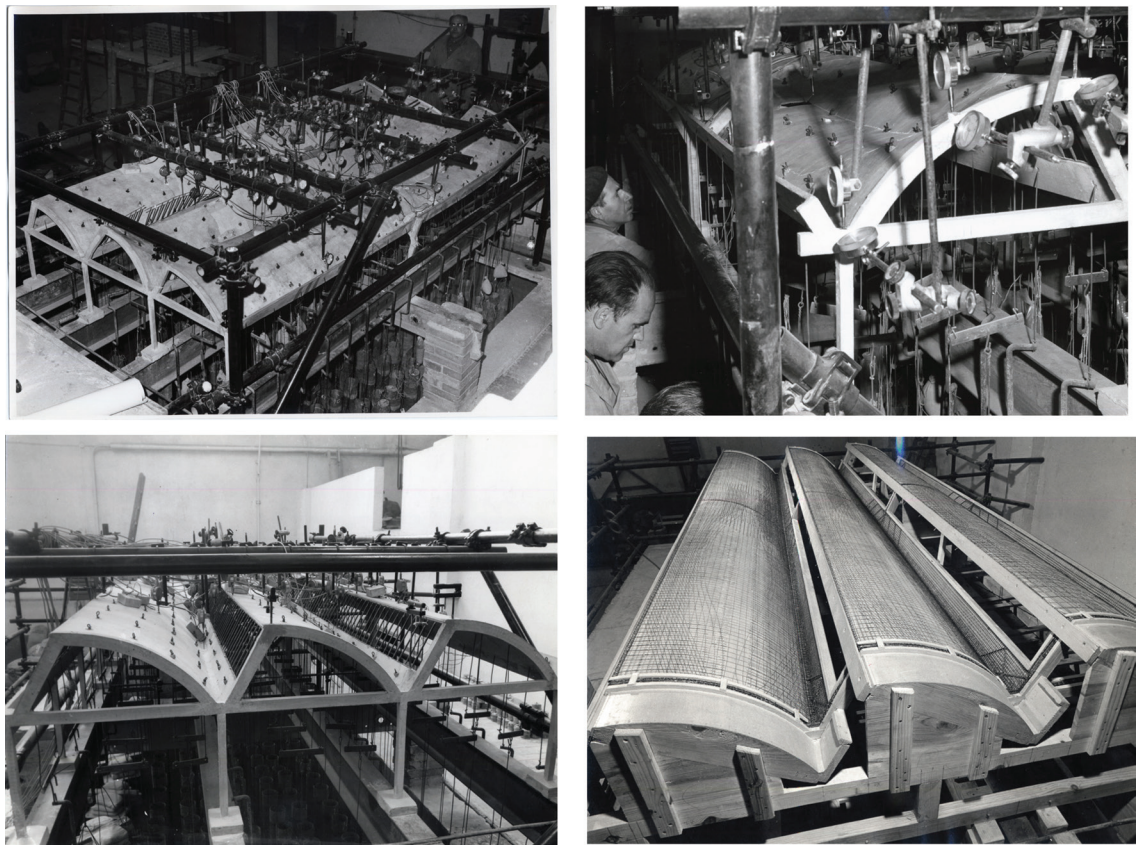


Figure 5.0.14 Torroja model testing for industrial ship factory in Seville (Naves Para HYTASA (1961)

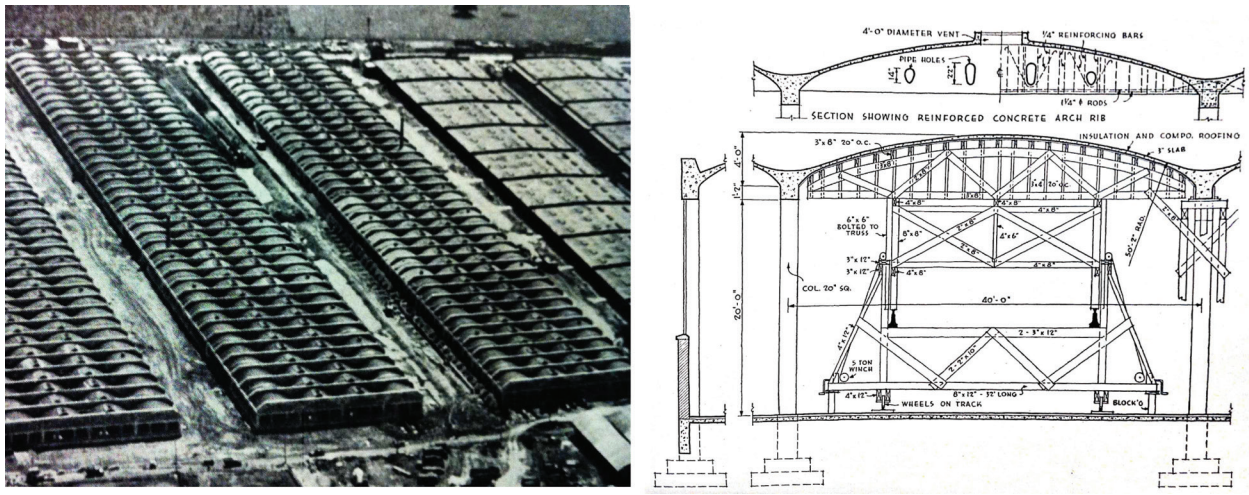


Figure 5.0.15 Army warehouse storage facility by Tedesko with movable formwork drawings for rapid construction (Columbus, 1941)

During World War II, steel scarcity and the increased need for long spanning, durable wartime buildings, shells' use grew rapidly as they were embraced by military engineers. Shells require very little reinforcing, which minimized the use of rationed steel, so the resulting forms were simple and materially over-designed. However, their builders refined their construction methods, allowing shells to be built quickly (relatively) and repeatedly with minimal materials, typically with movable and reusable formwork. (Figure 5.0.15)

In the mid-1950s, shells still weren't common choices for designers in the U.S. Because they are monolithic, their hyper-static force flow defied normal analytic skills. Analyzing shells was mathematical and it was known to be dense and difficult. Most practitioners still had no training or experience with them. The engineering community tried to advance shells by producing design manuals with detailed explanations and formulas showing how shells could be designed and assessed; these included tables, charts, and drawings that established rules of thumb (including height, span, thickness, etc.) for domes and vaults. This manual, and those that soon followed, negated the proprietary nature of shell design and opened it up to a broader range of designers.

Three lasting impacts resulted: Shells became viable choices for "common" buildings that needed long spans with minimal material. Shell construction became more economical by conjoining form and assembly in practice. Finally, these efforts were democratized; the ability to design shells was no longer limited to proprietors of knowledge and experience. (Figure 5.0.16)

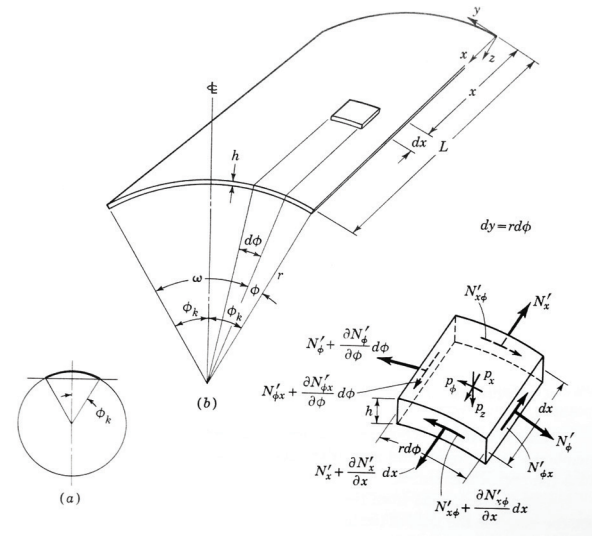


Figure 5.0.16 Typical graphic for vault design standards (adapted from Billington's *Thin Shell Concrete Structures*, 1965)

Shells that Weren't Shells: Free-Form Shapes and Failures

One of the earliest high-profile architectural projects to use a concrete shell was the Kresge Auditorium

(1951–1955) on MIT's campus in Cambridge, MA, designed by Eero Saarinen with Ammann & Whitney Engineers. Not only was the scale of the project unprecedented for a concrete shell (160ft (48.8m) clear span), there was no relevant example for how it would be drawn, engineered or even built—there was nothing in common design manuals to help. Although it was designed to look like a thin shell, the form deviated from conventional logic and structural principles. The shape (an ⅛th segment of a sphere) didn't match with the structural logic. Despite warnings expressed by the engineers, the form remained unchanged. (Figure 5.0.17)

In choosing not to match the form with forces, Kresge's roof didn't get the benefits associated with shells—except the expressive qualities—yet it endured the expense of construction and the inefficiencies of non-shell behavior.

The consequences of inexperience were profound. When the scaffolding was removed, the shell failed to perform as intended. Instead of being efficiently transferred to the ground, the shell's internal stresses were consolidated in the roof's stiffest elements—its three perimeter arches—which became overloaded. After six weeks of unabated sagging and fears of cracking and collapse, additional steel supports were placed under the arches around the perimeter to help support the roof. This high-profile project set an *unfortunate* precedent for thin shell design.

Immediately following Kresge's completion, Saarinen and Ammann & Whitney designed another "shell" project: The TWA Terminal (1956–1962) in New York. Although it was sculptural and expressive like a concrete shell, its form wasn't designed for natural resistance and transmittance of forces. Instead of being thin like a shell, the roof and support elements were massive—up to 4ft (1.2m) thick at points—because it was designed and analyzed like a series of cantilevered beams and slabs. Upon completion, it was celebrated as uniquely formed, but its lack of compliance with a more rigorous structural form was a source of common criticism including N. Keith Scott's statement that, "In many cases (Saarinen) has relied upon the sheer ingenuity

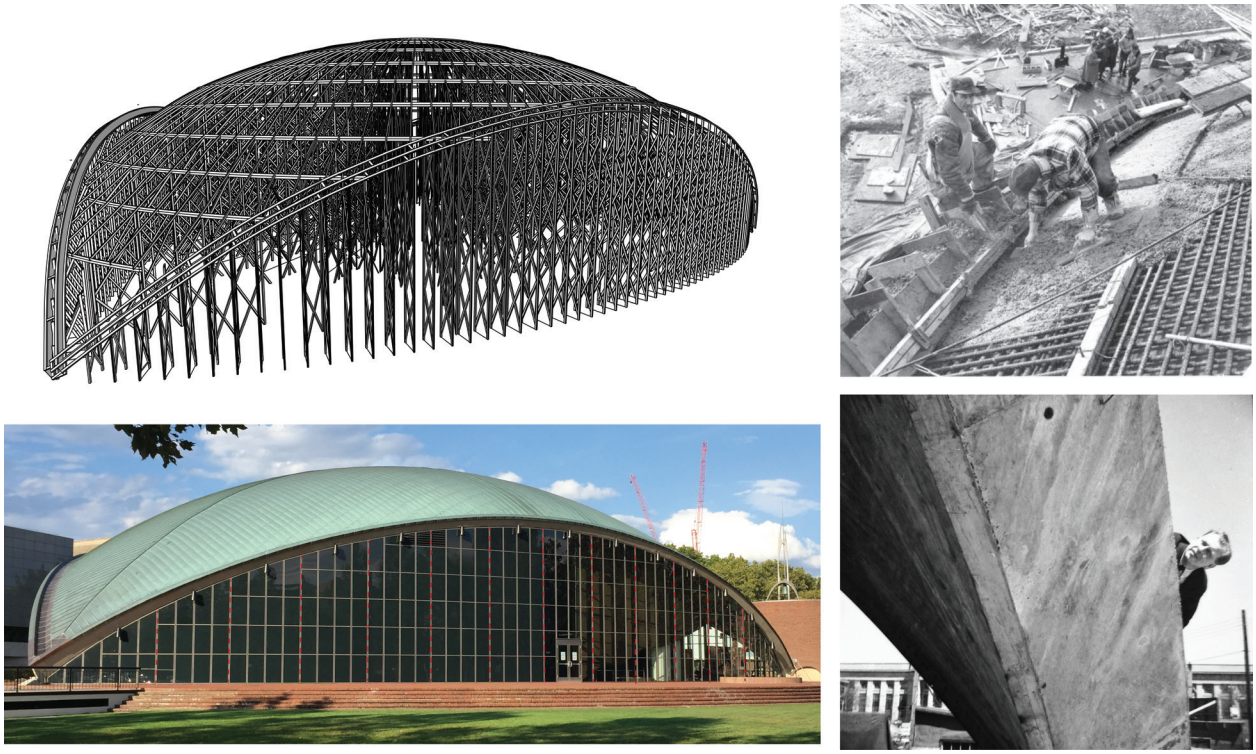


Figure 5.0.17 Kresge Auditorium, (left to right, top/down): Drawing of the complex formwork, complications of dome under construction, completed building (with steel column locations indicated), and Saarinen inspecting the shell (1955)

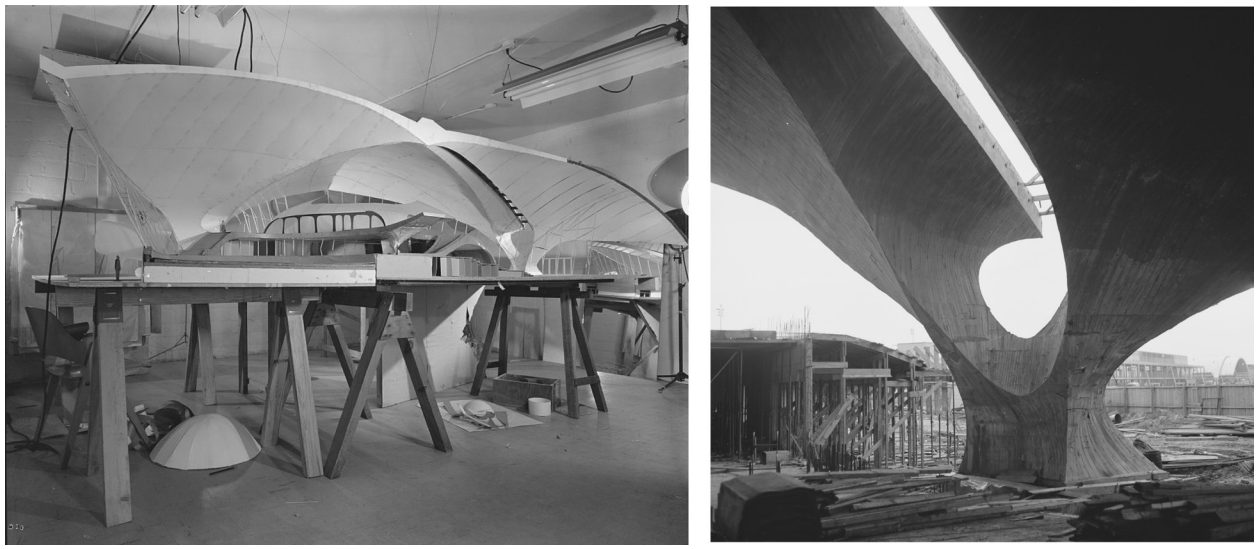


Figure 5.0.18 Form-finding and presentation models were used to refine geometry of the TWA Terminal and to assist in documentation of the shell (1962)

of modern technology to get him out of difficulties that would have presented insurmountable obstacles a quarter of a century ago." (Figure 5.0.18)

During the same decade, a third highly celebrated "concrete shell" project (that wasn't actually a concrete shell), designed by architect Jørn Utzon and engineers Ove Arup & Partners, was being built in Sydney, Australia (1959–1974). The problems were caused by the same issues as those at Kresge—a lack of correspondence between an architectural idea and a valid structural shell form. Although shells are curved and stiff, not all curved and stiff structures act as shells. It took nearly fifteen years of experiments, model testing, and full-scale prototypes for Arup's office to find a solution that worked.

These projects are touchstones in architectural history and are celebrated expressions of form, but they should be regarded as warnings, too. As they became more complicated, they became more expensive, and drifted farther away from shell's inherent benefits. These weren't shells. The carefully crafted mathematically based geometries of early domes and vaults gave way here to free expressions dictated by architects. Instead of celebrating the potential of shells to enclose large spaces with minimal material, these high-profile projects gave a different impression of shells as complicated, expensive, and risky. They nearly stunted the expansion of shells just as it had begun.

Efficient Shells: Hypars, Folded Plates, and Construction Economics

Fortunately, with every action comes an opposing reaction. Despite these high-profile lapses in the 1950s and 1960s, dedicated engineers demonstrated shells' advantages by refining the design of domes and vaults and creating new, efficient surface-resistant types. The most exemplary of these shell projects shared one trait: construction considerations were combined with architectural utility and structural form.

In this era, "form-finding" as a creative exercise was rarely discussed. Instead, the focus was on practical considerations of documentation, analysis, and construction—analyzing how many times formwork needed to be reused for vaults before it became cost-effective, for instance. Because of their forms' regularity, domes, vaults and folded plates offered the greatest potential. It was commonly assumed that these shells could become viable solutions *if* engineers could find reliable ways to design them and to construct them more effectively.

Figure 5.0.19 Using advanced principles of design and construction, this double-curved shell form was digitally generated then built when fabric formwork was cast over the cable-netting (Block Research Group, ETH Zurich 2014)

We need to answer three intra-related questions when designing double-curved shells: *What is the form*? *How does it work structurally*? *How can it be built*? When complexity of form becomes complexity in construction, the economic benefits that shells offer (e.g., enclosing large space with very few materials and supports) disappear. When these challenges are met, high-performing and expressive architectural forms emerge. (Figure 5.0.19)

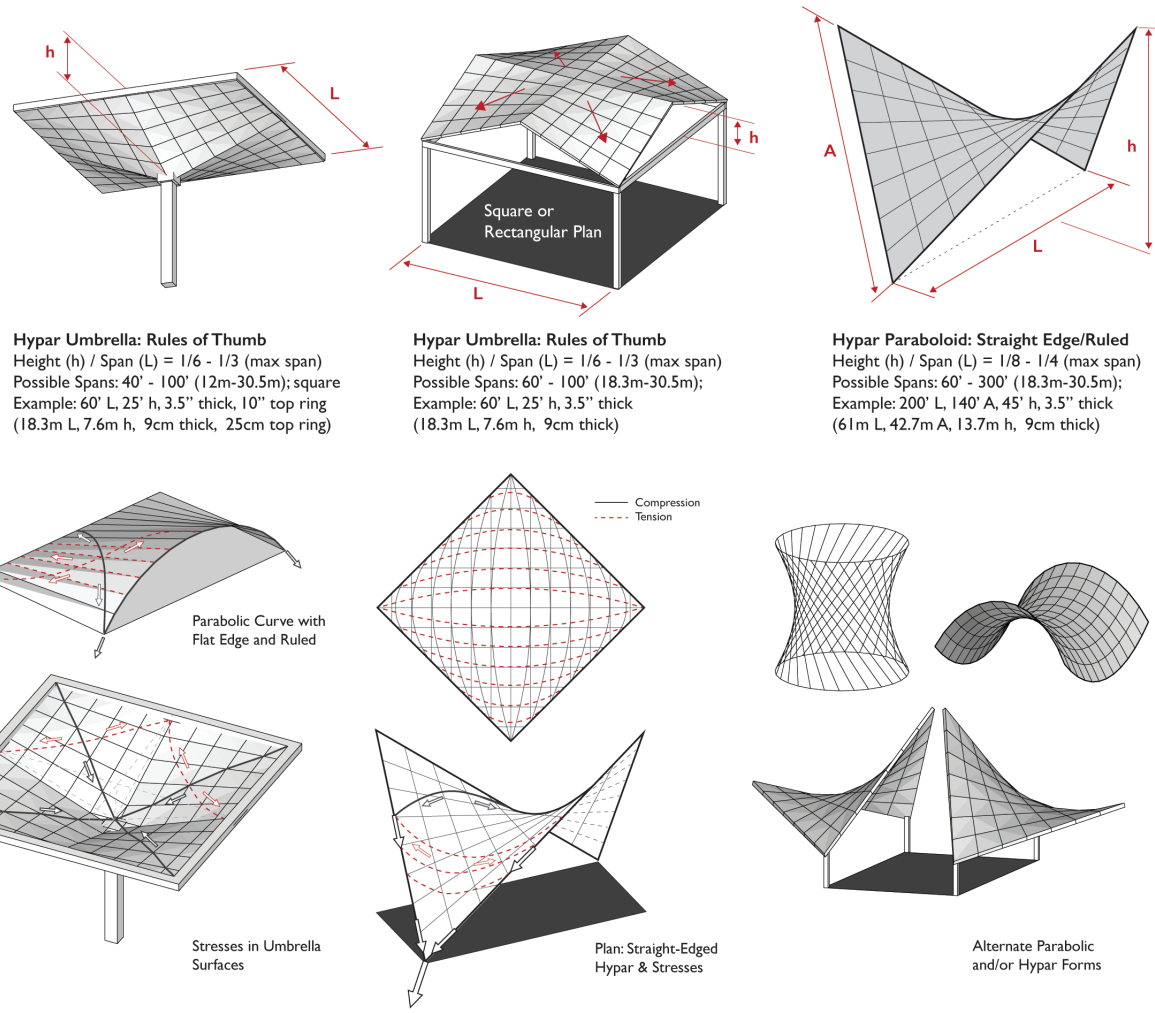
Shells' economic viability is primarily constrained by the labor required to build them, so leading designers developed ways to closely align structural form with construction methods. Shell formwork is more expensive to build than the shell itself. Unsurprisingly, many of these designers carried on the tradition established by Guastavino by serving as engineers and contractors for their shell projects. One of these designers, Felix Candela (1910–1997), criticized the separation between designers and contractors as a fundamental problem and the root cause of poor shell design, sarcastically stating: "The design of shells is conveniently protected by a respectable curtain of mystery and highly mathematical abstruseness."

Regular Forms for Double-Curvature

Engineer-builders devised many ways to make their shells more economically viable. They all used naturally resistant forms with geometries that allowed high rates of material utilization, but these efforts weren't enough. They also had to focus on reducing labor costs. Efficient construction relies on repeated operations with minimal crews, inexpensive formwork, or formwork that can be easily set, demounted and reused many times, leading to several different strategies. Nervi relied on regular and repeated geometries that could be built from smaller prefabricated ferrocement elements. The Uruguayan engineer and builder Eladio Dieste (1917–2000) minimized the amount of material he used by relying on smaller construction units—terracotta tiles—and carefully reused forms, which simplified construction and reduced the expertise needed.

Candela believed that if shell design and construction could be simplified and made more accessible, then more shells could be constructed. He argued that very few formulae, if any, were needed to create a shell. All that was really needed was a double-curved surface, a rigid membrane, and knowing how to build it. To simplify shell forming, Candela used *hyperbolic-paraboloid* forms that could be built using straight-line generators and corresponding straight-lined formwork. These forms, also known as *hypars*, are a general classification for shells that use straight lines to create curved, or "ruled" surfaces. Every point on the surface lies at the intersection of two straight lines that run across the surface. Horizontal sections taken through the surface are hyperbolic while vertical sections are parabolic. (Figure 5.0.20)

All hypars are doubly curved, which means that with proper support and shaping of the membrane, stresses in the structure will be low and the shell's



Hypar Umbrella: Rules of Thumb
Height (h) / Span (L) = 1/6 - 1/3 (max span)
Possible Spans: 40' - 100' (12m-30.5m); square
Example: 60' L, 25' h, 3.5" thick, 10" top ring
(18.3m L, 7.6m h, 9cm thick, 25cm top ring)

Hypar Umbrella: Rules of Thumb
Height (h) / Span (L) = 1/6 - 1/3 (max span)
Possible Spans: 60' - 100' (18.3m-30.5m);
Example: 60' L, 25' h, 3.5" thick
(18.3m L, 7.6m h, 9cm thick)

Hypar Paraboloid: Straight Edge/Ruled
Height (h) / Span (L) = 1/8 - 1/4 (max span)
Possible Spans: 60' - 300' (18.3m-30.5m);
Example: 200' L, 140' A, 45' h, 3.5" thick
(61m L, 42.7m A, 13.7m h, 9cm thick)

Figure 5.0.20 Types of hypars and general proportional guidelines

membrane will be thin. Their forms encourage primarily compressive stresses. The boundaries of hypars can be straight or curved, which allows for many possible forms. Hypars can be double-cantilevered (as saddles and inverted umbrellas), vaulted (with gabled and/or curved groin), supported on the ends, or supported continuously along their bases (a hyperboloid "skewed" cylinder). Hypars provide enclosure over spaces; Candela often used umbrella-like shells, repeated and arranged side-by-side, to enclose large spaces with few columns. Candela believed that, "of all the shapes we can give to the shell, the easiest and most practical to build is the hyperbolic paraboloid." (Figure 5.0.21)

Non-regular Forms and Isler's Three New Methods for Form-Finding

Thin shells of the 1950–1960s that were creative and economical were exceptions—not the rule. Most U.S. shells during this time were conservative formally because they were based on manuals and rules of thumb. Engineers had become wary of "free-formed" shell-like projects (e.g., Kresge Auditorium, Sydney Opera House, etc.) so there was a well-justified skepticism about proposals for shells that were formed differently—particularly forms that were generated by architects only. After all, compared to domes and vaults,

Figure 5.0.21 Candela combined geometry with construction considerations as a design generator (Church of San Jose Obrero (Monterrey, 1959)

how could geometric irregularities in shells be created, documented, accurately analyzed, or even built?

In 1959, at the inaugural conference of International Association for Shell and Spatial Structures (IASS), Swiss engineer and builder Heinz Isler (1926–2009) presented a paper entitled, "New Shapes for Shells" that challenged these traditions. He demonstrated how hanging fabric could establish accurate geometries for shells and how, when hardened, the inverted shell could be used for confirmative testing and analysis. This process stemmed from years of frustration trying to accurately generate, graphically document, and analyze geometrically complex double-curved (or *anticlastic*) concrete shells using traditional means. Isler described how the continuous, naturally occurring double-curvature of his pillow helped him realize that a physical model of tensed surface under pressure "was for 3D problems what the catenary line is for arches." The physical model, he argued, was an immediate and relatively accurate representation of an idealized structural form, and was based on the centuries-old "hanging chain" models method used by Hooke and Gaudi. Isler described his revelation about physical prototypes and shell forms being found in a pillow: "I saw that it was the solution to the problem. . .I saw the way to build a pillow, a technical pillow of the highest possible precision." (Figure 5.0.22)

Isler argued that these shells would be economical because of the minimal amount of material used to cover large spaces compared to other systems that also needed an enclosure system. But he could now also use savvy form-finding strategies to eliminate costly features like thickened slab edges on the slab. Most of the 90+ shells he designed relied on a framework of glue laminated wooden arch pieces

Figure 5.0.22 Isler's analysis model for Gips Union SA factory (Bex, 1968). Points along the shell were subjected to loading (hanging below) with sensitive electronic sensors attached to measure deflection of each point

Figure 5.0.23 Norwich Sports Village shell designed by Isler (1991)

below the shell to define the curvature of the shell. Isler employed light, tubular scaffolding that could be easily erected, demounted, and reused. Finally, he placed insulation onto the formwork that remain on the shell, providing an additional benefit. Isler occasionally used pneumatic bladders for projects that had repeated forms. The inverse of a tensioned surface (like a pneumatic) would become the purely compressive shell surface. (Figure 5.0.23)

Although the testing and analysis process using this method is painstaking and outdated in the light of more accurate digital techniques, there are few more effective form-finding methods for shells to generate immediate formal possibilities. Beginning shell designers should use variations on these form-finding methods to better understand connections between form and forces. (Figure 5.0.24)

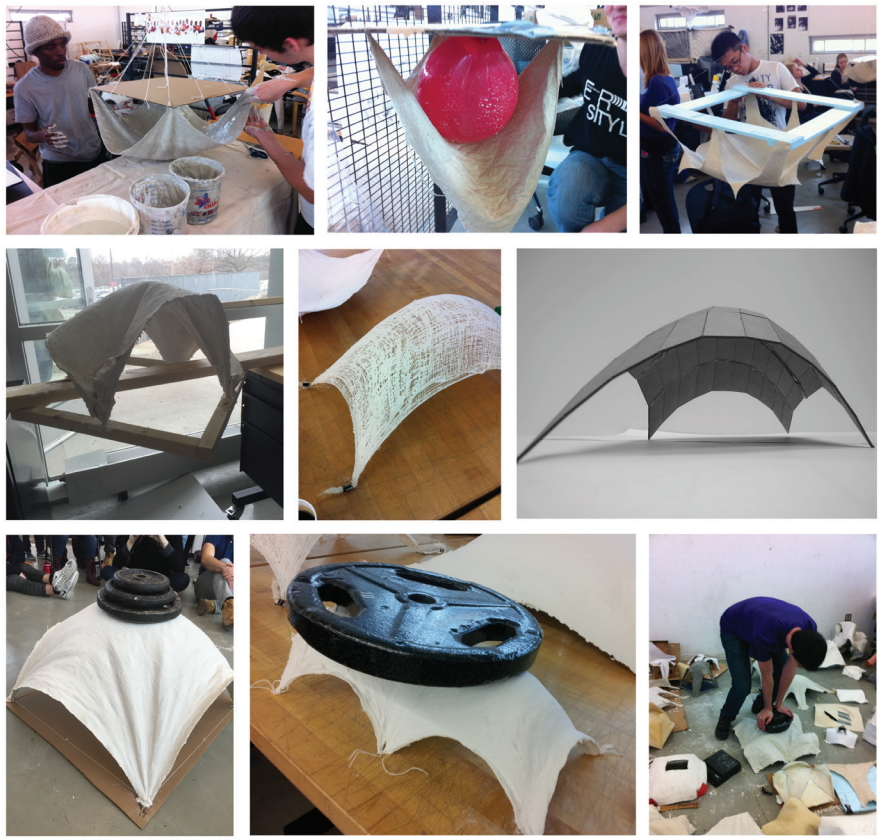


Figure 5.0.24 Examples of student work (SxD, Iowa State University)

FOLLOW THE FAILURES: BINISHELLS

In the 1960s, a proprietary construction system called Binishells was introduced by Dante Bini that used pneumatic bladder for formwork. In his system, reinforced concrete thin shells were cast over inflated rubber forms. The pneumatic formwork held until the concrete surface cured, leaving a large-scale space below. Hundreds of Binishells were built but a few of them failed.

To Do: Find examples of how Binishells were built and what caused their failures. What happened to the membrane's integrity when windows, doors, and other openings were cut into the shell or when multiple shells were joined? (Figure 5.0.25)

Figure 5.0.25 Cutting an opening into the Ashbury Primary School, New South Wales, NZ (1977)

Frei Otto and the Evolution of Digital Design and Analysis

Although Isler's method didn't lead to wider development of anticlastic shells, it mimicked the aggressive and creative formal exploration being undertaken by Frei Otto in the 1960s–1970s. Like Isler, Otto used physical models to find atypical structural forms, document their double-curved geometries, and estimate their elasticity and deformation under loading. Otto aspired to get accurate information from the models but this was cumbersome work, requiring larger models with more accurate gauges. He eventually recognized its limits in providing accurate information for full-scale buildings. Otto's collaborators at the Institute for Lightweight Structures (ILS) developed a digital system that recorded the models' geometry using a remedial computer-aided drafting program based on a photographic comparison method (*Photogrammetry*) to develop three-dimensional data. Irregularly-shaped structures could now be defined and analyzed geometrically as accurately as mathematically-derived forms. (Figure 5.0.26)

In 1972, Otto worked with architect Carlfried Mutschler and Ove Arup & Partners on a competition-winning entry for a large timber grid shell for the Bundesgartenschau Multihalle in Mannheim, Germany. Wood lattice-based grid shells don't function like concrete shells—like lattice shells from a century before, their stiffness is relative because their material distribution isn't uniform across their surfaces. Determining the diamond-shaped spaces' sizes between the members (known as "lamellas") and the material's thickness is a complicated analytical endeavor, so the team used physical models. Together with traditional structural analysis, Arup's office built a hanging chain model to find the final curved tension surface's geometry. They photographed this model, surveyed it, and the dimensions were converted into a computer program that produced an accurate 3D model of the hanging structure—the first "computer model" of its kind. This model was used for manual and computer analysis for the shell, but it was also useful to present three-dimensional visual information about the building. (Figure 5.0.27)

The Decline of Shells: Cost versus Value

This golden-age of "traditional" concrete shell construction lasted between 1950 and 1970. During this time, significant advancements were made in the shell design, analysis, and construction methods. Advanced methods of constructing shells made them more economical to build. These included: Movable/reusable formwork, pneumatic bladders, precasting/off-site fabrication of shell components, fabric-casting, and other strategies that regularized the formwork or reduced the amount of formwork needed. (Figure 5.0.28)

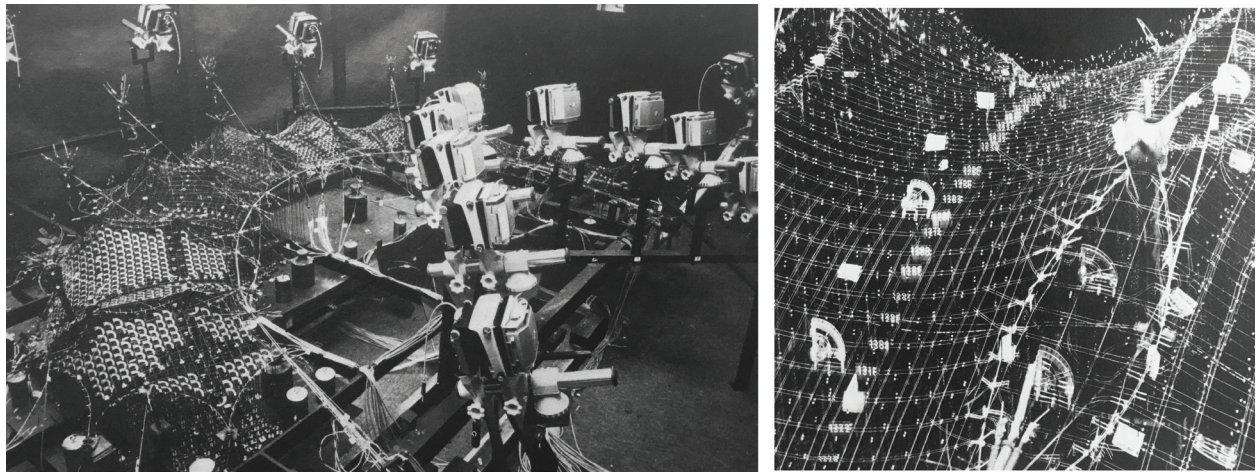


Figure 5.0.26 The form-finding and analysis process used for Olympic stadium roof. Note the composite before and after images show cable displacement of critical points (1971)

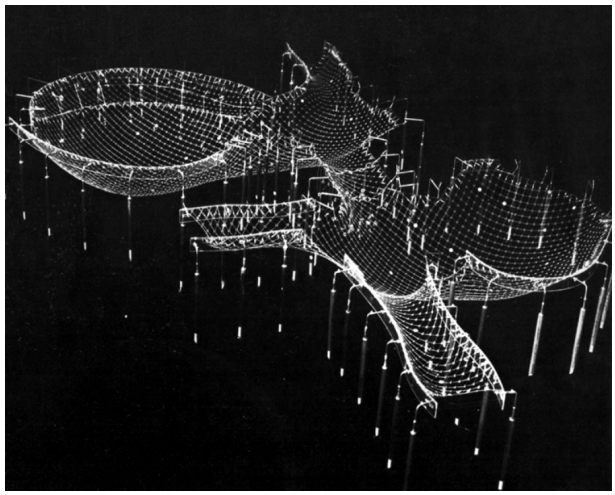
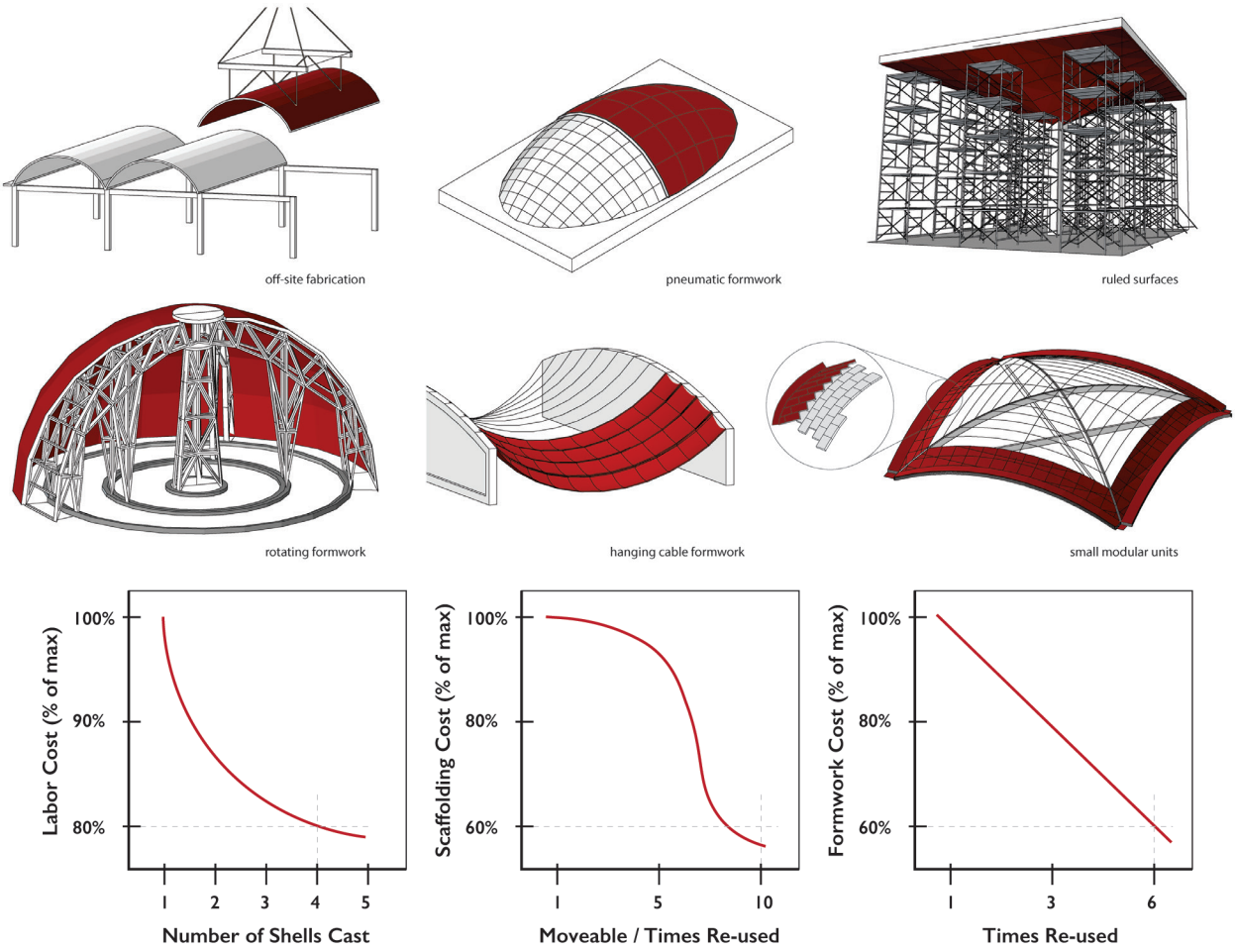


Figure 5.0.27 Inverted hanging chain form-finding model for the Bundesgartenschau Multihalle roof form

Some completed concrete shells had left an unimpressive legacy. Double-curved or folded surfaces didn't age well. They were difficult to waterproof, leaking at valleys and through cracked surfaces, they were often un-insulated, triggering thermal transfer issues, and many had poor acoustics. Shells enclosed space with minimal material but created more problems—sometimes because their newly-simplified design inspired some architects and engineers who weren't up to the task. (Figure 5.0.29)

Paradoxically, the number of shells constructed in the U.S. diminished while the computational software needed to analyze shells emerged. Exacerbating shells' decline were their higher construction costs. Despite many advances in strategies to reduce costs in shells, labor and certain material costs increased



Figures 5.0.28a and 5.0.28b Construction strategies for improving the economy of shell construction (left to right, top/down): Pre-cast vault sections, pneumatic (inflatable) formwork, ruled surface forms (hypar umbrella), rotating/reusable dome falsework, hanging cable and fabric formwork, and small modular units

Figure 5.0.29 Unfinished concrete vault from Ten Concrete Buildings installation (Judd, 1980–1984)

in the 1970s and 1980s, and shells fell out of favor. Tedesko and Candela had argued for decades that shells *aren't* expensive—they enclose the most space with the least material and rest on the fewest supports. Instead of blaming the structural system, they faulted poorly designed shells created by inexperienced engineers and priced by over-cautious contractors. Fewer innovations and experiments to make concrete shells cheaper or improve their performance developed in the era than hoped. Instead, design and construction industries looked to other long spanning spatial enclosure systems, including steel-grid shells, tensegrity systems, pneumatics, etc., all lighter systems that also minimized material but that were easier to construct.

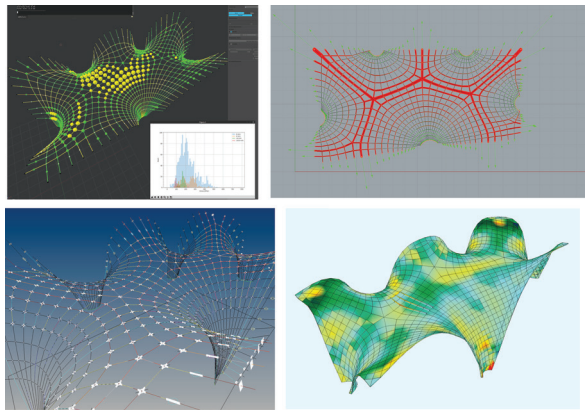
During the 1990s, advancements in computational analysis methods, material behavior, and construction technologies allowed new forms that were, at best, agnostic towards the importance of building form in providing structural resistance. These weren't as constrained formally by mathematical models or physical analogies. Complex surfaces could be built from braced, truss-like frames as they were a century before. But, unlike the early grid shells of the 1890s, these new forms weren't compelled to follow the geometry of form-resistant shapes. Because these weren't meant to act like surface-resistant structures—they weren't resisting and transmitting through their surfaces' membrane actions—they no longer had to rely on structural lessons of building form that traditional shells established.

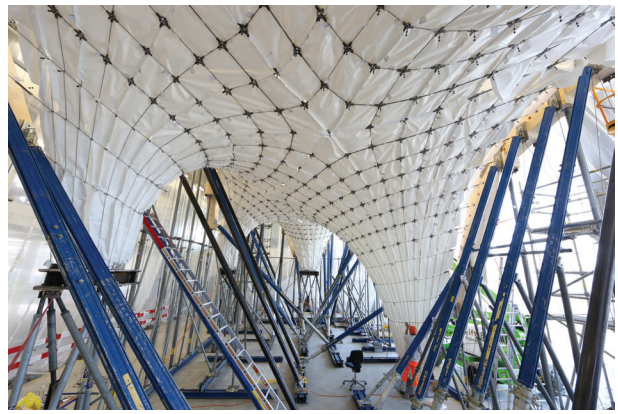
Contemporary Shells and Computational Forms

Fortunately, there has been a resurgence of shell design and construction. Digital models have increased the ability to design, document, and analyze complex double-curved shells. The main innovations have come from the ability to document the forms and to develop algorithms that help define optimal physical characteristics. New programs act like digital versions of physical models. Within a digital model, shells are treated as smaller, connected elements; each piece is assigned characteristics based on a defined geometry and loading condition and analyzed for compliance. The overall digital model has different parameters assigned to it (form, boundary conditions, material stiffness, etc.). If these parameters are changed, the digital model shows the consequences. Thankfully, these powerful tools have also been used to bolster the work of engineers and builders making building shells. Once structural and spatial data is digitized, it can increase efficiencies in material utilization, structural performance, and construction efficiency. (Figure 5.0.30)

At their most advanced, digital technologies allow designers to explore form-finding and to analyze shells. Once we establish boundary conditions (span, height, support locations, etc.), we can use computer simulations to create digital versions that behave like hanging fabric "thrust models" to find overall form options. We can then manipulate these models to simulate the effects of different "fitness criterion" on the structural form (e.g., what happens if we maximize

Figures 5.0.30a and 5.0.30b The Landesgartenschau Exhibition Hall is a prototype building that showcases the potential for robotically fabricated lightweight timber shells. The digital design and fabrication process were streamlined to coordinate the difficult geometric relationships (Achim Menges and University of Stuttgart, 2014)





Figures 5.0.31a, 5.0.31b and 5.0.31c a: The digital design and analysis methods use for the NEST HiLo Shell. Note the similarities between the digital representations and Otto's physical models (ETH Zurich/Block Research Group), b and c: Construction process and final view of the NEST HiLo cable-net, fabric formed shell

stiffness or minimize weight?). The feedback loop is nearly instantaneous and the visually-enhanced results allow clear comprehension of the consequences—even to non-engineers.

These same digital tools are also useful in constructing shells. Shell designers experiment with alternative or experimental materials, innovative digital design, and fabrication techniques. Digital models' ability to accurately define shell geometry is useful in many ways. First, instead of relying only on shells with regular geometries (domes, vaults, conoids, hypars, etc.) digital models can provide the dimensional accuracy needed to design and construct more complex double-curved surfaces, since they can be defined easily and translated accurately into formwork. Second, and more impactful, is the increased efficiency of matching a shape to forces and in economically optimizing the construction process. Traditional shells are materially efficient, but can be expensive to build. Contemporary designs look for the same structural benefits without economic penalties.

The connection between digital modeling and fabrication in shells is useful because it challenges one of the fundamental constraints of shell construction—namely, that shells must be monolithic, poured-in-place structures. Constructing shells from smaller pieces allows designers to consider other time and cost savings measures including off-site fabrication, modularity and mass production, and even assembly lines for construction. Imagine creating forms as expressive as Nervi's without being limited to only domes and vaults! One of the main constraints on shells has always been the economic consequences associated with constructing the form—simplifying the form and the means of construction produces a cascading set of savings. Digital models allow designers to create shells from pieces that are fabricated off-site, to accurately anticipate the form of scaffolding, and to keep track of cost savings while modeling the project (materials, labor, etc.).

The most successful projects look to optimize all essential design considerations associated with shells. One example is the cable-net and fabric formwork system used to prototype the NEST HiLo shell roof system by the Block Research Group at ETH Zurich. The project's goal was to optimize form-finding, analysis, and construction techniques to create a lightweight shell with a flexible formwork system that reduces material waste. (Figure 5.0.31)

Creating double-curved shells more easily has substantial benefits. Double-curved shells offer the largest possible spans with the least total material—they are sustainable and customizable building types that do more with less. Digital design and fabrication are making it easier for architects and engineers to visualize and understand the relationships between structural form and efficiency earlier in the process while providing data that documents and analyzes the form when searching for optimal solutions. Because the forms are digitized and parametric, design data can be leveraged to increase construction economy through mass production of customized forms.

Selecting shells as a structural system may seem risky. After all, shells aren't used as often as other systems that we've explored. But architectural design isn't a spectator sport. Designing shells creates new responsibilities, including proper form-finding and integrating construction technologies. These tasks normally fall on the periphery of an architect's expertise and responsibility, but they don't need to. In the past, professionals that developed the specialized knowledge to create these structures benefited from this expertise. They still do today.

Frequently Asked Questions

If there isn't a reliable and easily accessible way for architects to calculate stresses in shells, how can we feel comfortable proposing them as structural solutions? Architects shouldn't feel compelled to provide numerical analysis to justify their designs, as long as the ideas have technical competency. The most effective way of confirming a shell proposal's initial validity is to follow basic guidelines we've covered in this section (form, material, etc.) and to find examples of structures with similar qualities and proportions that have been built. These designs should be developed with an engineer using common tools to help visualize and analyze them. Perhaps your initial forms will be incorrect—perhaps too much stress or movement—but by testing them you'll have a better idea of how they will behave. If concrete shells are prohibitive, you could also explore grid-shell surface-resistant forms from other materials (steel and timber). (Figure 5.0.32)

Figure 5.0.32 The steel and glass shell roof over the central courtyard at the National Maritime Museum in Amsterdam derives its design inspiration for the form and lines from historic maritime maps, structural principles, and experiential qualities

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Figure 5.1.0 South Pond Pavilion in Lincoln Park Zoo, (Studio Gang Architects, Chicago, 2010)

CHAPTER 5.1

Spanning Surfaces and Shapes

Vaults and Plates

Vaults and plates are expressive surface-resistant structural systems that enclose spaces with curved or folded surfaces. Their forms determine their structural behavior, functional conditions, aesthetics of experience, environmental performance, and the economic consequences of construction. In this chapter, we'll learn by designing several options for a long spanning building.

Surface and Spaces

Surface-resistant structures rely on their form and their rigid membrane surfaces to span long distances with minimal material and support. Different formal options are available—each with different spatial qualities and corresponding structural consequences. We'll learn about the unique structural and formal qualities of vaults, lamellas, and folded plates, and explore how to develop and refine them in correspondence with the larger project goals. We'll use drawings, physical models, and digital simulations as well as established rules of thumb for span, height, thickness, and geometric proportion to learn how to find these forms and to develop them critically.

Design Challenge: Innovation Center

A prestigious midwestern university campus is designing an Innovation Center where students will learn to use advanced digital design and manufacturing techniques to create, construct, and test large-scale industrial items. The schematic design includes a plan, two sections, and the following parameters for performance: The main work area inside will need to remain column-free and to be illuminated by indirect lighting. Equipment will need to move in and out of the work area through overhead doors on the east and west ends, and through a large bi-fold hangar door on the north. The university desires a maintenance-free roof that reflects innovative design and fabrication methods. (Figure 5.1.1)

Your team was selected for your ability to apply structural strategies to make beautiful, environmentally-responsible, and economical environments. There is a surplus of skilled concrete workers and suppliers available locally, but the team is concerned that a heavy material like concrete may not be a good match for the long span roof. You know concrete shell roofs can provide maximum volumetric enclosure with minimal material, and the team has entrusted you with the development of this portion of the project. You suggest vaults as a good place to start.

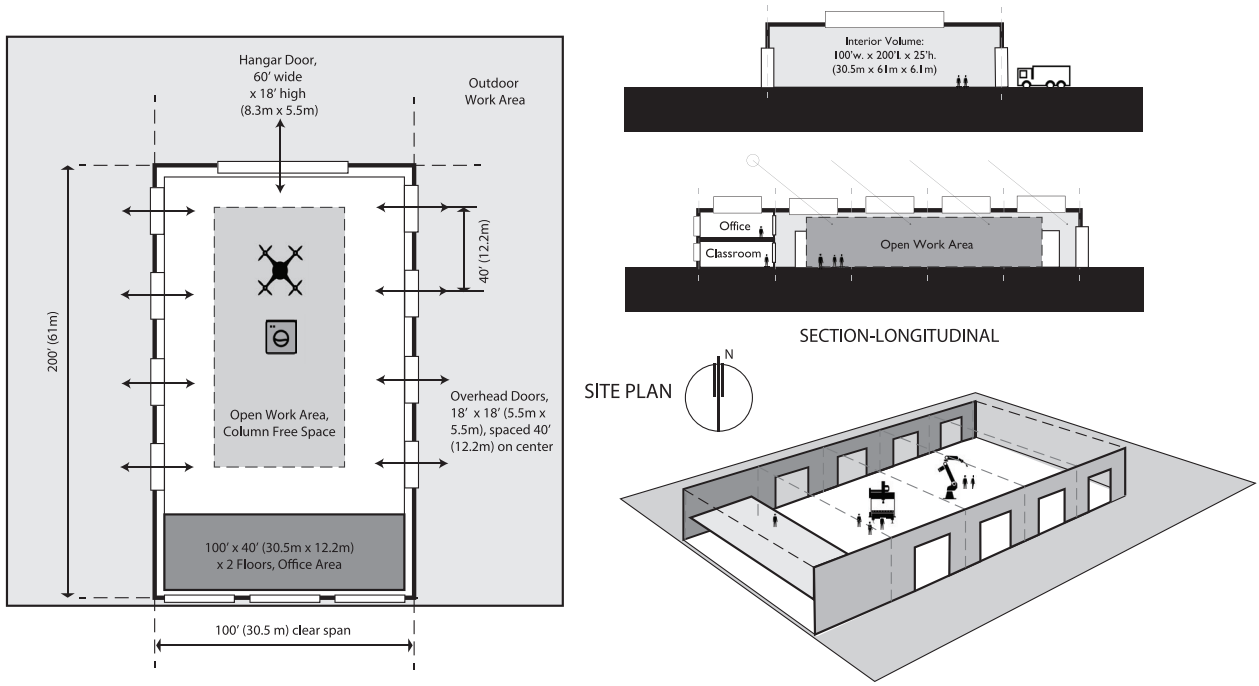


Figure 5.1.1 Design problem parameters: Plan, sections, and axonometric view

Basic Behaviors and Vault Forms: Tools for Thinking and Making

Form is the most important quality for surface-resistant systems and there are many possible options. Finding the form from a range of acceptable options is the most important initial phase. In 1962, Eduardo Torroja cautioned that, "Complex and abstruse mathematical calculations are not alone sufficient to lead to conception of a structure . . . intimate and intuitive comprehension of its working forms is also needed."

Surface-resistant forms can be single-curved (cylinders, cones, and barrel vaults) or double-curved (domes, saddles, hypars, etc.). Their geometry can be derived either from mathematical models (described by analytical functions), or from form-finding exercises (hanging cloths or chains), or from other free-formed shapes. Because of the plan's shape and the economic need to simplify the roof's geometry, we'll focus our efforts on singly-curved (or folded) roof forms with a "developable" geometry that can be extruded along an axis, namely, vaults, lamellas, and folded plates. (Figure 5.1.2)

Although most vaults have a regular sectional geometry developed from a single curvature, they are still complicated, time-consuming, and therefore expensive to build. To cast a monolithic surface in concrete high above the ground we must build up a sturdy system of curved formwork, leave it in place during the pouring and curing processes, and then—ideally—demount and reuse it for the adjacent vault. Formwork for common sectional geometries, like semi-circular arches, may be easier to build on site, but edge beams and end supports complicate any geometric configuration. There are ways to combat these inefficiencies if we address them in our work.

The vault's form is geometrically defined and the proportions of the vaults (width, length, height, thickness) are based on established rules of thumb, so we don't need to produce either a "form-finding" model or a "testable" model at this point. Because vaults provide support, enclosure, and expression, we'll need to evaluate them holistically, based on their compliance with design objectives. Instead of worrying about marginal differences in thickness and depth of members as we would with other systems, we'll primarily focus on form. Clear right or wrong answers may not be apparent, but evaluating designs on both quantitative and qualitative

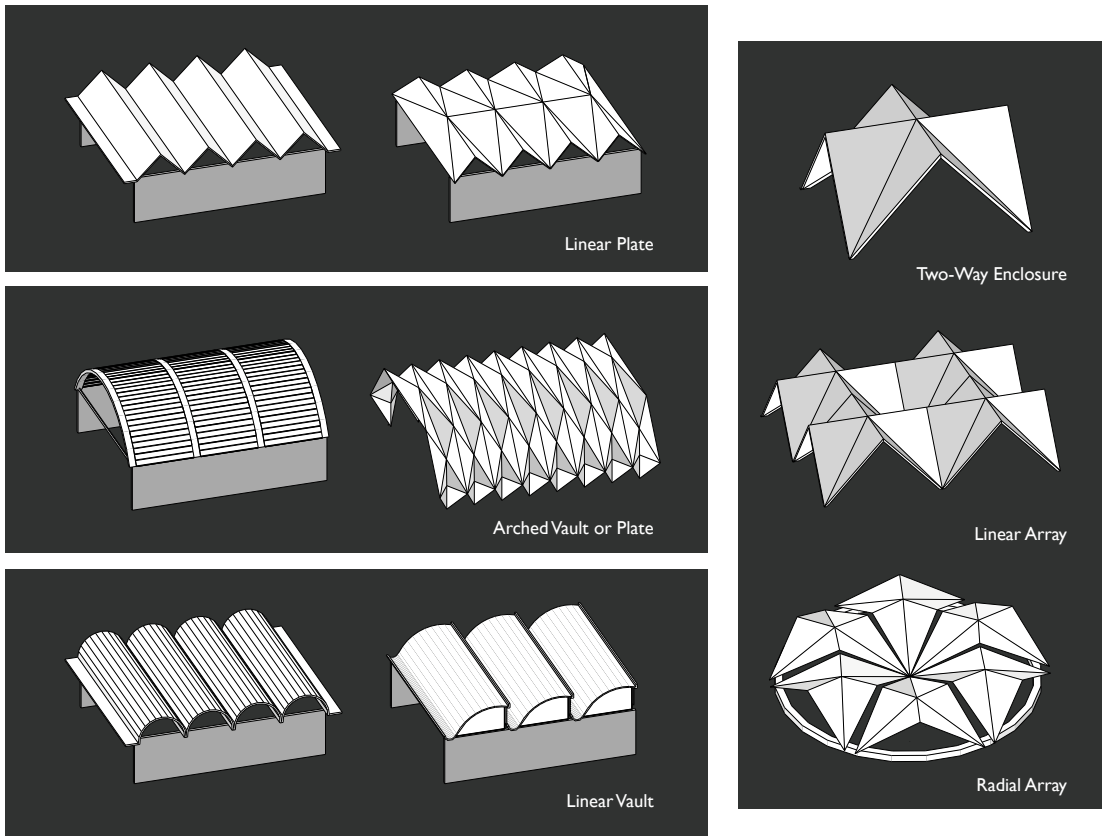


Figure 5.1.2 Vaults and plates generally categorized by shape, span, direction, and geometry

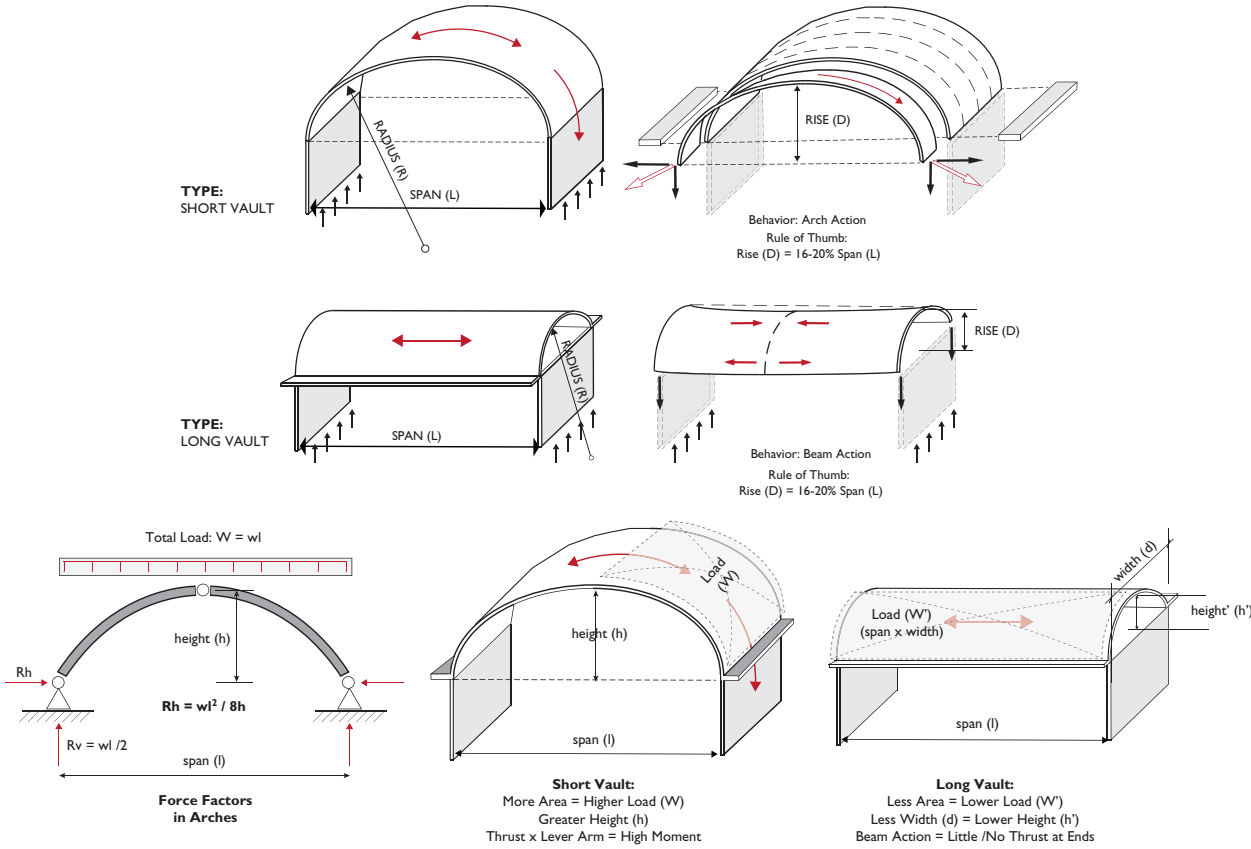
criteria will identify targeted improvements for ensuing options.

To define the actual geometry of these single-curved forms we'll decide on three primary considerations: *which direction the curved vault spans (relative to the supports), its cross-sectional shape, and how its form is stiffened to avoid deformations under loading.* We'll start with the most straightforward single-curved types of shells—barrel vaults. Because of their geometric regularity and their ability to extend along an axis, barrel vaults are well-suited to span rectangular bays.

Span Direction: Long and Short vaults

Despite their historic predominance, vault's behavior is still somewhat misunderstood. Because the top part of the vault is curved like an arch, it is common to believe that vaults behave like connected arches, but this isn't always the case. Vaults can behave like arches or beams, depending on their orientation. We'll need to determine which way the vault (or vaults) should span across the space. Sideways or lengthwise?

Vaults are classified as either "long" or "short." This is not based on how far the vault spans—a short vault can span a long way and long vaults can be short!). Rather, it describes how the vault curves relative to its span's direction. A short vault curves in the direction of the span, like an arch. It is supported on two sides, and developed or extruded along the long axis of the building. A long vault is curved perpendicularly to its span and is supported on its ends, not its sides. Short vaults look and act like arches, with outward thrust at the spring points, while long vaults look, and act like curved beams. Vaults may exhibit both behaviors depending on their proportions. (Figure 5.1.3)



Figures 5.1.3a and 5.1.3b Comparing basic characteristics of long and short vaults

THINKING AND THEORY: BUILT EXPERIMENTATION

Research biographical information about the Swiss engineer, Robert Maillart—specifically his work using reinforced concrete. Find an illustrative example showing how Maillart integrated materiality and form. In 1939, he completed his last project, the Cement Hall at the Swiss National Exhibition. This oval-shaped short vault shell spanned an impressive 88ft (27m) with only 2.5in (60mm) of material thickness (total dimensions:

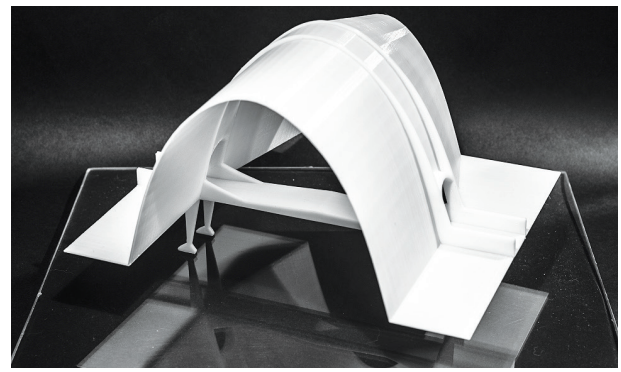


Figure 5.1.4 3D printed model of the Cement Hall shows the bridge, stiffeners, and column supports of the shell ("Structtura e Forma", 2015)

70ft long (21.4m), 88ft long (27m), and 50ft high (15.25m). It was supported on four slender columns on either side with an elevated bridge across the middle and large cantilevered flanges on the outer edges. The Cement Hall was a demonstration project to show how little concrete was required for a long span—what key strategies allow it to be so thin?

To Discuss: Comment on the elements used for support and stabilization—what was the purpose of the bridge and outer flanges? After the exhibition, the project was deliberately destroyed for testing purposes—what information were they looking for? (Figure 5.1.4)

Short Vaults: Rules of Thumb, Form, and Basic Behaviors

For the short vault scheme, we can use rules of thumb and knowledge about arch behavior to estimate widths and heights for the vault, and then evaluate these based on the problem statement. Because short vaults behave like arches their cross-sections should follow the lines of pressure derived from a funicular shaped cross-section (Chapter 2.0). Reinforced concrete short vaults are economical when used in spans between 40ft and 100ft (12m–30.5m). Although the span-to-rise ratio (L/d) in arched structures varies based on factors such as allowable thrust, a good rule of thumb is to assume the vault's height is between ⅕th and ⅛th of the span. If we follow the rule of thumb proportions for a short concrete vault across the 100ft (30.5m) space, our vault height will be between 12ft 6in and 25ft (3.8m – 7.6m).

Because a short vault acts like an arch, the largest issue to address is the thrust that it generates. Our vault rests upon support walls on its ends, so we'll have to consider ways of resisting these vaults' outward thrusts. These are low profiles, so they will generate large thrusts, particularly because they are also heavy. There are two strategies for resisting this: buttresses or tie-backs. Our vault needs to be a continuous surface, but we can lower the arch forms below the vault's surface and align these with buttresses that are angled to transmit diagonal loads into the ground. Or, we could add horizontal tie-backs at the spring-line of each arch. (Figure 5.1.5)

The arch's geometry is also a factor. The vault's height is, of course, related to the required 100ft (30.5m) span; even a shallow arch will make a tall interior volume. The higher roof volume isn't helpful for this project and doesn't introduce daylight into the space except at the ends. A higher arch creates less thrust (Chapter 2.1) but we can't extend the arch down to the ground continuously (like a Quonset hut) because the overhead doors on the two sides need a vertical wall surface. (Figure 5.1.6)

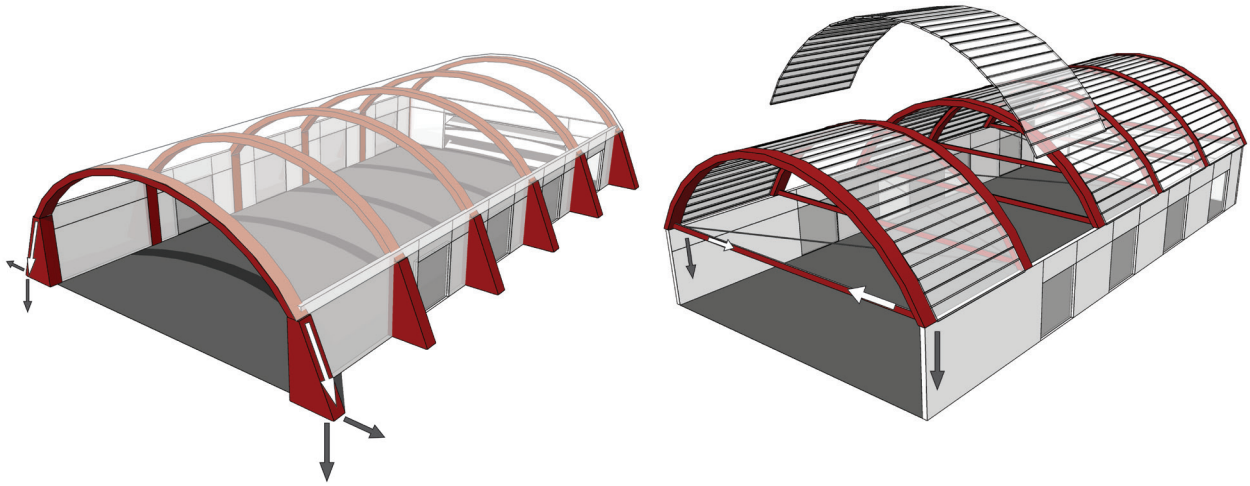


Figure 5.1.5 Two short vault options: Buttresses or tie-backs. Individual arches shown separated for clarity

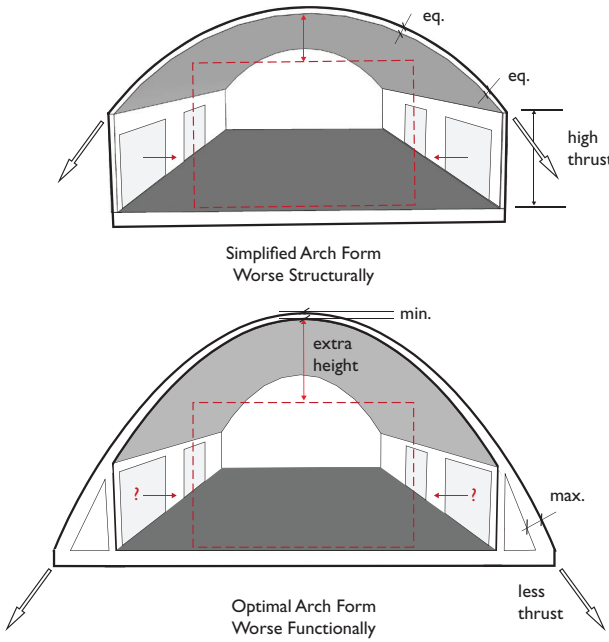


Figure 5.1.6 Short vault options comparing support, profile, and interior volume

We could develop an optimal cross-sectional shape for the shell using a "hanging chain" model, but this will create a more complex geometry compared to a single radius curve. Regular cross-sectional geometry will ease formwork construction. We could also try to make the vault more materially efficient by shaping its profile with a thin cross-sectional area in the top/middle and a thicker one at the ends. But irregular shapes and variable slab thicknesses aren't cost-effective to construct, particularly since concrete is relatively cheap compared to the labor of setting it. It would be more economical to pour the entire roof as the same thickness and adjust reinforcing to handle additional stresses. The minimum thickness for shells is determined by the need to provide cover around reinforcement in the concrete and not on the minimum thickness needed to resist the loads. When we evaluate structural and functional considerations together, we find that optimal structural conditions (funicular shape, variable thickness, arch set into the ground, etc.) all actually make the building worse.

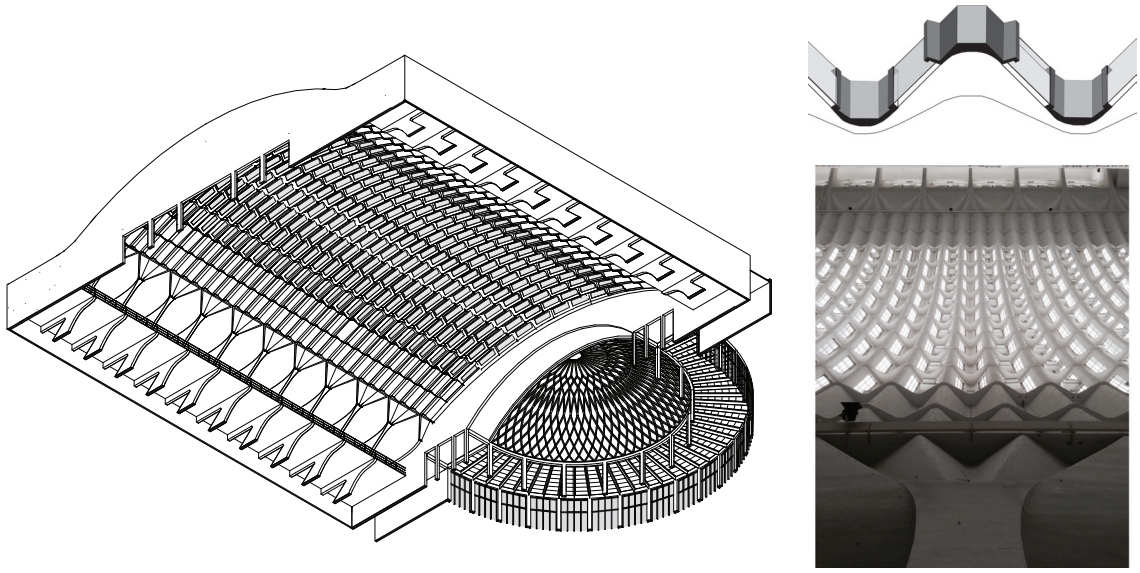
The Turin Exhibit Hall, by Pier Luigi Nervi, is a good example of how a long spanning short vault can be improved by responding to this form's inherent challenges this form. Nervi established the form of the arch based on the statical force flow to establish an efficient form-resistant geometry. He stiffened the surface by corrugating the transverse section of the shell surface. The resulting ridges and valleys were stiffened longitudinally and laterally, becoming form resisting waves. A surface-resistant structures' dead weight is its primary load, so Nervi lightened this roof by using pre-fabricated curved, V-shaped "ferrocemento" ribs with integral skylights. This prefabrication system and its assembly allowed this seemingly complex building to be assembled effectively and economically from smaller parts. Large angled buttress supports on the sides resist the curved roof's thrust over the 240ft × 310ft (73m × 94m) rectangular hall. The cross-section shows how this taller curved section makes these elements span more effectively, allows the surface to be thinner (2in thickness), stiffens the roof, and creates an effective framework for the clerestory skylights. (Figure 5.1.7)

MAKING MATTERS

Ferrocement is a prefabrication process by which thin wire mesh is folded over a jig to create a portion of the larger shell. The mesh is packed with cement so when it cures it acts as a precast structural element, albeit one that is more highly articulated in form than typical poured precasting methods. But how did it work, exactly?

To Do: Look at other projects by Nervi (Palazzetto dello Sport, etc.) or the de Menil Museum by Renzo Piano Building Workshop and consider how the ferrocemento system affected the choice of the overall form and the ability to construct the projects with such complexity.

To Discuss: Why were the precast elements shaped in the way they were? How did the shell maintain monolithic behavior when it was built from small pieces?



Figures 5.1.7a and 5.1.7b Axonometric view of the Turin exhibition hall from below (Illustration: T. Leslie), cross-section of a typical roof module, and interior view of the space

Short Vaults: Evaluations

The shell surface can resist some bending stresses and asymmetrical loads because it is made of reinforced concrete, but it's wise to minimize them or to stiffen the surface in other ways. One strategy to stiffen short vaults is to create regularly spaced, deeper arches underneath the shell surface—so the shell spans between the arches like a slab. The surface can also be stiffened by curving the surface between the arches perpendicular to their span, but this requires special methods of construction such as the centuries-old method of fabric-casting. (Figure 5.1.8)

Our 100ft span is on the outer range for economics, particularly when you consider that it transfers its thrust to the top of our support walls, creating a high overturning moment. Short vaults generate higher thrust forces because they have a span parallel with the arch profile, so they have more area to support, therefore more load, and therefore more height. Reviewing the formula for thrust in arches (from Chapter 2.0) we know that load, height, and span all contribute to higher thrusts, although span is the controlling factor.

Long vaults have much less thrust, are lower in height and may be easier to construct in series.

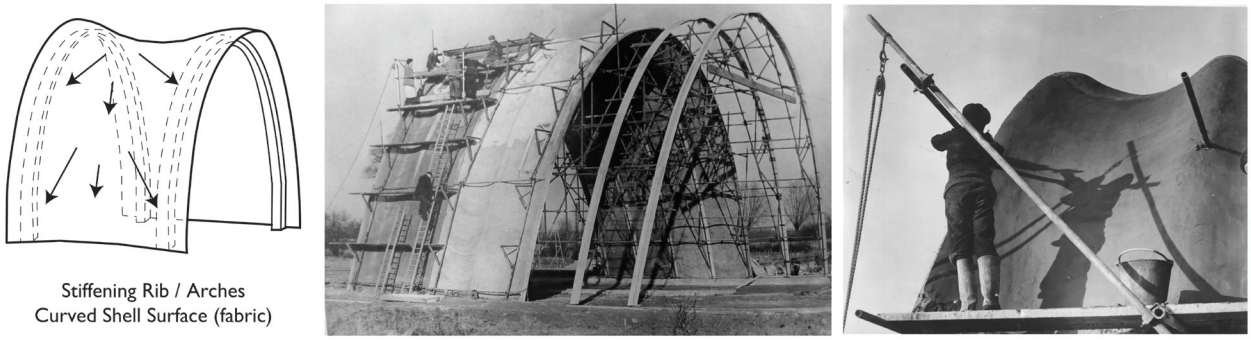


Figure: 5.1.8 James Waller's Ctesiphon shell design strategy and construction method (approximately 1950)

Long Vaults: Rules of Thumb, Form, and Basic Behaviors

Long vaults' curves run perpendicular to their spans, and they are supported on their two ends. In spite of the arched curvature, these shells' ability to span has little to do with the behavior of an arch. Instead, they behave like irregularly shaped beams. Long vaults are economical for spans between 40ft and 150ft (12m–46m) so they'd be a good match for us. Their ratio of span (length) to height is between *l*/10 and *l*/15. For example, a 100ft (30.5m) spanning long vault (like we have) would be between 7ft and 10ft (2m–3m). This creates a more modest interior volume than the short vault.

The stresses that develop in a long vault resemble the bending stresses found in beams: compression at the top, tension at bottom with highest stress levels in the middle of the span. Their stress diagrams look like those of arches and hanging cables with maximum intensity (represented by close spacing of lines) at the middle of their spans. These stresses act perpendicularly to each other, causing shear stresses that the surface of the shell resists like a diaphragm. Although this is similar behavior to a beam, the stresses here are being resisted by membrane action, so the thickness of the shell is still quite modest in comparison.

The long vault's cross-sectional curvature doesn't need to match the arch's funicular shape to be an effective spanning member. We can use elliptical, cylindrical, or cycloid curves. It simply can't be too flat. To maintain its load-transmitting effectiveness, the vault just needs to remain a curved, stiff, and stabilized form. This requires stiffening elements on the ends and sides to keep it from deforming or flattening out, typically with additional arches or shear wall *tympanums* underneath the shell. Without the stiffeners, the vault will flatten and fail.

The outer edges of a long vault *also* need to be restrained from deforming under stress. One strategy is to place the long vaults side-by-side in a series. Thrusts from adjacent vaults cancel each other out, leaving only the ends to be stiffened from outward thrust. These outer edges are often stiffened by extending a flat portion of the shell outward to act like a horizontal beam. This beam collects thrusts from the outermost vaults and transfers these into stiff vertical supports on the ends. Long vaults' economy is also increased when we place multiple shells side-by-side because the formwork can be reused. (Figure 5.1.9)

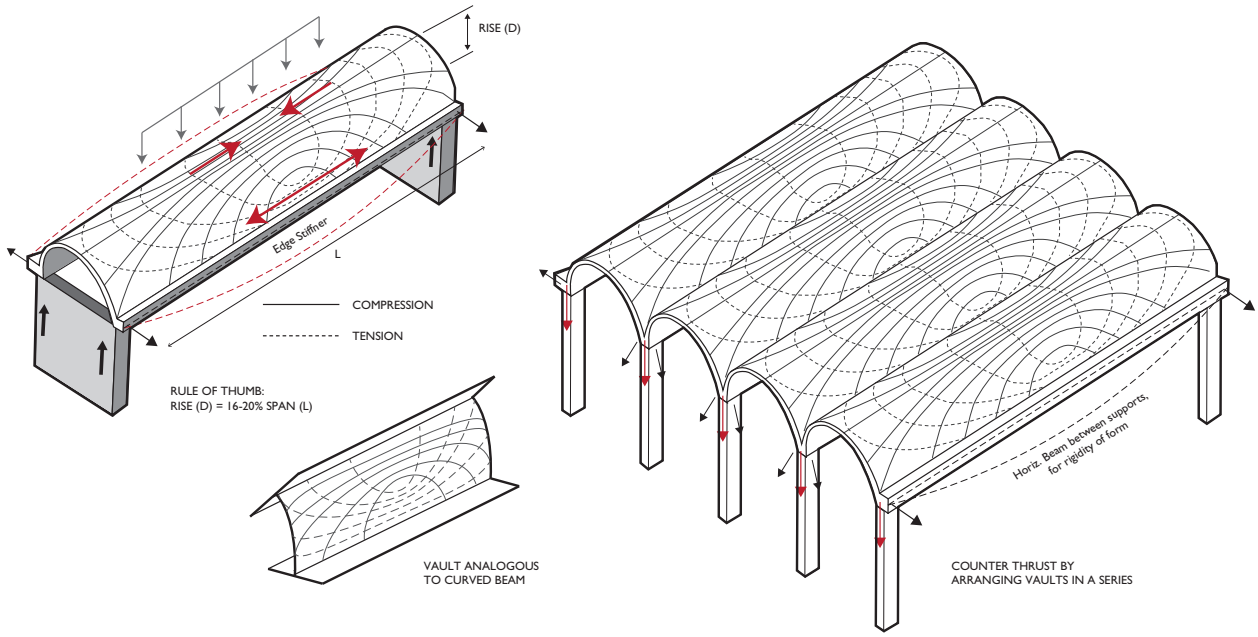


Figure: 5.1.9 Long vault stresses and form restraining strategies

LEARNING BY MAKING: PAPER VAULTS

Using only chipboard, create a curved structure that can span 18in (45cm) in one direction and support 2.5lbs (1.13kg) with minimal deformations. Stiffen and stabilize the form. Once it is stable, cut holes in the surface to see how discontinuity of the surface changes the overall performance. Stiffen around the openings as needed to maintain rigidity of the form.

Long Vaults

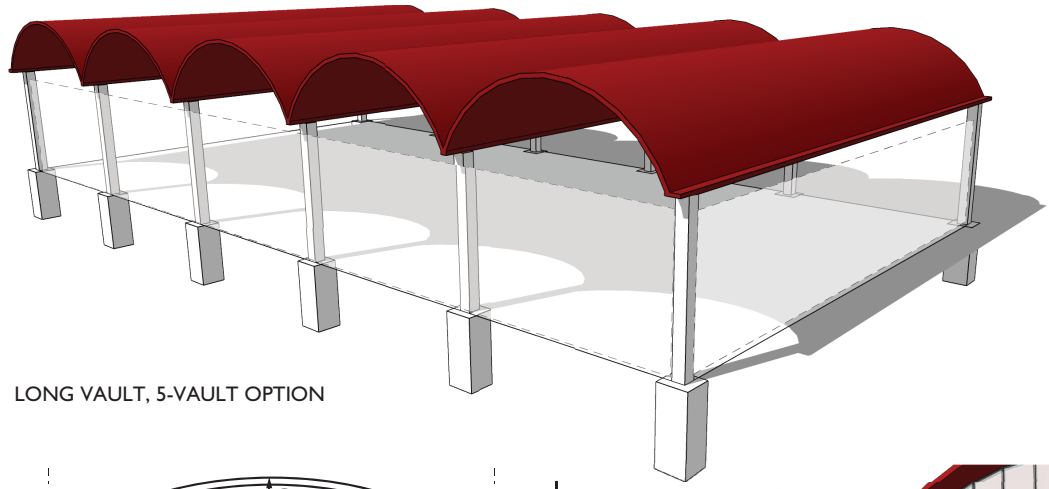
Long vaults' widths and heights have clear advantages: lowered roof heights, less outward thrust, and easier means to stiffen them. These can be enhanced if the roof is from multiple long vaults, repeated side-by-side with equal widths across the building's length. How do we determine this width? Because the long vault behaves like a beam and not an arch, this width isn't dependent on funicular forces. Instead, we can look at functional criteria. If we assume our vaults are 20ft (6m) wide each or 40ft (12m) wide, we can center the vaults over the overhead doors and still leave room for supports and stiffeners on vaults' ends.

First, we should determine the height of the vaults to make sure it can span across the space, since its capacity is determined by its height, like a curved beam. The width-to-height proportion of the vault can be established based on its transverse geometry (e.g., a 2:1 proportion of a circular arch) or a basic rule of thumb for more shallow vaults (e.g., a 3:1 ratio). There are a range of acceptable proportions because the vaults can be designed with different shell thicknesses and reinforced to respond to the bending stresses in different configurations. If we choose the shallower 3:1 proportion to reduce the amount of volume to condition, the 20ft (6m) wide vaults will rise 6ft 8in (2m) and the 40ft (12m) wide vaults will rise 13ft 4in (4m). We can check these heights against the longitudinal span for reasonable height-to-span ratio—typically between $\frac{1}{10}$ and $\frac{1}{18}$th of the longitudinal span. These proportions concur. (Figure 5.1.10)

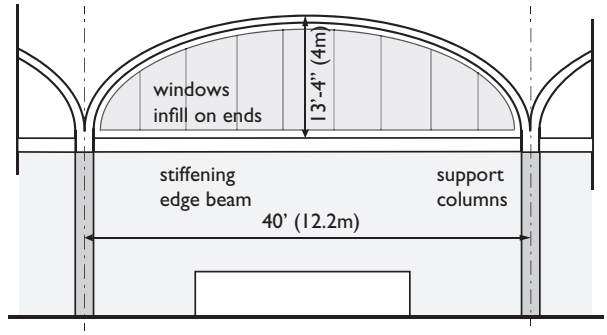
We can compare these vault proportions to a precedent with similar dimensions: Louis Kahn's Kimbell Art Museum in Fort Worth, Texas. This building uses a series of sixteen vaults that are each 100ft (30.5m) long by 20ft (6m) wide—similar spans to the option we've developed. The Kimbell's vault height, derived from a cycloid, rises 7ft from the edges—again similar to our results. These shells have a uniform thickness of less than 6in (15cm) and they are stiffened and supported on their short ends with concrete columns and arches. Along their long edges, they have a concrete channel between vaults and a horizontal extension on the outer vaults. But the Kimbell vault's look different than one might expect because of 3ft (1m) skylights that run down the centers of each vault. This seems counter-intuitive if you assume the vault transfers forces like an arch—the top of the arch is the essential keystone—but since this is a long vault it acts like a beam instead. This was an effective structural and architectural strategy; it centered the light within the space from above and it didn't adversely affect the load-bearing capacity. (Figure 5.1.11)

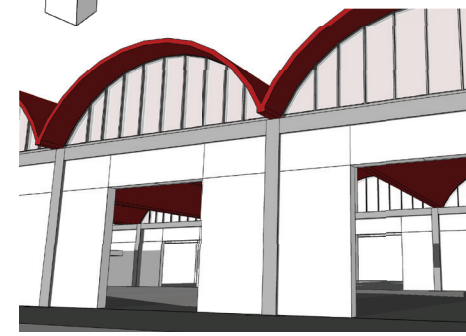
Both long vault options seem viable if the vaults are appropriately stiffened. The obvious difference between the two is the number of vaults (five versus ten) and the height difference. The number of vaults can be decided architecturally (as a formal issue), environmentally (the volume of air to condition), economically (building ten smaller vaults would be more expensive than building five larger vaults), and even structurally. We know that the vaults with less height will have a lower section modulus (analogous to shallower beams) and would therefore be less effective in resisting bending through form. They would have to resist higher levels of stress. The shorter height vault would need additional thickness and more reinforcing.

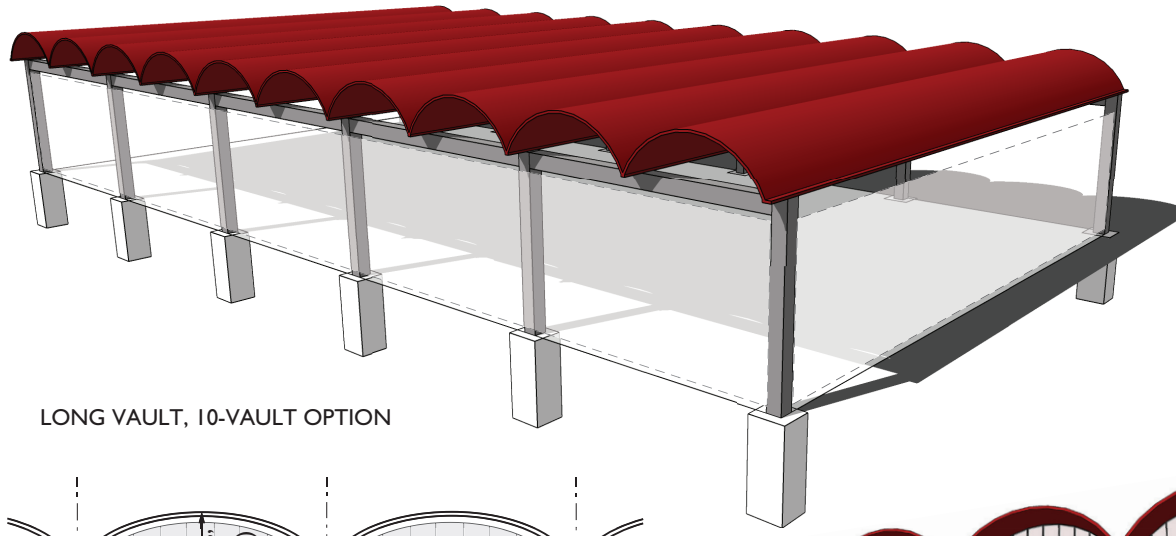
We could also create a digital simulation if we wanted a more detailed analysis to examine the distribution of stresses and deformations of the vaults. Like other digital analysis tools, this takes a set of indeterminate calculations, based on a parametric set of conditions, and converts them into a more accessible, visual representation. These show us that the stress' intensity is higher in the lower vaults (as a result of less



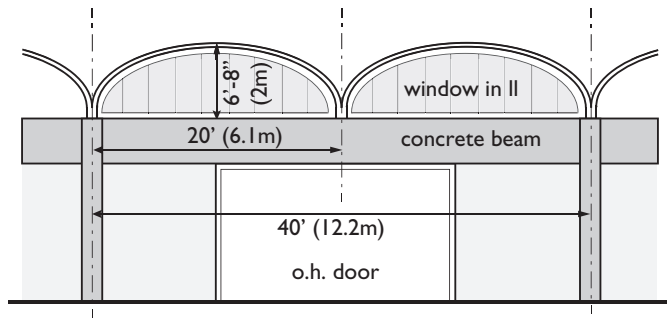
LONG VAULT, 5-VAULT OPTION

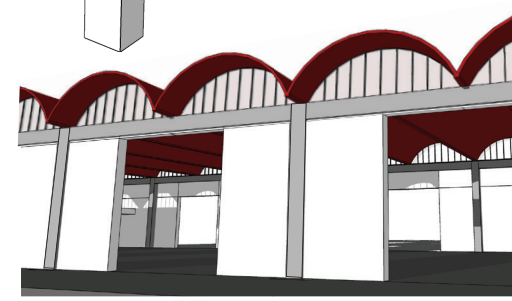






LONG VAULT, 10-VAULT OPTION





Figures 5.1.10a and 5.1.10b Long vault design options and dimensions

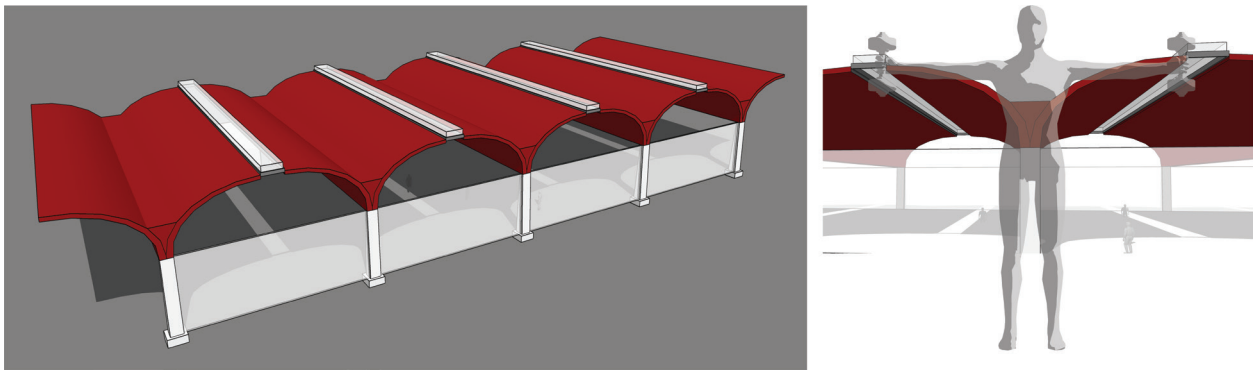


Figure 5.1.11 The Kimbell vaults were designed as symmetrically curved, Y-shaped beams supported in the middle on each end with a skylight in-between—like two arms reaching out

height) but that the larger vaults have a higher risk of displacement at the edges (particularly if unbraced). Both options seem structurally viable. Fewer vaults will have less displacement and more consolidated areas for stress distribution. (Figure 5.1.12)

The two schemes would need to be supported differently. The 40ft (12m) vaults could rest directly on columns, but the smaller 20ft (6m) vaults would need to be supported on a horizontal beam running between supports. This beam could also stiffen the ends of the

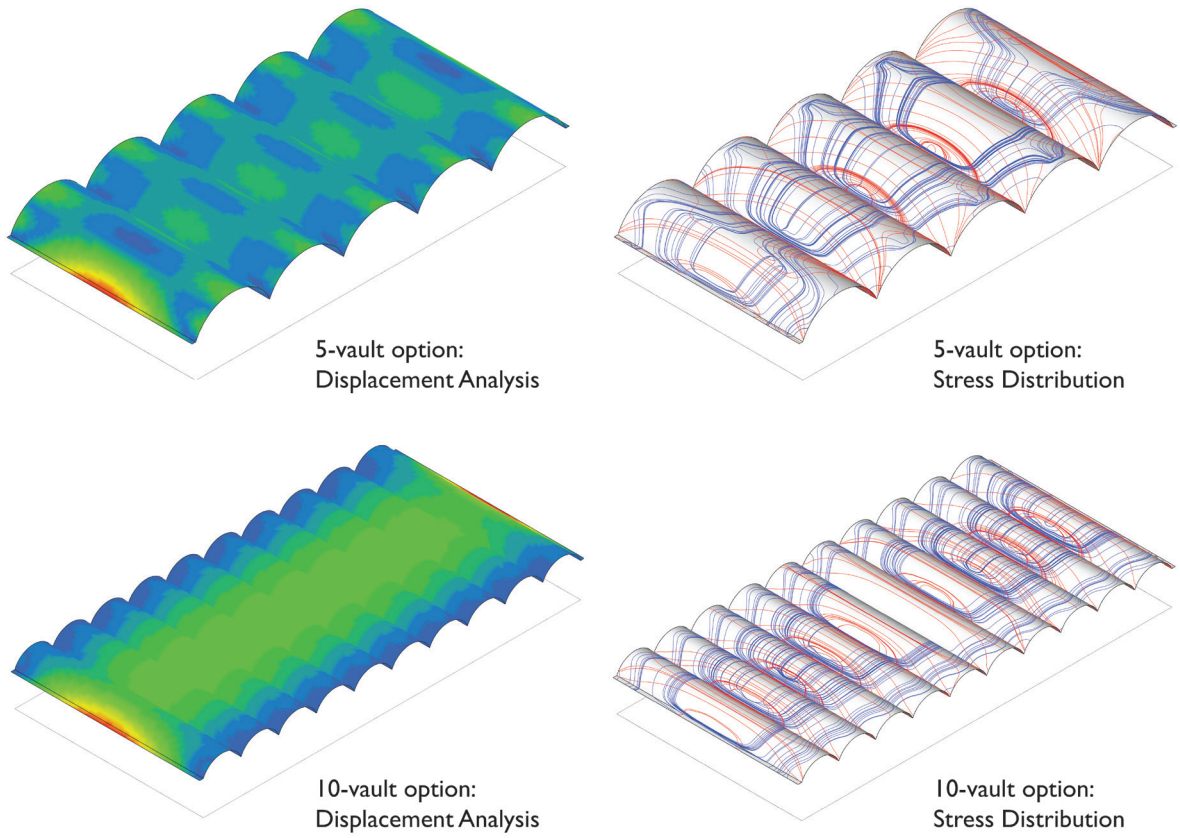


Figure: 5.1.12 Comparing force flow/stress distribution and displacement in both vault options

vault to help hold its shape and to allow daylight in from the east and west ends. The higher vault scheme would allow deeper light penetration, but neither of these roof designs would bring in a large amount of filtered daylight from above, like the program desired.

Also troubling is that these curves would funnel rainwater into the valleys between the vaults. There are ways to create a roofing insulation system that would divert this water to the vaults' edges, but this midwestern location also gets snow. If snow builds up on one side, it can create an asymmetrical point load on the curved vaults—a potentially dangerous loading condition that would need to be accounted for in the shell's thickness.

FOLLOW THE FAILURE: VAULTS AND SHELLS

What happens when a surface-resistant structure loses its stiffness and ability to transmit loads? Study the collapse at Terminal 2E, Charles de Gaulle Airport, Paris, France, (2004). Determine why the project failed and how its failure was related to form, materiality, assembly issues, or other factors. Was it intended to behave like a "shell" or a series of curved "beams"? What would you have done differently? Use drawings and images to support your conclusions.

Modified Long Vaults: Daylighting Options

A solid long vault has liabilities. If we modify the vault to introduce daylight from above, we affect its structural behavior—one of the dangers in a shell design is discontinuity in its surface. The most basic, albeit inelegant solution, involves just cutting holes for skylights into the shell surface. These openings would create localized stresses around them, so ideally, we would space them within a logical pattern of deeper gridded supports in the vault, like a curved waffle slab. Any modifications to the singular thickness of the surface would make forming and casting the vault more expensive.

We can explore other sectional shapes for the vault, besides an arch, as long as it is curved or folded, and stiff. One option is to create S-shaped "check mark" profiles for the vaults. In this section, clerestory windows bring light in along the length of one face for each vault—in our case facing north to bring in indirect light and to refract southern light, a common daylight strategy in industrial roof designs. (Figure 5.1.13)

Although the surface is curved, it isn't double-curved or continuous in two directions, so the roof would behave more like connected curved beams. Each vault

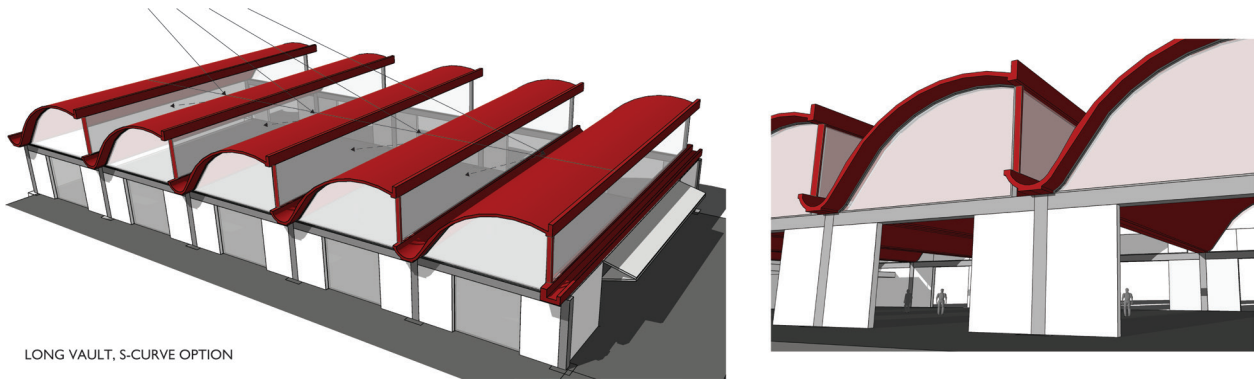


Figure 5.1.13 Modifying the section from continuous arch into an irregular S-shape allows for indirect northern daylighting through one side of the long vault

Figure 5.1.14 Industrial ship factory interior (Torroja, Naves Para HYTASA, Seville, 1961)

would need to be thicker and would need to be stiffened along its ends. It would also need to be supported on its four corners. This discontinuity produces anomalous behavior in the shell—so much so that physical models were often used to test these types of vaults in pre-computational engineering. (Figure 5.1.14)

Dieste and Brick Gaussian Vaults

We could keep these advantages while improving the structural performance by creating double-curvature in the vaults' surfaces. One option would be a mathematically defined shape, called a Gaussian curve, that curves in both the transverse and longitudinal sections, or eyelid-like arched waves that pop up through a flat surface. This is a highly efficient form that can span 180ft (54m) with only a few inches of material thickness.

These vaults act primarily in compression; the controlling curve (*directrix*) is a catenary arch, an ideal compressive form. Its primary form is the "short vault" catenary arch that spans across the space, but in cross-section it is made from S-shaped cross-sectional curves with varying rises. The vault's depth is highest at the center of its span, but diminishes at the ends, creating a large opening for natural light along one side. Unlike other short vaults, Gaussian vaults have a shorter height-to-span ratio (approximately 1:8–1:10) because they are stiffened by curved surfaces in two directions. Although the primary loads are axial (and resisted by the thickness of the surface, the S-shaped form stiffens the vault, resisting any bending caused by wind, rain, etc. One liability to the form is the thrust generated at the ends, because the overall form is arched. (Figure 5.1.15)

Eladio Dieste (1917–2000) was a Uruguayan engineer, architect, and builder who spent his career designing and constructing Gaussian vaults for utilitarian buildings like grain silos, markets, and factories. Dieste addressed thrust with thin exposed steel tie-backs on the interior of

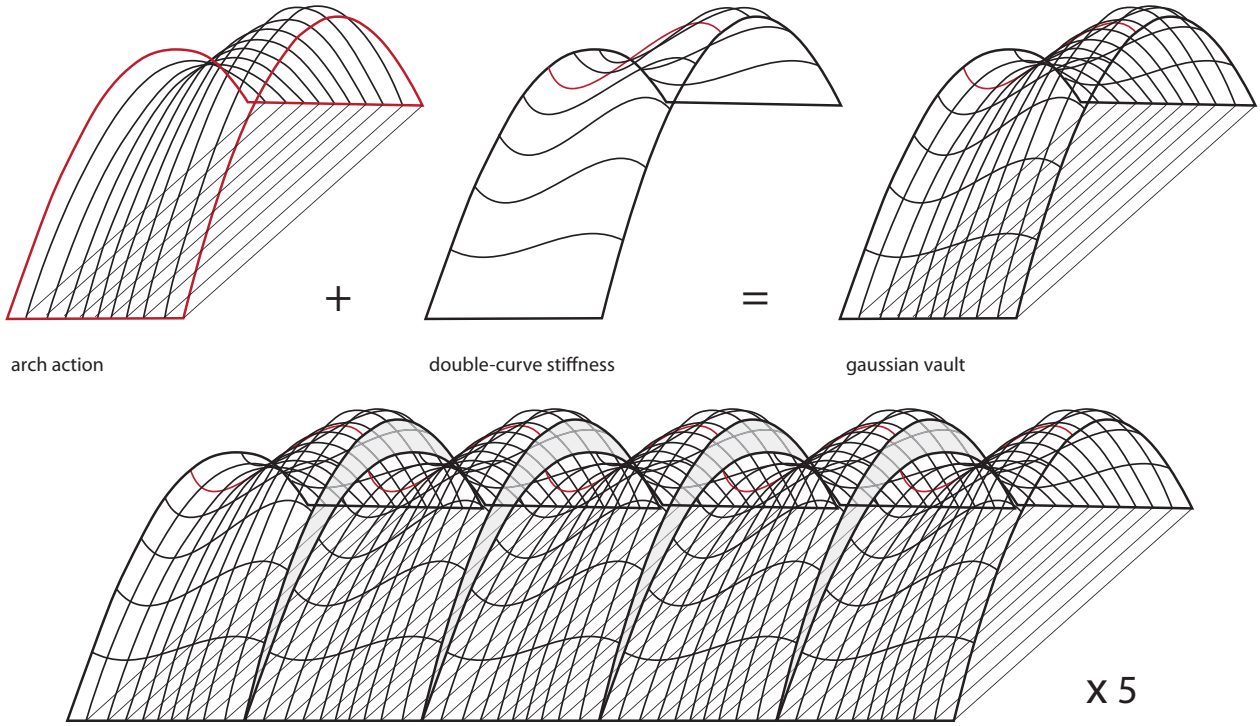


Figure 5.1.15 A Gaussian curve combines the form and benefits of arches in one direction with double-curved stiffening in the other direction

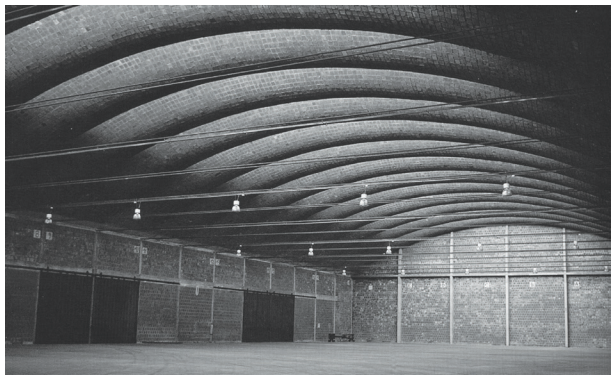
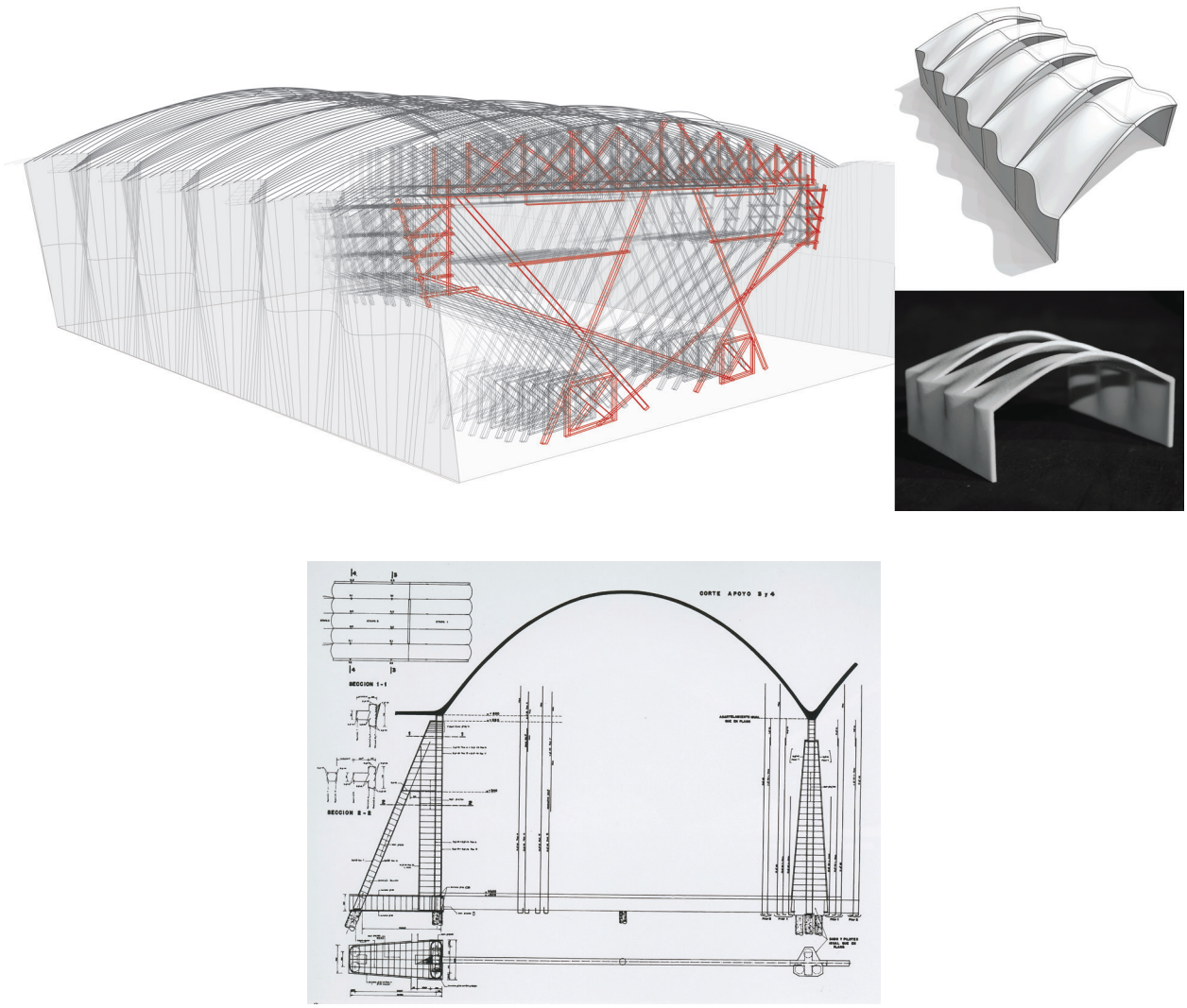


Figure 5.1.16 When arrayed, the vaults create indirect light that reflects off curved surfaces

the building. His shells were constructed from readily available and affordable brick and terra-cotta tiles instead of concrete. Bricks were more efficient because they weighed less than concrete, which meant that his vaults carried less weight and therefore needed less thickness. Their small modular size also improved the construction process: Bricks were laid upon formwork in a stacked bond pattern, spaced with ¾in (20mm) gaps between. Then a reinforcing mesh was placed between the joints, and the joints were filled with a cement/course sand mix (1:2.5 ratio). A topping of the same mixture was added atop it all to create continuity. (Figure 5.1.16)

Using bricks sped up construction because, unlike concrete vaults, most of a brick vault's surface area is already dried—the brick absorbs moisture and starts to stiffen immediately. Dieste could remove formwork only a day or two after setting the sand/mortar mix between the tiles. This rapid curing meant that these reinforced brick vaults could be built quickly. In traditional concrete shell construction, it takes time to set up and take down formwork and weeks for the concrete to cure. To save time, resources, and money, Dieste created a movable formwork that could be lifted up and down and rolled along the axis of the building. As soon as one vault was cured the formwork could immediately move to the next vault. Dieste pre-stressed the vaults by pinching together reinforcing wires set across their tops; by pulling them together and pre-stressing the vault's form, he was



Figures 5.1.17a and 5.1.17b Church of Christ the Worker (Dieste, 1960). Illustrations: (left): Formwork was movable, allowing the one vault to be set then lowered and moved over, (right): Digital design and rapid prototyping 3D printed model of the surfaces (Michael Sharman). 5.1.17b: By matching the form with the forces, Dieste's vaults can span long distances with minimally thick surfaces using edge stiffeners, buttress, and tie-backs at the edge (Dieste)

able to reduce tension in the vaults. The wires were then covered by a thin concrete screed. (Figure 5.1.17)

Lamella Structures and Braced Barrel Vaults

Continuously curved, monolithic vaulted surfaces made of concrete are difficult to construct, but *lamella vaults* offer a good alternative. A lamella vault takes advantage of curved vault behavior but it is easier to construct because it is made with smaller, intersecting straight elements. These short members are connected in a diamond-shaped pattern to make a vaulted form from intersecting skewed arches. These are still developable surfaces, but instead of being smoothly curved, they are made from a series of straight lines. Lamellas can also be curved on their ends into an ellipsoid volume.

Lamellas behave like other vaults: in arch-action when supported on their sides, in beam action when

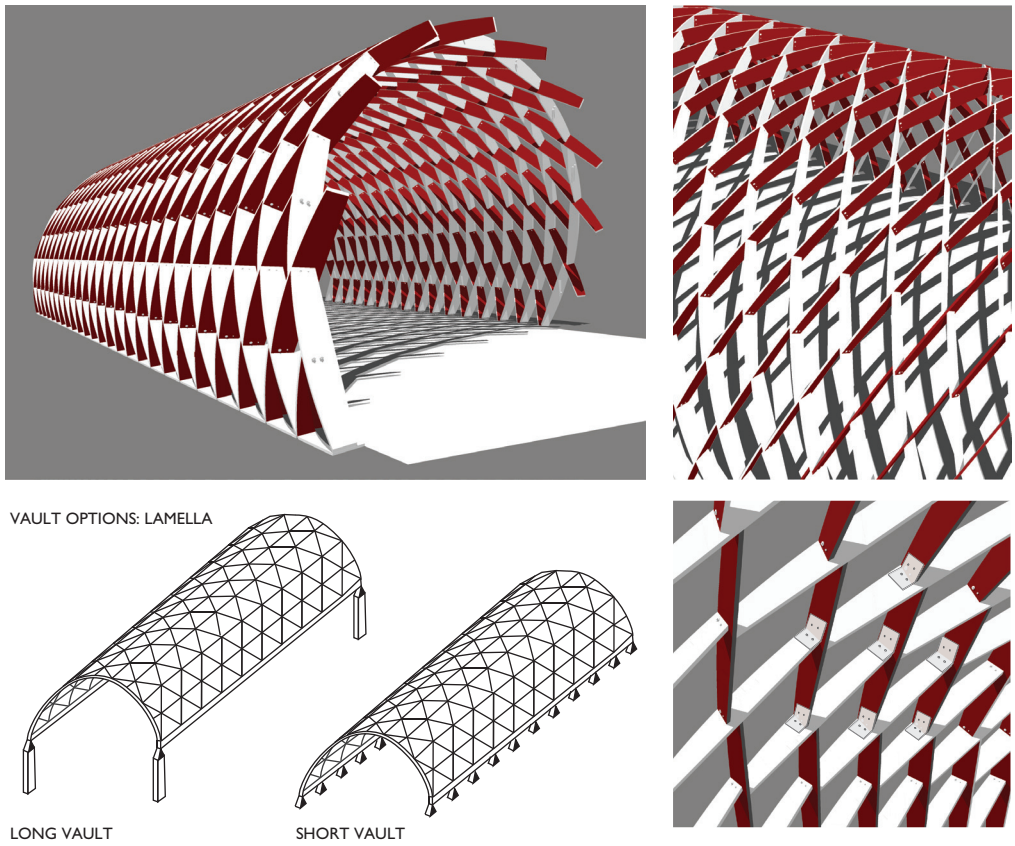


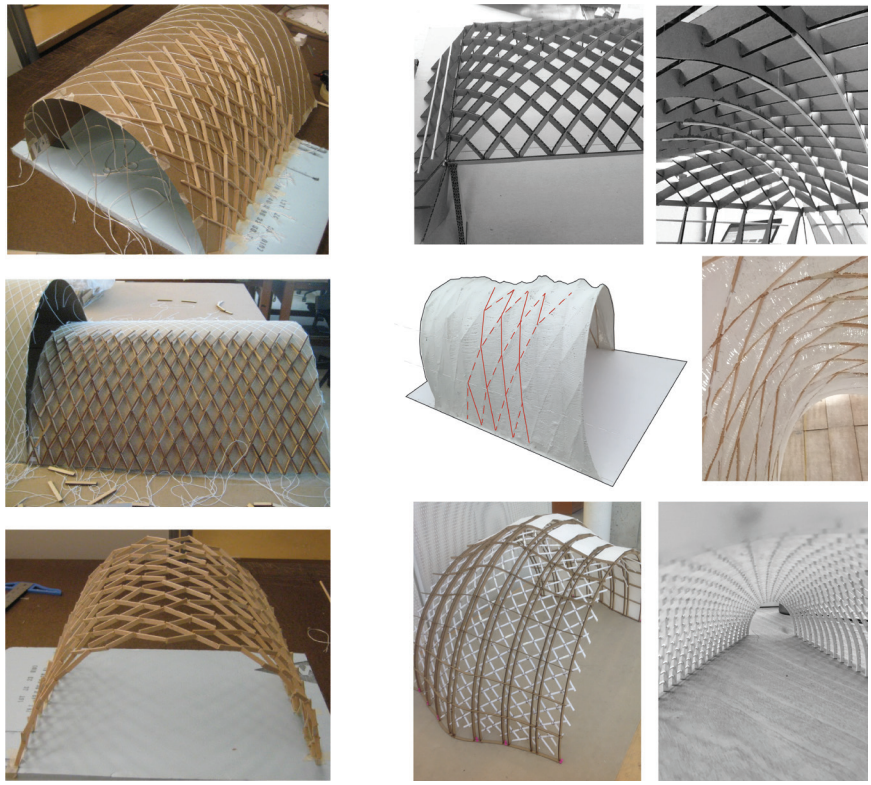
Figure 5.1.18 Smaller straight members can be overlapped and combined together in an arched vault form with a diagonal (or lamella) configuration

supported on their ends, and in both when supported on their four corners. Because they use smaller components throughout, there are more joints to secure throughout the structure—and therefore more potential weak points. Components can be assembled in the field—bolted together with brackets or assembled as larger sections of multiple prefabricated elements. Finding a material's limits, including ways of curving and prefabricating, can create elegant, contemporary design opportunities. (Figure 5.1.18)

MAKING MATTERS: LAMELLAS

Find examples of the earliest long span wooden lamella structures. Why was wood the preferred solution for these structures? Look at the three specific details of the structure: How are the straight-line boards staggered and arranged to form the lamella pattern? How are these pieces connected? What is the detail at the base (and how does this relate to fixed or pinned arches)?

To Discuss: Wooden lamella structures are relatively low-tech. Would they be easy or difficult to construct? Cite specific conditions of construction to defend your answer. (Figure 5.1.19)

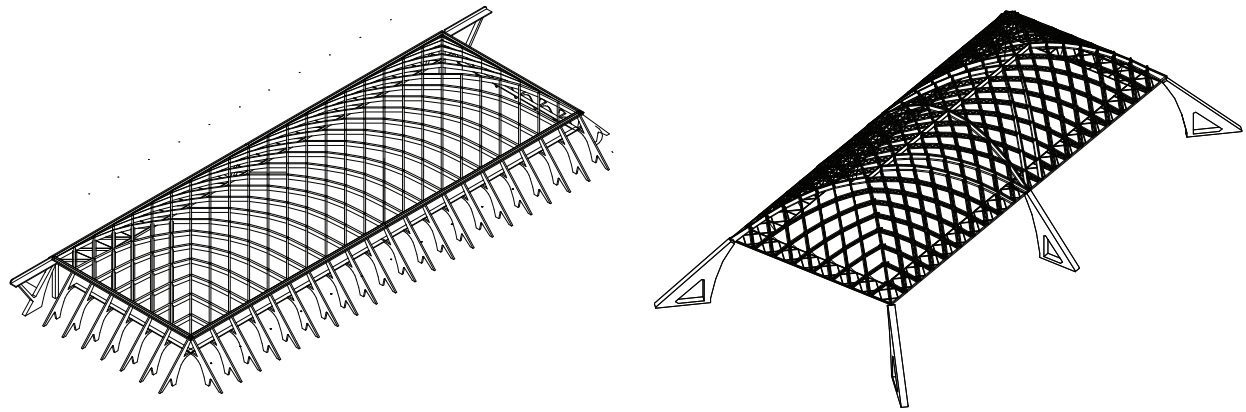


Figures 5.1.19a and 5.1.19b Student lamella experiments (SxD, Iowa State University)

Scarcity and economics clarify design intent, especially in the case of the lamella structures created by Nervi. The efficiencies and expressiveness of the work derived from shortages in labor, steel, and timber, as well as wartime economic and scheduling pressures in 1930s Italy. As an engineer and builder, Nervi understood that efficiency in shells was relative. Instead of selecting forms based on optimal structural geometry, he opted for forms that were "good enough" (like domes and vaults), especially if simplifying the form made building it more effective and economical.

In 1935, Nervi designed a cast-in-place lamella structure for airport hangers in Orvieto, Italy (1935). The 330ft × 135ft (100m–41m) structures looked like ellipsoid waffle slabs with supporting buttresses on three sides and large openings for airplanes on the fourth side. He used space framing principles to arrange the deep lamella members with shallower diagonal voids between them. The redundancy of resisting elements and the monolithic nature of concrete allowed loads to travel in many paths in case of bomb damage.

It was a daring departure from traditional shells in the 20th century, but four years later when Nervi submitted a proposal for a second round of hangars at Orbetello (1939–1942), he envisioned a similar form but with a new construction method. Instead of casting them as singular forms, these vaults were made from smaller precast concrete lamellas. Because of the shell's regular geometry, these pieces could be cast off-site and erected in place without heavy cranes. The precast pieces weighed less than the cast-in-place version so the structure carried less weight, meaning less material and labor. Scaffolding was dramatically reduced as only one portion of the building needed to be supported at a time. The resulting building improved upon the previous hangars and became the basis for the building's celebrated aesthetics. (Figure 5.1.20)



Figures 5.1.20a and 5.1.20b Comparing the form, supports, and relative solidity of the Orvieto (left) and Orbetello (right) lamella vaults (T. Leslie)

Folded Plates: Behavior and Patterns

As we've learned from our paper-folding experiments in Chapter 5.0, a folded, fan-like surface of adjoined planar surfaces can be a strong and efficient load-bearing surface structure. Folded plates are the real-world manifestation of this idea. They are more efficient than adjacent tall girders because they get significant structural advantage by joining adjacent plates to one another. Rigidly connected folds at the ridges and valleys help stiffen the structure so it can resist stresses in the most advantageous way—within the plane of the surface. When cast in concrete, the rigidity of the folds and concrete's monolithic nature allow folded plates to span up to 200ft (61m). Folded plates have many of vaults' advantages without the disadvantages associated with constructing curved surfaces. (Figure 5.1.21)

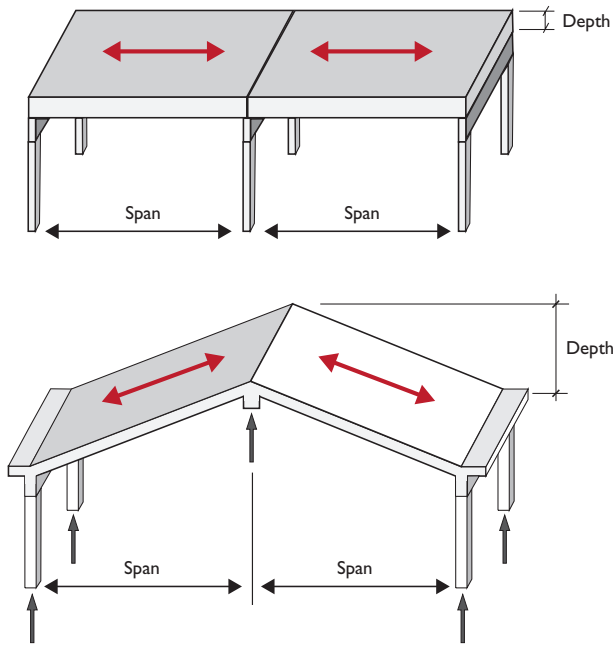
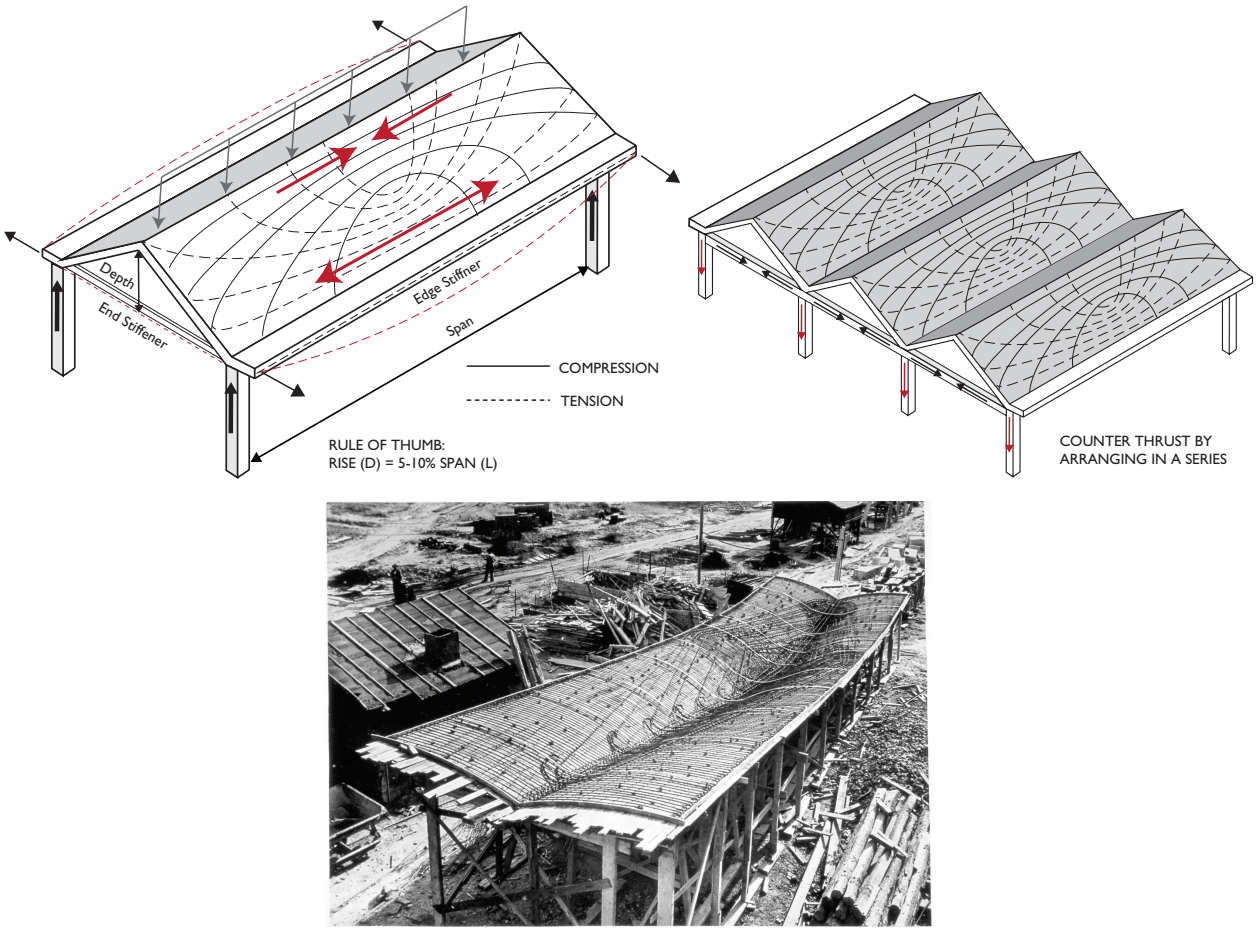


Figure 5.1.21 When folded, and rigid, the fold acts as another support point to reduce span. Additional height of plate makes it effective in transferring loads, like a beam

Longitudinally spanning plates should have a depth between 1/10 and 1/18th of the overall span. These plates' thickness and their reinforcing can vary depending on span and loading. They act like beams, so in the longitudinal direction folded plates have to resist compression along their top edges and tension stresses along their bottom edges. Folds at the ridges and valleys are subjected to bending forces as they transfer their loads to the supports along the length.

Like long vaults, folded plates can be arranged repetitively side-by-side. Their proportions and how they are placed establishes a compositional pattern that affects interior space and exterior expression. As long as the surface remains stiff and rigid, these planes can be configured in many ways and still maintain their structural efficiencies. Paper folding is a quick way to understand the relationships between formal geometry, materiality, and structural performance for a variety of surface structures. (Figure 5.1.22)



Figures 5.1.22a and 5.1.22b Folded plate stresses and rules of thumb. 5.1.22b: Accurate placement of reinforcing in locations with anticipated tension stress (Torroja, Zarzuela prototype, Madrid, 1935)

There are several applicable folding patterns: Parallel folds, Yoshimura patterns, and herringbone patterns. In these structures, straight and reversed folds are made to convert flat surfaces into triangulated planes that form bearing surfaces. Being able to manipulate the number of folds and the positions of ridges and valleys is advantageous for designers that want to use the surface as part of a larger strategy (e.g., formal aesthetics, correspondence with interior functions, daylighting, storm water management, passive ventilation, etc.). These options can be easily prototyped and evaluated by folding paper—pushing and pulling these folds apart can reveal options not easily uncovered using traditional methods.

MATERIAL MATTERS

Paper folding allows us to develop options for folded plate forms, but be aware of this tool's limits. Paper doesn't provide an inherently stiff surface and the folds aren't rigid. Any folded plate model will need to be held in place on its sides/ends to hold its shape under even moderate loading. Other materials are possible. Plywood or plastic sheeting are used for plate-structured pavilions because they are lighter.

(continued)

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One benefit of a folded plate structure is that it can be collapsible and deployable—ideally as a flat-pack that opens up into an enclosure. In these structures, joints are the most critical aspect—after all, they need to be flexible for deployment and then instantly rigid during construction. In a world where flimsy tents are the predominant deployable shelters, it is worth considering options for a deployable plate system as well. (Figure 5.1.23)

Figure 5.1.23 The Flexfold proposal for a deployable emergency relief shelter (Structures in Service interdisciplinary studio, Iowa State University, College of Design, 2017)

Folded Plates: Design Options

Plates could improve on the vault designs' daylighting and storm water management problems. The simplest pattern is a V-shaped accordion pattern of parallel folds. These mountain and valley folds create an extruded zig-zag line across the span. If we space the folds at 20ft (6m) on-center, as we did with one of the long vaults, we can take this rule of thumb for minimum slopes at the edges and use simple triangulation to solve for the plates' other dimensions. In this case, the peak rises only 5ft 5in (1.75m) from the support point. This system relies upon the plates to act as girders spanning longitudinally. These plates cannot be too shallow or else they will lose their effectiveness as inclined spanning members. A good rule of thumb is to maintain a minimum slope of 30 degrees. Comparing this height back to the rule of thumb height-to-span of 1:10–1:15 for folded plates, we find that this height is too shallow, so we need to adjust the angles of incline to something steeper. If we increase the angle to 45 degrees, and recalculate the dimensions, we find that the peak height increases to 10ft (3m)—within the range of compliance. (Figure 5.1.24)

The bottom folds of these plates can be prevented from spreading apart by fixing them to the supporting walls on the ends, by locating columns at each valley, or by creating Y-shaped columns centered on the ridges. Horizontal extensions can be added onto the two sides. To better control storm water, we can use one of two strategies. First, we could alternate the locations of the peaks and valleys on the east and west ends by using a counter fold scheme—instead of inclined parallel rectilinear plates, the roof would have a triangular pattern in plan that would define the plates' back-and-forth arrangement. Alternatively, we can keep a parallel fold scheme and add sloped surfaces into the valleys, like gabled roofs, that run from the peak to the edges, like a "short" span. Because this new surface interrupts the continuity of the parallel plates with a new inclined surface, this scheme would open up additional interior volume, which may be advantageous. This is the simplest of many options.

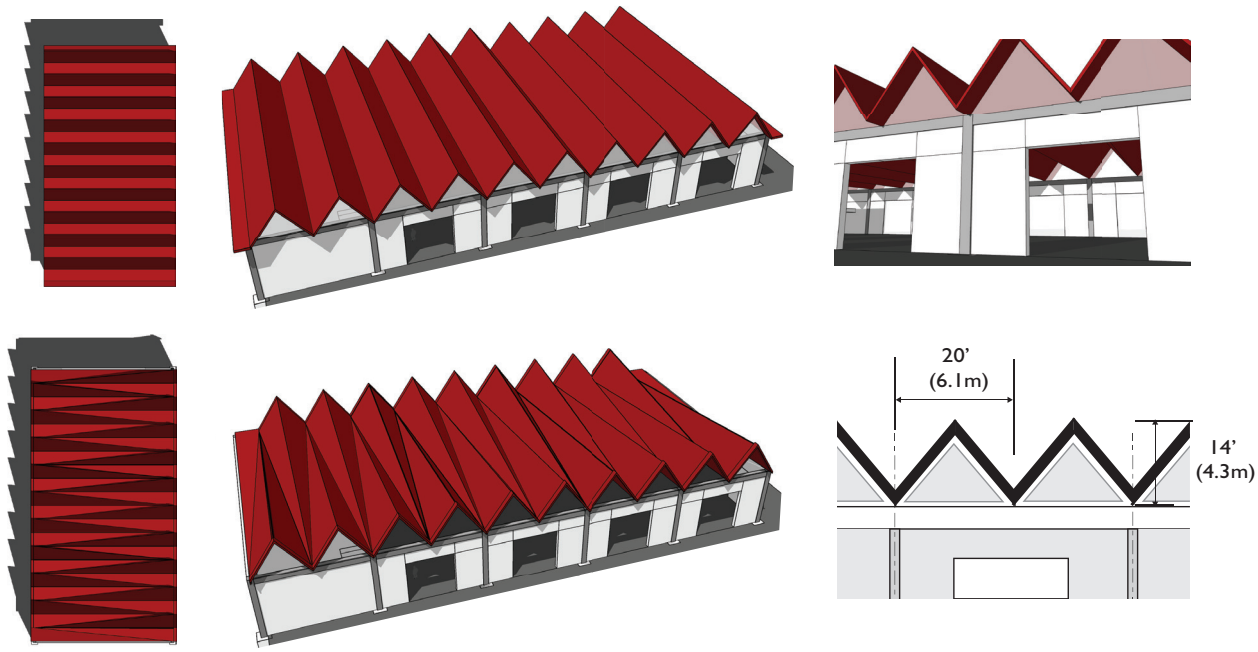


Figure 5.1.24 Flat plate basic schemes

Folded Plates: Evaluation and Modifications

Unfortunately, a parallel scheme doesn't improve on the daylighting strategy—light still only comes in from the two ends. But, unlike the difficult geometry of curved vaults that resisted modification, portions of the planar plate surface can open up with windows to allow light to come in. Like the vaulted clerestory window scheme, the folded plate can be modified in cross-section to integrate vertical windows.

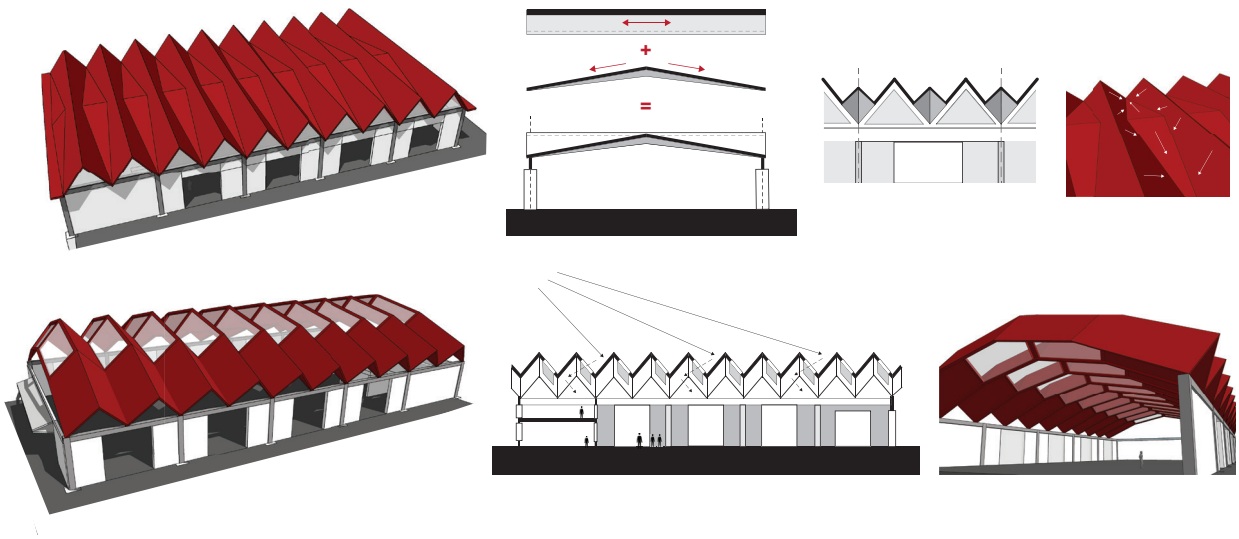
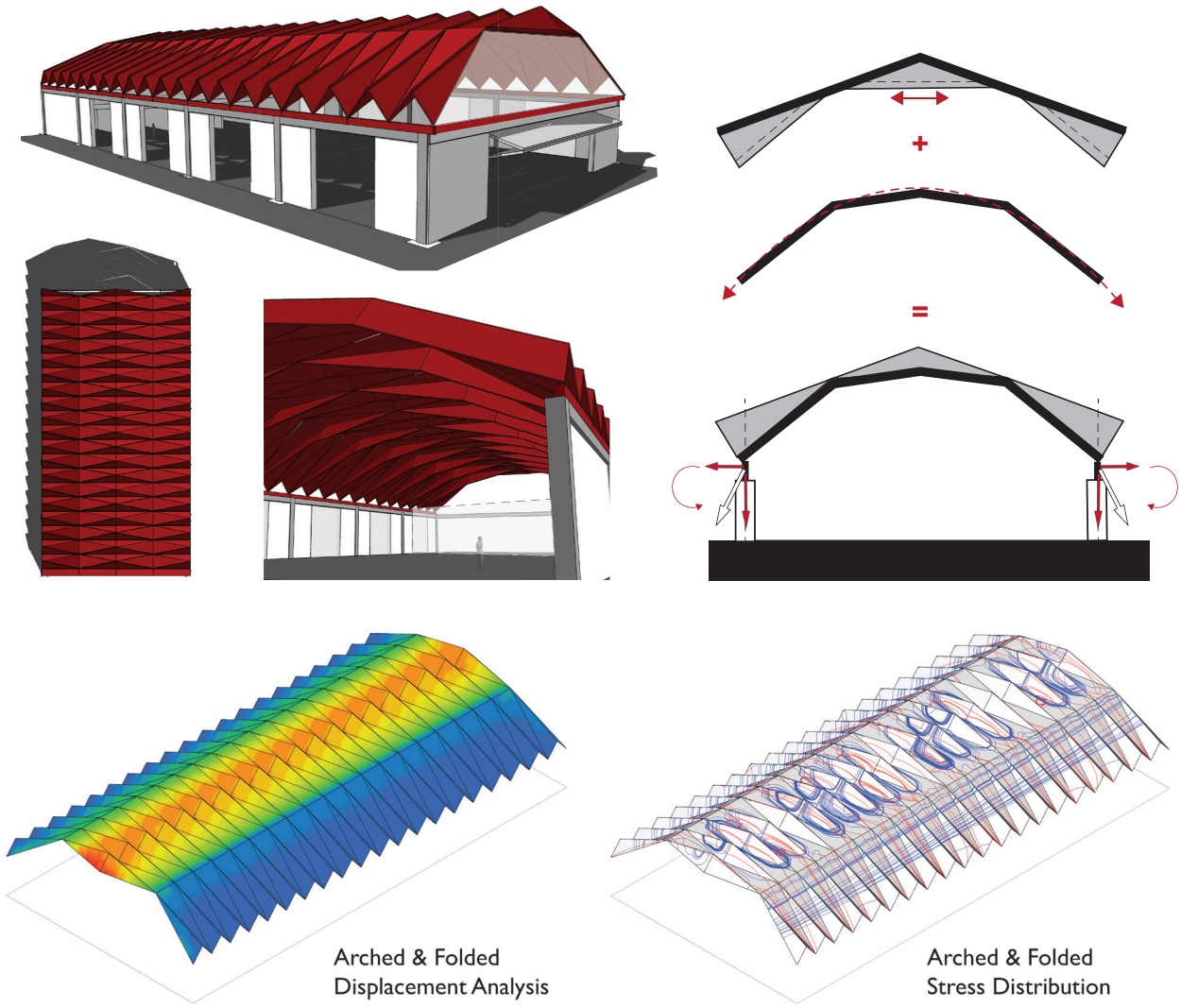


Figure 5.1.25 Modifications for environmental conditions of rain and sun. The discontinuity of shell surfaces for skylights is a potential area for problems like bending stress and large deformations

The most common arrangement for folded plate roofs is an inclined Z-shaped section. In this parallel fold arrangement, one of the planar surfaces can become a window or skylight. The Z-shape section provides stiffening to the top and bottom of the solid spanning plane. These stub walls would be useful for waterproofing and flashing. This cross-section is still a parallel fold arrangement, though it doesn't solve the storm water problem. In order to solve both issues, we could combine two of the schemes. Ideally the folded surfaces should maintain their east/west orientation, push water to the edges, and open up triangular vertical surfaces for daylighting. If the plates can maintain their continuity across the span, the roof forms used to channel water can be made of skylights. A variation of this scheme with smaller but more frequent folded plates (20ft on-center (6m)) is shown—skylights are finally incorporated into the scheme to distribute light throughout the roof. (Figure 5.1.25)

Arched Folded Plates and Frames

To increase efficiency, we can combine strategies. We can fold the roof like a long vault folded plate while

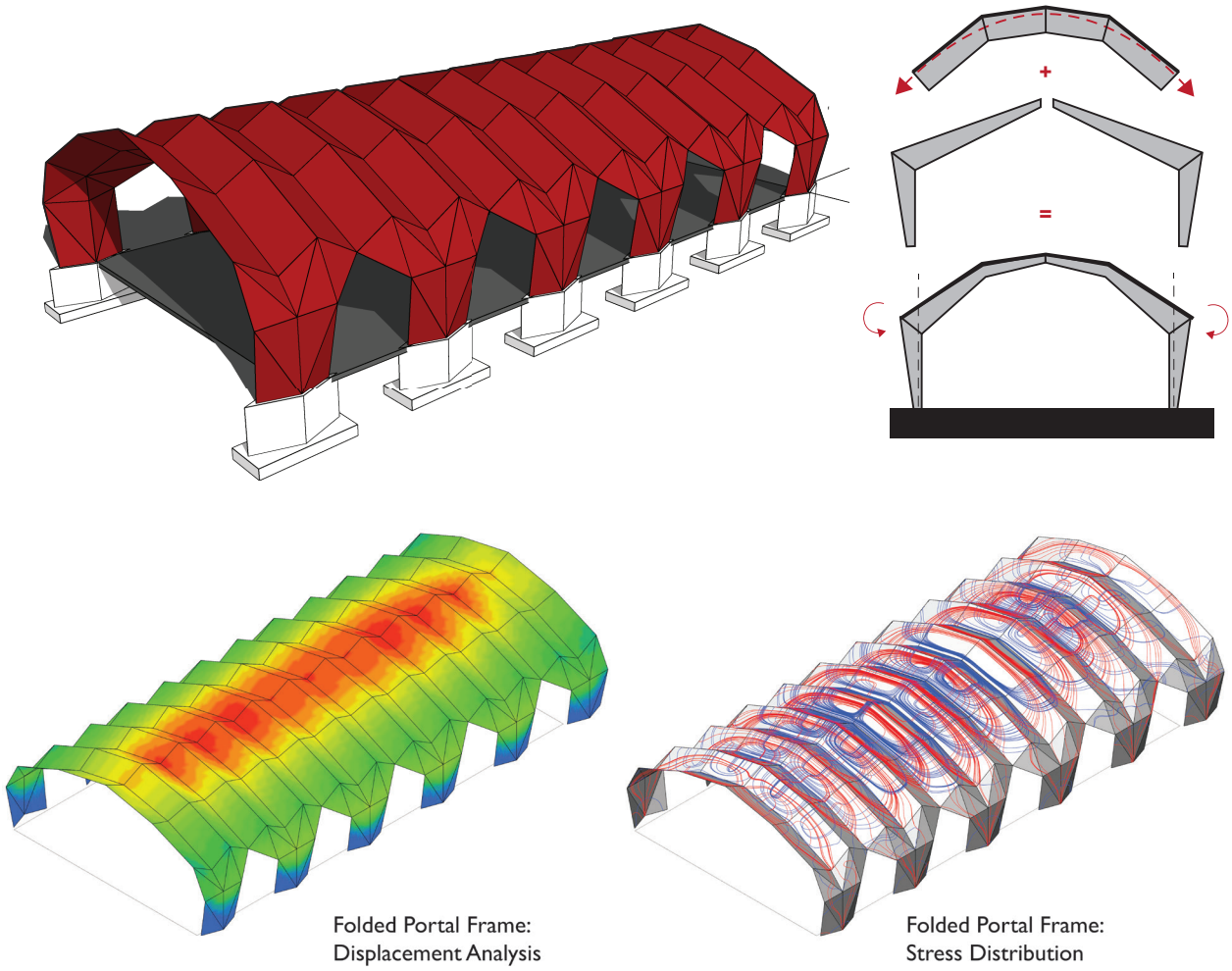


Figures 5.1.26a and 5.1.26b Increasing the effectiveness of the proposal structurally, functionally, and environmentally by arching the plate structure. 5.1.26b: Maximum intensity of tension stress and displacement occurs at the middle peak. Note the distribution of compression (red) and tension (blue) stresses along the sides

also arching the roof system, like a short vault. An arched profile delivers an advantage of force flow, but it also creates thrust at the spring points. If the overall roof form starts to look like a gabled roof (peaked in the middle and sloped to the sides) it will generate an outward thrust at its support points. This horizontal force creates a large moment force at the supports that will need to be resisted. When this occurs, a design opportunity arises that solves the stiffening problem on the ends and the thrust on the sides. By extending the folded plates down to the vertical supports, we can make the entire folded plate structure into a long span rigid frame, one that arches in one direction and folds in waves in the other. (Figure 5.1.26)

In this arrangement, the columns and spanning members are connected as a monolithic shell, so they work together to resist moment forces at the connection between the horizontal and vertical supports. A cast-in-place structure is already a monolithic moment connection, so we aren't adding to the complexity of the connections to exploit this. To increase the formal rigidity of the supporting walls, we can also fold these like the roof. The folded plates are a continuous surface simply turned down at the ends and tapered down to the ground—the forces would follow this flow.

Because these vertical folded plates align with (and adjoin) the horizontal plates' edges, both systems are



Figures 5.1.27a and 5.1.27b Combined strategies of an arched folded plate with a moment frame. 5.1.27b: Larger elements are heavier with greater displacement and higher and more intense stresses at the peaks and valleys

stiffened and stabilized. No additional edge stiffeners may be needed. The vertical plates can incline outward or remain upright and the fold can either rest in the same plane as the valley or the ridge above. These schemes folded plates as the predominant formal expression of the entire building enclosure—not just the roof. The structures look like short vaults, but because of the folding, they act like long vaults. (Figure 5.1.27)

Two folded patterns that would solve the storm water issue, allow ample daylighting, and produce interesting and efficient forms are the Herringbone pattern and the Yoshimura pattern. The Herringbone pattern has a zigzag corrugation in two directions—the arch-shaped profile goes up and down in a series of triangular planes and the transverse section alternates valleys and ridges. The pattern is made of symmetrical trapezoids that form a herringbone tessellation. There are plenty of vertical triangular surfaces atop the structure that could be to skylights. The perimeter would need to be supported by buttressed columns or the folds' spacing would have to be spread out enough to accommodate the overhead doors on the sides.

The Yoshimura pattern looks like folded diamond components connected end-to-end and side-to-side around a curved surface. The folds in the diamonds create stiffness and stresses follow stiffness—in this pattern we see that the resulting pattern of ridges and valleys looks like a lamella structure. The vertical surfaces can be made into skylights or clerestory windows across the top. The overall structure has an arched shape like a short vault, but the folds create discrete long vault fold behavior, too. Because it extends down to the vertical surface, the pattern would need to be modified for the overhead doors on the sides, but we could solve this with Y-shaped buttresses between the doors that would stiffen their edges and resolve the vertical and horizontal forces generated by the form. One of our final proposals can reflect these benefits. (Figure 5.1.28)

Simple folded plate structures aren't considerably more expensive than flat structural slabs. But

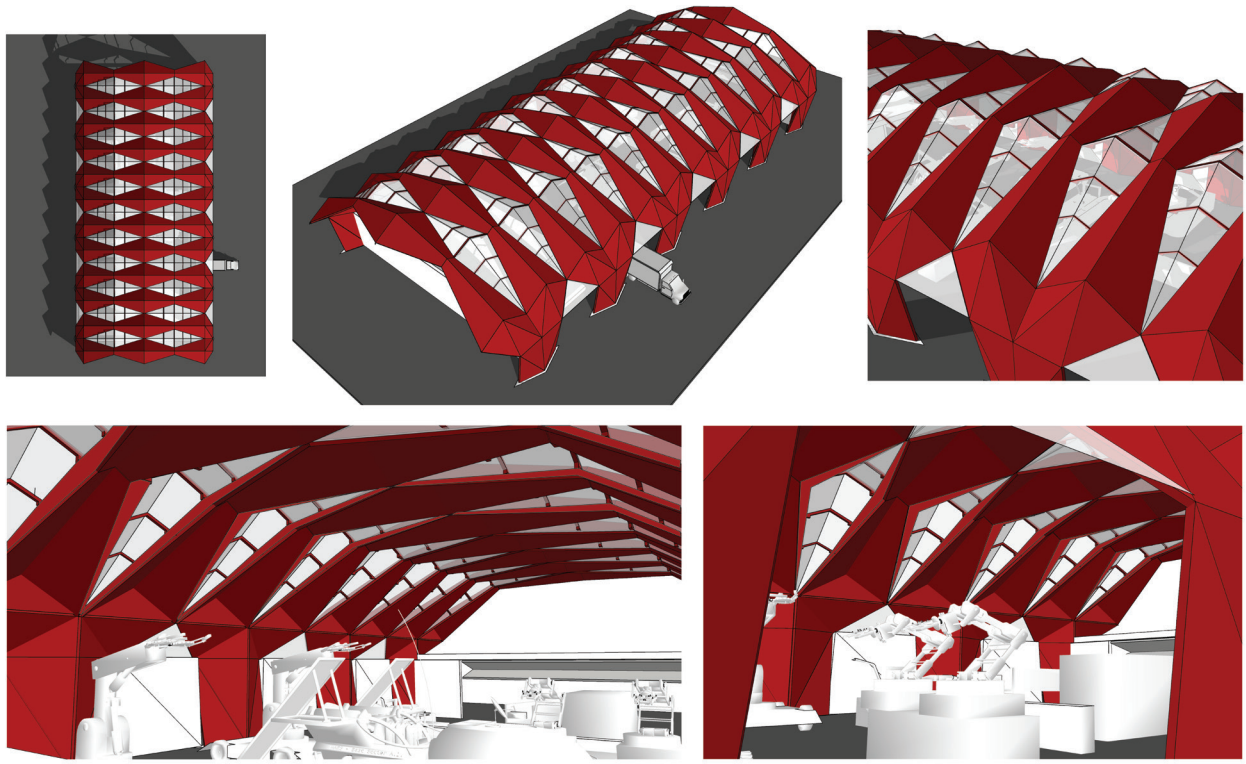


Figure 5.1.28 Arched moment frame scheme with skylights has a distinct aesthetic that reflects structural behaviors that are responsive to a multitude of performance standards

these more complicated patterns would require more complex formwork, which would increase labor costs. The formwork could still be planar and re-usable, which would help. Extending the folded plate surfaces to the vertical plane of the building would only work if the pattern spacing and geometry could be done in correspondence with the building's function. However, this modification's structural and architectural advantages (e.g., an efficient solution, lowest possible interior volume, effective options for diverse daylighting, positive storm water run-off, and an interesting building form), suggest that this option could be a viable alternative.

BREAKING: CONFIRMATIONS AND CALCULATIONS

Concrete shells are heavy and labor intensive to produce. Forms based on an assumption of concrete may be challenged once structural analysis and cost estimation are complete. Can a structure still act like a shell if it is made from trusses of steel? Contemporary shells look for alternative ways to translate surface-resistant formal principles into other materials. One example is the Yokohama Port Terminal building, by Foreign Office Architects (2002). As their design developed, they substituted solid concrete plates with a tessellated pattern of trusses.

To Discuss: How might this change in material resolve problems of design, detailing, and construction? Would this form still be regarded as a "continuous" surface with steel? How was the form stiffened on the ends and sides? How were forces resolved into the supports? (Figure 5.1.29)

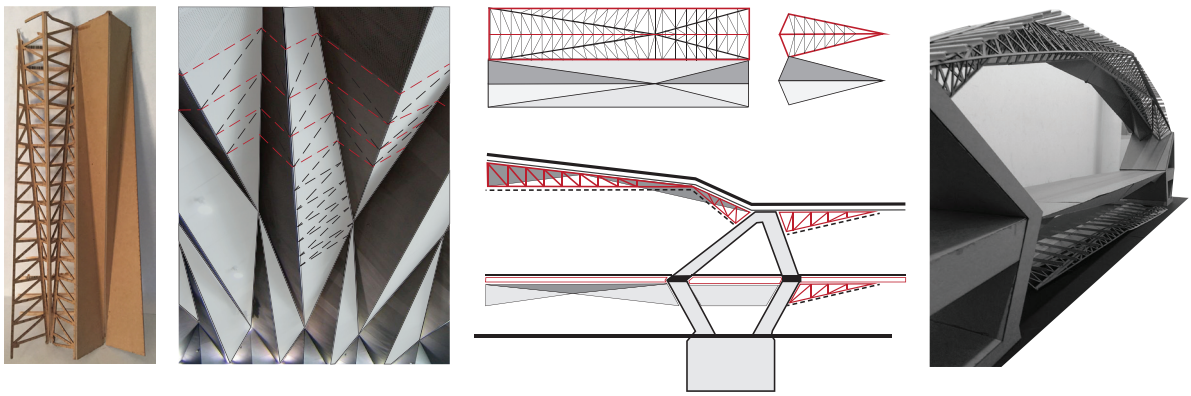


Figure 5.1.29 Building models and drawing analysis of the tessellated trusses used to create the Yokohama Port Terminal (FOA, 2002)

Summary of Schemes

The primary assignment was to explore whether or not concrete could be a viable option for the long span roof—clearly it can. Discerning between options is the challenge as vaults, lamellas, and plates, each have aesthetic, formal, and functional consequences. These structural proposals are also inherently architectural, so any assessments should be made from structural and aesthetic perspectives.

Some schemes were formally complex while others were conventional and simple. It is worth deciding if this a problem or if there is anything you can do to simplify them. To better understand these options' economic and practical consequences, consider whether you'd make the same choices if you had to prepare construction drawings for the schemes or to pay for them with your own money. Could you document the project in a way that that wouldn't penalize

"complexity?" Would scaffolding or construction drawings influence your designs (e.g., based on standard plywood sizes or proportions of standard concrete panel forms)? Shell design cannot be separated from the economic realities of use.

We don't need to follow the same analytical process that cost-estimators or engineers do to design vaults or plates, but we have to understand their perspective to argue for their use. Once we understand these structures' basic behaviors as extensions of the same logic we've studied in arches, beams, and frames, we can fold paper, build models, and develop sketches that test our ideas and communicate our intentions. If we evaluate this work by considering the important relationship between architectural form and structural performance—ideally by working in collaboration with engineers—the eventual solution should satisfy both disciplines.

Frequently Asked Questions

Is there an advantage to using new technologies of mass timber products, including glue laminated beams, to create lamella structures? Of course. The lamella is advantageous because it is curved as a larger form but also because of the triangulated lattice grid that makes up the shell and wood—laminated wood products can better match the curved surface and they can be fabricated off-site and assembled into larger pieces to reduce in-field connections at each lamella. Shigeru Ban has worked with celebrated engineers (Cecil Balmond, Frei Otto, etc.) to create double-curved grid shells using wooden members (Haesley Nine Bridges Club House, Japanese Pavilion, and the Centre Pompidou-Metz). Look at construction images to see the advantages of larger-scaled elements. (Figure 5.1.30)

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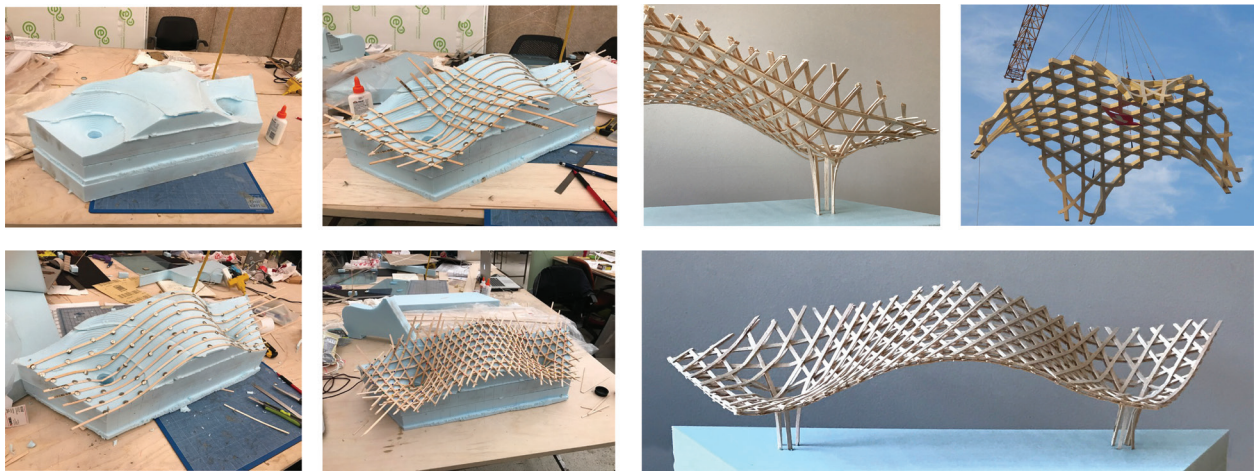

Figure 5.1.30 Student form-finding and analysis models of the timber grid shell Haesley Nine Bridges roof (SxD, Iowa State University)

5.1.17 17a: By Isabelle Leysens and David O'Brien, 17b and 17c: Michael Sharman, 17d: By Eladio Dieste.

5.1.20 Illustrations by Tom Leslie, "Beauty's Rigor: Patterns of Production in the Work of Pier Luigi Nervi," University of Illinois Press.

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PART 6

Resistance and Frames

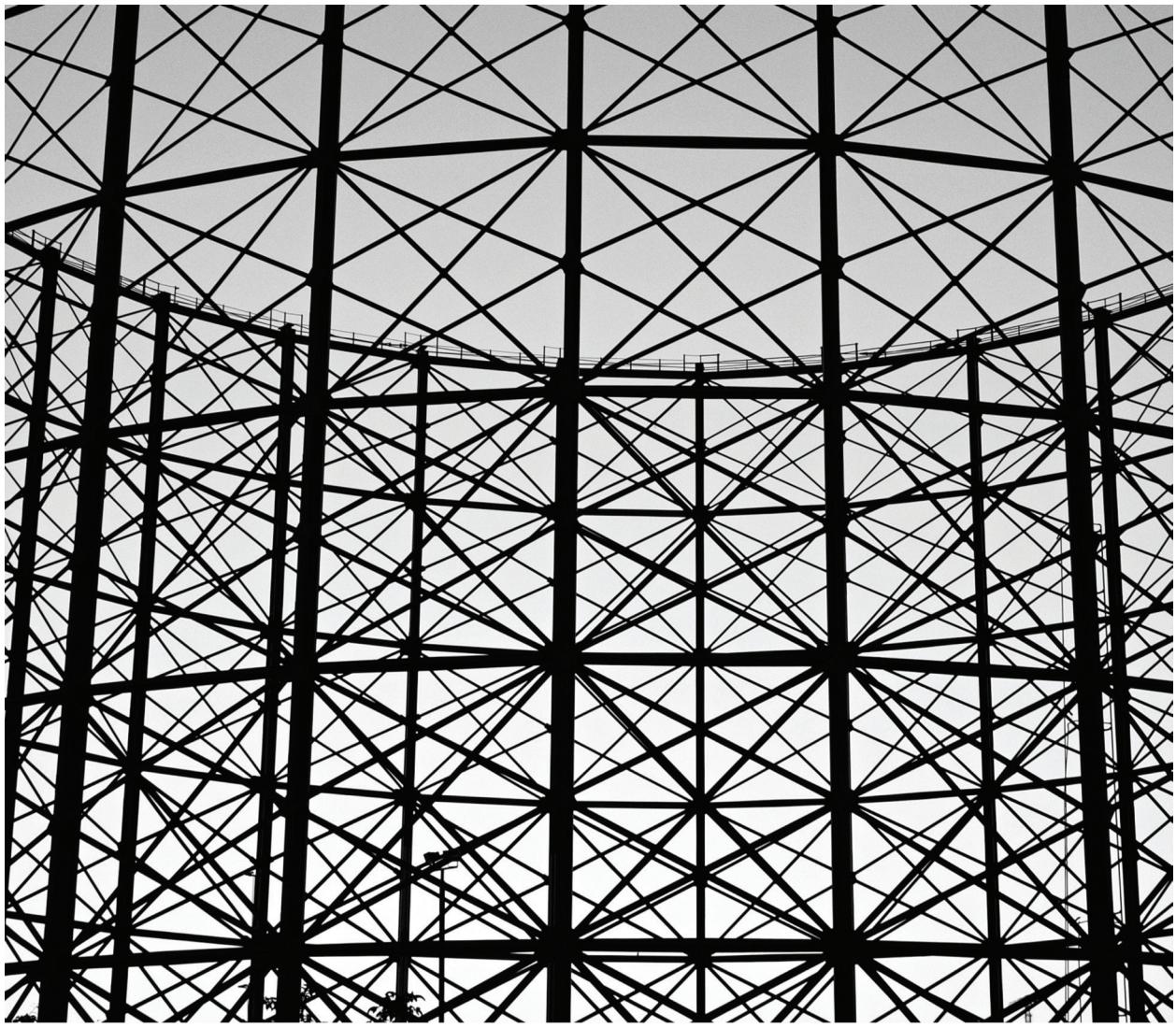


Figure 6.0.0 Stabilization tactic becomes the predominant expression in a power plant's fence enclosure

CHAPTER 6.0

FRAMES

Strategies for Stability

Buildings need to resist and transmit all the loads applied to them. Whether they do this through their form, section, vectors, surfaces, or frames they all share one requirement—stability. If buildings aren't adequately stabilized, they won't stand. This portion of the book will focus on the shared challenges of designing for stability, including structural connections and frames.

Connections and Stability

Stability is a fundamental requirement for structure. In the earliest chapters, we discussed how building structures could be evaluated based on the S-words of *strength, stiffness, stability, and suitability*. Whereas strength and stiffness are material qualities, stability and suitability are achieved through the arrangement of structural parts. We can size elements that are strong enough and stiff enough to resist loading *but* if the building doesn't also resist lateral/sideways loads—if it isn't *stable*—then it is no longer safe or serviceable.

Designing for stability is a three-dimensional challenge that integrates multiple elements in each plane of potential movement. When connected together, these *frames* collectively resist vertical, horizontal, and lateral forces.

This aspect of structural design is of crucial importance to the safety and welfare of building users; the failure to resist these forces creates sudden and catastrophic building collapses. We often think of structures as unmoving or static, but buildings are designed to move laterally when subjected to wind and seismic forces—hopefully not too much or in ways that haven't been anticipated. Because we are never sure which way

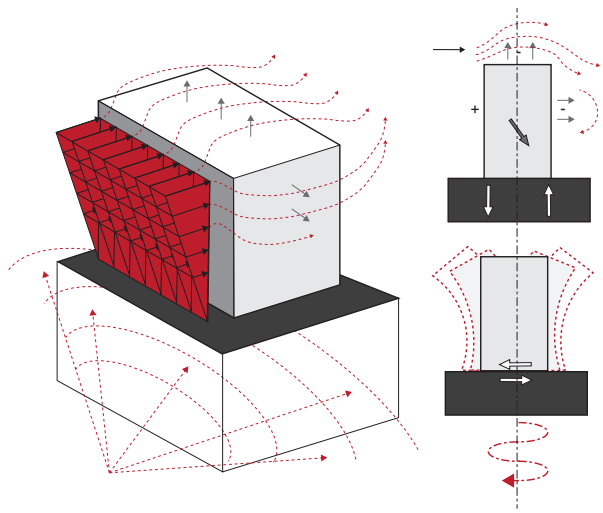


Figure 6.0.1 Wind and seismic forces and their potential effects on buildings: Wind creates pressure that pushes and pulls buildings. Seismic forces shake the ground and therefore the building

the wind may blow or the ground may shake, lateral resistance strategies must consider worst-case scenarios for each and create framing solutions that resist and transmit these forces effectively. (Figure 6.0.1)

Tools for Understanding and Testing for Stability

In order to select and place our load stabilizing elements throughout a structural frame, we'll need to first understand how these forces move through a structure. Professional designers use physical and/or digital models to visualize the three-dimensional force flows caused by lateral loads. We'll rely on physical models to help us stabilize various structures. This will allow us to see and feel how forces move through a system, particularly when we incorporate stiffening elements. In these exercises, we aren't trying to simulate the magnitude of the air pressure or the adequacy of the load stabilizers to resist these forces. Nor are we looking for a perfectly stiff building. We are looking for geometric stability in how we arrange our load stabilizers and their connections to resist lateral loading.

Principles of Stability

One of structural design's fundamental challenges is creating a state of static equilibrium between the forces that are applied to a structure and the resistance it generates. Some loads are fixed, like the dead load weight, while wind and seismic loads are dynamic in their location and magnitude. Gravitational loads may seem to be the most difficult ones to account for, but anyone that has carried a portfolio across a windy campus or been on a sailboat can attest to dynamic lateral loads' power.

The wind blows, the earth shakes, and our buildings need to stand in spite of this. These forces are energy imparted into normally static buildings that inflict dynamic loads of varying magnitudes and directions. Wind creates air pressure that bends and attempts to overturn a building while inducing a sliding shear stress at the base. Vibrations from seismic forces cause the base of the building to move suddenly and violently, which oscillates and shakes the entire structure. Stabilizing a building against wind and earthquakes involves difficult calculations that are beyond the scope of this book and the professional responsibilities of most registered architects, but there are fundamental design strategies that can provide this resistance. (Figure 6.0.2)

To resist lateral/sideways forces buildings must have horizontal and vertical elements integrated into their overall structural scheme. These load stabilizers may be dedicated elements, but as we'll see, some load *collectors* (e.g., slabs) and load grounders (e.g., columns) can also help stabilize building structures. Some building forms are naturally resistant to lateral forces (low, symmetrical, and naturally-curved forms like domes, cones, etc.) while others (like those with asymmetrical plans or re-entrant corners) have

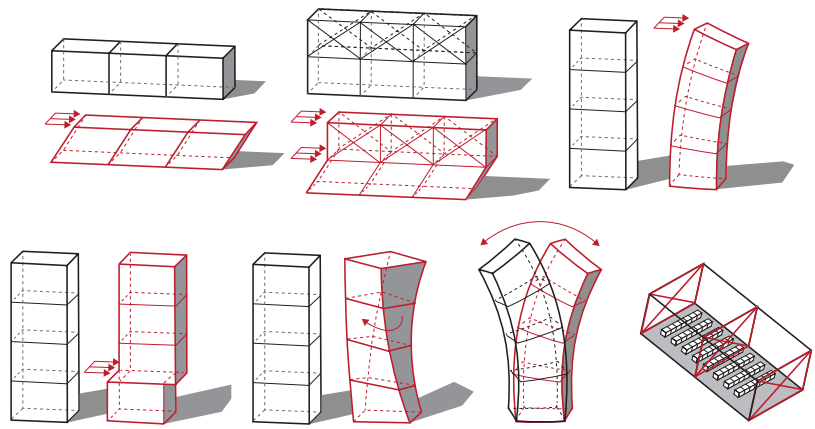


Figure 6.0.2 Typical lateral failures (left to right, top/down): Sideways deformation (racking), bracing only one-story (soft story), bending stress, base shear, twisting (torsion), excessive movement (drift), and mismatching lateral bracing with program

Figures 6.0.3a, 6.0.3b, and 6.0.3c *(continued)*

(continued)

Figures 6.0.3a, 6.0.3b, and 6.0.3c Designing for stability helps to prevent common failures and distortions caused by gravitational, lateral, and seismic forces. Shown: Rural Iowa, Amsterdam, and Kathmandu

inherent complications. Every rectangular structure needs to be braced against racking. Because forces follow stiffness, load stabilizers need to be strategically placed in plan and elevation to avoid twisting or irregular movements. (Figure 6.0.3)

Stability, therefore, is a fundamental constraint to the design process. Charles Eames described the ability of a designer to recognize and address constraints as, "one of the few effective keys to the design problem."

Designers use different methods to resist against these forces and look for ways to coordinate the placement and utility of load stabilizing elements with formal or architectural goals. This chapter will look at several approaches: stabilizing with connections, bracings, and/or forms with active learning exercises for each. But first, to understand the ways that a building moves when subjected to lateral loads, it is easiest to start by understanding the movements allowed by different connection types.

Types of Connections and Frames

Creating stability is often a matter of selecting proper connections. Stresses consolidate and forces move from one element to another at connections. The type of connection determines how the load is transferred and how the structure will move under loading. By designing with connections in mind, we can assess the macro and micro behaviors of the building under loading to determine how the load stabilizers can be integrated.

Connections can be *pinned*, *fixed/rigid*, or *rollers*. These are defined by the differences in how they transmit loads from one element to another. *Pinned*, or simply supported, connections allow rotation. Pinned supports

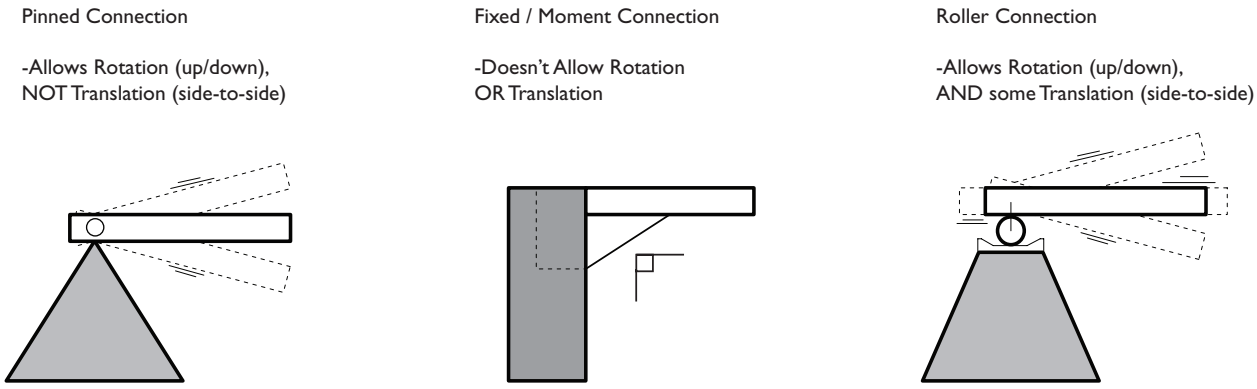


Figure 6.0.4 Three types of connections and allowable range of movement for each

for a beam allow it to bend and rotate to resist the horizontal forces applied to them, but they only allow the beam to transmit vertical forces at the connections. Pinned supports cannot resist *any* moment forces so they only allow force transfer in a single direction, which allows us to find equilibrium in our beam loading and shear/moment forces. Understanding elements behavior when they are pinned together is simple, because each element can be sized discretely with a simplified load transfer. (Figure 6.0.4)

THINKING: STRUCTURES AROUND YOU

Structural connections determine how forces are transferred from one element to another. Find four examples of pinned or hinged connections in everyday elements (natural and/or fabricated) and explain how they are designed to either rotate or remain fixed.

Pinned connections are used in many different applications. We've assumed these types of connections in our examples of beam framing, trusses (within the truss and in its framing connections), and our body structures (our shoulders, ankles, etc.). Pinned joints can look like the small triangles used in our representations (e.g., an axle and bearing), they can be actual pins connecting two elements together (e.g., truss connection points or at the end of X-bracing rod), or look nothing like a pin at all but still allow rotation to occur, albeit to a lesser degree (e.g., a connection between a beam and column with a multitude of bolts through a plate). (Figure 6.0.5)

Fixed/rigid (or moment) connections do not allow any rotation to occur. They maintain a "fixed" angle

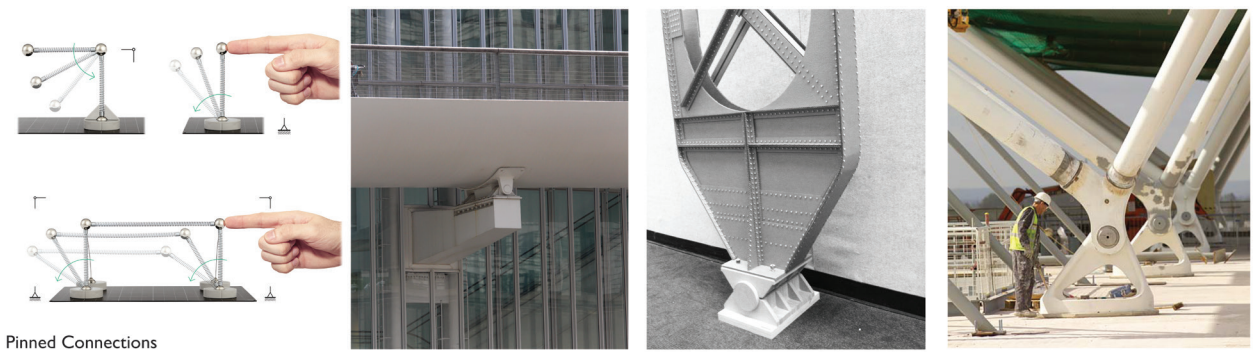


Figure 6.0.5 Examples of pinned connections (left to right): Mola Structural Kit, Art Institute of Chicago, (Renzo Piano Building Workshop), Gallery of Machines (student model), and Heathrow Airport Terminal 5 (Rogers Stirk Harbour + Partners)

between the connected elements. As a result, these elements share the resistance of the applied loading. For example, in a beam/column connection with a fixed connection, a column helps the beam to resist bending. This shared resistance is complicated. Structures with fixed connections are *indeterminate* and require more advanced methods for analysis. In structural frames with moment connections, inflection points in the middle of beam spans and column heights mark places where no moment force occurs—seen where the deformed shape switches from one curvature to another. This makes a good location for construction splices. (Figure 6.0.6)

Fixed connections vary based on the material: Wood-to-wood connections can't be fixed together unless steel plates are added or additional diagonal "knee-braces" triangulate the connection. Steel connections can be made rigid with welded connections between plates at the top and bottom flanges. Cast-in-place concrete connections are *always* fixed/rigid connections, while precast connections are not unless they're cast together. Fixed joints are labor and material intensive because they must be fully affixed. The overall structure relies on this shared resistance to loading, so fixed connections need to be tested and confirmed in-field.

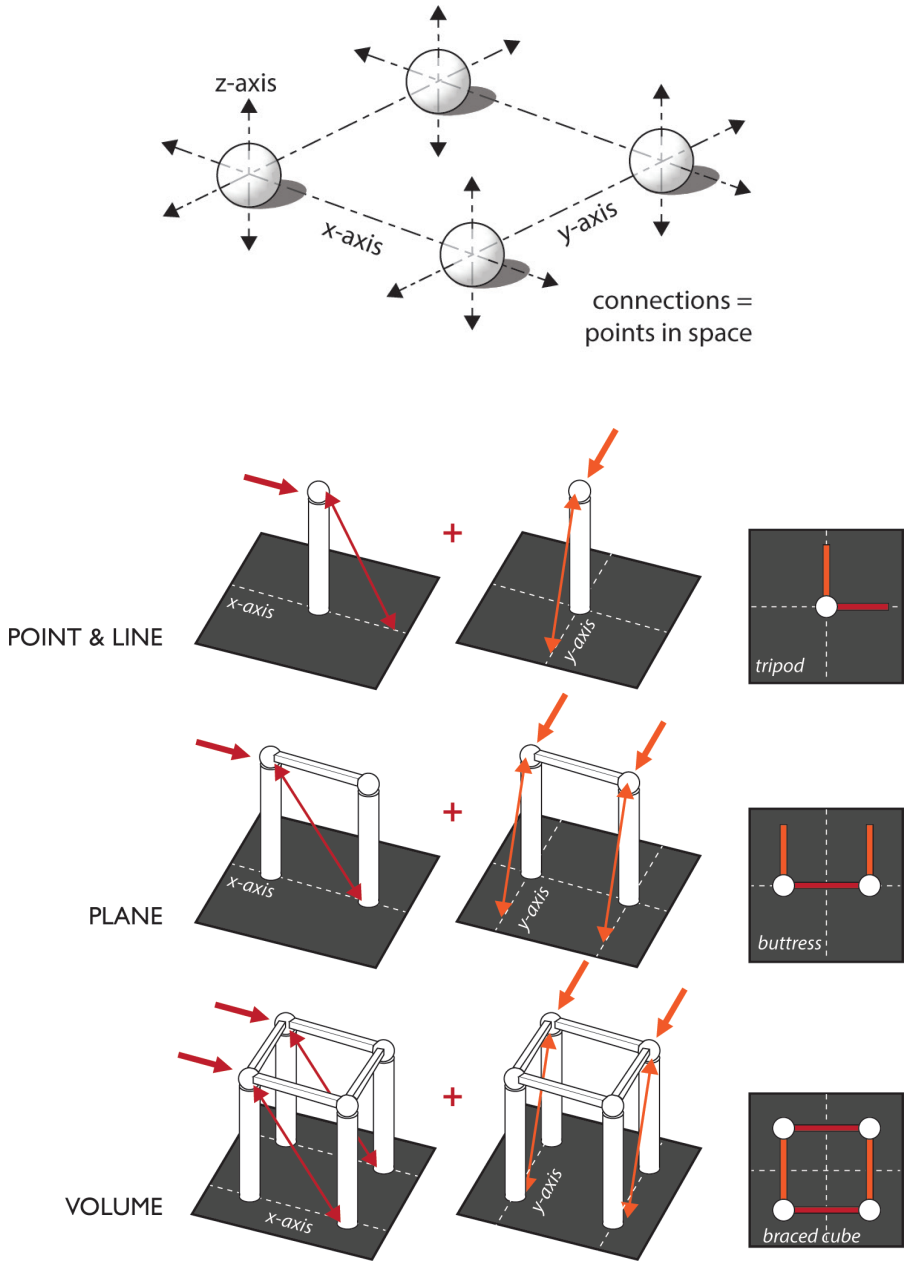
Rollers are rarely used in buildings but they are found in long span structures like bridges that are subject to thermal expansion or shifting load locations. These connections allow *both* translation (side to side) and rotation (up and down) between the spanning and supporting members. Horizontal forces are usually modest and have to be constrained by the roller's dimension—so these connections can't be used for structures with high thrusts, like arches. (Figure 6.0.7)

Load Stabilizing Strategies: Braces, Diaphragms, and Fixed Connections

Designing for stability can be simplified if you picture the connections as "points in space" that need to be fixed in place (x, y, and z axes) and then place

Figure 6.0.6 Examples of fixed connections (left to right): Mola Structural Kit, Barn at Bloedel Reserve (Bainbridge Island), demonstration frame at Cal Poly San Luis Obispo, and portal framed building under construction (Ames)

Figure 6.0.7 Examples of roller connection and a pin/roller hybrid (New Haven and San Sebastian)



Figures 6.0.8a and 6.0.8b Each connection point can move independently unless connected and triangulated to other points. When stabilizing lines, planes, and volumes, diagram the push/pull direction of forces and locate bracing within that plane

stabilizing elements within the same axis you are trying to stabilize. For example, if we place a point on top of a vertical wooden dowel, we've fixed the point from falling down (z-axis), but it can still move along the x- and y-axis. We'll need a "tripod" of stabilizing members to adequately stabilize a single point against movement in all three axes. (Figure 6.0.8)

A plane with pinned connections has several points that need to be fixed in place or else it will rack sideways under loading. We can stabilize planar frames in three ways: by adding additional load stabilizing members (commonly called bracing), with a stiff plane (a diaphragm), or with fixed rigid connections. (Figure 6.0.9)

EXERCISE #1: SIMPLE STABILITY

Model a horizontal beam spanning between two columns that are glued down to a base. Use ductile elements (wooden dowels) and connect them with hot glue (try using a flexible circular element, like a solid candy). Start with pinned connections at each intersection, then experiment with fixing the connections between the top of the column and base and between the columns and beam. Apply lateral forces (gently push side-to-side) to the frame to see how behavior changes before and after your modifications.

Bracing is the most used lateral resisting system. It typically consists of a single or double (X-formed) diagonal brace across a plane. A single brace will need to resist tension and compression, since the stress will change based on which direction the load is applied. We can improve the bracing's efficiency by using two cross braces (X-bracing). In this system, one element will sag under compression while the other tenses, again depending on the force's direction. Full X-braces only need to resist tension, which makes them smaller than braces sized to resist compression buckling. A *chevron* (or inverted V-brace) is a modification of an X-brace that is useful when we need to place doors or openings within a frame, but it inflicts a bending load on the top member.

Another option to brace the frame is to fill the entire plane with a rigid wall, called a *shear wall*—the wall physically affixes all the points together within the plane, doesn't allow lateral movement, and transfers lateral load diagonally across its plane into the ground. Shear walls must be stiff and strong enough to resist and transmit loads and they must be firmly connected to other lateral resisting systems like floor slabs.

Finally, we can change our connections to fixed/rigid connections. These stabilize the frame by not allowing the frame to rotate and collapse. Fixed connections induce high bending moments at the beam and column connections and are therefore not as efficient as shear wall or braces. Using a physical model, we can observe how the physical frame changes behavior when we fix the top connections, or the bottom connections, or both. (Figures 6.0.10 and 6.1.11)

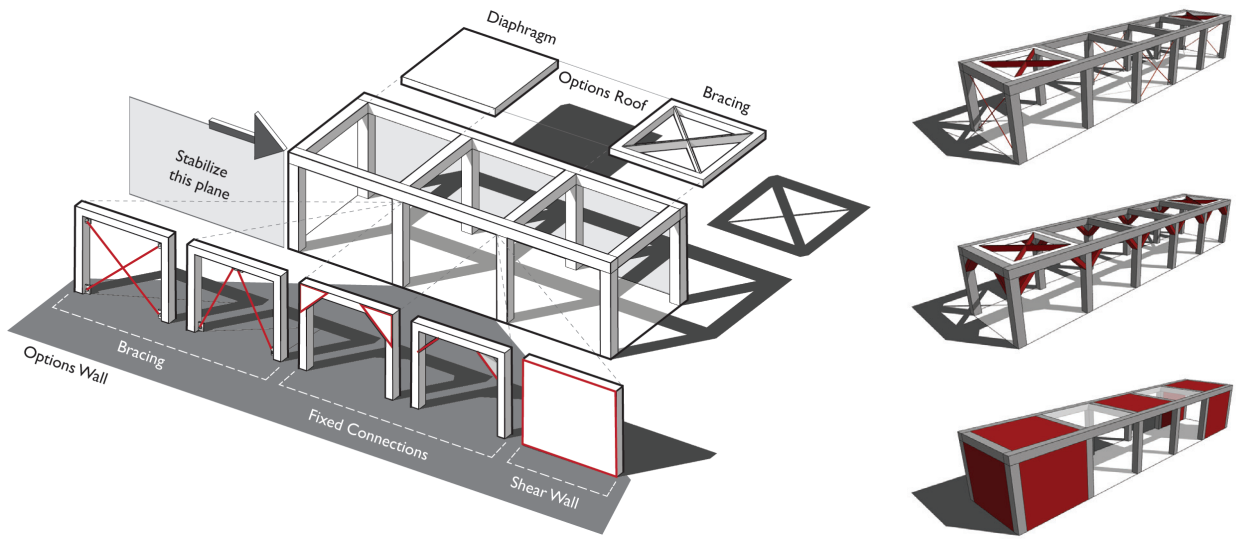


Figure 6.0.9a and 6.0.9b Types of stabilizing options within a frame. Consider the spatial, experiential, and aesthetic consequences of each potential choice

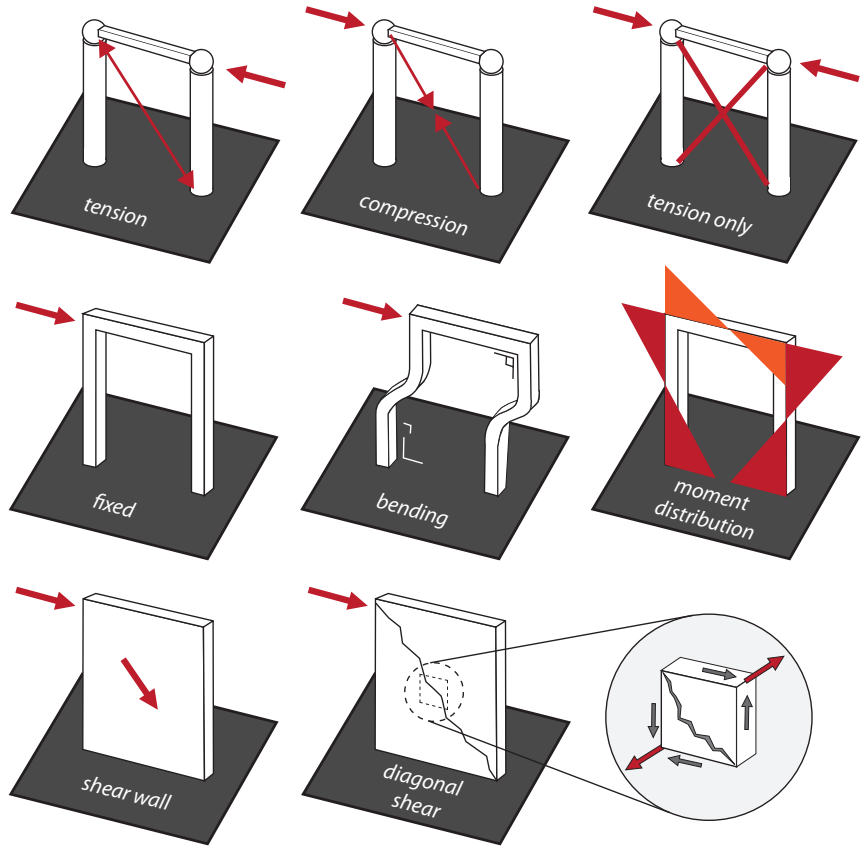


Figure 6.0.10 Stabilizing options have different types of stresses as a consequence to their resistance: axial stresses (compression and tension), bending stress (moment connections), and shear stress in walls

	BRACING	MOMENT FRAMES	SHEAR WALLS
ADVANTAGES	-Bracing locations can be coordinated with program. -Very stiff, good for wind resistance. -Predictable behavior. -Part of visible expression.	-Bracing locations don't adversely affect program spaces (like walls and braces). -Flexible to use, aesthetically clean expression possible. -Ductile enough to reduce seismic forces.	-Very stiff; Good for wind resistance. -Helps define program spaces (e.g., stair, elevator, etc.). -Can also serve as support element.
CHALLENGES	-Less flexible planning than fixed frames (occupies entire bay). -Expression on exterior needs to be coordinated. -Stiffness may be bad for seismic events.	-Expensive and time consuming to build and test. -Flexibility may cause problems in high-rise buildings. -Complicates force flow through structure (beams and columns both resist bending).	-Stiffness is bad for seismic events. -Hard to change / modify if program changes. -Needs to be stacked with other walls and braced in both plan directions.

Figure 6.0.11 Challenges and advantage comparison of different stabilizing options

Stabilizing Multiple Adjacent Cubes

What happens if we stabilize multiple cubes that are connected side-to-side or end-to-end like a typical one-story framing grid? Without a stabilizer, pinned connections would allow the cubes to rack sideways into a parallelogram and fail, falling like a set of dominos. Would we have to stabilize *every plane*?

If you think about stabilizing frames as fixing points in space, you'll realize that we wouldn't need to stabilize every frame within this grid. If we fix one point from moving horizontally, then the rest of the connections are also fixed in that direction because of the horizontal "beams" between them. This is the principle of transmissibility, which tells us that forces move through a system, from one frame to another, until they are redirected or resisted. In this case, horizontal forces don't have to be resisted at the exact point where they act upon the frames, they just have to be resisted *somewhere* along the line of action/plane. Sometimes horizontal loads will follow a diagonal member down into the ground (in compression) but if the load direction is switched, the horizontal load will be pulled back (and down), placing the member in tension. (Figure 6.0.12)

We could stabilize hundreds of frames placed side-by-side with a single stabilized frame anywhere along their length. In practice, however, stabilized bays need to occur regularly to avoid consolidated stresses and differential stiffness that could cause torsion. The bracing's frequency varies depending on the stabilizing system used, but in general, braced frames need to occur *every fourth bay*. Of course, this doesn't stabilize the frame against any forces applied perpendicularly to this plane—for that we have to add braces in the same plane as the forces that are applied.

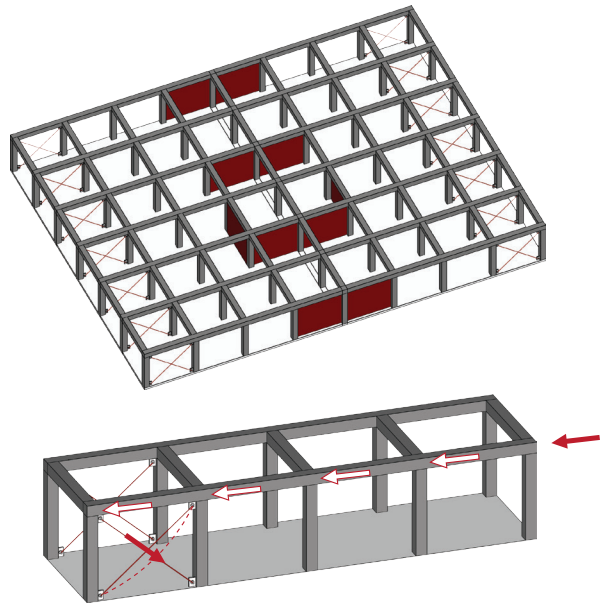


Figure 6.0.12 Stabilizing one bay fixes other bays within the same plane. This is useful in large, multi-bay framed buildings. Rule of thumb is that one out of four bays per plane should be stabilized for adequate stiffness

When a plane becomes a geometric solid, like a cube, it's easiest to stabilize it by working plane-by-plane around the perimeter. Because they are set perpendicularly to each other, one braced plane helps to stabilize the others from perpendicularly applied forces. Eventually, when a cube has all the planes adequately stabilized, it has twelve sticks (vertical, horizontal, and diagonal framing and bracing components) and four connection points—the same 3:1 ratio of sticks to point connections as a tripod.

EXERCISE #2: 12-NODE EXERCISE – SINGLE-STORY

Stabilize this frame against lateral forces from all directions. Start by establishing a grid (A, B, C, and 1, 2, 3, etc.) and stabilize it plane-by-plane. Test it by pushing and pulling at each point until it is stable. Draw the arrows as they move through the system, starting with the applied load. You need one load stabilizer within each grid/plane. Make sure forces have a path to the ground (either pushing down or being pulled back up). Remember, forces don't take right angles as they move through a frame, so load stabilizers placed perpendicularly to the load direction aren't effective. There is a difference between structures that are stable and stiff; if your physical model still moves a bit, it is understandable. (Figure 6.0.13)

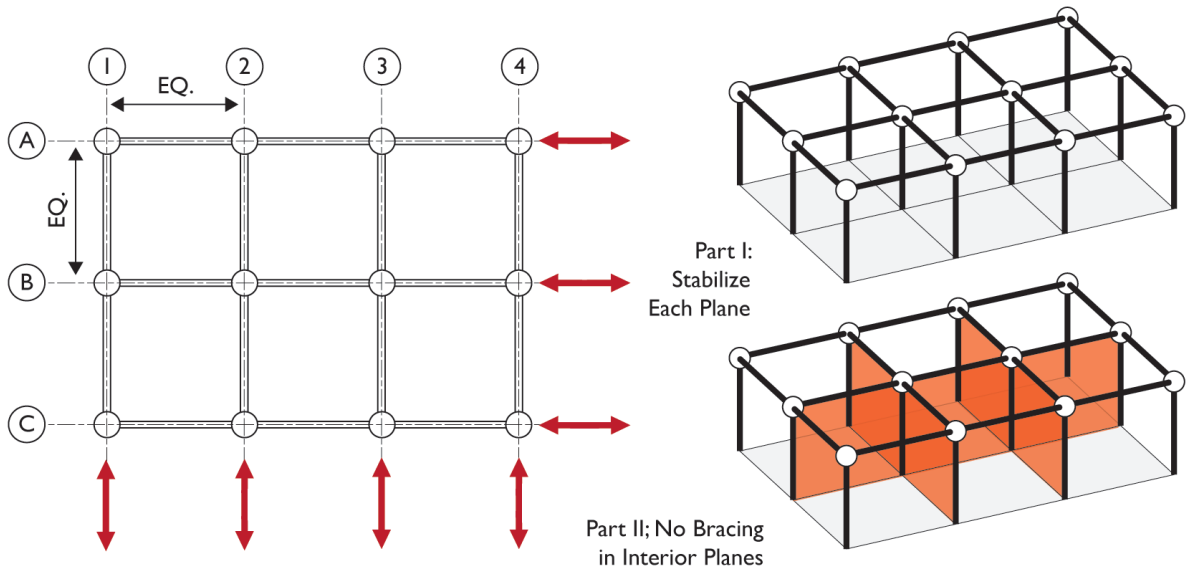


Figure 6.0.13 Bracing 12-node stability exercise

MODIFICATION #2A: HORIZONTAL DIAPHRAGMS

The roof and/or floors can be used to reduce interior braces.

Try it. Remove the braces from the interior planes and replace them with braces in the top/roof plane instead. Notice how this stiffens the roof and allows lateral stability to be transferred from one plane (grid line B) to a parallel plane that is already stabilized (grid A or C). By moving the lateral stability into a horizontal plane, we are creating a horizontal *diaphragm*. Diaphragms are used to collect and transfer lateral forces across multiple bays to vertical stabilizing elements. These diaphragms, generally floor slabs and roofs in buildings, free up interior frames from lateral bracing. It is helpful to think of diaphragms as flat horizontal beams that span between the vertical load stabilizing supports. Like beams, diaphragms with reduced depth, minimal thickness, and/or large penetrations become weaker with consolidated points of stress. These horizontal shear planes transfer lateral resistance only to braces that are parallel to the applied load. (Figure 6.0.14)

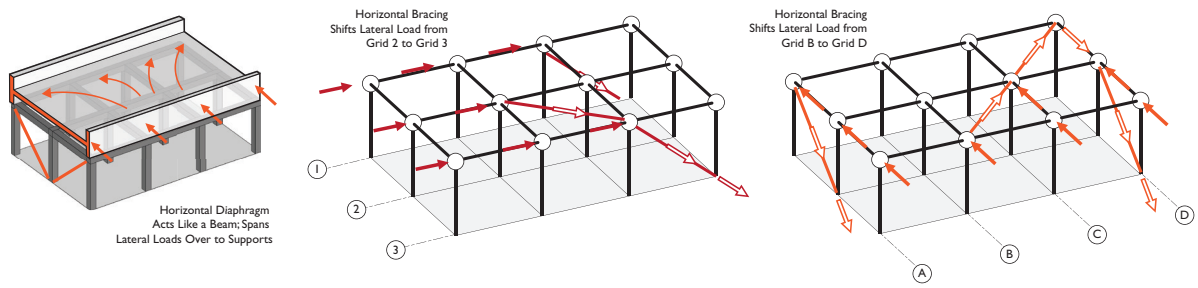


Figure 6.0.14 When interior frames can't be braced, bracing in horizontal planes transfers forces to *parallel planes* that are braced

Placement of Stabilizing Elements

These exercises demonstrate how braces and connections can stabilize single or multiple frames. Although stabilizers could be placed anywhere within a plane of framing to stabilize it, it does make a difference (both architecturally and structurally) where they are placed. Finding locations for lateral bracing should always occur early in a design process.

Consider a basic square grid building that uses shear walls for bracing, but only on one end. These walls' placement has structural consequences, but architecturally you'd like to locate the walls along one end of the plan if possible. Can you align the two priorities? The plan is theoretically stable as each plane has a stabilizing element, but the asymmetry of the wall placement makes one end of the plan stiffer than the other. Uneven lateral resistance can cause the entire building to twist. As a consequence, this plan is prone to rotating around its stiff end—creating *torsional instability*. (Figure 6.0.15)

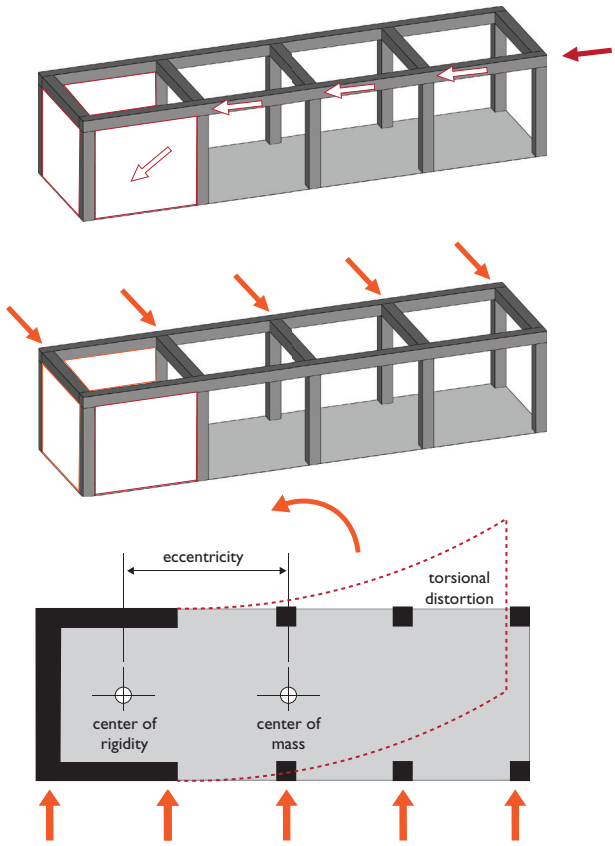


Figure 6.0.15 This frame is effectively braced in one direction but not the other. This results in differential stiffness (center of rigidity and center of mass off-set by large eccentricity) and torsional distortion

Because torsion is a moment force about the center of mass in the footprint, the most effective lateral bracing locations are ones that are bi-axially symmetrical. Stabilizing systems for buildings with multiple frames perform best when lateral resistance is evenly distributed throughout the footprint. Bracing can be located continuously at the perimeter of the building, in the corners, ends, or middle, or even as a centralized building core. Guidelines suggest optimal structural behavior and not rules. We can resist plan torsion in different ways, but it is worth exploring options that satisfy multiple objectives. These potential locations correspond well with a proper functional placement of stair and elevator walls in multi-story buildings. Because stairs and elevators typically use shear walls for load-grounding and fireproofing, they are frequently used as load stabilizing elements too. (Figure 6.0.16)

Multi-Story Building Frames: Soft Stories

Plan torsion is only one risk of incorrectly placing stabilizing elements. In multi-story buildings it is important to also apply bracing in the same locations at every story, ensuring continuity of the bracing down to the foundation. If some stories are braced but others are not, a condition known as a *soft story*, the building risks collapse. All levels must be braced. Differential stiffness between floor levels is very real problem for buildings with large, tall, open ground floors and shorter, stiffer, upper floors.

Strategies and Tactics for Stabilization

There are three organizational strategies for load stabilizers in a multi-gridded building frame: *Bay/Grid*, *Casing*, and *Core*. *Bay/Grid* relies on a bracing system that stabilizes all the bays throughout the building (similar to our physical models). This is the most common, economical, and functional approach because any type of stabilizing system would work well. *Casing* relies on stabilizing elements along the building's outer perimeter. There are ways of embracing visual impact of the lateral bracing strategy and incorporating it into the design of the building (e.g., with braced frames, diagrids, or rigid connections). The *Core* method leaves the interior bays

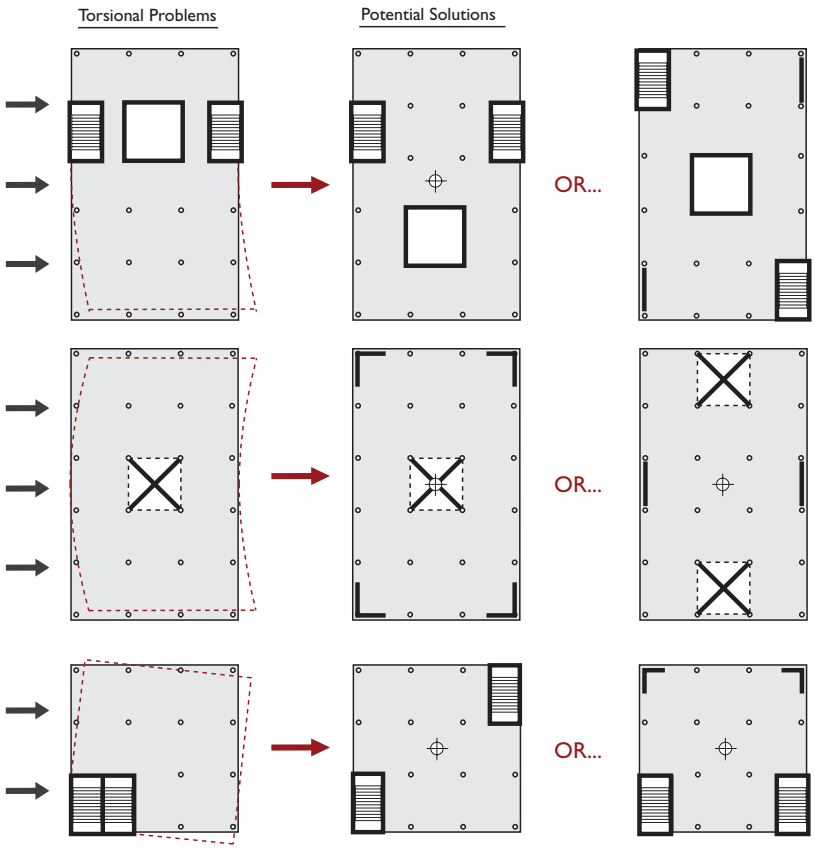
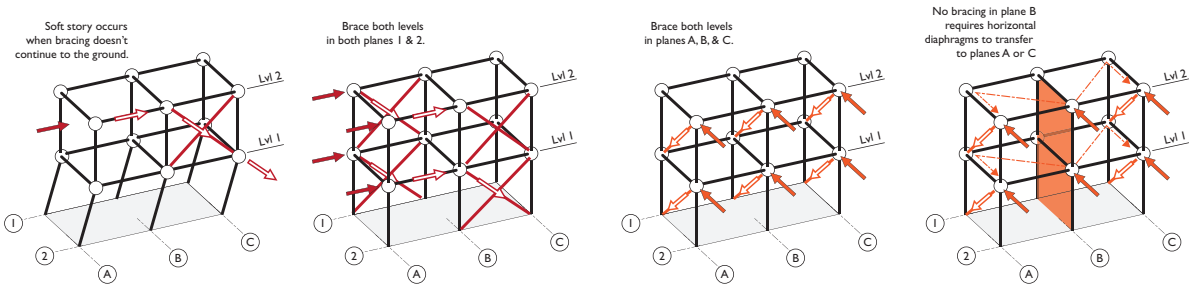


Figure 6.0.16 In multi-story buildings, the placement of stair/elevator circulation core shear walls can create plan torsion. Consider bi-axially symmetrical arrangements of stiffening elements, if possible

EXERCISE #3: 12-NODE – TWO-STORY

Using physical models, create a stable two-story frame based on the grid of columns shown without bracing the interior frame (using bracing or shear walls only in the external planes). Horizontal diaphragms at the roof and floor are allowed, but you are challenged to leave a multi-story open space between floors. It is recommended that you start from the top, working plane-by-plane around the perimeter until the top is stabilized. As you move to the floor below, visualize how the lateral forces from the upper floor are resisted and transmitted. Brace the lower frame, test for stability and torsional instability in plan. (Figure 6.0.17)



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Figures 6.0.17a and 6.0.17b Two-story bracing exercise. Avoid soft-story collapse by ensuring a continuity of force flow through bracing from the top to the bottom. Consider the spatial consequences of using bracing, fixed connections, or walls to do so. 6.0.17b: Mola structural kit model demonstration of multi-story bracing options

EXERCISE #3A: 12-NODE – THREE-STORY

Next, create a stable three-story frame based on the grid shown, leaving a two-story open space. Before you stabilize it, notice how much sway and torsion the upper floors have. This is a result of the larger moment arm between where the load is applied (at the top of the tower) and the stable base. Because the tower is essentially three stacked cubes, notice how few options you have in stabilizing it.

EXERCISE #3B: 16-NODE – THREE-STORY WITH CANTILEVER

Add an additional cube to the middle of the three-story frame so that it hangs off the side (like a tilted T-shape) and re-stabilize the frame using a minimum number of braces. You will now have to stabilize this new cube against gravity and lateral forces. Is there a way to fix these connection points in place that resists both types of loads? (Figure 6.0.18)

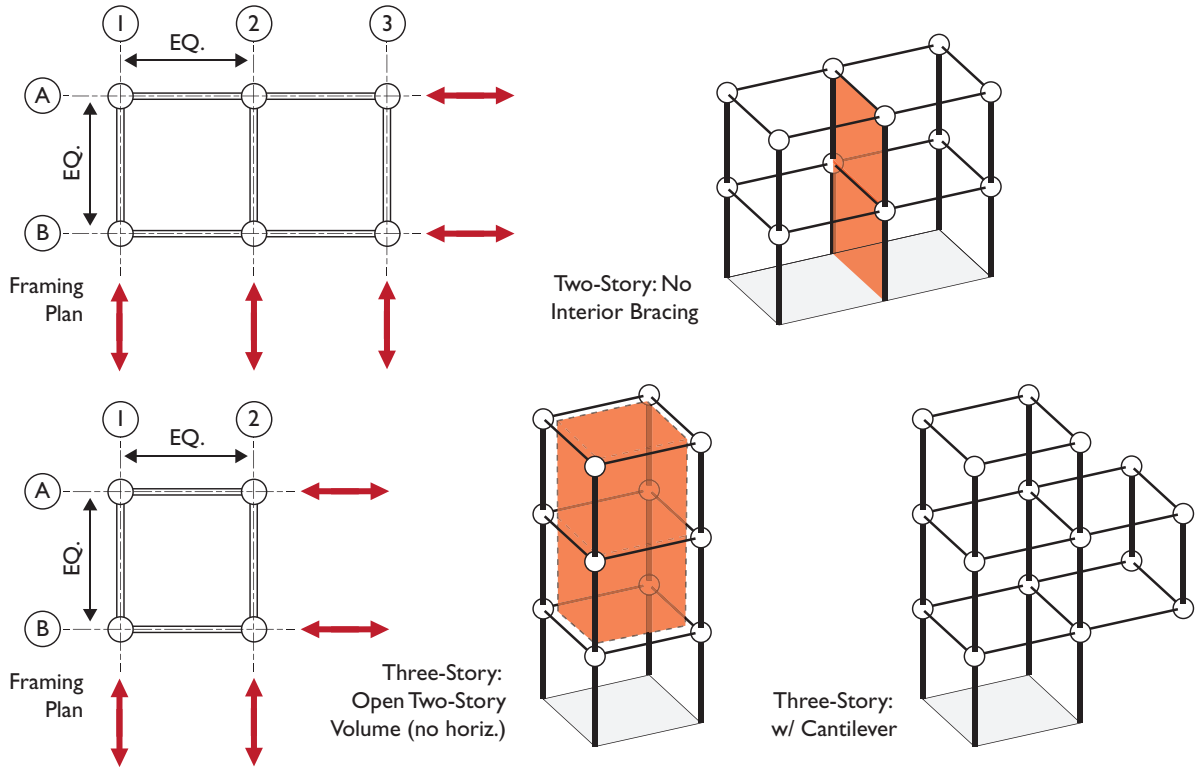

Figure 6.0.18a Problem set-up for Exercises #3A and #3B

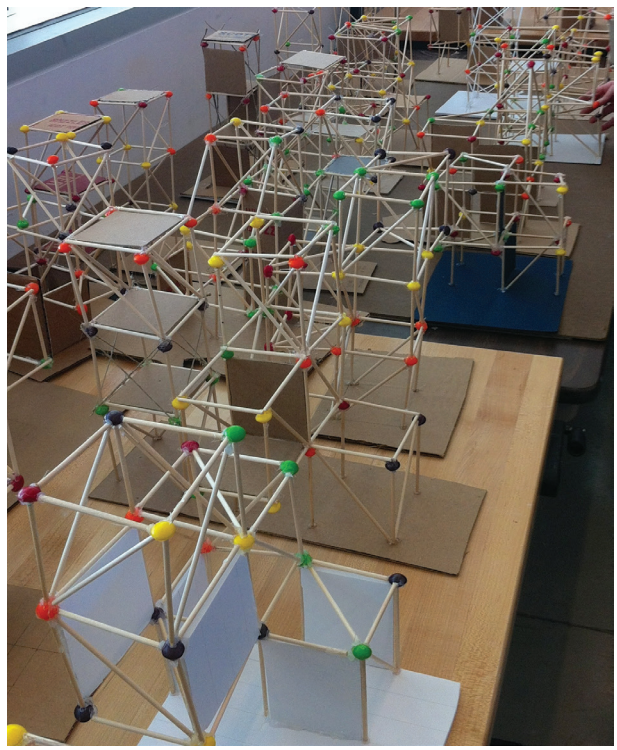

Figure 6.0.18b Examples of student models (SxD, Iowa State University)

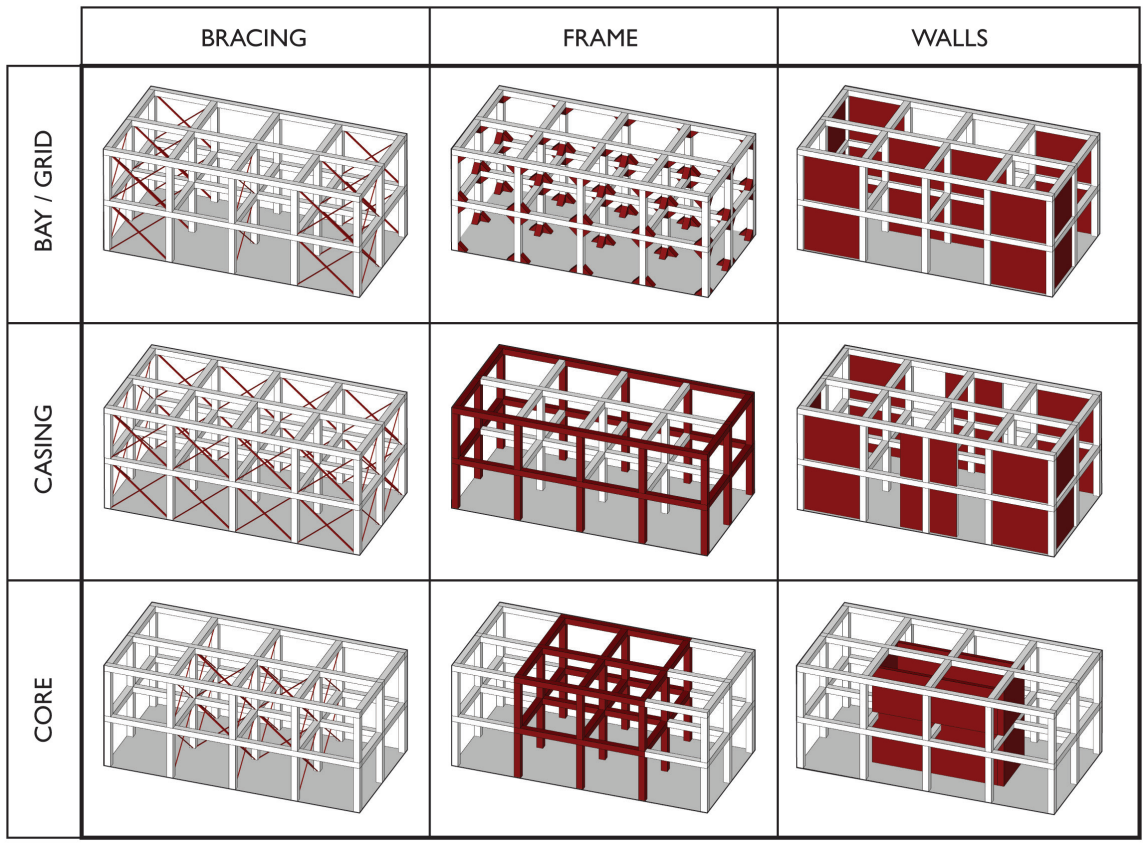

Figure 6.0.19 Graphic matrix of the nine common bracing strategies and tactics

and the building perimeter free of visible bracing. Cores can be trussed frames, but they can also serve a double purpose as diaphragm walls for stairs and elevators. (Figure 6.0.19)

Every strategy needs to be paired with tactics for actually achieving stability: *braces*, *rigid frames*, *diaphragms*, and/or *shear walls*. By considering structural and architectural performance, you can determine the stabilization method that is best suited for each application and where this bracing should occur (e.g., bracing in a bay/grid orientation or shear walls as core and casing). In the following chapter, we'll look at how stabilizing strategies are design generators for high-rises.

Stability and Expression Through Form

Relying on the building's form to create naturally-resisting shapes is the most effective overall approach. The same rule applies when trying to stabilize a building. Some irregularly shaped buildings, particularly with L-, H-, or T-shaped plans, have re-entrant corners that collect torsional stresses because their wings have different centers of mass. Alternatively, certain forms are naturally resistant to overturning, including those that have a wide base-to-height ratio, are geometrically symmetrical in plan (e.g., domes, cones, etc.), or stiffened with a curved or folded surface. The triangulated configurations of truss frames are inherently resistant to racking. Some buildings that aren't inherently stable embrace their lateral resisting systems as an integral part of their design expression, while other laterally-stable building forms are formally responsive to their stresses and support conditions, such as the moment frame, hinge frame, and three-hinged portal frames. (Figure 6.0.20)

Moment frames are distinguished by the rigid connections between their columns, beams, and foundations. Each element, including the foundation, is recruited to resist bending, requiring larger members to resist these stresses. We can relieve the foundations

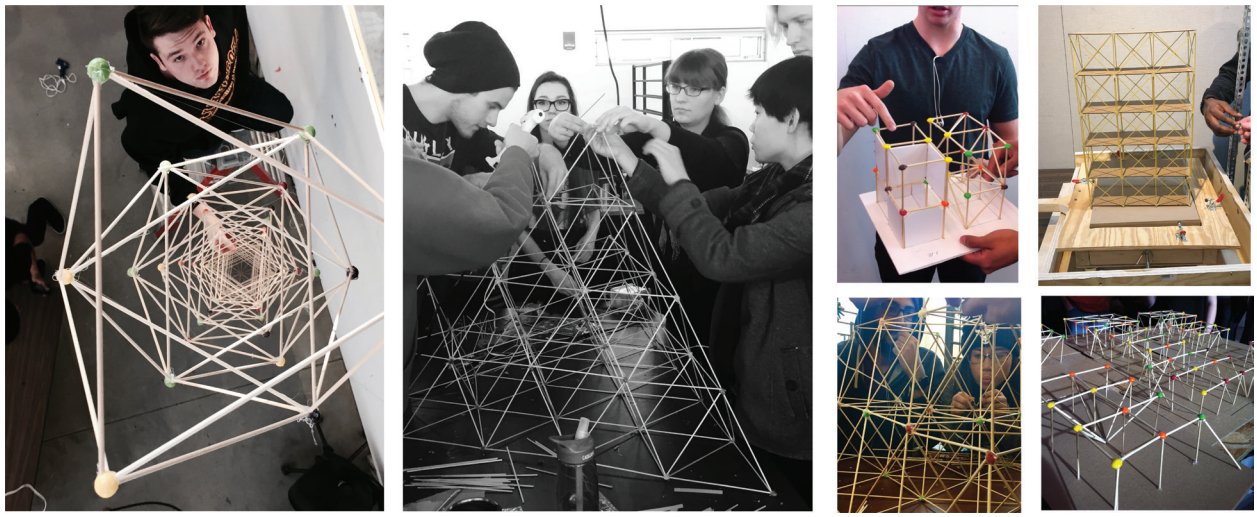

Figure 6.0.20 Stabilizing structures as a potential design generator (SxD, Iowa State University)

from bending stress if we add pinned connections at the base, creating a hinged-frame, but the column and beam will both have high bending stresses across their length. We can then relieve some of these stresses and consolidate these moment forces more by adding a third hinge in the middle of the beam (like the three-hinged arch) to create a three-hinged portal frame.

If we draw a moment diagram of the resulting forces, we'll see that we've created two hybrid "column-beams" meeting in the middle of the span—essentially two halves of an arch. This creates an enormous amount of moment force at the connection points. To resist this, both the columns and beams need to get physically larger at this location, using area and orientation to resist high localized bending, but they can also be reduced in profile at the middle of the span and at the column base, where no bending occurs. Moment connections make this frame stable against lateral stresses along this axis, but it would still need bracing in the longitudinal direction. Because the profile matches the moment diagrams and the shape is resistant to lateral forces, rigid frames are materially efficient and are typically the only type of section-active structural systems that are effective for long span buildings. (Figure 6.0.21)

Design Standards and Lateral Forces

We've now looked at different ways of stabilizing a structure laterally, but we haven't discussed the forces that will impart these loads or how to design resistance to them. There are two types of dynamic lateral forces that buildings must resist: Wind and seismic. We may not know where these loads will come from or when they'll occur, but we have to plan for the worst-case scenario. Building codes determine the magnitude of loads that need to be resisted based on the geographic location of the building, height, occupancy, and historical record of extreme wind and seismic events. Unsurprisingly, a high-rise hospital in a high seismic (or hurricane) zone will have a more stringent set of

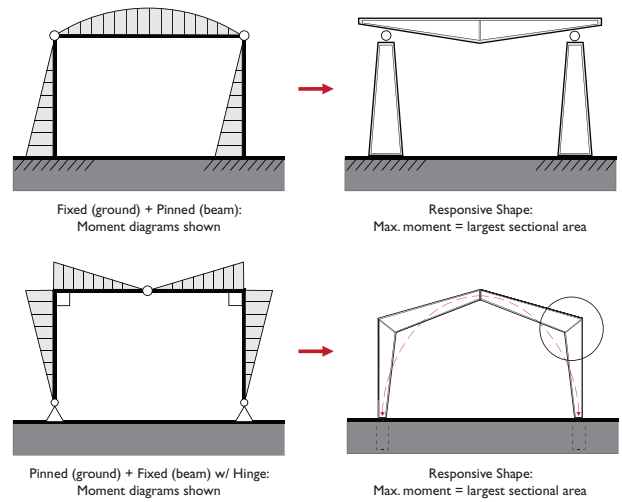

Figure 6.0.21 Connections create different stresses in columns and beams. To make a responsive form, match the structural profile with internal stresses

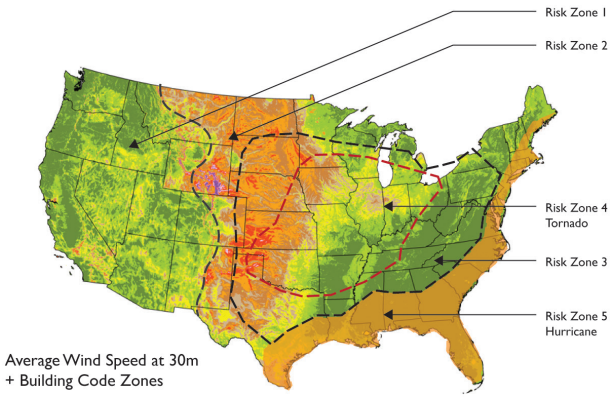

Figure 6.0.22 Representation of a typical wind speed map and design risk zones for extreme weather conditions

design standards than a single-story warehouse in the Midwest. The two forces have different effects on the design and require different strategies. (Figure 6.0.22)

Designing for Wind Resistance

Buildings are obstacles that deflect or otherwise impede this mass of airflow caused by wind. They have to resist these forces and transmit them to the ground. This pressure is exerted on the surface of the building either directly (windward pressure pushes "in") or indirectly (leeward is a negative pressure that pulls out). Maps will show the wind force a building has to resist based on the exposure of the local topography (coastal, flat, hilly), historical data, and risk factors. Wind pressure increases as height above grade increases and friction from the ground diminishes. This creates a profound challenge as the highest wind forces are exerted at the top of the highest towers, thereby causing the highest overturning moments.

A building's aspect ratio (height-to-width) is the single largest determinant of the its structural design. Tall and narrow buildings are governed by their need to resist lateral forces, while lower and wider buildings are most vulnerable to gravitational forces. Wind forces cause three potential types of failure: Base shear (sliding), overturning moment, and the drift from deflection. One passive solution is to create a building that is heavy enough to withstand sliding or overturning. Alternatively, the building frame can be made

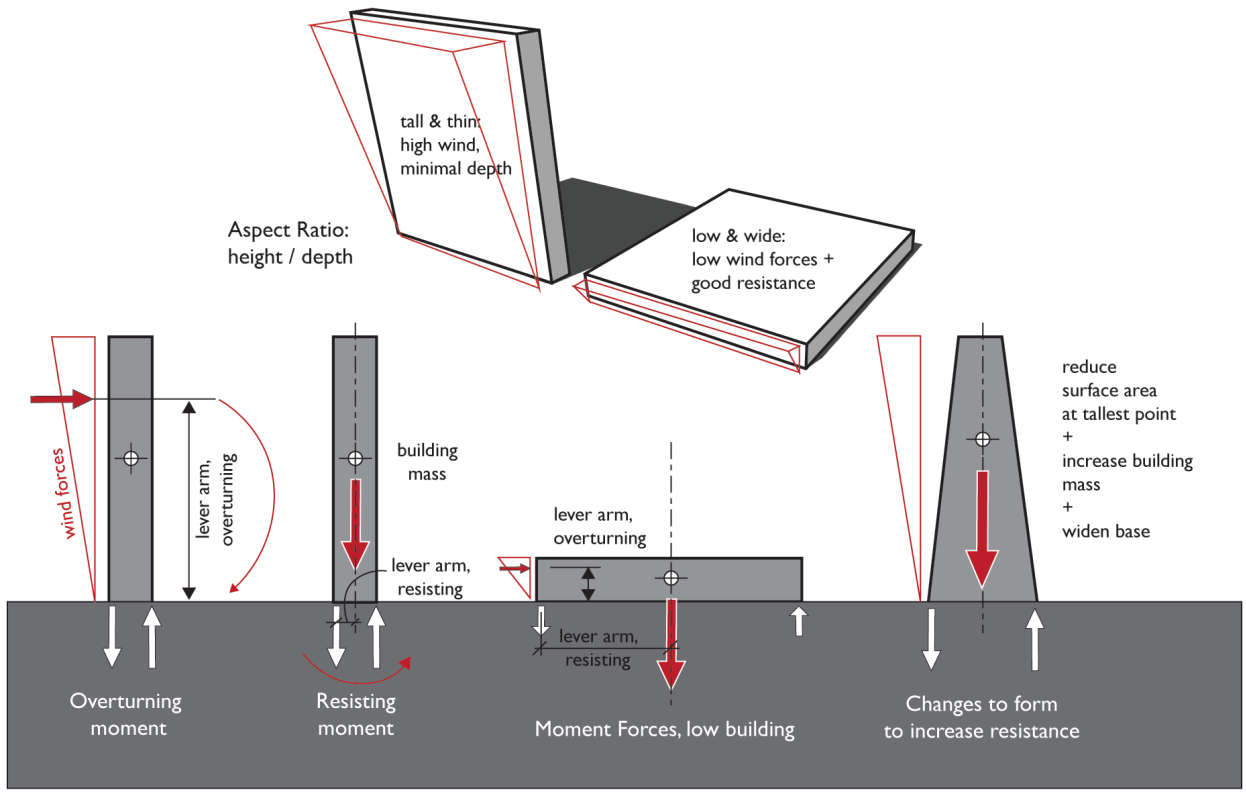

Figure 6.0.23 Effective resistance to wind design considers aspect ratio (base-to-height) and dead load weight of building versus overturning moment from wind

FOLLOW THE FAILURES

Structural collapses are rare occurrences in completed buildings, but when buildings are under construction and temporary bracing is used to stabilize the system until the framing is complete, the risks increase. Even in completed buildings, lateral wind forces can induce movements that trigger failures. Research one of the following structural stability failures: Indiana State Fair Stage Collapse (2011), De Grolsch Veste Stadium Roof in Enschede, Netherlands (2011), University of Washington Football Stadium Addition (1987), Tennessee River Bridge (1995), and New York Coliseum (1955).

To Do: Determine what forces caused the failure, describe why it failed, and suggest ways that the failure could have potentially been avoided with additional bracing.

stiff enough to resist bending stress without incurring too much drift (side-to-side sway). Unfortunately, a heavy and stiff building isn't an efficient or economical solution, and its weight is a liability for seismic stress. (Figure 6.0.23)

There are three stages of design for lateral resistance. First, a general strategy needs to be established as to how the structure remains stable (employing connections, bracing, shear walls, either in a bay/grid, casing, or core arrangement). Second, lateral forces are resolved at the foundations, so a general calculation for overturning moments and base shear establishes how a building has to connect into its foundation. Light, tall buildings with narrow footprints need to resist tension stresses pulling the building off its foundation on the windward side. Finally, individual elements that resist the lateral stresses need to be sized and analyzed to make sure they can resist these. To do this, we return to tributary area to define the amount of stress that each framing element is resisting—the further apart the lateral braces are, the more stress it will have to collect and transfer. (Figure 6.0.24)

Because wind applies positive and negative pressures to the building skin, architects also have to select building enclosure systems that account for these forces and how these systems transfer their loads into the structural framing (i.e., a curtainwall framing). Negative pressure and uplift on parapets, overhangs, canopies, and even lightweight structural systems like tents/membranes can cause significant problems.

In practice, conventional buildings can integrate lateral stability strategies without any special tools. As long as elements' placement is done with common sense, it is no more difficult to determine resistance to lateral loads than it is to gravitational. But special circumstances will require more advanced design and analysis, particularly with major wind events, such as hurricanes, where forces can exceed the minimum design standards. Because wind loads are dynamic and potentially damaging to a structure, digital simulations are useful to determine which locations will have the highest risk and stress. The taller the project, the more important an efficient and affordable lateral design strategy becomes. When the building form is unconventional or the surrounding terrain is atypical such as urban landscape of high-rises, wind tunnels can test physical models. (Figure 6.0.25)

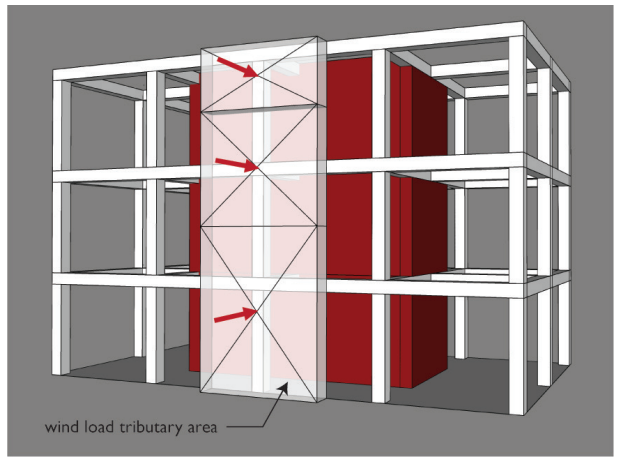

Figure 6.0.24 Wind loads can be calculated like gravity—total force (load per area) × tributary area (shown in white). These forces are transferred to the building structure and compared to capacity of stabilizing element to provide resistance and force transfer

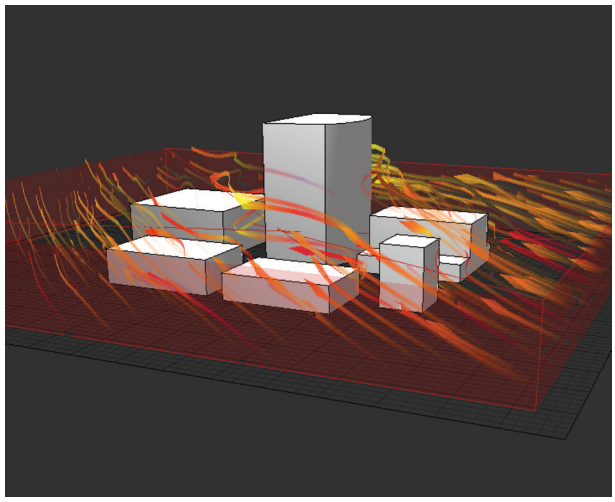

Figure 6.0.25 Specialized design tools for wind include wind tunnels and digital modeling/animations. Shown: Autodesk Vasari

Designing for Seismic Resistance

Seismic forces are caused by the release of waves of energy in tectonic plates. Like wind forces, the risk factors are determined by geographical location and historical data. Shear waves produce a sideways or up/down motion that shakes the earth and causes the most damage. Unlike wind forces, the direction, amplitude,

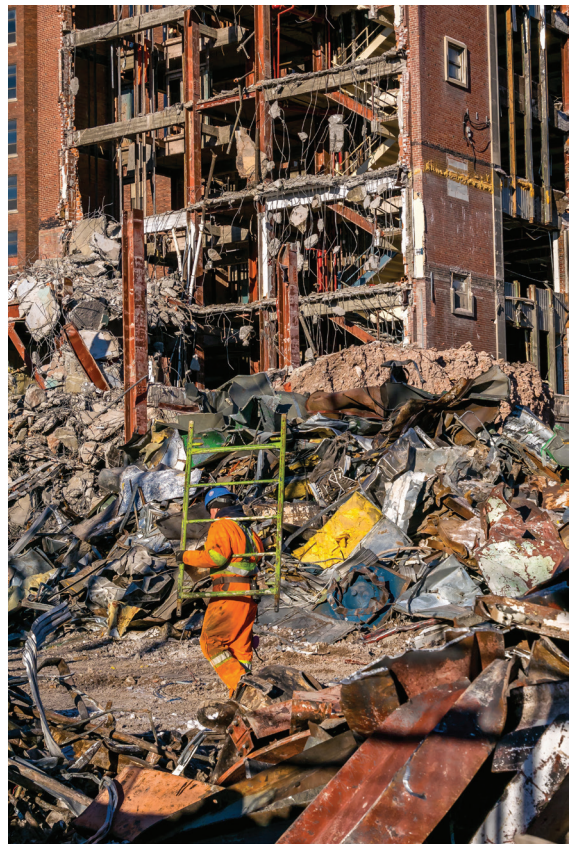

Figure 6.0.26 Seismic forces can cause tremendous damage to buildings. The goal for seismic design isn't to resist damage, it is to keep the buildings standing

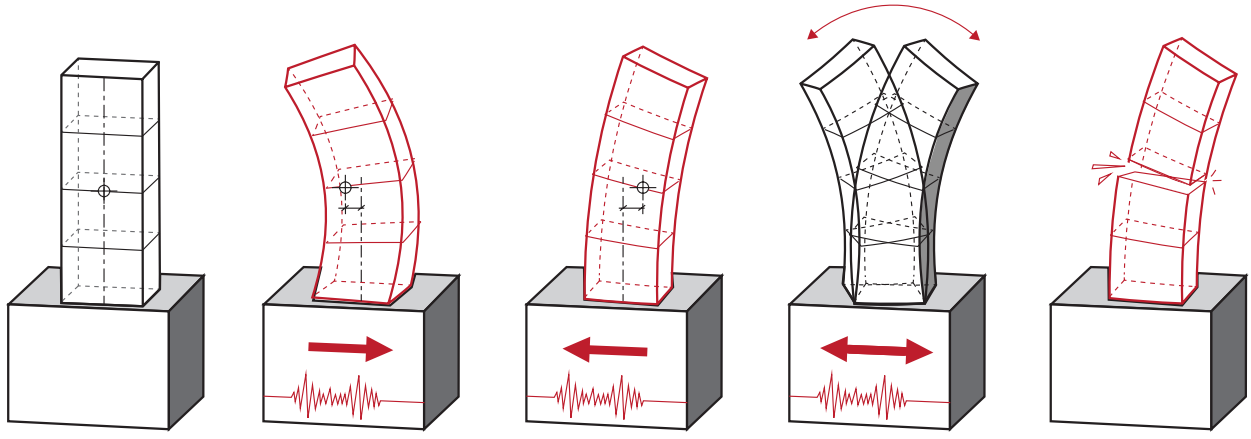

Figure 6.0.27 Seismic forces and potential failures

and acceleration of earth movement is unpredictable. These are fast and violent dynamic movements that can cause enormous damage to buildings. Horizontal forces pose particularly grave threats. (Figure 6.0.26)

During an earthquake, the building's mass initially resists the sideways movement through inertia, but eventually relents and moves. This creates a wave-like motion in the building. Sideways ground movement is measured by the time period it takes for a building to complete one back-and-forth oscillation, the *period*. This period is dependent on the mass, or weight of the building and the rigidity of the supports. (Figure 6.0.27)

Stiffer and heavier buildings have a shorter period, which subjects them to higher shear forces and greater damage. Instead of dissipating the seismic energy through movement, energy in stiff buildings is released *dangerously* by the supporting elements breaking in the back-and-forth motion. Inertial movement caused by seismic forces increases with the building's weight. Whereas heavy and stiff buildings were good solutions to wind forces, more flexible buildings with longer oscillation periods, like steel structures, are better for seismic forces. Increasing a structure's stiffness by using concrete or masonry buildings, increases the seismic load and risks greater damage.

It is both paradoxical and problematic that a good strategy for lateral wind resistance (heavy/stiff structures) is a poor solution for lateral seismic resistance.

If you consider both wind and seismic as forces created by a release of energy, then one common solution to resist both would be one that absorbs this energy and safely transfers it to the ground. This can be done with base isolation systems, tuned mass dampers, or shock-absorbing X-bracing. Unlike wind forces, the goal for seismic design isn't to resist loads without damage (the forces are too great). Instead, seismic codes and analytical methods determine acceptable ranges of resistance to the forces with their primary focus on making sure the building doesn't collapse. (Figure 6.0.28)

The building form and the location of its bracing elements both impact a structure's resistance to seismic forces. The ideal seismic-resistant building shares many characteristics with ideal wind-resisting forms. The frames' layout should be symmetrical in plan and elevation with equal distribution of stiffening elements three-dimensionally throughout.

Lateral Design: Making and Breaking

Because wind and seismic forces are dynamic and unpredictable, we may test their effects with digital simulations and/or physical prototypes. Physical testing is often necessary because of the complicated nature of dynamic movement. Wind

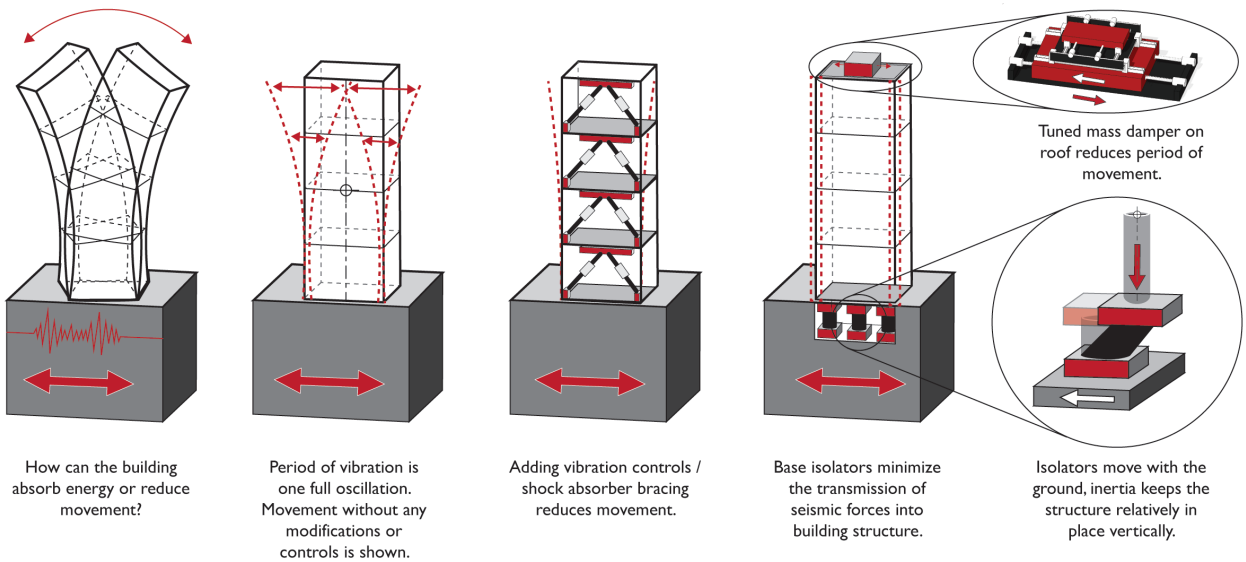

Figure 6.0.28 Strategies for seismic resistance and absorption

testing for high-rise buildings is done in wind tunnels, while seismic connections are often tested on shake-tables. Although shake tables aren't useful tools for architects to measure actual displacement, they demonstrate physical movement and differences in relative stiffness. Physical models and testing on a small shake table is improved when ductile materials (wooden dowels) are used with restrictions on the types of bracing allowed (e.g., compare shear wall bracing to X-bracing in same building frame), and various options are tested and recorded. (Figure 6.0.29)

Figures 6.0.29a and 6.0.29b Using lightweight, flexible (but brittle) materials like spaghetti in models reveals behavior under induced lateral loads. Compare casing, walls, bracing, strategies to reveal location and extent of stress (SxD, Iowa State University)

Engineers and product manufacturers use physical models for seismic testing more commonly than architects. Very large modeling, including full-scale buildings, can be placed upon a giant moving plane that simulates seismic movement—a larger and more accurate version of a shake-table. These tests help us understand how materials, connections, or energy-absorbing elements perform under seismic loads. We may use information gathered from these tests to create digital simulations. Advancements in dynamic digital modeling have enabled structural designers to better understand the effects of different magnitude earthquakes on buildings. Despite this information, seismic events still have devastating effects, particularly when buildings are constructed of heavy, non-reinforced masonry or concrete—typically a consequence of economic hardship and lack of access to more ductile structural material. The magnitude of seismic forces is measured in the Richter scale; a logarithmic scale system in which each whole number represents a ten-fold increase in amplitude, or nearly 32 times the amount of energy released, from the whole number below. Because a potential earthquake's magnitude is never known, we design buildings to resist a range of potential forces. But sometimes earthquakes exceed what codes anticipate.

Designing frames for stability isn't simply a matter of adding something to our existing methods of framing and construction. Consider how these structural factors could lead to new material and assembly options. Imagine a semi-active digital dampering system that absorbs wind forces on the curtainwall before they are transferred to the building frame, or a masonry block alternative that absorbs seismic movement without fracturing or damaging the building. Imagine the creative ways of integrating these performance goals into your building designs. (Figure 6.0.30)

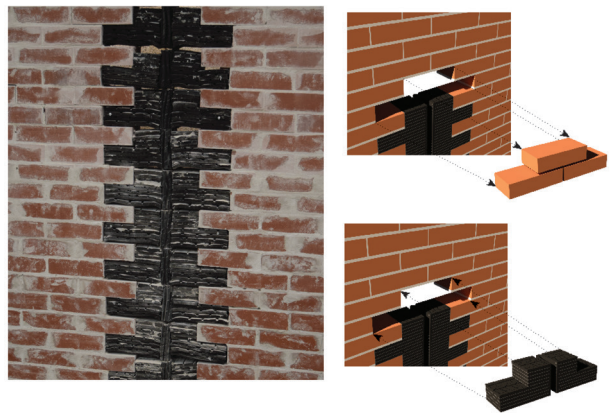

Figure 6.0.30 Retro-Brick prototype for recycled tire brick replacement for seismic resistance (Structures in Service, Iowa State University (Gonzalez, Ramirez, Bardiji-Izard, Whitehead))

Stability in Summary

Stability is fundamental for any structural system. Designers have options for which lateral stability tactics to select (connections, braces, etc.), where these stabilizers are placed, and how they fit in with the overall building form. These choices determine how lateral loads are transferred and how the overall structure behaves. They can be applied to buildings of different heights, forms, and structural systems. Continuity of stiffness and lateral resistance must be maintained down to the foundation. The geographic location of a project, its height, and building occupancy determine the load that needs to be resisted. Some solutions for wind resistance may not be useful for seismic zones, but designers have access to tools that model and simulate these forces in order to determine the optimal response to both threats.

MAKING AND BREAKING: DESIGN EXERCISES

Revisit one of the design challenges from a previous chapter (arches, cables, beams, and/or trusses) and subject them to a deeper scrutiny in relation to their lateral stability design strategies.

To Discuss: How would height, program, or location of the structure change the considerations for wind design? What stabilizing strategies work better with certain structural systems than others (e.g., are shear walls better in beams than trusses? X-bracing in arches?)?

Frequently Asked Questions

Do designers really need to know, or decide, the manner of each joining method within a building? This depends on whether the designer intends to architecturally emphasize each joint or not and, by extension, what the joint reveals about the overall structural strategy. For some, these considerations show a commitment to craft and consideration at multiple scales (imagine the Centre Pompidou in Paris without beautifully articulated connections) or an aesthetic "celebration" of how a building works (typified by the high-tech movement). Designers can't determine the manner of connection for aesthetic reasons. Some connections *must* be fixed or pinned. But this doesn't mean that the connections cannot be aesthetically interesting. For traditional framing, especially where the structure isn't exposed, these are determined in conjunction with the structural engineer and fabricator to determine efficient and effective connections. How elements are connected needs to be evaluated in economic terms as well. (Figure 6.0.31)

Figure 6.0.31 Integration of structural expression, including bracing, defines the aesthetic at the Centre Georges Pompidou (Renzo Piano and Richard Rogers, Paris, 1971)

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Special thanks to: Mola Structural Kit for model images (and a great learning tool). 12-Node stabilizing exercises courtesy of Prof. Kevin Dong and Emeritus Prof. Jake Feldman from Cal Poly San Luis Obispo.

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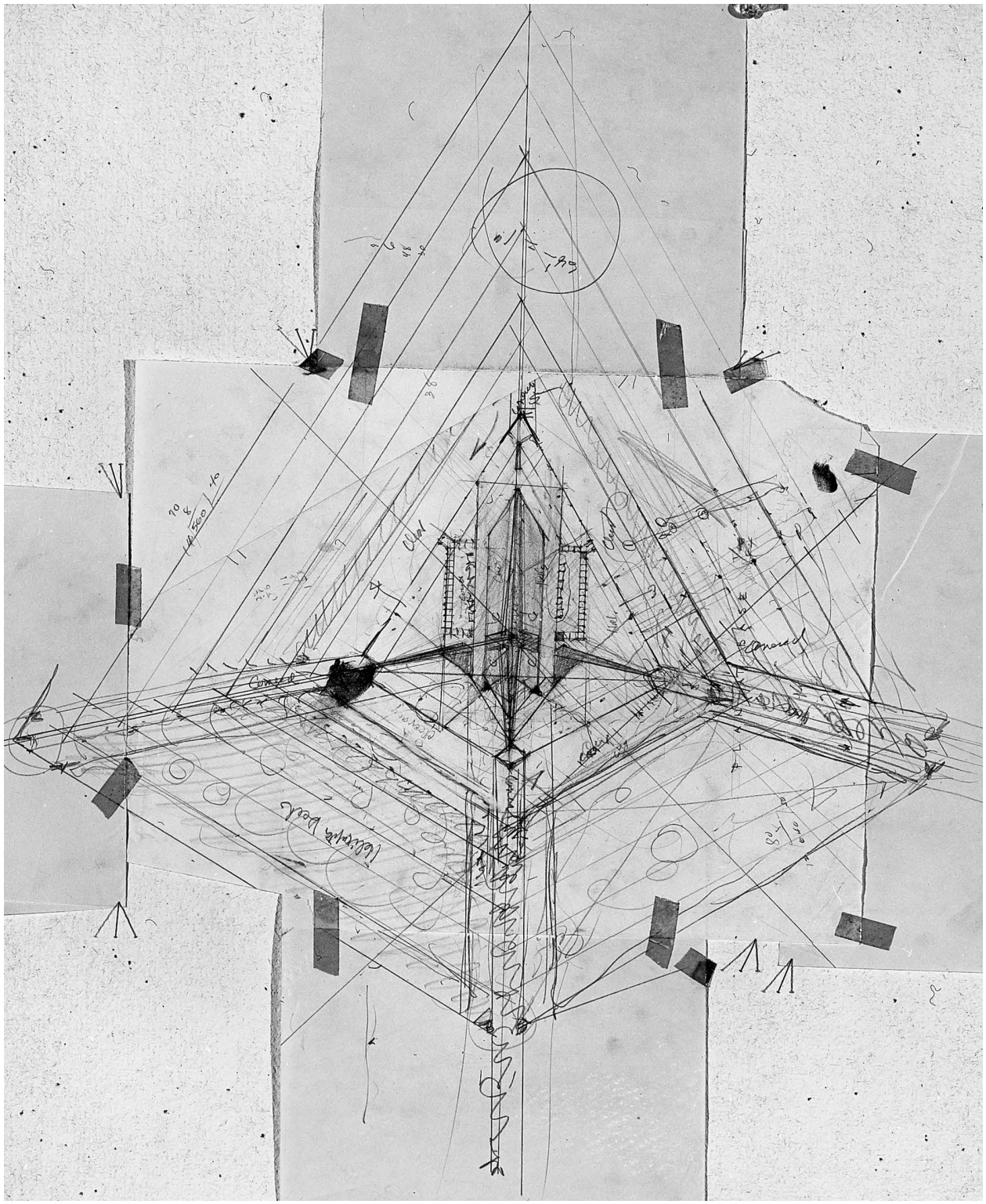


Figure 6.1.0 Plan sketch of "Mile-High Illinois" tower (Wright, 1957)

CHAPTER 6.1

High-Rise Structures

Taller, Thinner, and Smarter

Structural ingenuity and material advancements allow us to create structures that rise to remarkable heights. These high-rise structures face unique design challenges that demand a holistic approach to economic, architectural, and structural principles.

In his 1957 book, *A Testament*, Frank Lloyd Wright described the nature of his work as a combination of engineering and architecture, writing that, "I plead guilty to the tough impeachment . . . So the poet in the engineer and the engineer in the poet and both in the architect may be seen here working together, life-long." The book contained an ambitious proposal for the "Mile-High Illinois" skyscraper. This 5,280ft (1,730m) tall, 528-story building (18.5 million square feet) would be twice as tall as the tallest building in the world today. Was this fantasy or foresight? (Figure 6.1.1)

Before we begin, ask yourself:

- Why are structures' heights limited?
- What factors cause these limitations?
- Would something this tall even be technically possible?
- If it were technically possible to build, what else would need to be considered to make it viable?

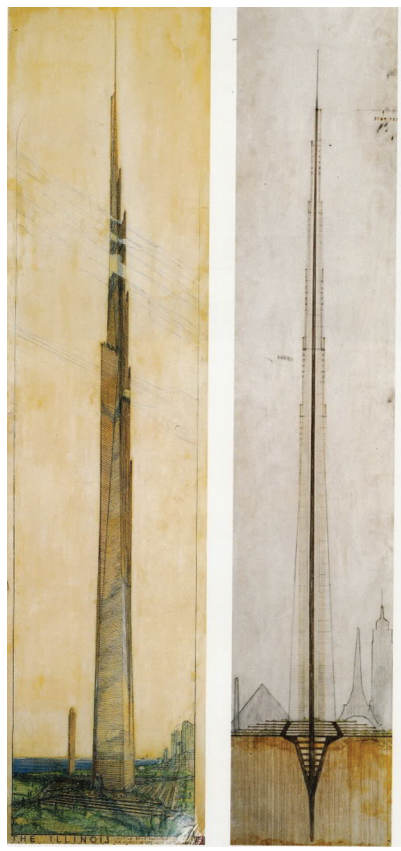


Figure 6.1.1 Elevation and section drawings for the Mile-High tower proposal (Wright, 1957)

Structural Height and History

Structural designers have always been tasked with creating strong and stable frameworks to enclose architectural volumes. Across the centuries, patterns of structural types emerged, based on the ways that

Figure 6.1.2 A comparison of two divergent solutions for getting high above the ground and their contrasting materials and forms (1931)

materials and forms can work together to solve "spanning." As material performance, building technology, and scientific information evolved, our ability to build larger structures that enclosed more space increased. As did the potential to build taller.

Although we can look back across two millennia of long spanning structures, there were few successful experiments of habitable building structures that were quite tall. As structures grew taller, resisting lateral/wind forces became the primary challenge, but ancient building materials and framing methods weren't able to resist these forces easily. To resist overturning forces of the wind, tall structures tapered from a wide base to a smaller peak—for centuries, the Giza Pyramids were the tallest structure in the world at 481ft (147m). (Figure 6.1.2)

There are illustrative examples, particularly in Gothic cathedrals, of structures that developed a more responsive structural form to become "taller." Rouen Cathedral's towers rose to 495ft (151m), for instance, incorporating wind resistance into its overall form as buttresses. However, because these structures were made from materials like stone that could only resist

Figure 6.1.3 Engraving showing relative heights of important large buildings in the world in 1850 (Reynolds and Emslie)

compressive forces reliably, they relied on arches, vaults, or large bearing walls to achieve these heights. The material stiffness, the weight of the structure, and the curvature of the surfaces all resisted wind.

Despite the aspirations to build tall there were limits to how high a compression-based structure could be built. In fact, when the Eiffel Tower was completed in 1889, its height of 986ft (300m) was nearly twice as tall as any other existing structure. It was an illustrative example of iron framing's efficiency and potential for new forms, as well as a testament to professional engineering's benefits. It changed the fundamental structural challenge from "spanning" to "stacking." Instead of looking for ways to enclose larger spaces side-by-side, structures could enclose more space by becoming taller. (Figure 6.1.3)

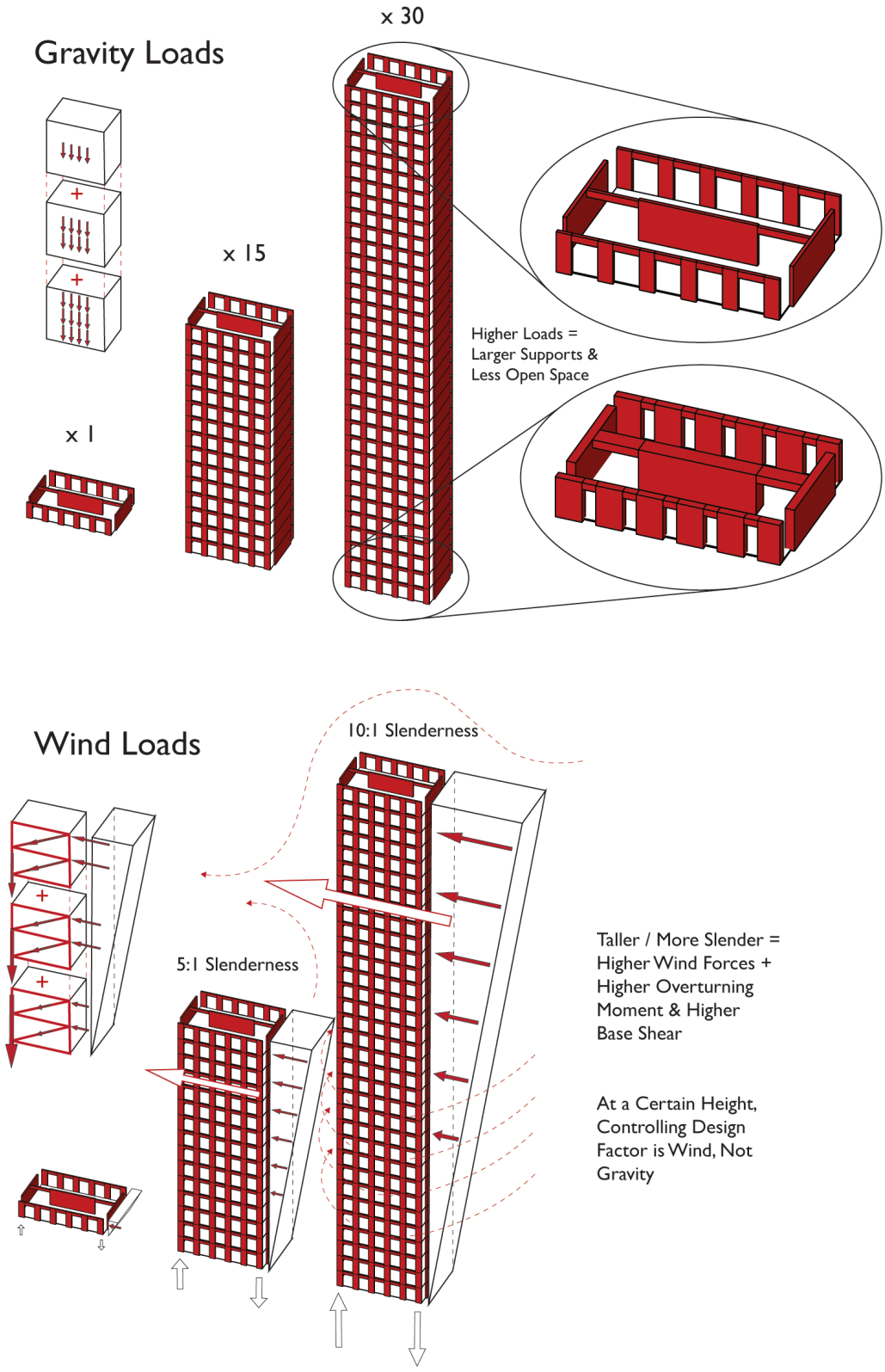
THINKING: CONCEPTS BEFORE CALCULATIONS

Collect a series of plastic cups and bottles of different heights and proportions. Fill some of them up to various levels with materials of different densities (water, sand, etc.). We'll subject these objects to basic pushing/pulling in an attempt to better visualize the behavior of high-rises when subjected to wind forces. A greater range of differences in height and weight of the conditions that you create will generate better results.

To Do: First test an empty bottle then a full bottle of the same size—notice how the lighter but stiffer objects easily overturn while the heavy objects are more prone to sliding or shear at their base. Notice how lighter and less stiff materials bend and deflect instead of simply overturning—refer back to the different movements you observed in the stacked cube exercises from Chapter 6.0. Record your observations about the relationship between density and stiffness under testing.

To Discuss: Under what conditions would a heavier and stiffer high-rise be better? (Figure 6.1.4)

Figure 6.1.4 Sample of potential model experiments to associate mass, height, and overturning moments



Figures 6.1.5a and 6.1.5b High-rises have to resist gravity and wind forces. Highest gravity loads occur at the base, highest wind forces occur at the top creating highest overturning moment at the base

What Makes High-Rises Different Structurally?

The taller a building gets, the more exacerbated its structural problems become. Increasing the height of a structure increases the magnitude of both gravitational and lateral loads. The loads from all the stacked floors accumulate into large vertical loads at the base that must be resisted by larger columns and foundations. Because wind forces increase based on their height above the ground, they exert large forces on the top of the structure, bending and threatening to overturn it or shear it off its foundations. As buildings grow taller, both challenges increase. Wind and gravity loads are collected, grounded, and stabilized in distinct ways. In projects with thin height/width proportions, lateral loads govern design decisions. Resisting these loads is an essential structural challenge, one that determines the project's form, arrangement, location of structural framing, type of materials, and type of connections. (Figure 6.1.5)

Because tall structures need to resist increased gravitational loads and high-level lateral forces, high-rise buildings have more material than other structures—material that needs to follow restrictive fire-code protections. Because these are structurally-dominant buildings, our challenge is how to define how and where structural elements are placed to efficiently resist gravity and lateral forces. These have consequences because each floor's framing is repeated throughout the structure. It needs to be coordinated with life-safety issues of egress, and the functional and economic issues associated with their high use volume. These considerations invited particular scrutiny during the onset of high-rise design.

Skyscrapers: The Emerging Indivisibility of Architectural and Structural Principles

In the late 19th century, intense economic growth in Chicago and New York City led to innovations in design and construction—specifically the "structural frame"—which allowed habitable multi-storied buildings to be completed at heights previously unseen. Instead of relying on the masonry and timber framing that spread across larger areas of land, both cities needed affordable, rapidly constructed, fireproof building construction that consolidated large usable volumes into small areas.

Early "skyscrapers" didn't reach heights comparable to ancient towers, cathedrals, or monuments. They were typically only ten stories or about 130ft tall, but they were responsive to functional and economic goals. These were occupied spaces, not simply monuments or bell-towers. Compelled by these developments, an emerging profession of high-rise designers strove to understand the technical challenges of building bigger and taller buildings.

In the 1880s, John Root (a partner in Chicago's architecture firm, Burnham and Root) summarized these technical challenges in a manifesto of engineering goals that still resonate today:

1. *The design should provide the largest floor area and most spacious building consistent with financial success*;
2. *The floor plan must provide maximum daylight for rooms*;
3. *(Elevators) should be located centrally*;
4. *The buildings services . . . should be located to allow easy use, maintenance, and alternation*;
5. *The height of each story should be standardized at the optimum—10ft 6in (3.2 m)*;
6. *Walls should contain as many openings as are consistent with their structural function*;
7. *The structural steel frame and fireproofing of the columns and beams should reflect not only the loads they need to carry, but also the soil conditions beneath*;
8. *foundations for walls or columns should be built on a grillage of steel rails embedded in concrete*;
9. *Construction of a building should progress with equal speed over the whole area of foundations to avoid unequal settlement*.

Root identified three major building elements that needed to be re-considered for the increased forces associated with high-rises: load-bearing walls, foundations, and wind-resisting elements. In traditional buildings, load-bearing masonry walls were used for enclosure and support, but they became larger and thicker as buildings grew taller. Load accumulation from these heavy materials resulted in inefficient floor plans and smaller openings at the base of the building—not ideal circumstances. Eventually iron (or steel)

framing was moved to exterior walls to support the weight of each story of masonry, avoiding the problem of accumulated loads and thicker walls. (Figure 6.1.6)

As buildings grew taller, their gravitational loads increased while wind loads became controlling factors. In response, high-rises had to integrate load stabilizing frames into their overall structural systems. Traditional buildings used perimeter load-bearing masonry walls for wind resistance (i.e., shear walls), but the size of these walls that resulted as buildings grew taller required new systems for wind bracing.

Diagonal braces took up valuable floor space, so designers also explored rigid portal frames. Like solid walls, these frames supported gravity loads *and* resisted lateral forces, but they were lighter and more compact. Designing these frames was only possible because engineers could now estimate the bending loads and moment forces in indeterminate structures. These advancements led to less weight, more open space, and better resistance to wind forces as structures grew taller. (Figure 6.1.7)

The third structural element that needed re-engineering was the foundation. Traditional buildings grounded their loads using walls or columns that rested upon foundations made of rubble. Walls spread these loads across a continuous trench footing, avoiding large consolidated point loads. When columns were used, particularly in taller buildings with higher loads, these concentrated loads could be too great for the bearing capacity of the rubble and soil below, unless a caisson was built down to the bedrock. New "grillage" foundations stacked iron rails or I-beams atop each other, in a pyramid shape, to spread out points load from columns to wider areas below. (Figure 6.1.8)

Root's assessment established one other important factor: he linked architectural and engineering considerations. Structural strategies need to serve architectural and economic goals, especially daylight and floor area, and architecture needed to respect the physical constraints of the structure, particularly its core and framing. He felt this would resolve, "our architectural designs into their essential elements." Of course, these buildings were more than just "structures." Other functional challenges needed to be integrated in order to make the structures work as

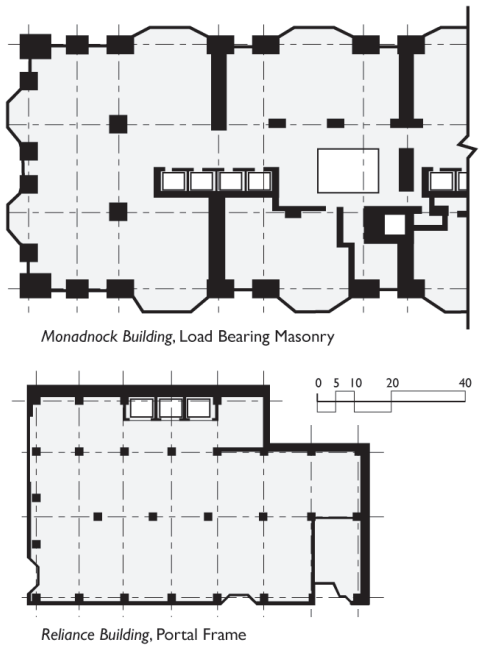


Figure 6.1.6 Plan comparison of ground level structural member sizes comparing load-bearing masonry of sixteen-story Monadnock Building (1893) versus portal frame of fifteen-story Reliance Building (1895)—both projects by Burnham and Root in Chicago

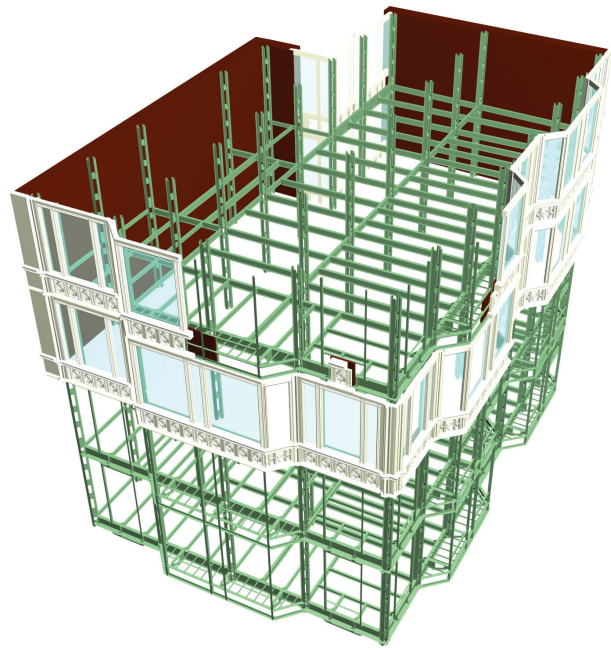


Figure 6.1.7 Reliance Building frame. By separating the façade from load-bearing obligations and bracing the framing laterally with fixed connections, buildings became lighter, more open, and taller (T. Leslie, *Chicago Skyscrapers, 1871–1934*)

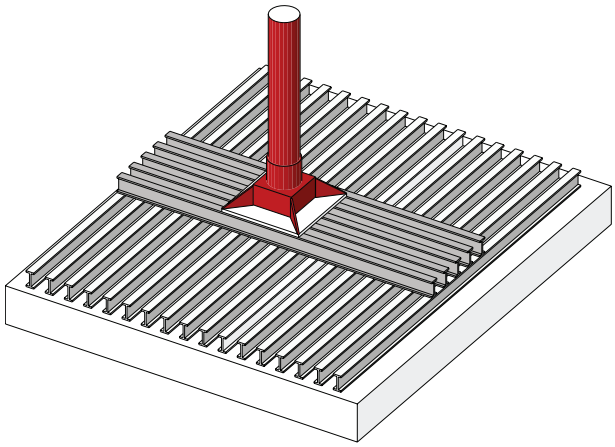


Figure 6.1.8 Grillage foundations used multi-layered systems of beams and concrete pads to spread the point line forces from columns across a broader area

intended, including elevators, building services, electric lighting, air conditioning, water pressure, etc.

As innovations in building systems merged with improvements in materials—especially structural steel—construction, and structural design, buildings became larger and taller. In 1931, less than fifty years after the first high-rise buildings were completed (e.g., the Home Insurance Building in Chicago (138ft (42m)), the Empire State Building (1931) became the first building to have a hundred floors, topping out at 1,250ft (381m) as the tallest building in the world—a record it would hold for nearly forty years. The ensuing decades have seen the height of tall towers increase incrementally (Sears Tower, 1450ft (442m) in 1974, Petronas Towers, 1,483ft (452m) in 1998, Taipei 101, 1,671ft (509m) in 2004, and the Burj Khalifa, 2,717ft (828m) in 2010). (Figure 6.1.9)

Figure 6.1.9 The Petronas Towers appear braced together at mid-height, but the bridge is for observation and egress (Kuala Lumpur, Cesar Pelli, 1993)

Although height is a celebrated feature of these buildings, it is hardly the most relevant measure or consideration. A taller building doesn't automatically represent "progress." Our initial question about Wright's Mile-High tower is perhaps misleading. There *are* technical limits as to how high structures can be built (strength, stiffness, stability, and serviceability), but there are also issues of economics, feasibility, functional utility, life-safety, etc. that are often more fundamental and, ultimately, deciding factors.

As Root predicted, high-rise design decisions are commonly based on economic factors that require architectural and structural solutions. Instead of comparing "tall, taller, or tallest," we could evaluate material utilization or ways of improving the functional utility of the spaces. As such, high-rises compel theoretical and practical questions:

- Why do we need high-rises?
- If we do need them, how can we design them well?

If the "how" doesn't work (e.g., little leasable space, small windows, life-safety hazards, etc.) then we risk sacrificing all the reasons "why" the building was built in the first place.

High-Rises: Definition, Motivations, and Materials

High-rises' ubiquitous presence in urban centers around the world suggests their many potential benefits. They create dense locations of people and resources—factors linked to a more sustainable resource and energy model than sprawling solutions. They create vertical cities of mixed-use programs and concentrated economic activity. On an experiential level, they can offer unique views and ample daylight. They can also establish an identifiable profile to a city skyline, and help

MAKING MATTERS: EMPIRE STATE BUILDING

The final height of the Empire State Building (and its corresponding role as a cultural phenomenon) was not only its only remarkable feature. It was constructed in a staggeringly short period of time: 410 days, three months ahead of schedule.

To Do: Find images of the project under construction. Research the building's construction process and identify the factors that allowed this speed.

Discuss: What issues were related to the design geometry/form/framing plan (was it intended to be efficiently built)? What factors were related to the steel industry and/or the construction climate in the U.S. in 1930–1931? (Figure 6.1.10)

Figure 6.1.10 Empire State Building with timeline of floor assemblies indicated and construction photos

define the image of their respective settings and the era in which they were created. (Figure 6.1.11)

Despite these advantages, there are still distinct challenges. Concentrating people and resource into slender structures that are subjected to high forces makes them high-risk buildings. These are resource-intensive buildings that are economically speculative. Despite our ability to design and build high-rise projects easily compared to a hundred years ago, they are still a somewhat rare building type.

What Is a High-Rise? Definitions and Basic Behaviors

We can define a "high rise" by several standards: the number of stories (typically over ten), the overall building height (100ft (30.5m)), or a height determined by local code officials based on fire access limitations. These classifications are necessary for codes, but they are all imperfect in defining structural characteristics (e.g., the pyramids at Giza would

Figures 6.1.11 New York City skyscrapers: Flatiron Building, 22 floors (Burnham, 1902), Chrysler Building, 77 floors (Van Alen, 1930), and 56 Leonard Street, 60 floors (Herzog & de Meuron, 2016)

be "high-rises" based on height). We'll use some less-specific criteria.

To focus on the most useful high-rise design strategies we'll base our study on physical characteristics and proportions. Specifically, when a building's "tallness" or "thinness" becomes the defining structural behavior, it acts as a high-rise. As the Council on Tall Buildings explains, "height creates different conditions in the design, construction, and operation from those that exist in 'common' buildings." Slenderness is relative. Typically, buildings with a height-to-width ratio of 10:1 or higher require special measures. Resisting the additional stresses of a slender tower depends on framing strategies and materials. In these cases, we'll need another set of structural principles that we haven't had to use yet.

High-rises are built only with strong and stiff materials like steel and reinforced concrete. The materials must be fireproofed and fixed into a stable frame, either with braces or rigid connections (cast concrete is always a fixed connection). Steel framed high-rises generally have a height-to-width ratio around 7:1 (e.g., World Trade Center towers, Hancock Building). Because concrete is stiffer, high rises relying on concrete can have proportions closer to 11:1 (e.g., 401 Wabash Avenue, Chicago, Burj Khalifa tower, Dubai etc.). Mass timber high-rises have become a realistic option, but they rely on steel and composite concrete connections to stiffen the building and generally require special approval from code officials; wood hasn't been typically approved as high-rise material because of its combustibility). But because wood is lighter than steel or concrete while having a superior environmental profile, wood towers could offer unique benefits. (Figure 6.1.12)

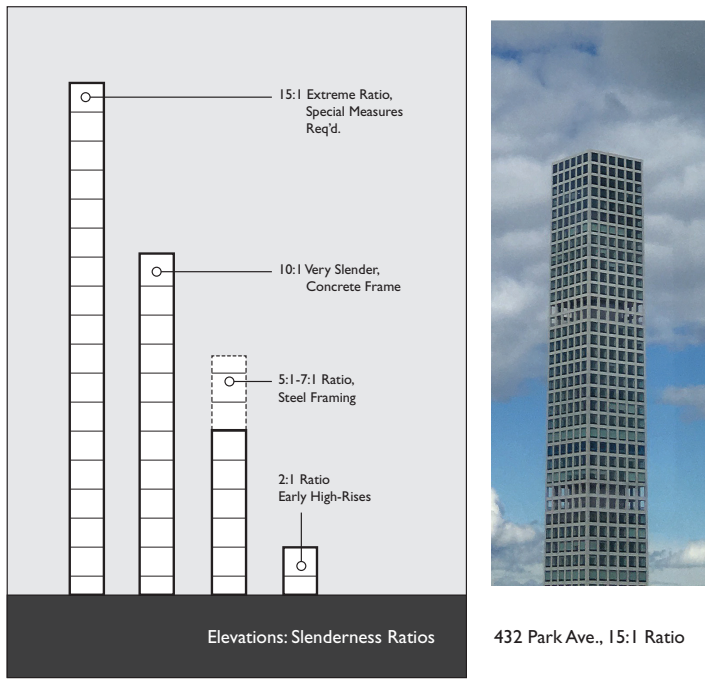


Figure 6.1.12 Diagram of slenderness ratios for high-rises and corresponding potential construction systems. Shown: 432 Park Avenue, New York (eighty-five floors, Vinoly, 2015)

MATERIAL AND MAKING MATTERS: TIMBER TOWERS

Part I: The Timber Tower was a speculative project completed by the designers at Skidmore Owens and Merrill (SOM) in 2014. The project's goal was to understand the potential benefits and limits of a high-rise tower built primarily from mass timber.

To Do: Review their initial drawings and report.

To Discuss: What are the benefits of using mass timber in high-rises? What are the liabilities? What specific challenges do high-rise buildings create for this material? Part II: What building is currently listed as the tallest mass timber tower in the world? How does its height-to-width ratio compare to steel or concrete towers? Is it constructed solely from timber or does it have a hybrid connection system like SOM's Timber Tower proposal?

The Paradox of Efficiency in High-Rises

High rises aren't simply engineering challenges. As the Chicago structural engineer Fazlur Khan (SOM) explained, "Today, without any real trouble, we could build a 150-story building. Whether we will, and how the city will handle it is not an engineering question, it is a social question." It is also an economic question of resources.

High rises are huge and resource-intensive so there is a challenge to be efficient in all of their design choices. But "efficiency" as a design strategy is fraught with paradoxes, compromises and conflicts. Inefficient schemes have more weight to resist, more materials to purchase, and more difficulties during construction. But heavier buildings are naturally more resistant to swaying caused by the massive wind loads so their weight isn't completely wasteful.

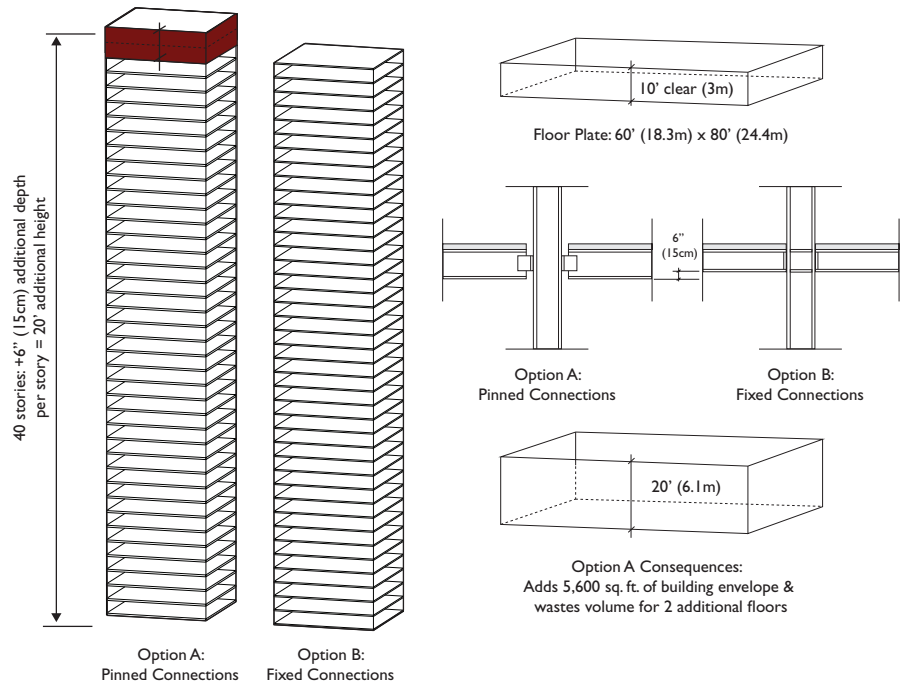


Figure 6.1.13 Small changes in floor-to-floor heights or depth of structural members can have significant economic consequences in construction cost, leasable area, building enclosure surface area, etc.

A second common paradox is the economy of floor depth and width of the column spacing. Widely spaced columns carry greater loads, so they must become larger and have to be paired with deeper floor systems that span farther. Deeper floors mean taller buildings or fewer floors (e.g., a modest 2in difference in additional depth for a sixty-story tower is the equivalent of an additional story). Alternatively, we could reduce floor depth by adding more columns. But too many columns reduces the available floor area and functional utility. It may also force smaller window openings. (Figure 6.1.13)

One solution to help stabilize drift in high-rises, the mass-tuned damper, seems counter-intuitive as it involves placing hundreds of tons of non-load-bearing mass high above the ground. Although we usually avoid adding extra weight to our structures, in this case it helps. In these active-systems, a giant mass "floats" near the top of the building; when the wind blows, the building moves but the mass of the damper remains in position. Based on basic Newtonian physics, this system reduces the overall period of sway caused by the wind because of the inertial resistance of the damper's mass. One of the tallest towers in the world, Taipei 101 (1,671ft (509m)) has a 728-ton damper installed near the top of the building that reduces building movement caused by winds or seismic events. (Figure 6.1.14)

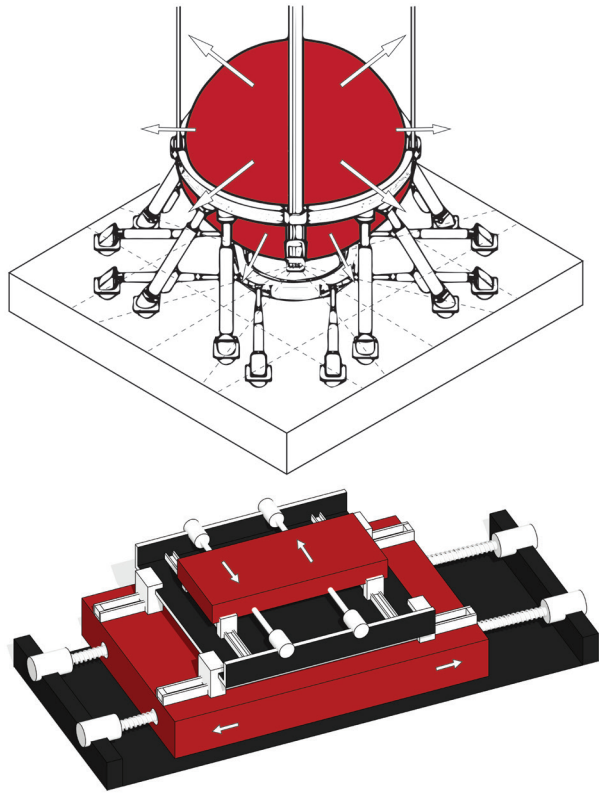


Figure 6.1.14 Drawings of dampers atop Taipei 101 (top) and Citibank Tower (bottom)

TOOLS OF THE TRADE: WIND TOWERS

To create viable high-rises, engineers need to ensure that both gravitational and lateral loads are resisted. In the 1970s, structural engineers began to use custom-created computer algorithms to assist their analysis. These programs helped perform complicated calculations, particularly for large buildings with indeterminate connections. These early programs couldn't generate a useful digital model or simulate dynamic conditions like wind. As such, other types of models were better suited for evaluation—including wind tunnel tests.

Designing for wind testing can be difficult, particularly in urban settings where the laminar flow of wind through surrounding buildings causes unpredictable wind forces. These projects require a thorough testing of dynamic loading, so they rely on both computational and physical models. Engineers want to ensure that the buildings won't move too much ("drift") because movement is uncomfortable for occupants and can cause damage to the building enclosure. Drift can also jam elevators, crack interior walls, or cause creaking noises. Tests help to confirm engineering calculations and to suggest ways to make the building framing more efficient. Recent advancements in dynamic computational modeling have reduced the number of required wind tunnel tests. (Figure 6.1.15)

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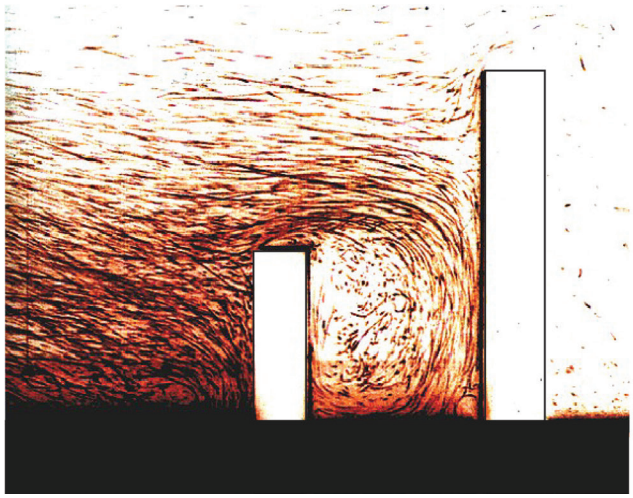


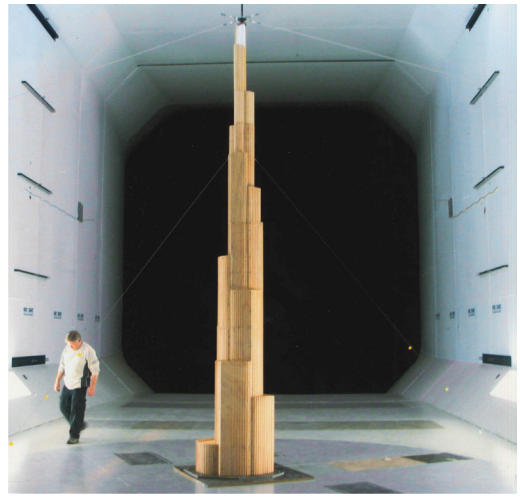


Figure 6.1.15 Wind testing simulations can use smoke and particles to demonstrate interactions between building surfaces. Tall buildings can be tested using a wind tunnel. Shown: prep for Burj Khalifa tower (SOM), testing by RDMI

Let's try to better understand the structural principles of high-rises from both macro and micro scale by looking at three different types of physical analogies.

Analogy #1: High-Rises as Stacked Floors

One common strategy is to stabilize the taller building by first stabilizing (and then repeating) individual floors. This follows the idea that if we get one floor of framing resolved (load collectors, grounders, and stabilizers) and align this layout with architectural goals for utility and aesthetics, we could theoretically repeat and stack them. There is a useful logic to this approach based on a regular and repeated building form. Architecturally it allows us to place building cores in a location that supports life-safety, building systems, and elevator access with a structural grid that is compatible with the function. (Figure 6.1.16)

Using this method, we can start to determine how the loads are collected and distributed from the horizontal slabs to the vertical bearing elements (e.g., one-way or two-way framing; composite decking, structural slabs, etc.). Once we determine the structural floor's depth based on the vertical supports' locations, we can determine the overall floor-to-floor height and estimate the number of floors and total height of our building. As long as each floor has adequate lateral bracing that aligns vertically and continuously throughout the building, this approach should lead to a viable schematic. Once one floor layout is done, we can extrapolate this scheme to other floors (top, middle, bottom) to see what the consequences of accumulated vertical loads is on the lower floors and foundations.

This approach isn't looking at the big picture of the building—it assumes a regularly stacked-and-repeated building form. But gravitational loads accumulate as they move down while wind pressure grows in the opposite direction, becoming greatest at the highest level. This "floor-by-floor" approach is a helpful in initial planning and sizing, but if every floor was the same, each one would be overstructured to resist the gravitational and lateral loads based on the worst-case level!

However, depending on the building height and materials used, designing each floor for the "worst-case" gravity and lateral scenarios may not add a significant amount of extra materials when compared

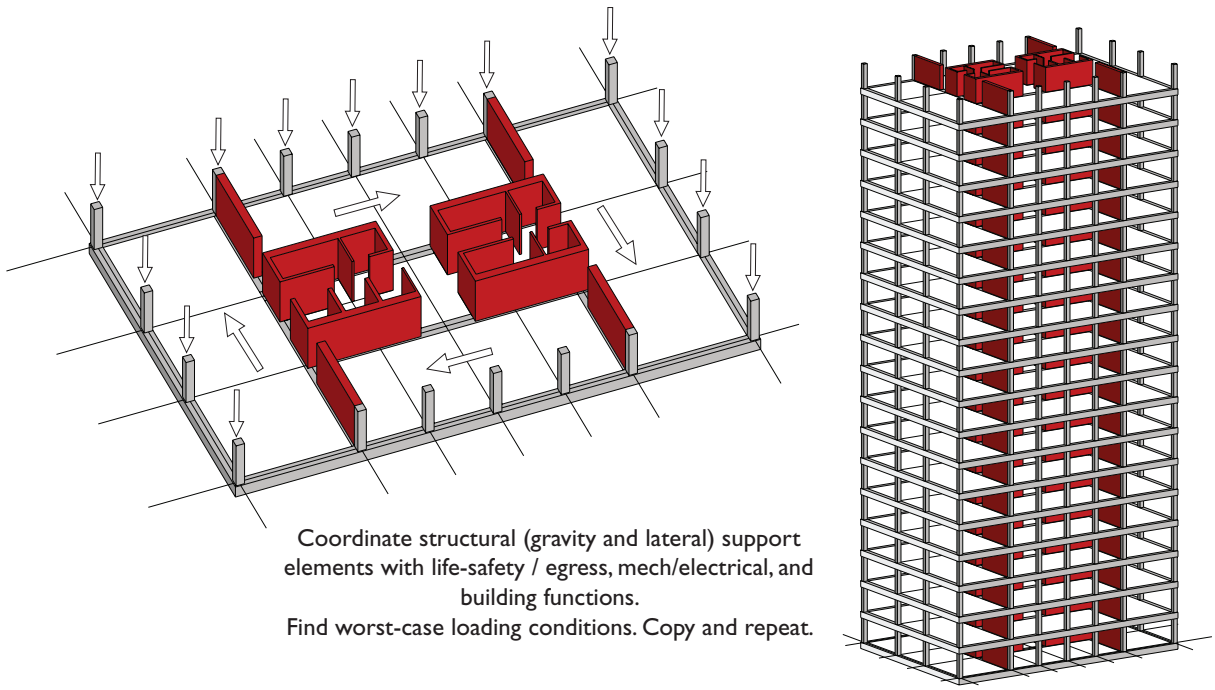


Figure 6.1.16 'Stacked-and-repeated' floor analogy. Despite changing gravitational and lateral loads, each floor is designed the same (worst-case scenario)

to the economic benefits of standardizing formwork and other building elements. There may be an aesthetic uniformity in size and depth as well. For example, the ability to repeat and stack floors often becomes part of a tower's expression, particularly when the floor slabs (or the building mass itself) extend out from the face of the building. (Figure 6.1.17)

Figure 6.1.17 Articulating the floor-by-floor repetition can be a central aesthetic priority (Marina City, Goldberg, Chicago, 1968)

Analogy #2: High-Rise as a Column

A high-rise has a height/width ratio that is analogous to a column that is fixed to the ground and unrestrained at the top. Therefore, we can adapt strategies that improve a column's performance to high-rise design.

The primary threat to most columns is buckling, especially if they are too thin. Buckling can be resisted by increasing the radius of gyration and the moment of inertia; in other words, by moving material to the column's outer edges, preferably symmetrically. A hollow tube resists buckling more efficiently than a solid one, similar in principle to the *casing* strategy discussed in the previous chapter. This analogy isn't exact. High-rises are subjected to lateral loads while columns typically resist only gravitational ones, but this solution is still effective. Because it is unbraced at one end and fixed at another, a high-rise "column" would also be more effective with a wider base and a narrower top—a common strategy for high-rise forms. (Figure 6.1.18)

High-rise "columns" can also reduce their risk of buckling if their effective (or unbraced) length is reduced

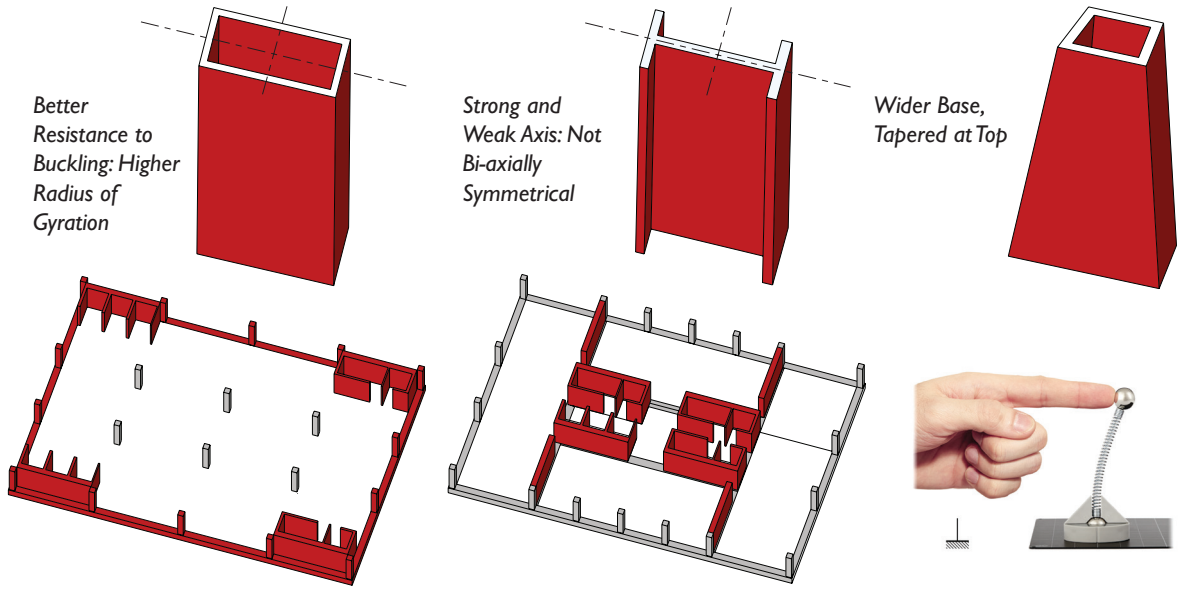


Figure 6.1.18 The column/high-rise analogy helps explain effectiveness of mass distribution to the edges and the corresponding increase in buckling resistance

Figure 6.1.19 Bracing is a predominant part of the expression in the CCTV Headquarters under construction (Office of Metropolitan Architecture (OMA), Beijing, 2012)

or braced like a flying buttress. The most common approach is to use a "bundled tube" where the tower acts like several columns bound together, usually wider at the base and thinner at the top. Tubes can also be connected. For example, the CCTV Headquarters in Beijing (2012) consists of two inclined towers (each 768ft (234m) tall) that are connected at the top with a multi-story portion of the building that cantilevers out from each tower. The project, designed by OMA and Cecil Balmond, creates an expressive structure by solving a simple idea. By fixing the tops of the towers together, the building acts as a stiff Mobius-strip tube. (Figure 6.1.19)

Analogy #3: High-Rise as a Cantilever Beam

A more apt analogy for the stresses that high-rises are subjected to, and their deformations, is to think of a high-rise not as a tower, but as a cantilevered beam

BREAKING: VISUAL EVALUATION

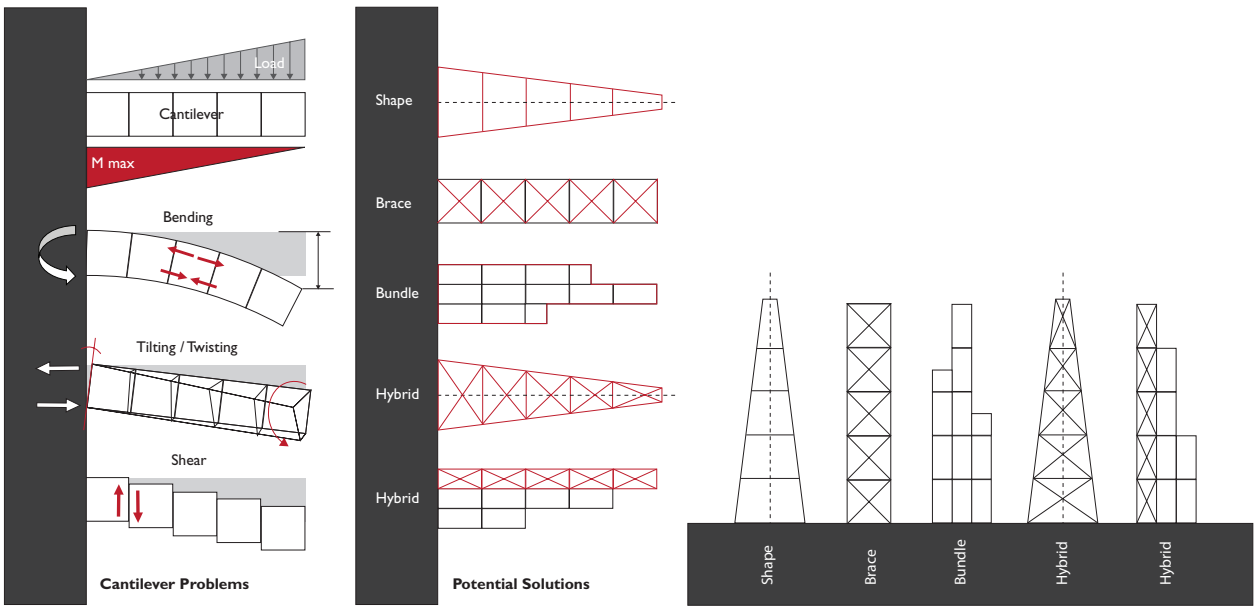
Looking closely at the CCTV tower scheme, we can see patterns of diagonal bracing expressed irregularly across the façade. What does the varying density of these diagonals signify? What tools were used to generate these patterns?

To Discuss: What sort of construction issues would you expect to arise in making two large separate towers connected together at the top?

sticking out of the ground. If we draw a cantilever beam and rotate it horizontally, we can imagine what would happen to it as a high rise. Remember that the loading increases across the length to its maximum level at the point farthest from the support. The cantilever beam would risk being overturned, or being bent under wind loads, or twisting under torsion, or even failing in shear. What solutions would help a cantilevered beam become more resistant to these forces?

If the form matched the moment diagram, it would be wider at the base and tapering towards the end—like a shelf bracket. But if we started to think three-dimensionally we'd know that a purely planar form wouldn't be effective when the load position changed—the wind can come from any direction, after all. A planar solution would also risk torsional twisting. If the form became more volumetric, like a cone, or tapered tripod, or even a bundle of sticks, it would be more naturally resistant to these forces. We could make a similar form by bundling several tubes together—wider at their connection and thinner at their ends. We could also reduce deflection by adding triangulated stiffening elements along the beam, making it into a cantilevered truss.

Adding more perpendicular belts around the perimeter of the tube at the middle and ends would stiffen it further. As long as the stiffening elements are placed within the tube in a balanced



Figures 6.1.20a and 6.1.20b *(continued)*

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Figures 6.1.20a and 6.1.20b Towers are essentially cantilevers out of the ground so we can flip the cantilever beam diagrams to the side and we'll see common high-rise forms. 6.1.20b: Transamerica Building, 48 floors (Pereira, San Francisco, 1972)

and symmetrical way, the applied forces wouldn't be able to twist it. Finally, we could develop hybrid solutions, such as cones with bracing or braced bundles, depending on our needs for reducing bending. (Figure 6.1.20)

High-Rise Strategies and Tactics: Internal versus External Bracing

These three analogies—high-rise as stacked floors, columns, and cantilevers—present holistic strategies

FOLLOW THE FAILURES: CITIGROUP TOWER, 601 LEXINGTON AVENUE

When the fifty-nine-story Citicorp Tower (now Citigroup) was completed in 1977, it was the seventh tallest building in the world (915ft (278m)). Due to an air-rights arrangement, the building design team, architect Hugh Stubbins and engineer William LeMessurier, created a tower that balanced precariously on four massive stilts nine floors above the site. These cores weren't underneath the corners of the tower but were off-set to the middle of each face. A lightweight but efficient bracing system stabilized the tower. It was a cutting-edge design. After an undergraduate engineer student contacted LeMessurier with a question—she wondered why the columns weren't on the corners and how this affected the wind calculations (specifically with quartering winds), it set forth dramatic events and clandestine repairs. Without remediation, the building could have collapsed. The story of this near-failure is required reading for all architects and engineers.

To Do: Find information about what LeMessurier discovered about the risks the building faced. Using diagrams, explain how the bracing was designed to resist and transfer loads and why it wasn't adequately constructed.

To Discuss: Consider the ethics of the situation and discuss how the relationship between the architect, engineer, the client, and the general public affected the process and the eventual outcome. (Figure 6.1.21)

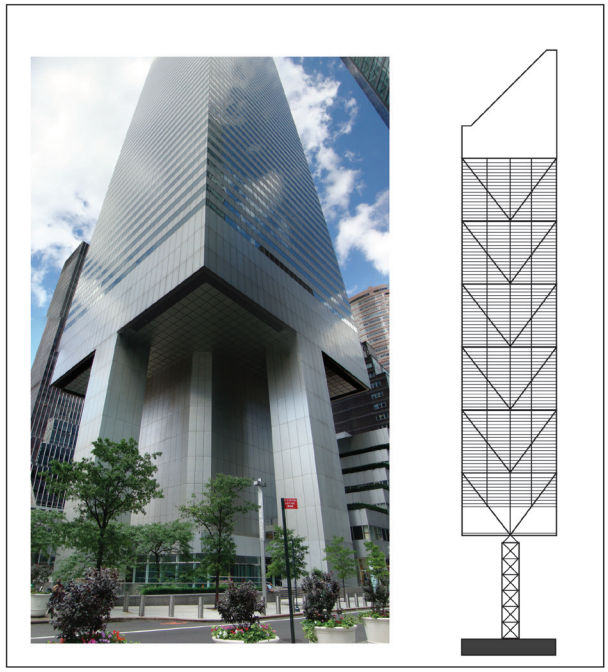


Figure 6.1.21 Ground level view, section of Citigroup Tower showing multi-story lateral bracing and unique column placement at ground level, and mass-tuned damper on roof (by LeMessurier)

for designing high-rises. As always, relying on responsive physical characteristics and proportions is most effective. After all, some forms are more naturally resistant to wind forces than others.

Whatever their forms, all high-rises need to have explicit strategies to resist lateral forces. These lateral-resisting systems can be "internal or external" to the building. Internal strategies involve a structural frame like the "bay-grid" approach in Chicago skyscrapers and a structural core, or cores. External solutions place lateral resistance at the perimeter, like the "casing" approach in our analogous column, and stabilize it with diagonal braces, tubes, rigid connections, etc. Most high-rises may employ a combination of the two. Stair and elevator cores are continuous solid and load-bearing "internal" core walls that run throughout most high-rises.

There are three main tactics for lateral resistance: frames (rigid or moment resisting), bracing (diagonals), or shear walls. Applying these tactics to the internal or external strategies describes potential schemes: *Internal bay/grid scheme with rigid frames, Internal shear walls (otherwise known as a core scheme), external diagonal bracing, external shear walls, etc.* High-rises resist elevated levels of both gravitational and lateral forces. In some cases, this requires different structural elements assigned to each task (e.g., load grounders versus load stabilizers) but in other cases, the same structural element can resist both types of load.

Some structural schemes cannot resist lateral loads above a certain height. Before you begin it is helpful to know what is feasible or not. Schemes that rely on the "internal" systems for resistance (Core Schemes, Rigid Frames and Braced Frames) are at the lower end of the chart for allowable height. Taller structures rely on "external" systems to resist higher loads (Tubes and Diagrids). The highest combine overall form and external bracing for their maximum heights (Bundled Tubes and Buttressed Cores). (Figure 6.1.22)

Structure, Stiffness, and Heights: Understanding Possible Options

For the remainder of the chapter, we'll look at these strategies in more detail. Although it is not the only important

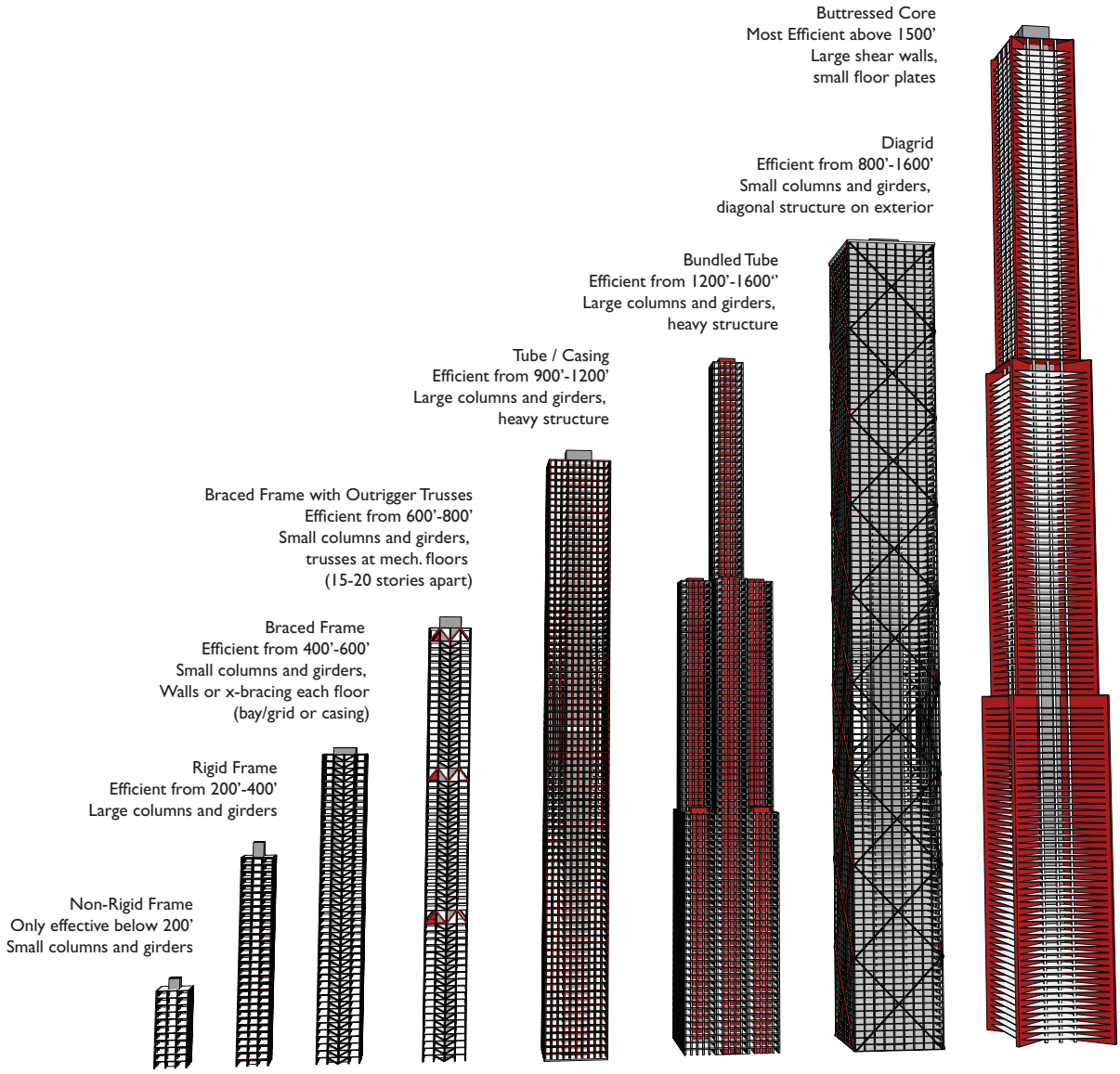


Figure 6.1.22 Options for high-rise framing classifications and allowable heights for each (T. Leslie and R. Whitehead)

criterion in high-rise design, we'll use maximum height as the basis for our system selection to see which systems are most effective at which heights and why.

Internal Core Schemes: Effective up to 200ft (61m)

Internal core schemes provide lateral resistance with shear walls enclosing the stair, elevator, and mechanical cores. High-rises can have consolidated or multiple cores depending on layout. These are typically continuous vertically and usually of concrete to provide lateral resistance. They are assumed to act as vertical cantilevered beams fixed at their foundation. Because the cores are the stiffest element, they should be distributed symmetrically in plan to avoid torsion. They must be securely connected to the horizontal diaphragms at each floor, so concrete is typically the material of choice for the entire structure. One good example is Wright's Johnson Wax Research tower,

Figure 6.1.23 The Johnson Wax tower was an exemplary high-rise tower with floors cantilevered from the central core (Wright, Racine, 1936)

which uses a massive central concrete core like a stayed mast, with reinforced concrete floors cantilevering from the core. This eliminates columns from the perimeter, allowing a free façade. (Figure 6.1.23)

Internal Rigid Frame: Effective between 200ft and 400ft (61m–122m)

Internal rigid frames rely on moment resisting rigid frames for all the bays on all the floors within the building. This was the predominant structural system for steel high-rises until the 1960s. The system allows the internal columns and girders to share both gravitational and lateral loads—because they are rigidly connected, they bend instead of rotating relative to one another. Columns and girders are larger in this system, and construction is costlier as a consequence. Because these elements are tasked with resisting both types of loads, columns get larger at the building's base. These structures are effective, but ultimately limited in heights by efficiency. Any increase in height also increases the gravitational and rotational stresses on the lower level columns and framing. This makes them functionally and economically prohibitive. Larger columns and girders reduce window heights and floor-to-floor height clearances. However, if the rigid frame is combined with a core system (a logical evolution) the core can take on more lateral resistance and the towers can increase to up to sixty stories, or between 200ft and 500ft (61–152m)). (Figures 6.1.24 and 6.1.25)

Internal and/or External Braced Frame: Effective between 400ft and 600ft (122m–183m)

These framing systems integrate diagonal bracing to form vertical trusses. Diagonals are effective at stiffening a structure. Consequently, these structures

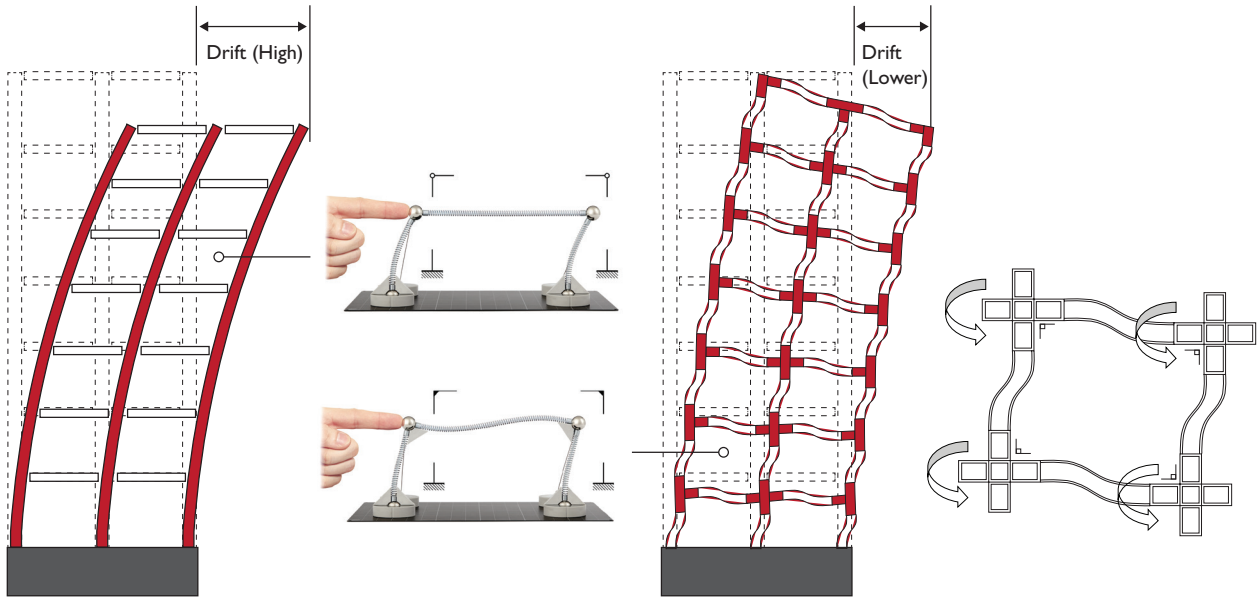


Figure 6.1.24 Comparing drift between pinned and fixed framing connections. Fixed/rigid connections help reduce drift but add bending stresses to columns

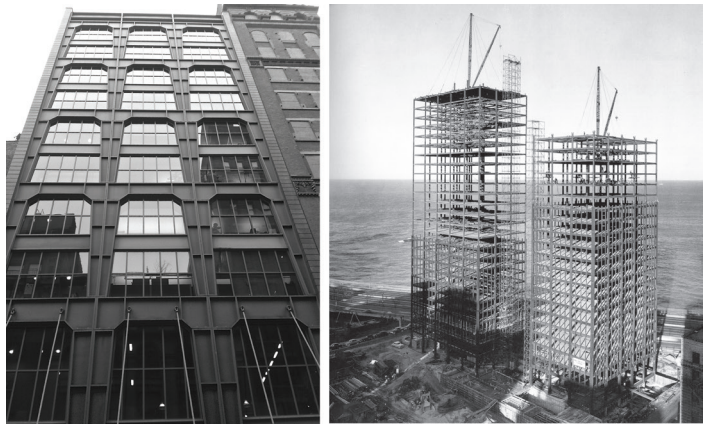
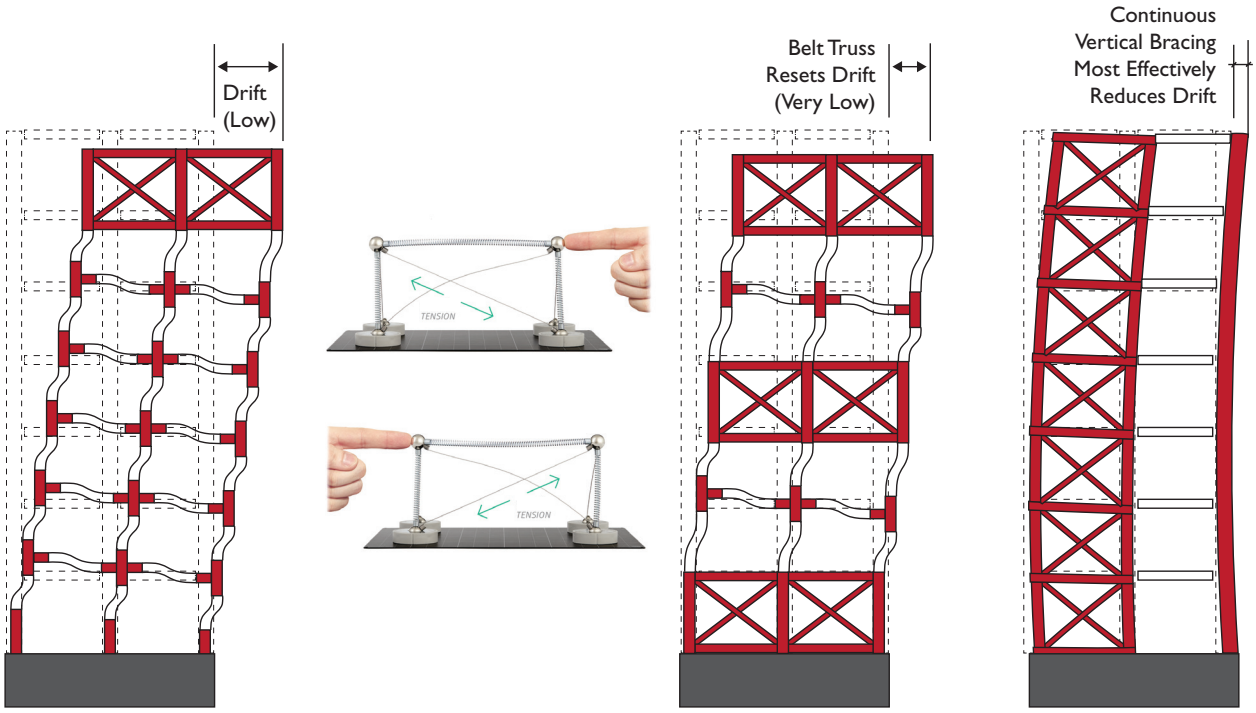


Figure 6.1.25 Moment connections can be incorporated into the building's expression in different ways (Scholastic Building, Rossi, NYC, 2001 and Lake Shore Drive apartments, Van der Rohe, Chicago, 1949)

have less drift and can be built taller. The amount of allowable drift is often a controlling factor.

Vertical trusses are integrated with the vertical columns in these schemes, and the columns serve as chord members for the truss. Because columns are braced against lateral movement, this scheme eliminates the bending forces on columns, which only have to resist gravity. Because the diagonals provide lateral bracing, the interior connections don't need to be fixed/moment connections. They can just be pinned, which simplifies construction and makes fabrication more economical. Small columns and girders are possible because they aren't bending under lateral loads.

These vertical trusses have diagonals that typically extend one story in height (forming a K- or V-shaped truss). These are the structures' stiffest elements so they will ideally line up vertically on every floor to avoid torsion. They must continue into the foundations to avoid soft-story behavior. These braces can be internal or external, but either way has consequences. Internal bracing takes up valuable floor space and may limit the flexibility of floor layouts—it also isn't



Figures 6.1.26a and 6.1.26b Triangulating bays with a braced frame reduces sideways drift. The location and frequency of bracing (horizontal versus vertical) determines amount of drift. 6.1.26b: The Hotel Arts Building braces the four corners with continuous vertical trusses/bracing and outrigger trusses in the middle and top (SOM, Barcelona, 1994)

as effective in resisting lateral loads. Locating this bracing on the perimeter improves the building's performance, but it may obscure windows or reduce suitability of the building's functions. (Figure 6.1.26)

External Braced Frame with Outrigger Trusses: Effective between 600ft and 800ft (183m–244m)

Above a certain height, the vertical trusses of a braced frame, even when they are located on the perimeter, begin to allow too much deflection. Braced systems can gain stiffness by adding horizontal trusses (or "belts") around their perimeters. These outrigger trusses have diagonals fixed to the vertical columns and horizontal floor connections, and they wrap all around the perimeter. The full-story trusses fix this "belt" in place to keep it from drifting side-to-side. Because of its triangulated members, each floor's drift is stopped so the building "resets" to vertical. A single outrigger truss can brace 15+ floors of a building until the accumulated drift becomes too great and needs to be reset again with another outrigger truss. The vertical spacing of these floors throughout the overall building height is often a convenient location for an intermediate mechanical floor. Placing a belt truss at the top-most floor is effective at reducing the overall drift. (Figure 6.1.27)

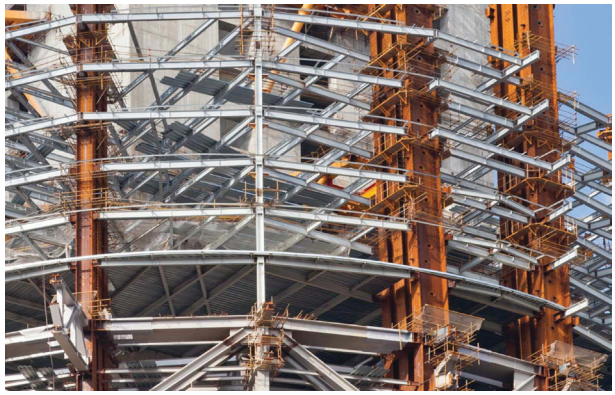


Figure 6.1.27 Outrigger truss for Shanghai Tower (128 floors) bracing the exterior bracing back to the central core (Gensler, Shanghai, 2014)

External Tube Structures: Effective between 900ft and 1200ft (275m–366m)

High-rise structures would more effectively resist side-to-side bending if they distributed their resisting elements equally around the perimeter, analogous to a column becoming wider to better resist buckling. Fazlur Khan (SOM) developed the design ideas and analytical tools needed to create the first tube-structured high-rises. Khan realized that by moving a moment resistant frame to the building exterior (using either concrete or steel), it became more effective at resisting bending. Importantly, this scheme assigns both gravitational and lateral resistance capabilities to the exterior columns and girders. They don't bend as much as a simple rigid frame system because they are spaced closely together—just 15ft apart on the Willis (Sears) Tower. Although windows at the perimeter of tube structures are smaller as a result, the interiors become more flexible without interior bracing.

Figure 6.1.28 With the external tube structure providing gravitational and lateral resistance, the interior of the World Trade Center ("Twin Towers") was nearly see-through during construction (photo by project engineer, Leslie Robertson)

MAKING AND BREAKING: TALLEST TOWER CHALLENGE

How tall a tower can you built using only thin sticks? The challenge seems elemental, but coordinating and constructing a tower with the proper geometry to remain stable isn't simple. The tower must be free-standing without being tethered to anything else. This challenge is best completed with a larger team during a constricted time frame. (Figure 6.1.29)

Figure 6.1.29 This twenty-one foot tall free-standing tower was designed and assembled within two-hours. The geometry of the framing stabilizes the tower and allows it to be assembled in smaller modules (SxD, Iowa State University)

Of course, these projects still have shear walls and building cores on the interior to support the floors. Engineer Leslie Robertson used this approach to make the World Trade Center towers the tallest buildings in the world (1,368ft (442m)), until another tube structure—the Sears Tower, albeit bundled tube, was built a few years later. (Figure 6.1.28)

Bundled Tube Structures: Effective between 1200ft and 1600ft (366m–488m)

Form matters. Towers that are wider at the base and narrower at their top better resist overturning. This is the basic idea behind the bundled tube. Khan first integrated this strategy in the Willis (Sears) Tower in Chicago in 1974. His initial intent wasn't to build the tallest tower but to simply provide an efficient way of arranging space and constructing a high-rise. The resulting plan bundled nine tubes (each 75ft × 75ft in plan (23m × 23m) together for the first fifty floors, dropping tubes away until it reached the final record-breaking height of 108 floors (Chapter 1.2). These structures have the same large columns and girders as other tube structures, a result of their resisting gravity and lateral loads simultaneously. To reach their maximum heights, bundled tubes can integrate outrigger (or belt) trusses where tubes end. Because column and girder weights increase as the building grows taller, there is a maximum height for bundled tube structures (120+ stories). To build higher requires a lighter, but stiffer, arrangement. (Figure 6.1.30)

External Diagrid Bracing: Effective between 800ft and 1600ft (244m–488m)

Diagrids are an increasingly popular option for high-rises. These are diagonal meshes that wrap around a building's perimeter, but they are different than

Figure 6.1.30 The bundled tubes of the Willis Tower also incorporated belt trusses as mechanical rooms at the top of each bundle.

the outrigger trusses. The effectiveness of high-rises depends on the material's stiffness, arrangement, and economy and diagrids are useful in all regards. When the price of steel rose dramatically in the 1970s and 1980s, tubes and bundled tubes fell out of favor because of the amount of material they required. The added weight in tube structures comes from the obligation of columns and girders to resist lateral loads and gravity. These can become lighter if diagonal braces are used, since braced frames are lighter than moment frames. Engineers discovered that they could combine the efficiency of the tube and braced frames by wrapping a large braced truss around the building perimeter.

To lighten the structure, gravitational and lateral resisting elements should be the same. Instead of using rigid connections, gravitational loads can be resisted and transmitted by angled columns that also resist lateral loads due to their triangulated configuration. These create stiff but lightweight structures; besides their cores, these structures need no other internal supports. When the structure's proportions become slender (more than 7:1) it is actually advantageous to have variable angles on the diagrids (steeper at the base and flatter at the top). These diagrids are primarily of steel because of the difficulty in fabricating and erecting these diagonal connections in concrete. (Figure 6.1.31)

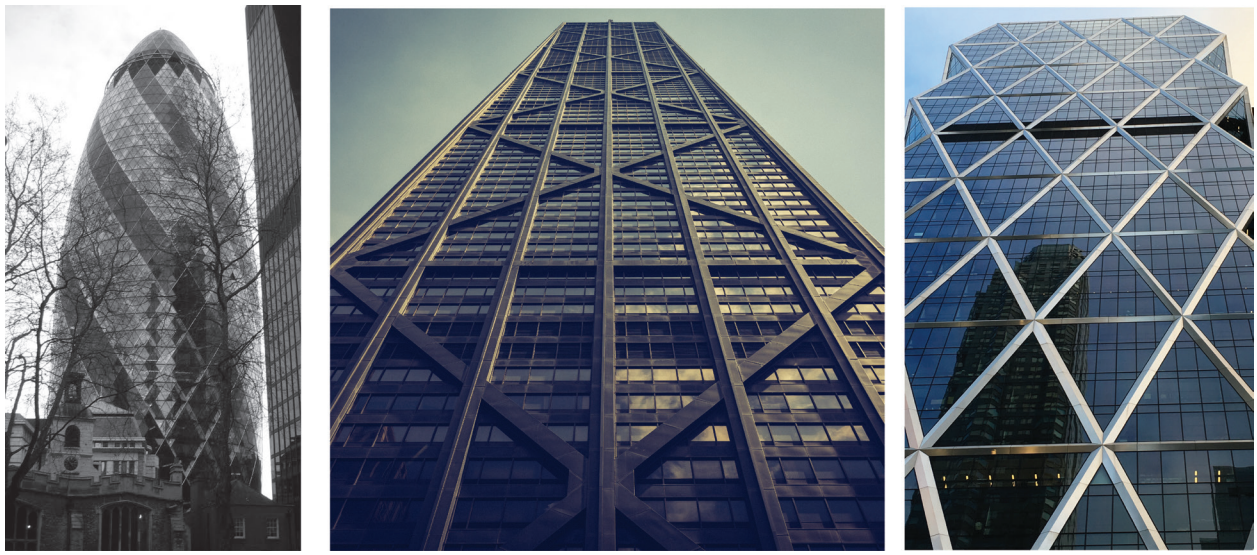


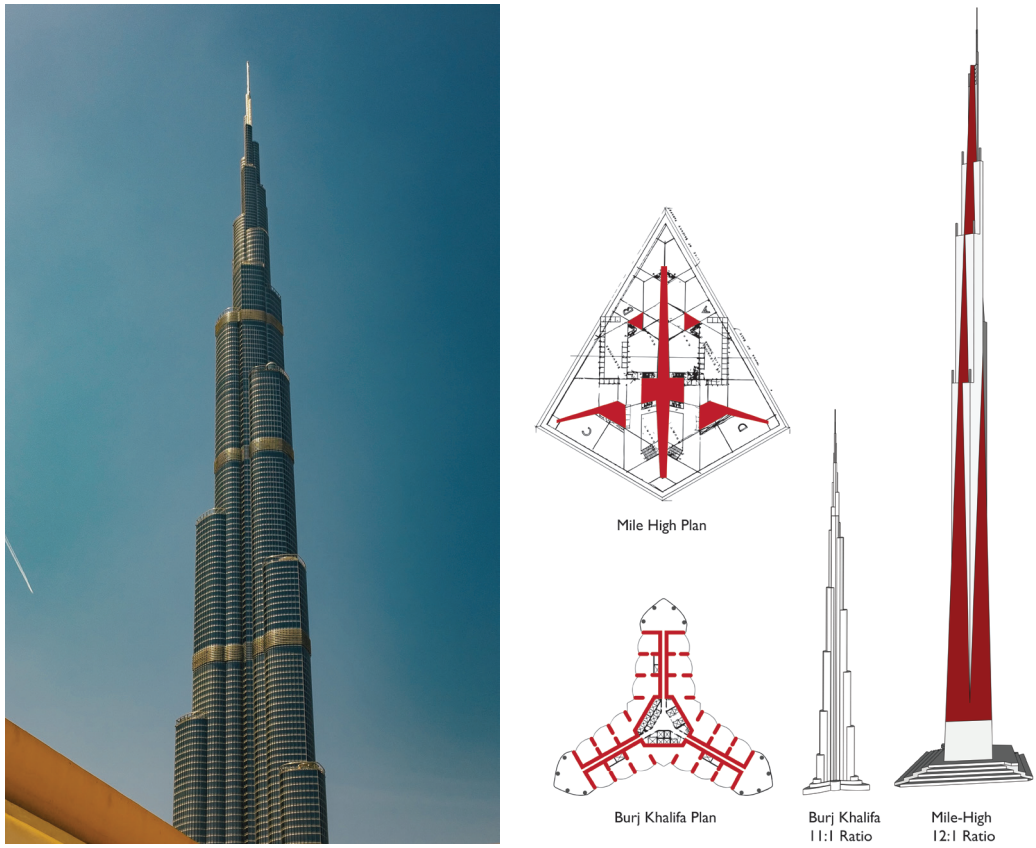
Figure 6.1.31 Three examples of diagrids: The "Gherkin" Tower, 30 St. Mary Axe (Foster + Partners, London, 2003), Hancock Tower (SOM, Chicago, 1969), and Hearst Tower (Foster + Partners, New York, 2003)

Buttressed Core: Effective between 1600ft and 2000ft (488m–609m)

Because advancements in admixtures have increased their strength and stiffness, very tall and slender towers can be constructed out of concrete. To achieve these, the building form and bracing systems need to be optimized. In plan, you can see how the current world's tallest tower, the Burj Khalifa, 2,722ft (830m), uses buttressed concrete walls arranged into a triangle with a hexagonal core at their center. Along each buttress, shear web walls brace it perpendicularly and shear walls act like "noses" at the end of each leg. Unlike other high-rise structures, this building is almost entirely residential. It doesn't require the same flexible open spaces that we saw in commercial projects. Its upper floors barely have any leasable area. This brings us back to where we started the chapter, with questions about Wright's Mile-High tower proposal. (Figure 6.1.32)

Back to the Beginning: A Mile High or High Enough?

Structures' heights are limited by many factors. There are material limits as to how much weight a structure can carry, limits to how stiff a material can be or how stiff an assembly of braces can become, and increasingly burdensome loads as height increases. There are functional limitations too. Even if the Mile-High tower could stand one can imagine the massive drift at the top, the long waits at the elevator to travel 328 floors, and the compounded life-safety issues of housing and evacuating 130,000 people (Wright's estimate) in one tower. Ultimately, the limitations for these projects are also economics and feasibility. High-rises shouldn't be monuments. Instead they should be an expression of all that is logical, righteous, inspiring, and aspirational about structural design.



Figures 6.1.32a and 6.1.32b Comparing Burj Khalifa to Wright's proposal shows the similar structural strategies (tapered buttress core with bundles) and aspect ratios (height/base)

Figure 6.1.33 Exhibition of tower models designed by Skidmore, Owens, & Merrill. The lowest models are all over twenty stories. (2017 Chicago Architecture Biennial exhibition)

Frequently Asked Questions

What is the future of skyscraper designs? Have we reached maximum allowable heights? We've reached a theoretical limit in building heights of about one kilometer. These limits aren't caused by the capacity of steel or concrete for building frames, but rather by the capacity of elevators and restrictions on their cable lengths—there are associated concerns with vertical egress and travel time as well. A more pronounced limitation is always economics. Every record for "tallest structure" has been compelled by economic speculation. (Figure 6.1.33)

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Appendix

Spanning Tables, Charts, and Additional Examples

Load Types

D = Dead Load	S = Snow Load
L = Live Load	E = Earthquake Load
W = Wind Load	T = Thermal Effect

Determining Dead Load from Materal Weights

Weight = Density × Volume

W (total weight) = (Weight/unit area) × Area (A)

w (weight per unit length) = (Weight/unit volume) × Area (A)

Load and Resistance Factor Design

Structures and foundations designed to exceed the factors loads in the following combinations (worst-case scenario); ASCE-7 (2010)

Combination 1	1.4D
Combination 2	1.2D + 1.6L + .5 (Lr or S or R) Worst-case
Combination 3	1.2D + 1.6(Lr or S or R) + (L or .5W)
Combination 4	1.2D + 1.0W + L + .5(Lr or S or R)
Combination 5	1.2D + 1.0E + L + .2S
Combination 6	0.9D + 1.0W
Combination 7	0.9D + 1.0E

Determining Wind Loading: Based on IBC requirements

Wind Load (W) = Area (A) × Wind Pressure (P) (measured in lbs or kg)

Area = Length × Width of surface resisting wind (Tributary Area, ft^2 or m^2)

Wind Pressure (P) = Combined height, exposure, and gust factor (Ce) × Pressure/Drag Coefficient (Cq) × Wind Stagnation Pressure (Qs) × Importance Factor (Iw)

- Combined Factor (Ce) : Code given value based on terrain exposure (Exposure B, Exposure C, or Exposure D (worst-case))
- Drag Coefficient (Cq): Code given value
- Wind Stagnation Pressure (Qs): Code given value OR Qs = .00256 × Velocity $(V)^2$ (measure in pressure: lb/ft^2 (kg/m^2))
- Importance Factor (Iw) = Code given value, essentially a factor of safety protecting certain building types with higher I-value

Table A.1 Design Loads in Building Codes

WEIGHT OF BUILDING MATERIAL BY VOLUME		
	(lb/ft^3)	(kg/m^3)
Concrete/Masonry		
Brick	120	1922
Conc. Block (CMU)	100	1602
Concrete	150	2403
Metals		
Aluminum	165	2643
Steel	485	7769
Iron	450	7208
Stainless steel	500	8009
Copper	550	8810
Wood		
Douglas Fir	35	561
Cedar	23	368
Oak	50	801
Pine	25	400
Stone		
Granite	175	2803
Limestone	165	2643
Miscellaneous		
Gravel	120	1922
Soil	80–120	1280–1922
Water	62.4	1000
Straw / Hay	20	320
Rubber	100	1600

WEIGHT OF MATERIALS / ASSEMBLIES BY AREA		
	(lb/ft^2)	(kg/m^2)
Wall/Ceiling/Floor Assemblies		
Gyp. Bd. Ceiling, 5/8"	15	73.2
Acoustic Tile Ceiling	6	29.3
Ceramic Tile floor	60	292.8
Wood Floor Assembly	15	73.2
Window/Glass		
Curtain wall w/ Glass	65	317.2
Sheet Glass, 1/4"	3	14.6
Insulated Glass, 1"	6.5	31.7
Glass Block, 4"	20	97.6
Steel Floor/Wall Assemblies		
Deck and Conc. Slab, 6"	60	98–293
Steel Framing*	14–65	68–317
Partitions w/ Gyp.	35	170.8
Concrete Assemblies		
Cast slab, 6"	60	292.8
Precast plank, 6"	40	195.2
Wood Framing		
2x4 wall w/ Gyp. Bd	55	268.4
2x6 floor joist w/ plywd.	25	122.0
2x12 floor joist w/ plywd.	34	165.9
Platform Framing	14–25	68–122
Roof Materials/Assemblies		
Metal w/ sheathing	12	58.6
Asphalt Shingle w/ sheath.	10	48.8
Built-up Roof w/ insul.	30	146.4
Clay Tile	60	292.8
Sheathing, 3/4"	3	14.6
Insulation, Loose, 4"	6	29.3
Rigid Insulation, 4"	6	29.3

*Steel framing members vary—see manufacturer information

Table A.2 Weight of Material by Volume and Area

Usage	Live Load (lbs/ft.$^{2)}$	Live Load (kg/m$^{2)}$
Assembly Areas, Fixed Seats	60	300
Assembly Areas, Lobbies	100	500
Assembly Areas, Movable Seats	100	500
Stages and Platforms	150	750
Balconies	100	500
Balconies, single-family	60	300
Offices	50	250
Lobbies, meeting rooms	100	500
Offices where filing is intensive	125	600
Habitable attics	30	150
Bedrooms	30	150
Residential other than sleeping	40	200
Apartments and Hotel Rooms	40	200
Apartment and Hotel Buildings, Public Areas	100	500
Catwalks	25	125
Corridors, main floor	100	500
Storage, Light	125	600
Storage, Heavy	250	1200
Dance Halls, Dining Rooms, Bars	100	500
Library Stacks	150	750
Library Reading Rooms	60	300
Garages, Cars Only	50	250
Garages, trucks	150	750
Light Manufacturing	125	600
Heavy Manufacturing	250	1200
Rest Rooms	60	300
Classrooms	40	200
Stadium Seating	100	500
Retail, Main floor	100	500
Retail, Upper Floors	75	400

Table A.3 Minimum Uniform Live Loads: IBC requirements

	Density		Allowable Working Stresses								Modulus of Elasticity E		Yield Strength (f'y)				Thermal Coefficient (10–6)	
Material			*Bending (f'b)*		*Tension (f't)*		*Compression (f'c)*		*Shear (f'v)*				*Tension*		*Compression*			
	kg/cm3	lbs/in3	kg/cm2	psi (lb/in2)	kg/cm2	psi (lb/in2)	kg/cm2	psi (lb/in2)	kg/cm2	psi (lb/in2)	kg/cm2	psi (lb/in2)	kg/cm2	psi (lb/in2)	kg/cm2	psi (lb./in2)	in/in * R	m/m * K
Concrete	0.0023	0.0830	.45 * f'c	.45 * f'c	—	—	141–422	2,000–6,000*	Varies	Varies	210,930	3,000,000	42	600	422	6,000	13	7.22
Brick Masonry	0.0021	0.0750	—	—	—	—	12	175	—	—	105,465	1,500,000	56	800	141	2,000	5	2.75
Wrought Iron	0.0078	0.2810	2,025	28,800	844	12,000	844	12,000	703	10,000	1,898,370	27,000,000	2,812	40,000	3,375	48,000	10	5.27
A36 Structural Steel	0.0078	0.2830	1,547	22,000	1,547	22,000	1,547	22,000	844	12,000	2,038,990	29,000,000	4,922	70,000	2,531	36,000	11	6.11
Aluminum (Cast)	0.0027	0.0980	1,687	24,000	1,055	15,000	1,055	15,000	—	—	632,790	9,000,000	422	6,000	2,812	40,000	21	11.7
Plastic (Poly. Laminate)	0.0026	0.0903	337	4,800	3,058	43,500	2,531	36,000	—	—	70,310	1,000,000	141	2,000	562	8,000	40	22
Douglas Fir, Select	0.0006	0.0230	102	1,450	70	1,000	120	1,700	7	100	133,589	1,900,000	1,055	15,000	274	3,900	3.07	2
Southern Pine, Select	0.0006	0.0230	141	2,000	141	2,000	105	1,500	9	128	112,496	1,600,000	1,055	15,000	281	4,000	5	2.75
Structural Glass	0.0026	0.0930	408	5,800	408	5,800	10,195	145,000	—	—	674,976	9,600,000	652	9,280	16,312	232,000	6	3.3
Bamboo (Varies Age/Species)	0.0012	0.0430	350–1500	5,000–20,000	1,266	18,000	703	10,000			1,687,440	24,000,000					5	2.7
Diamond	0.0035	0.1256	—	—	—	—	—	—	—	—	11,952,700	170,000,000	—	—	—	—	1.1	0.6
Carbon Fiber	0.0018	0.0640	—	—	35,150	500,000	—	—	—	—	2,327,261	33,100,000	38,741	551,000	—	—		

Table A.4 Properties of Typical Building Materials (Strength And Stiffness)

NOTE: These figures are averages, intended for illustration only. Depending on manufacture and processing, materials may vary significantly from these figures.
*Compression measured parallel to grain for woods
**Concrete compressive strength varies
Thermal Stress (f) = Thermal Coefficient * Temperature Different (Delta T) * Modulus of Elasticity, E
Thermal Strain (Change in Length) = Thermal Coefficient * Change in Temperature * Original Length

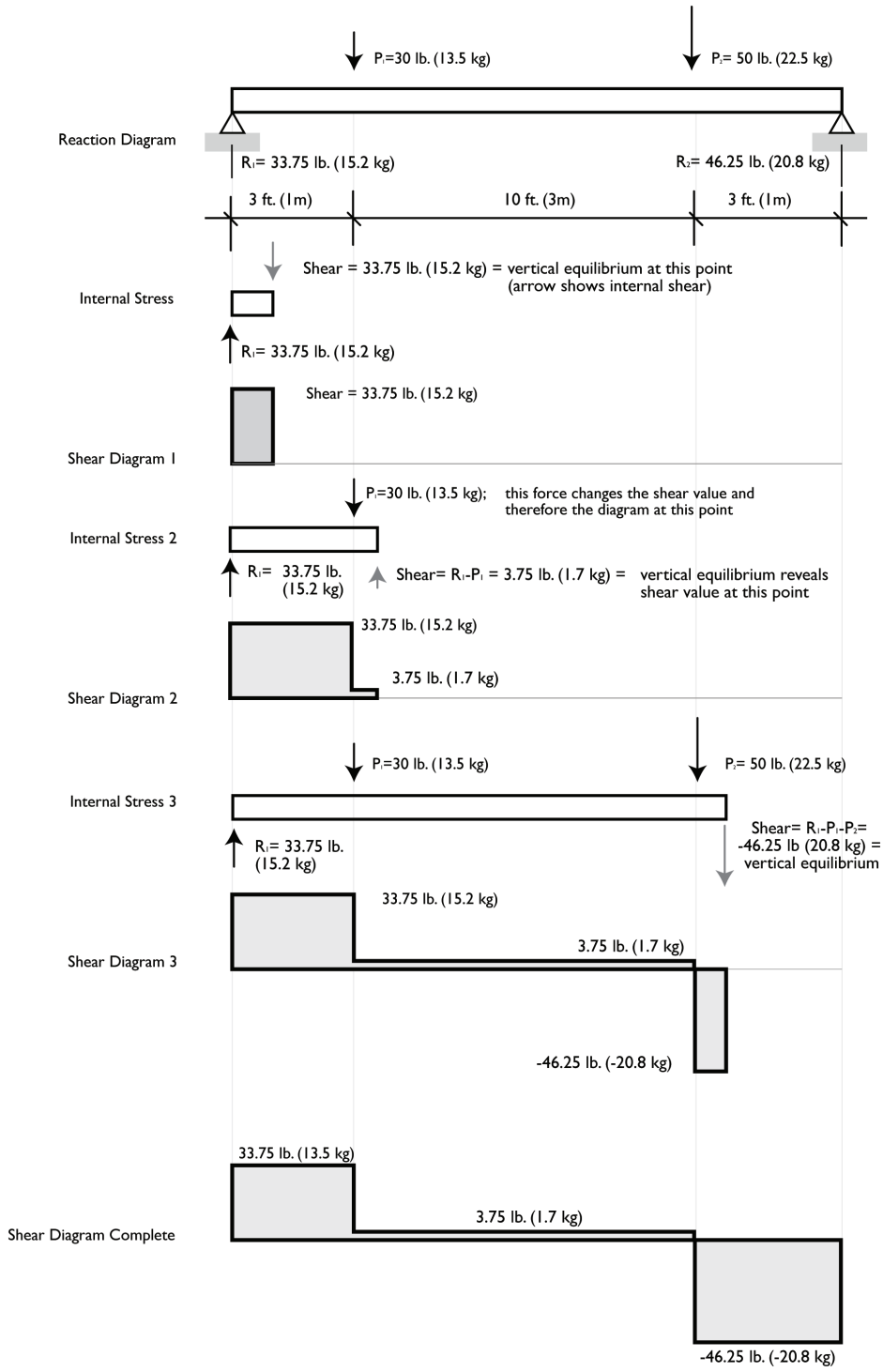
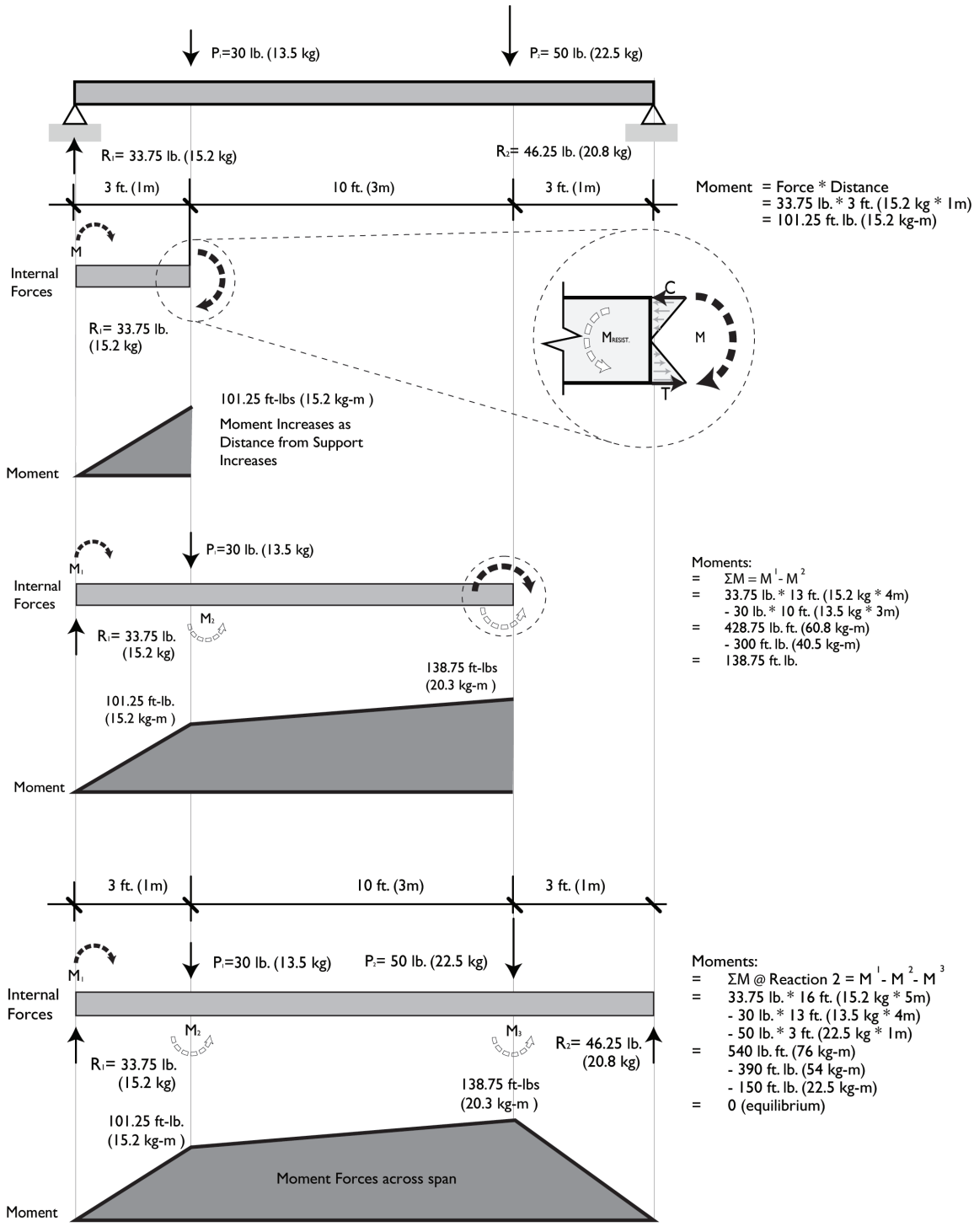
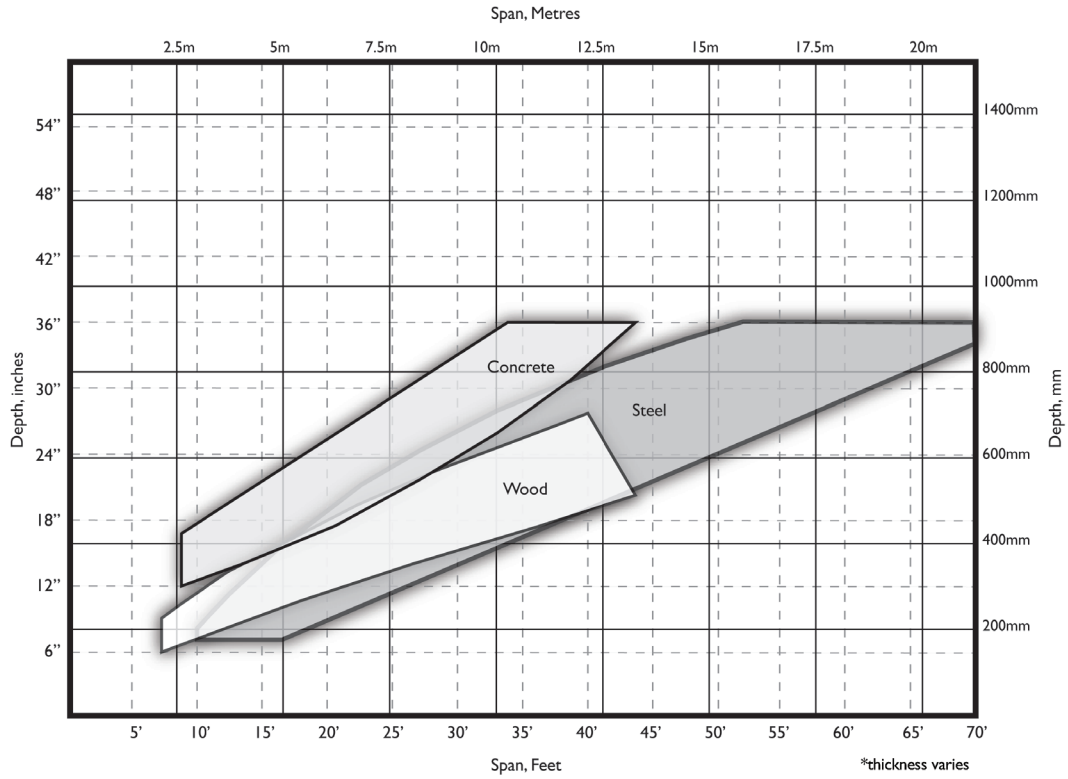


Figure A.5 Shear diagram example



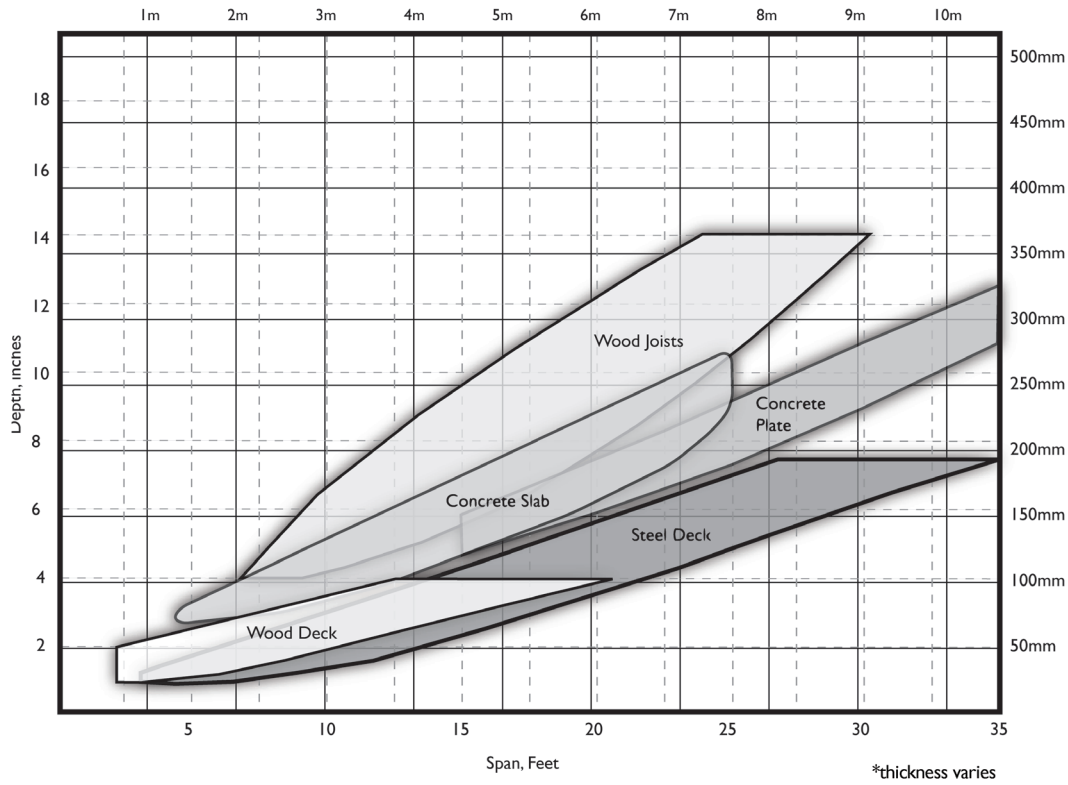
Moment Diagram Example: Step-by-step process

Figure A.6 Moment diagram example



BEAMS (BY MATERIAL): SPAN & DEPTH ESTIMATION

Figure A.7 Beams (by material): Span and depth estimation



FLOOR STRUCTURES: SPAN & DEPTH ESTIMATION

Figure A.8 Floor structures: Span and depth estimation

Designation: Nominal Sizes		Section Module (Sx–x)		Weight per linear foot	
Metric	U.S.	cm^3	in^3	kg	lbs
25 × 75	1 × 3	12.80	0.78	0.70	0.47
25 × 100	1 × 4	25.10	1.53	0.95	0.64
50 × 75	2 × 3	25.61	1.56	1.40	0.94
75 × 75	3 × 3	42.68	2.60	2.26	1.52
50 × 100	2 × 4	50.19	3.06	1.90	1.28
25 × 150	1 × 6	61.97	3.78	1.49	1.00
75 × 100	3 × 4	83.66	5.10	3.17	2.13
25 × 200	1 × 8	107.69	6.57	1.96	1.32
100 × 100	4 × 4	117.12	7.15	4.43	2.98
50 × 150	2 × 6	123.95	7.56	2.97	2.00
25 × 250	1 × 10	175.30	10.70	2.51	1.69
75 × 150	3 × 6	206.58	12.60	4.97	3.34
50 × 200	2 × 8	215.37	13.14	3.93	2.64
25 × 300	1 × 12	259.29	15.82	3.05	2.05
100 × 150	4 × 6	289.22	17.65	6.96	4.68
50 × 250	2 × 10	350.59	21.39	5.01	3.37
75 × 200	3 × 8	358.96	21.90	6.56	4.41
150 × 150	6 × 6	454.48	27.73	10.93	7.35
100 × 200	4 × 8	502.54	30.66	9.17	6.17
50 × 300	2 × 12	518.59	31.64	6.10	4.10
75 × 250	3 × 10	584.32	35.65	8.36	5.62
50 × 350	2 × 14	719.37	43.89	7.18	4.83
100 × 250	4 × 10	818.05	49.91	11.57	7.78
150 × 200	6 × 8	845.11	51.56	14.91	10.03
75 × 300	3 × 12	864.32	52.73	10.17	6.84
200 × 200	8 × 8	1152.42	70.31	20.33	13.67
75 × 350	3 × 14	1198.95	73.15	11.97	8.05
100 × 300	4 × 12	1210.04	73.83	14.23	9.57
150 × 250	6 × 10	1355.93	82.73	18.88	12.70
75 × 400	3 × 16	1588.21	96.90	13.78	9.27
100 × 350	4 × 14	1678.52	102.41	16.77	11.28
200 × 250	8 × 10	1849.00	112.81	25.75	17.32
250 × 250	10 × 10	1874.95	114.40	32.62	21.94
150 × 300	6 × 12	1986.95	121.23	22.86	15.37
100 × 300	4 × 16	2223.49	135.66	19.30	12.98
200 × 300	8 × 12	2709.47	165.31	31.17	20.96
150 × 350	6 × 14	2738.15	167.06	26.84	18.05
250 × 300	10 × 12	3432.00	209.40	39.48	26.55
150 × 400	6 × 16	3609.56	220.23	30.81	20.72
200 × 350	8 × 14	3733.85	227.81	36.60	24.61
300 × 300	12 × 12	4154.52	253.48	47.79	32.14
150 × 450	6 × 18	4601.15	280.73	34.63	23.29
250 × 350	10 × 14	4729.54	288.56	46.35	31.17
200 × 400	8 × 16	4922.12	300.31	42.02	28.26
300 × 350	12 × 14	5725.23	349.31	56.10	37.73
250 × 400	10 × 16	6234.69	380.40	53.22	35.79
200 × 450	8 × 18	6274.30	382.81	47.44	31.90
300 × 400	12 × 16	7547.25	460.48	64.43	43.33
250 × 450	10 × 18	7947.44	484.90	60.09	40.41

Table A.9 Beam Sizing: Selected Timber Sections Sorted by Section Modulus (S)

Note: Section Modulus values vary between types of heavy timber elements (e.g., Glulam, LVL, etc.)
See manufacturer information for specifics

Designation: Manufacturer Sizes		Section Modulus (Sx–x)		Weight		Depth (d)		Flange Width (w)		Web Thickness		Flange Thickness	
Metric	U.S.	cm^3	in^3	kg/m	lbs/ft	cm	in	cm	in	cm	in	cm	in
W 250 X 25	W 10 X 17	265.52	16.2	25	17	25.7	10.1	10.19	4.01	0.61	0.24	0.84	0.33
W 250 X 28	W 10 X 19	308.13	18.8	28	19	26.0	10.2	10.21	4.02	0.64	0.25	1.02	0.40
W 250 X 39	W 10 X 26	457.28	27.9	39	26	26.2	10.3	14.66	5.77	0.66	0.26	1.12	0.44
W 310 X 39	W 12 X 26	547.43	33.4	39	26	31.0	12.2	16.51	6.50	0.58	0.23	0.97	0.38
W 250 X 49	W 10 X 33	573.65	35.0	49	33	24.7	9.7	20.22	7.96	0.74	0.29	1.12	0.44
W 310 X 45	W 12 X 30	632.65	38.6	45	30	31.3	12.3	16.56	6.52	0.66	0.26	1.12	0.44
W 360 X 45	W 14 X 30	688.38	42.0	45	30	35.2	13.8	17.09	6.73	0.69	0.27	0.99	0.39
W 250 X 58	W 10 X 39	690.02	42.1	58	39	25.2	9.9	20.29	7.99	0.81	0.32	1.35	0.53
W 360 X 51	W 14 X 34	796.55	48.6	51	34	35.5	14.0	17.15	6.75	0.74	0.29	1.17	0.46
W 310 X 60	W 12 X 40	850.64	51.9	59	40	30.3	11.9	20.35	8.01	0.76	0.30	1.32	0.52
W 250 X 73	W 10 X 49	894.89	54.6	68	46	25.3	10.0	25.40	10.00	0.86	0.34	1.42	0.56
W 410 X 53	W 16 X 36	926.04	56.5	54	36	40.3	15.9	17.75	6.99	0.76	0.30	1.09	0.43
W 310 X 67	W 12 X 45	952.26	58.1	67	45	30.6	12.1	20.45	8.05	0.86	0.34	1.47	0.58
W 360 X 64	W 14 X 43	1027.65	62.7	64	43	34.7	13.7	20.32	8.00	0.79	0.31	1.35	0.53
W 410 X 60	W 16 X 40	1060.43	64.7	59	40	40.7	16.0	17.78	7.00	0.79	0.31	1.30	0.51
W 460 X 60	W 18 X 40	1121.08	68.4	59	40	45.5	17.9	15.29	6.02	0.81	0.32	1.35	0.53
W 360 X 72	W 14 X 48	1152.22	70.3	71	48	35.0	13.8	20.40	8.03	0.86	0.34	1.52	0.60
W 310 X 79	W 12 X 53	1157.13	70.6	79	53	30.6	12.1	25.40	10.00	0.89	0.35	1.47	0.58
W 410 X 67	W 16 X 45	1191.55	72.7	67	45	41.0	16.1	17.88	7.04	0.89	0.35	1.45	0.57
W 460 X 68	W 18 X 46	1291.53	78.8	68	46	45.9	18.1	15.39	6.06	0.91	0.36	1.55	0.61
W 410 X 74	W 16 X 50	1327.59	81.0	74	50	41.3	16.3	17.96	7.07	0.97	0.38	1.60	0.63
W 250 X 115	W 10 X 77	1407.90	85.9	114	77	26.9	10.6	25.91	10.20	1.35	0.53	2.21	0.87
W 310 X 97	W 12 X 65	1440.68	87.9	97	65	30.8	12.1	30.48	12.00	0.99	0.39	1.55	0.61
W 460 X 74	W 18 X 50	1457.07	88.9	74	50	45.7	18.0	19.05	7.50	0.91	0.36	1.45	0.57
W 530 X 74	W 21 X 50	1548.86	94.5	74	50	52.9	20.8	16.59	6.53	0.97	0.38	1.37	0.54
W 310 X 107	W 12 X 72	1596.39	97.4	107	72	31.1	12.3	30.58	12.04	1.09	0.43	1.70	0.67
W 460 X 82	W 18 X 55	1611.14	98.3	82	55	46.0	18.1	19.13	7.53	0.99	0.39	1.60	0.63
W 250 X 131	W 10 X 88	1614.42	98.5	131	88	27.5	10.8	26.09	10.27	1.55	0.61	2.51	0.99
W 360 X 101	W 14 X 68	1688.17	103.0	101	68	35.7	14.0	25.50	10.04	1.07	0.42	1.83	0.72
W 460 X 89	W 18 X 60	1770.12	108.0	89	60	46.3	18.2	19.20	7.56	1.07	0.42	1.78	0.70
W 530 X 85	W 21 X 57	1819.29	111.0	85	57	53.5	21.1	16.66	6.56	1.04	0.41	1.65	0.65
W 360 X 110	W 14 X 74	1835.68	112.0	110	74	36.0	14.2	25.58	10.07	1.14	0.45	2.01	0.79
W 460 X 113	W 18 X 76	2392.94	146.0	113	76	46.3	18.2	28.02	11.03	1.09	0.43	1.73	0.68
W 530 X 109	W 21 X 73	2474.89	151.0	109	73	53.9	21.2	21.08	8.30	1.17	0.46	1.88	0.74
W 610 X 101	W 24 X 68	2524.06	154.0	101	68	60.3	23.7	22.76	8.96	1.07	0.42	1.50	0.59
W 310 X 179	W 12 X 120	2671.57	163.0	178	120	33.3	13.1	31.29	12.32	1.80	0.71	2.79	1.10
W 460 X 128	W 18 X 86	2720.74	166.0	128	86	46.7	18.4	28.19	11.10	1.22	0.48	1.96	0.77
W 530 X 123	W 21 X 83	2802.69	171.0	123	83	54.4	21.4	21.23	8.36	1.32	0.52	2.13	0.84
W 610 X 113	W 24 X 76	2884.64	176.0	113	76	60.8	23.9	22.83	8.99	1.12	0.44	1.73	0.68
W 310 X 202	W 12 X 136	3048.54	186.0	202	136	34.1	13.4	31.50	12.40	2.01	0.79	3.18	1.25
W 360 X 179	W 14 X 120	3114.10	190.0	178	120	36.8	14.5	37.26	14.67	1.50	0.59	2.39	0.94
W 610 X 125	W24 X 84	3212.44	196.0	125	84	61.2	24.1	22.91	9.02	1.19	0.47	1.96	0.77
W 360 X 196	W 14 X 132	3425.51	209.0	196	132	37.2	14.7	37.39	14.72	1.65	0.65	2.62	1.03
W 690 X 125	W 27 X 84	3491.07	213.0	125	84	67.8	26.7	25.30	9.96	1.17	0.46	1.63	0.64
W 690 X 140	W 27 X 94	3982.77	243.0	140	94	68.4	26.9	25.37	9.99	1.24	0.49	1.91	0.75
W 760 X 147	W 30 X 99	4408.91	269.0	147	99	75.3	29.7	26.67	10.50	1.32	0.52	1.70	0.67
W 360 X 262	W 14 X 176	4605.59	281.0	262	176	38.7	15.2	39.75	15.65	2.11	0.83	3.33	1.31
W 760 X 173	W 30 X 116	5392.31	329.0	172	116	76.2	30.0	26.67	10.50	1.45	0.57	2.16	0.85

Table A.10 Beam Sizing: Selected Steel Sections Sorted by Section Modulus (S)

Note: Consult AISC Handbook for full range of structural shapes

Allowable Axial Loading For Selected Southern Pine/Douglas Fir
Column Sizes Based on Modulus of Elasticity of 400,000psi Grade Wood

		Effective Unsupported Length (k * L) Metres (feet) in kips and kg × 1000															
		1.80M (6')		2.4M (8')		3.0M (10')		3.6M (12')		4.2M (14')		4.8M (16')		5.4M (18')		6.0M (20')	
Nominal size (metric), cm	Nominal Size (US), inches	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)
100 × 100	4 X 4	5.0	11.1	3.3	7.3	2.2	4.9	1.6	3.5	1.2	2.6						
100 × 150	4 X 6	7.8	17.4	5.1	11.4	3.5	7.8	2.5	5.5	1.9	4.1						
100 × 200	4 X 8	10.3	22.9	6.8	15.1	4.6	10.2	3.3	7.3	2.9	6.5						
150 × 150	6 X 6	12.4	27.6	11.2	24.8	9.4	20.9	7.6	16.9	6.0	13.4						
150 × 200	6 X 8	16.9	37.6	15.3	33.9	12.8	28.5	10.4	23.1	8.2	18.3	6.6	14.6	5.4	11.9	4.4	9.8
150 × 250	6 X 10	21.4	47.6	19.4	43.0	16.2	36.1	13.1	29.2	10.4	23.1	8.3	18.5	6.8	15.0	6.0	13.4
200 × 200	8 X 8	24.3	54.0	23.2	51.5	21.6	48.1	19.6	43.5	17.1	38.0	14.5	32.3	12.3	27.4	10.4	23.1
200 × 250	8 X 10	30.8	68.4	29.4	65.3	27.5	61.0	24.8	55.1	21.6	48.1	18.5	41.0	15.6	34.7	13.2	29.3
200 × 300	8 X 12	37.3	82.8	35.6	79.0	33.2	73.8	30.0	66.7	26.2	58.2	22.3	49.6	18.9	42.0	15.9	35.4
250 × 300	10 X 10	39.8	88.4	38.7	85.9	37.4	83.0	35.6	79.0	33.1	73.6	30.2	67.0	27.0	60.0	23.8	52.9
250 × 300	10 X 12	48.2	107.0	46.8	104.0	45.0	100.0	43.0	95.6	40.1	89.1	36.5	81.2	32.7	72.6	28.8	64.0
250 × 350	10 X 14	56.7	126.0	54.9	122.0	53.1	118.0	50.4	112.0	47.3	105.0	42.9	95.3	38.4	85.3	33.8	75.1
300 × 300	12 X 12	58.5	130.0	57.6	128.0	56.3	125.0	54.9	122.0	52.7	117.0	50.0	111.0	46.8	104.0	43.0	95.6
350 × 350	14 X 14	81.0	180.0	80.1	178.0	79.2	176.0	77.4	172.0	75.6	168.0	73.4	163.0	70.2	156.0	66.6	148.0
400 × 400	16 X 16	107.1	238.0	106.2	236.0	105.3	234.0	103.5	230.0	101.7	226.0	99.9	222.0	97.2	216.0	93.6	208.0

Table A.11 Column Design Chart: Heavy Timber (Southern Pine/Douglas Fir)

*Chart options sorted by allowable radius of gyration

Nominal Size—Metric	Nominal Size—Imperial	Effective Unsupported Length (k × L) Metres (feet) in kips and kg × 1000																																			
		1.80M (6')		2.10M (7')		2.4M (8')		2.7M (9')		3.0M (10')		3.3M (11')		3.6M (12')		3.9M (13')		4.2M (14')		4.5M (15')		4.8M (16')		5.1M (17')		5.4M (18')		5.7M (19')		6.0M (20')		6.3M (21')		6.6M (22')		7.0M (23')	
		kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)
W 100 × 19	W 4 X 13	28	62	26	57	23	51	20	45	18	39	14	32	12	27	10	23	9	20	8	17	7	15														
W 130 X 24	W 5 X 16	37	83	36	79	33	74	31	69	29	64	26	58	23	52	21	46	18	39	15	34	14	30	12	27	11	24	9	21	9	19	8	18				
W 150 X 24	W 6 X 16	38	85	36	81	35	78	33	74	32	70	29	65	27	60	25	55	23	50	20	45	18	39	16	35	14	31	13	28	11	25	10	23	9	21	9	19
W 130 X 28	W 5 X 19	44	97	41	92	39	87	36	81	34	75	31	69	28	62	25	55	21	47	18	41	16	36	14	32	13	29	12	26	10	23	9	21				
W 150 X 30	W 6 X 20	49	109	47	105	45	101	43	96	41	91	38	85	36	80	33	74	30	67	27	61	24	54	22	48	19	42	17	38	15	34	14	31	13	28	12	26
W 150 X 37	W 6 X 25	62	137	59	132	57	126	54	120	51	114	48	107	45	100	42	93	38	85	35	77	31	69	27	61	25	55	22	49	20	44	18	40	16	36	15	33
W 200 X 46	W 8 X 31	80	178	78	174	76	169	74	164	72	159	69	154	67	148	64	142	61	136	59	130	55	123	53	117	50	110	46	102	43	95	39	87	36	79	32	72
W 250 X 49	W 10 X 33	85	189	83	184	81	179	78	173	75	167	72	161	70	155	67	149	64	142	61	135	57	127	54	120	50	112	46	103	43	95	39	86	35	78	32	72
W 200 X 52	W 8 X 35	90	201	89	197	86	191	84	186	81	180	78	174	76	168	73	162	70	155	67	148	63	141	60	133	56	125	53	117	49	109	45	100	41	91	37	83
W 250 X 58	W 10 X 39	101	224	98	218	95	212	93	206	90	199	87	193	83	185	80	178	77	170	73	162	69	153	65	145	61	135	57	126	52	116	48	106	43	96	40	88
W 200 X 59	W 8 X 40	104	230	101	225	99	219	96	213	93	206	90	200	87	193	83	185	80	177	77	170	72	161	69	153	65	144	60	134	56	125	52	115	47	105	43	96
W 200 X 71	W 8 X 48	125	277	122	270	119	264	116	257	112	249	108	241	105	233	101	224	97	215	93	206	89	197	84	187	79	176	74	165	69	154	64	143	59	131	54	120
W 250 X 73	W 10 X 49	130	289	128	284	126	279	123	273	121	268	118	262	115	256	112	249	109	242	106	235	103	228	99	221	96	213	92	205	89	197	85	188	81	180	77	171
W 310 X 79	W 12 X 53	140	312	138	307	135	301	133	295	130	288	127	282	124	275	120	267	117	260	113	252	110	244	106	235	102	227	98	218	94	209	90	199	85	189	81	179
W 250 X 80	W 10 X 54	144	319	141	313	139	308	136	302	133	296	130	289	127	282	124	275	121	268	117	260	114	253	110	244	106	236	102	227	98	218	94	209	90	200	86	190
W 200 X 86	W 8 X 58	151	335	147	327	144	319	140	311	136	302	132	293	127	283	123	273	118	262	113	251	108	239	103	228	97	215	91	202	85	189	79	175	72	161	66	147
W 310 X 86	W 12 X 58	154	342	151	336	149	330	145	323	142	316	139	309	136	302	132	294	129	286	125	277	121	269	117	260	113	250	108	241	104	231	99	221	95	210	90	199
W 250 X 89	W 10 X 60	160	355	157	349	154	343	151	336	148	329	145	322	141	314	138	307	135	299	131	290	126	281	122	272	118	263	114	254	110	244	105	234	100	223	95	212
W 360 X 91	W 14 X 61	162	359	158	352	155	345	152	338	149	331	145	323	142	315	138	306	134	297	130	288	126	279	121	269	117	259	112	248	107	237	102	226	97	215	91	203
W 200 X 100	W 8 X 67	174	387	171	379	167	370	162	360	158	350	153	339	148	328	142	316	137	304	131	292	126	279	119	265	113	251	106	236	99	221	93	206	86	190	78	174
W 310 X 97	W 12 X 65	175	389	173	384	171	379	168	373	165	367	162	361	160	355	157	348	153	341	150	334	147	326	144	319	140	311	136	303	133	295	129	286	125	277	121	268
W 360 X 101	W 14 X 68	180	400	177	393	173	385	170	377	166	369	162	360	158	351	154	342	149	332	145	322	140	311	135	301	130	289	125	278	120	266	114	253	108	241	102	227
W 250 X 101	W 10 X 68	181	402	178	395	175	388	171	381	168	373	164	365	161	357	157	348	153	339	149	330	144	320	140	310	135	299	130	289	125	278	120	267	115	255	109	242
W 310 X 107	W 12 X 72	194	431	191	425	189	420	186	413	183	407	180	400	177	393	174	386	170	378	167	370	163	362	159	354	155	345	151	336	147	327	143	318	139	308	135	299
W 360 X 110	W 14 X 74	196	436	193	429	189	421	185	412	181	403	177	394	173	384	168	373	163	363	158	352	153	341	148	329	143	317	137	304	131	292	125	278	119	265	113	250
W 250 X 115	W 10 X 77	204	454	201	447	198	439	194	431	190	422	186	413	182	404	177	394	173	384	168	373	163	362	158	351	153	339	147	327	142	315	136	302	130	289	124	275
W 250 X 115	W 10 X 77	205	456	202	448	198	440	194	432	191	424	186	414	182	405	178	395	173	385	168	374	163	363	158	352	153	340	148	328	142	316	136	303	131	290	124	276
W 310 X 117	W 12 X 79	213	473	210	467	207	461	204	454	201	447	198	439	194	432	191	424	187	415	183	407	179	398	175	389	171	380	167	370	162	360	158	350	153	339	148	328
W 360 X 122	W 14 X 82	217	482	213	474	209	465	205	456	201	446	196	435	191	425	186	413	181	402	175	389	170	377	163	363	158	351	152	337	145	323	139	308	132	293	125	277
W 250 X 131	W 10 X 88	234	521	231	513	227	504	223	495	218	485	214	475	209	464	204	453	199	442	194	430	188	417	182	405	176	392	170	378	164	364	157	349	151	335	144	319
W 310 X 129	W 12 X 87	235	522	232	515	229	508	225	501	222	493	218	485	215	477	211	468	207	459	203	450	198	440	194	430	189	420	184	409	179	398	174	386	169	376	164	364
W 360 X 134	W 14 X 90	246	547	243	541	241	536	239	530	236	524	233	517	230	511	227	504	224	497	220	489	217	482	213	474	210	466	206	458	202	449	198	440	194	432	190	422
W 310 X 143	W 12 X 96	259	575	256	568	252	560	248	552	245	544	241	535	237	526	232	516	228	506	223	496	219	486	214	475	209	464	203	452	198	440	193	428	187	416	181	403
W 310 X 158	W 12 X 106	286	636	283	628	279	620	275	611	271	602	266	592	262	582	257	572	252	561	248	550	242	538	237	526	231	514	226	502	220	489	214	475	208	462	202	448
W 360 X 162	W 14 X 109	297	661	294	654	291	647	288	640	285	633	282	626	278	618	274	609	270	601	266	592	262	583	258	574	254	564	249	554	245	544	240	533	235	523	230	512
W 310 X 179	W 12 X 120	324	721	320	712	316	702	311	692	307	682	302	671	297	660	292	648	286	636	281	624	275	611	269	597	263	584	256	569	250	555	243	540	236	525	222	493

Table A.12 Wide Flange (W) Column Design Chart: Selected A36 Steel Column Shapes (Axial Loading)

Note: sorted by acceptable least allowable radius of gyration on weak axis
Consult AISC *Steel Construction Manual* for full range of structural shapes

Nominal Size (Metric)—mm	Nominal Size (US)—inches	Effective Unsupported Length (K × L) Metres (feet) in kips and kg × 1000																																					
		WEIGHT		1.80M (6')		2.10M (7')		2.4M (8')		2.7M (9')		3.0M (10')		3.3M (11')		3.6M (12')		3.9M (13')		4.2M (14')		4.5M (15')		4.8M (16')		5.1M (17')		5.4M (18')		5.7M (19')		6.0M (20')		6.3M (21')		6.6M (22')		7.0M (23')	
		kg/m	lbs/ft	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)	kg (000)	lb (000)
75mm Pipe	3" Pipe	11.30	7.58	17	38	16	36	15	34	14	31	13	28	11	25	10	22	9	19	7	16	6	14	5	12	5	11	5	10	4	9								
89mm Pipe	3.5" Pipe	13.58	9.11	22	48	21	46	20	44	18	41	17	38	16	35	14	32	13	29	11	25	10	22	9	19	8	17	7	15	6	14	5	12	5	11	5	10		
76 × 76 × 6.4	3 × 3 × 1/4	13.13	8.81	24	53	22	49	20	44	17	38	15	33	12	27	10	23	9	19	8	17	7	15	6	13	5	11	5	10										
75mm Pipe	3" Extra Strong Pipe	15.28	10.25	23	52	22	48	20	45	18	41	17	37	15	33	13	28	11	24	9	21	8	18	7	16	5	12	5	11										
102mm Pipe	4" Pipe	16.09	10.79	27	59	26	57	24	54	23	52	22	49	21	46	19	43	18	40	16	36	15	33	13	29	12	26	10	23	9	21	9	19	8	17	7	15	6	14
102 × 102 × 4.8	4 × 4 × 3/16	14.04	9.42	29	64	27	61	26	58	25	55	23	51	21	47	19	43	18	39	16	35	14	30	12	27	11	24	9	21	9	19	8	17	7	16	6	14	6	13
89mm Strong Pipe	3.5" Extra Strong Pipe	18.63	12.50	30	66	28	63	27	59	25	55	23	51	21	47	19	43	17	38	15	33	13	29	11	25	10	23	9	20	8	18	7	16	6	14				
102mm Strong Pipe	4" Extra Strong Pipe	22.33	14.98	36	81	35	78	34	75	32	71	30	67	28	63	27	59	24	54	22	49	20	44	18	39	16	35	14	31	13	28	11	25	10	22	9	21	9	19
75mm Ex. St. Pipe	3" Double Ex. St. Pipe	27.70	18.58	41	91	38	84	35	77	31	69	27	60	23	51	19	43	17	37	14	32	13	28	11	24	10	22												
127mm Pipe	5" Pipe	21.79	14.62	37	83	36	81	35	78	34	76	33	73	32	71	31	68	29	65	29	64	26	58	25	55	23	51	21	47	19	43	18	39	16	36	14	32	13	30
102 × 102 × 7.9	4 × 4 × 5/16	22.11	14.83	45	100	43	95	41	90	38	84	35	78	32	72	29	65	26	58	23	51	20	44	18	39	15	34	14	31	13	28	11	25	10	23	9	21	9	19
152 × 152 × 4.8	6 × 6 × 3/16	21.66	14.53	48	107	47	105	46	102	45	99	43	96	42	93	41	90	39	87	37	83	36	80	34	76	32	72	31	68	29	64	27	60	25	56	23	51	19	43
152mm Pipe	6" Pipe	28.28	18.97	50	110	49	108	48	106	46	103	45	101	44	98	43	95	41	92	40	89	39	86	37	82	36	79	34	75	32	71	30	67	28	63	27	59	25	55
127mm Strong Pipe	5" Extra Strong Pipe	30.98	20.78	53	118	51	114	50	111	48	107	46	103	45	99	43	95	41	91	39	86	36	81	34	76	32	71	29	65	27	59	24	54	22	48	20	44	18	41
102mm Ex. St. Pipe	4"Double Ex. St. Pipe	41.06	27.54	66	147	63	140	60	133	57	126	53	118	49	109	45	100	41	91	36	81	32	70	28	62	25	55	22	49	20	44	18	40	16	37	15	33		
152mm Ex. St. Pipe	6" Extra Strong Pipe	42.59	28.57	75	166	73	162	72	159	70	155	68	151	66	146	64	142	62	137	59	132	57	127	55	122	52	117	50	111	47	105	45	99	41	92	39	86	36	80
203mm Pipe	8" Pipe	42.56	28.55	77	171	76	168	75	166	73	163	72	161	71	158	70	155	68	152	67	149	65	145	64	142	62	138	61	135	59	131	57	127	55	123	54	119	52	115
203 × 203 × 6.4	8 × 8 × 1/4	38.49	25.82	88	196	87	193	86	190	84	187	83	184	81	180	79	176	78	173	76	169	74	165	72	160	70	156	68	151	66	147	64	142	62	137	59	132	57	127
152 × 152 × 9.5	6 × 6 × 3/8	40.97	27.48	90	201	88	196	86	191	84	186	81	180	78	174	76	168	72	161	69	154	66	147	63	140	59	132	56	124	52	115	48	107	44	98	40	89	34	75
127mm Ex. St. Pipe	5" Double Ex. St. Pipe	57.47	38.55	97	216	94	209	91	202	88	195	84	187	80	178	77	170	72	160	68	151	63	141	59	130	54	119	49	108	44	97	39	87	36	80	32	72	30	67
254mm Pipe	10" Pipe	60.35	40.48	111	246	109	243	108	241	107	238	106	235	104	232	103	229	102	226	100	223	99	220	97	216	96	213	94	209	92	205	90	201	89	197	87	193	85	189
203mm Ex. St. Pipe	8" Extra Strong Pipe	64.68	43.39	117	259	115	255	113	251	111	247	109	243	108	239	105	234	103	229	101	224	99	219	96	214	94	209	91	203	89	197	86	191	83	185	81	179	78	173
203 × 203 × 9.5	8 × 8 × 3/8	56.05	37.60	129	286	126	281	125	277	122	272	120	267	118	262	115	256	113	251	110	245	107	238	104	232	101	225	99	219	95	212	92	205	89	197	86	190	82	182
152mm Ex. St. Pipe	6 " Double Ex. St. Pipe	79.25	53.16	138	306	135	299	131	292	128	284	124	275	120	266	116	257	111	247	107	237	102	227	97	216	92	205	87	193	81	181	76	168	70	155	64	142	59	131
310mm Pipe	12" Pipe	73.88	49.56	136	303	135	301	135	299	133	296	132	293	131	291	130	288	128	285	127	282	125	278	124	275	122	272	121	268	119	265	117	261	116	257	114	254	113	250
254 × 254 × 7.9	10 × 10 × 5/16	60.15	40.35	140	311	139	308	137	305	135	301	134	297	132	293	130	289	128	285	126	280	124	276	122	271	120	266	117	261	115	256	113	251	110	245	108	240	105	234
254mm Ex. St. Pipe	10" Extra Strong Pipe	81.60	54.74	149	332	148	328	146	325	144	321	143	318	141	314	139	309	137	305	135	301	133	296	131	291	129	286	126	281	124	276	122	271	119	265	117	260	114	254
310mm Ex. St. Pipe	12" Extra Strong Pipe	97.53	65.42	180	400	179	397	177	394	176	390	174	387	172	383	171	379	169	375	167	371	165	367	163	363	161	358	159	353	157	349	155	344	152	337	150	334	148	329
203mm Ex. St. Pipe	8" Double Ex. St. Pipe	107.96	72.42	194	431	191	424	188	417	185	410	181	403	178	395	174	387	170	378	166	369	162	360	158	351	153	341	149	331	144	321	140	310	135	299	130	288	124	276
305 × 305 × 9.5	12 × 12 × 3/8	86.61	58.10	204	453	202	449	200	445	198	441	197	437	195	433	193	428	190	423	188	418	186	413	184	408	181	403	179	397	176	391	174	386	171	380	168	373	165	367
254 × 254 × 12.7	10 × 10 × 1/2	93.11	62.46	216	481	214	476	212	471	209	465	207	459	203	452	201	446	198	439	194	432	191	424	188	417	184	409	180	401	176	392	173	384	169	375	165	366	161	357
356 × 356 × 9.5	14 × 14 × 3/8	101.83	68.31	241	536	240	533	238	529	236	525	234	521	233	517	231	513	229	508	227	504	225	499	222	494	220	489	218	484	215	478	213	473	210	467	208	462	205	456
305 × 305 × 12.7	12 × 12 × 1/2	113.40	76.07	267	593	265	588	262	583	260	577	257	571	255	566	252	559	249	553	246	546	243	540	240	533	237	526	233	518	230	511	226	503	223	495	219	487	215	478
356 × 356 × 12.7	14 × 14 × 1/2	133.69	89.68	317	704	315	699	312	694	310	689	308	684	305	678	302	672	300	666	297	660	294	654	291	647	288	641	285	634	282	627	279	619	275	612	272	604	269	597
406 × 406 × 12.7	16 × 16 × 1/2	154.00	103.30	367	815	365	810	362	805	360	800	358	795	356	790	353	785	351	779	348	773	345	767	342	761	340	755	337	748	333	741	331	735	328	728	324	721	321	713
406 × 406 × 15.9	16 × 16 × 5/8	189.88	127.37	451	1003	449	998	446	992	444	986	441	979	438	973	435	966	432	959	428	951	425	944	421	936	418	928	414	920	410	912	406	903	403	895	399	886	395	877

Table A.13 Pipe Column Design Chart: Selected A36 Steel Column Shapes (Axial Loading)

Note: Consult AISC *Steel Construction Manual* for full range of structural shapes
Structural tubing (square sections) are typically graded to 46ksi (shown here) instead of 36ksi

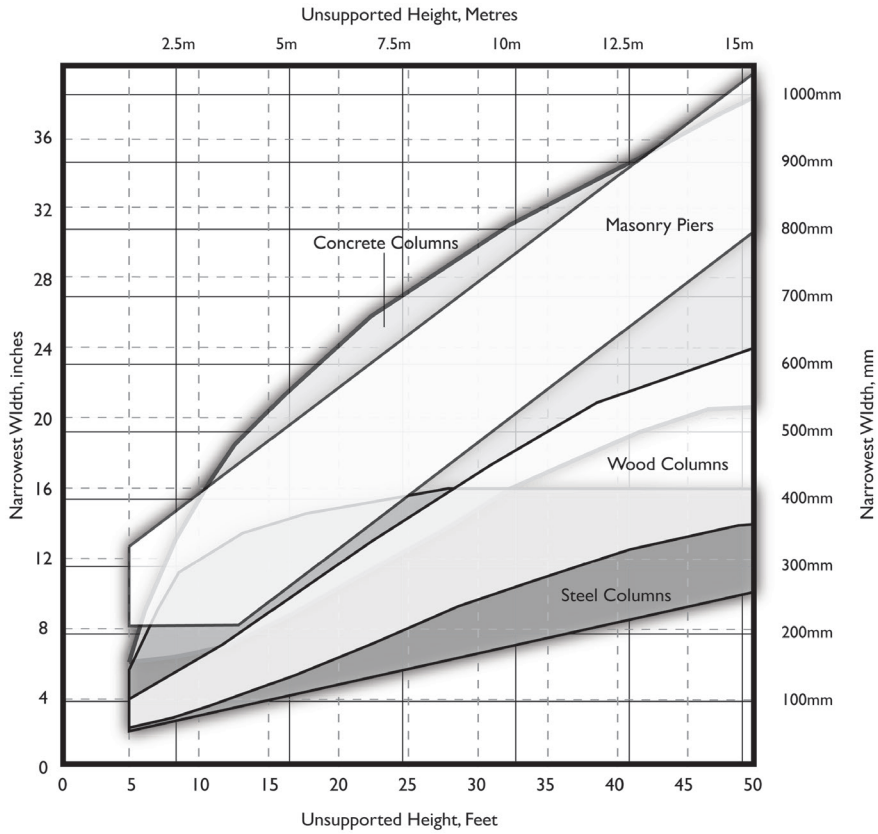
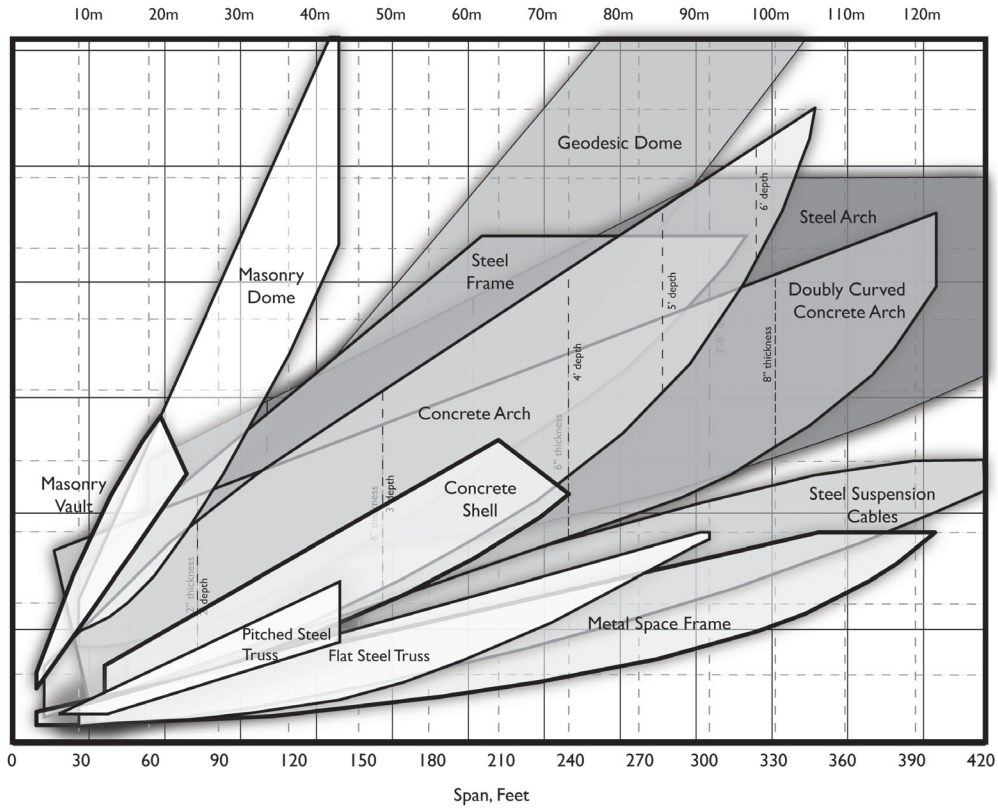


Figure A.14 Column size: Height and width estimation

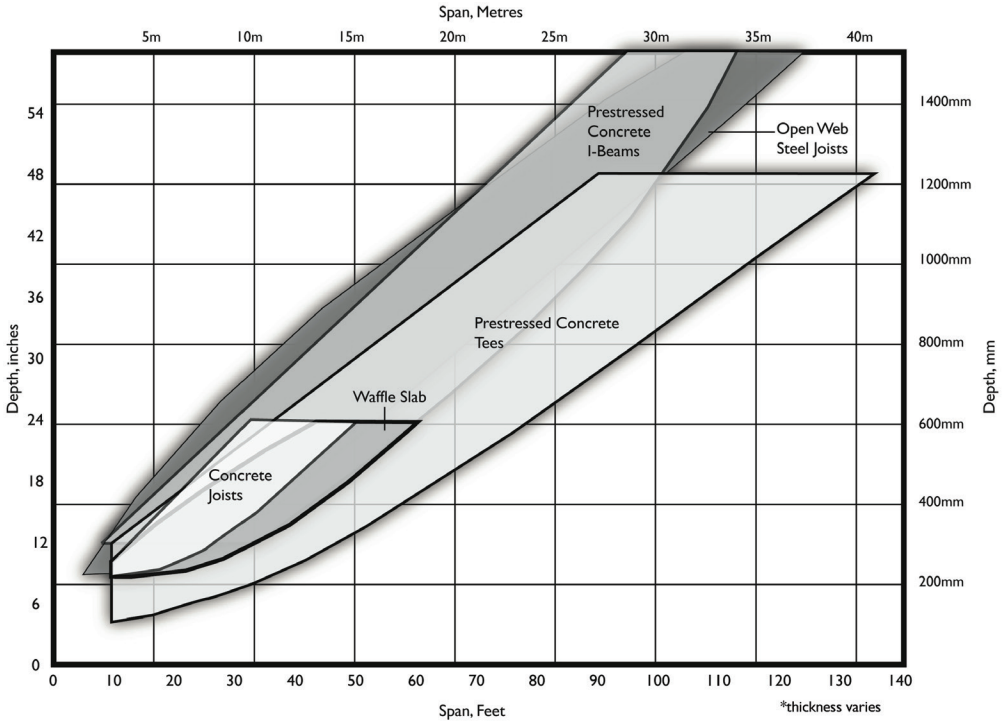
Soil Type	Allowable Bearing, kg/m²	Allowable Bearing, tons/sq ft.
Sound Rock	585,900	60
Medium Rock	390,600	40
Intermediate Rock	195,300	20
Well-cemented Hardpan	117,180	12
Compact, well-graded gravel	97,650	10
Poorly-cemented Hardpan	78,120	8
Compact gravel	78,120	8
Loose Gravel	58,590	6
Weathered or Porous Rock	19,530 to 78,120	2 to 8
Coarse Sand	29,295 to 58,690	3 to 6
Hard Clay	48,825	5
Gravel/Sand mix	39,060	4
Fine Sand	19,530 to 39,060	2 to 4
Fill	19,530 to 39,060	2 to 4
Dense Silt	29,295	3
Medium Clay	19,530	2
Medium Silt	14,647	1.5

Table A.15 Allowable Bearing Loads on Various Soil Types



LONG SPAN SYSTEMS: POSSIBLE SPANS

Figure A.16 Long span systems: Possible spans



LONG SPAN SYSTEMS: SPAN & DEPTH ESTIMATION

Figure A.17 Long span systems: Span and depth estimation

Span Range: Feet																	
	Type	Material	System	20	40	60	80	100	120	140	160	180	200	220	240	260	280
One-Way Spans	Section Resistant, Beams	Timber	*Planks*														
			Joists														
			Laminated														
			Box Beams														
		Reinforced Concrete	*Slabs*														
			Beams														
			Precast plank														
			Precast channel														
			Precast tees														
		Steel	*Decking*														
			Wide-Flanges														
			Plate Girders														
	Form Resistant, Arches	Timber	*Laminated*														
		Steel	*Built-up*														
		Concrete	*Cast-in Place*														
	Form, Cables	Steel	*Suspended*														
			Cable-Stayed														
	Vectors, Trusses	Timber	*Trussed Rafters*														
			Special Design														
		Steel	*Open Web*														
			Curved/Special														
		Concrete	*Vierendeel*														
	Surface, Shell	Timber	*Folded Plate*														
			Lamellas														
		Concrete	*Folded Plate*														
			Vaulting														
Two-Way Spans	Section, Slabs	Concrete	*Flat Plate*														
			Waffle Slab														
	Vector Truss	Steel	*Space Frame*														
	Surface, Shells	Concrete	*Dome*														
			Hypar														
			Anticlastic														
		Steel	*Dome*														
			Special Design														
					12		24		36.5		50		61		73		85

Table A.18 Allowable Span Ranges by Structural Type and Material

Span range: meters

Taylor & Francis
Taylor & Francis Group
http://taylorandfrancis.com

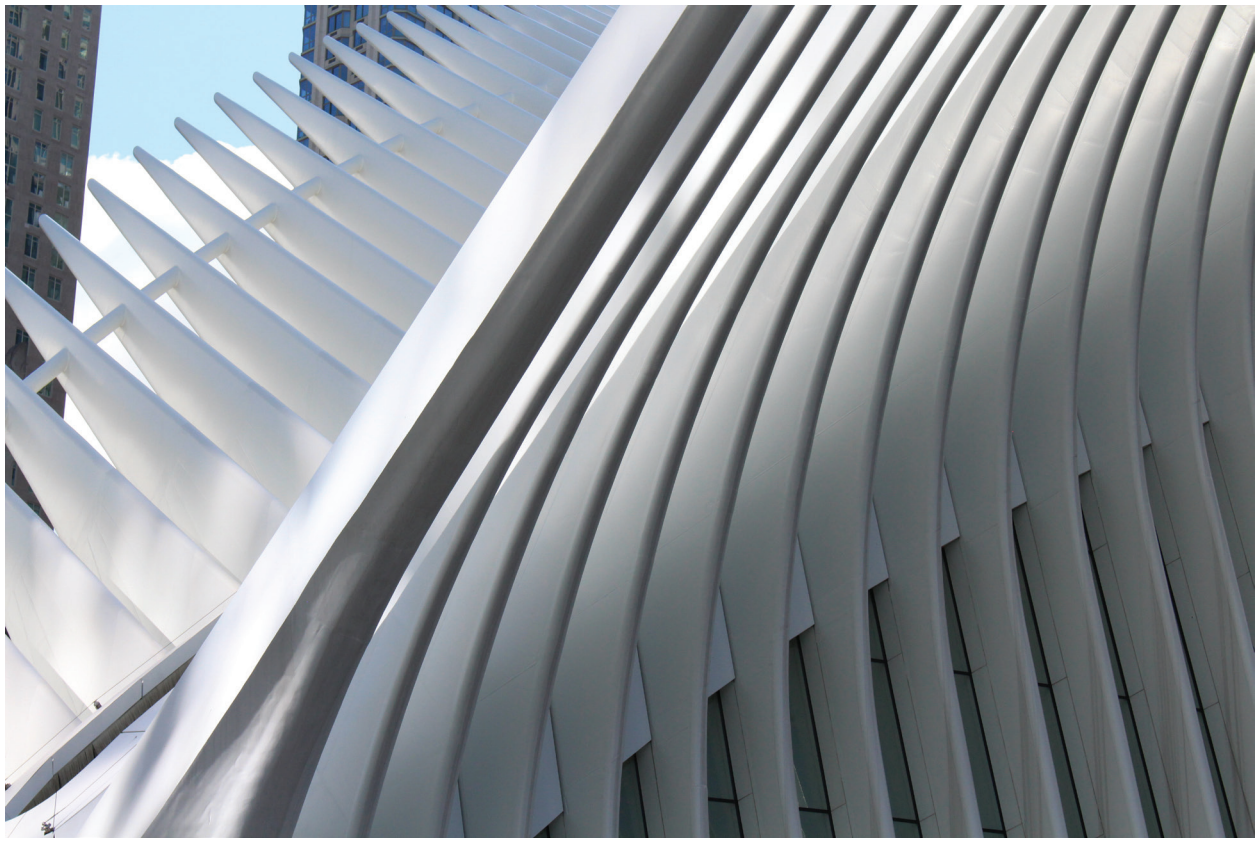


Figure AR.0 The Oculus PATH Station, New York City

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Glossary

Allowable Stress A generally accepted safe load per unit of area for a given material. Often dictated by codes or industry standards.

Anisotropic Material with a consistent structural behavior in all directions. Steel and concrete are examples of anisotropic materials.

Anticlastic Double-curvature at a given point: convex along a longitudinal plane and concave along the perpendicular section.

Arch Line The compressive line of an arch demarking the ideal funicular path of stress. Also known as meridians. A series of vertical load path in domes for compressive stresses.

Beam A structural member that resists bending.

Bending An internal state of stress in which loads are carried perpendicular to their direction.

Boring A test drilling that procures a long cylindrical sample of earth.

Braced Frame Planar, vertical panels designed to provide lateral stiffness to a structure. They can be braced with fixed connections, as a shear wall, or with bracing members.

Cable-Stayed A structural system that uses linear cables to directly support a roof or beam.

Caissons Foundation type that creates a vertical drop in the ground. Retaining walls must be designed as cantilevers to resist the overturning moment of the "held-back" soil.

Castellated Beam A beam, usually steel, that has been cut lengthwise near the neutral axis and reassembled so that it gains depth without gaining weight.

Centroid A measure of a shape's "average" point in space or, more accurately, its center of gravity. This will correspond with the shape's neutral axis if it is employed as a beam, and is thus an important characteristic.

Column Technically, any axially loaded member. Usually a vertical member carrying gravity loads and designed to resist both compression and buckling.

Column Cap A conical or rectilinear element between a column and a floor plate that is designed to spread the shear force between the two over a wider cross-sectional area.

Compression An internal state of stress in which a material's fibers tend to be pushed into one another.

Concentrated Load A load that is applied at a discrete point.

Concurrent Forces The condition when three or more lines of force acting within the same plane intersect at a single point.

Cross Bracing A method of providing lateral resistance to frames by triangulating the corners of a rectangular bay.

Dead Load Loads that will not change over the course of a structure's life.

Determinate Structure A structure whose supporting conditions cannot be calculated algebraically, typically because fixed connections offer the capacity to take moment stresses.

Developable Curve Referring to a surface that is singularly curved. They are straight in one direction, curved in the other, and can be formed by bending a flat sheet. Cones, and barrels are developable.

Diaphragms Planar surfaces (vertical or horizontal) used to stabilize against lateral loads, often by transferring loads to parallel planes. Essentially, shear walls work like cantilevers tipped up on their ends, absorbing lateral loads by going in to bending and transmitting the loads to a foundation.

Distributed Load A load that is spread out over a line or surface.

Dome A surface that is curved in more than one plane. Such surfaces provide great resistance to deformation through their material strength. A dome is the simplest of these.

Double-Curved A surface that is curved in more than one plane. Such surfaces provide great resistance to deformation through their material strength. A dome is the simplest of these.

End Conditions How a column is connected to the beams or supports at its ends. Because these play a role in how much or little the column can move,

they are important considerations in how well the column will resist buckling.

Environmental Load Loading caused by environmental factors (snow, rain, wind, etc.). The loads change direction and magnitude frequently. The loads can be either gravitational or lateral loads. Codes are used to determine magnitude of loads based on geographic location.

Equilibrium The condition of being at rest. Any forces acting on the object must be balanced by forces adding up to equal and opposite reactions. There are two major states of equilibrium: translational and rotational.

Fixed (or Moment) Connection A connection that does not permit a structural member to rotate or translate.

Flanges In beam design, top and bottom elements designed to put the most possible material in the most efficient places—far from the neutral axis.

Flexure Formula A simple formula that relates allowable stress, section modulus, and maximum allowable moment: $M = fS$. The maximum allowable moment in any beam (M) is equal to the material's maximum allowable stress (f) multiplied by the section modulus of its cross-sectional shape (S).

Folded Plate A thin, planar structural system that relies on "folds" or bends in its cross-section to develop bending resistance, allowing it to span as a beam.

Force That which tends to produce a change in the motion of a physical body.

Measured in units of weight, i.e. 100 pounds.

Force Couple The development of axial forces perpendicular to an external load in a beam. Usually consists of an internal compressive force on the side toward the load, and an internal tensile force on the side away from the load.

Frame A system of horizontal and vertical structural members that collectively resist gravity and lateral forces more effectively than they would on their own.

Free Body Diagram A mathematical abstraction of the forces at work on a single, monolithic structure in which the object in question is reduced to a single point.

Geodesic Dome A name given to a type of a framed dome. Named for a word denoting the shortest possible line between two points on a surface.

Gravity Load Loads acting on a structure along a line between the structure and the earth's center.

Hoop Lines Also known as parallels. The horizontal sections of a dome; the largest parallel is the equator. In a shell dome outward thrust is resisted by tension along the hoop lines below about 45° above the horizontal.

Indeterminate Structure A structure whose supporting conditions cannot be calculated algebraically, typically because fixed connections offer the capacity to take moment stresses.

Inflection Point The point in a bending structure at which the resulting curvature changes from one direction to the other.

Lateral Load Loads acting on a structure along lines other than those between the object and the earth's center, e.g. wind, earthquakes, etc.

Lateral Stability The second order of magnitude of structural design, after gravity resistance. Simply put, the ability of a structure to resist sideways forces.

Lever Arm The distance between the action of a force and its pivot point.

Live Load Loads that may change over the course of a structure's life.

Load Any external force acting on a structure.

Mat Foundation A foundation type that uses the entire footprint of a building to spread its load over the soil below. Typically used for large volume, low-rise buildings where the foundation can be combined with the ground floor slab.

Membrane A structural member that relies on surface stresses in tension and shear only to achieve its span. Compare with a shell, which uses tension, compression, and shear.

Moment The resultant force about a point, measured by multiplying the quantity of the force by its lever arm. Expressed in kilogram-meters, pound-feet, or similar units.

Moment Connections Another term for fixed connections, more accurately describing the fact that they can carry bending moment from one member to another due to their stiffness.

Moment of Inertia Simply put, a weighted measure of a shape's area, accounting for both quantity of area and average distance of each point from the centroid.

Neutral Axis A line through a beam that represents the "axle" of a force couple. This line undergoes no tension or compression while the beam is in bending.

One Way Slab (or Plate) A thin, planar structural system that gains most of its structural performance from simple bending resistance along the axis of the span.

Panel Point Related to trusses. A panel point is an intersection of diagonal and/or vertical web member with the top and/or bottom chord.

Pile Foundation A foundation type that uses long, usually cylindrical rods (piles) to either reach a firmer soil below, or develop bearing through friction with the surrounding soil.

Plate A thin slab. Slab denotes a heavy, thick material, while plate is more descriptive of metal or wood.

Pneumatic A structural system that relies on air pressure to keep a thin membrane in constant tension, enabling it to span considerable distances.

Pre-Stressing The use of tensioned steel in concrete beams to absorb much of the beam's tensile stress.

Racking The tendency of a frame to change shape by skewing when undergoing lateral loading.

Radius of Gyration Basically, a measurement of a column shape's efficiency. Radius of gyration is defined by: $r = \sqrt{(I/A)}$ where I is the shape's moment of inertia and A is the shape's area. Note that we must usually find r for the weakest axis.

Reaction A force that develops in resistance to an applied force. Can be active (e.g. pulling on a cable), or passive (e.g. a pedestal bearing the weight of something).

Retaining Walls Foundation type that creates a vertical drop in the ground. Retaining walls must be designed as cantilevers to resist the overturning moment of the "held-back" soil.

Section Modulus A refinement of Moment of Inertia that provides a more usable number for structural calculations by counting depth one fewer times. It is found by dividing a shape's moment of inertia by the distance from its centroid to its "farthest fiber" (c): $S = I/c$.

Settlement The tendency of a structure to work its way down through the ground, often the result of a change in composition of the soil (e.g., from flooding) or inadequately designed foundations.

Shear An internal state of stress in which the particles of a material tend to slide past one another.

Shell A three-dimensional compression structure whose geometry ensures that all loads are resolved through surface stresses, thus requiring little in the way of buttressing.

Slab A structural member that gains some of its performance by two-way resistance to bending.

Slenderness Ratio A measure of a column's length divided by either its narrowest width or by its radius of gyration in the weakest axis, defined by formula: kl/r.

Space Frame A network of trusses arranged at an angle to one another and interconnected to span in two dimensions.

Spread Footing A foundation type that works like an inverted slab, taking the point load of a column or pier and distributing it over an area of foundation "pad" (usually concrete) so that the soil is not stressed past its capacity.

Statics The physics of things at rest.

Substructure The part of a building's structure below the ground floor.

Suspension A structural system that uses a main cable and smaller "hangars" to support a roof or beam. The main cable in suspension structures will take on a characteristic funicular shape, approximating a catenary shape as more hangars are added.

Synclastic Surfaces are doubly-curved with similar curvature in each direction; like revolved arch forms.

Thrust A horizontal/outward force. An inevitable consequence of two-dimensional arched structures or cable structures. The geometry of these systems creates a resultant force—outward in compression structures, inward in tension structures—that must be countered for the structure to stand.

Tributary Area The total area of floor plate that "flows" into a column. In multi-story buildings, this will include the weight and loads of every floor plate

above the column in question. Figured on each floor by assuming each column will carry an area defined by the centerlines of each surrounding bay.

Truss A bending member that replaces the solid web of a beam with a network of axially loaded members, usually arranged in triangular panels.

Two Way Slab (or Plate) A square-proportioned bay defined by a thin, planar structural system that gains its structural performance from a combination of bending along and across the axis of the span. Loads are distributed to support members around the perimeter.

Waffle Slab A two-way concrete slab system made by pouring concrete to form a network of intersecting concrete joists.

Web In beam design, an element designed to space flanges apart from one another, and thus from the neutral axis. This may be a solid plate, as in steel W-shapes, or a much lighter element, such as the bent metal bar in open web joists.

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