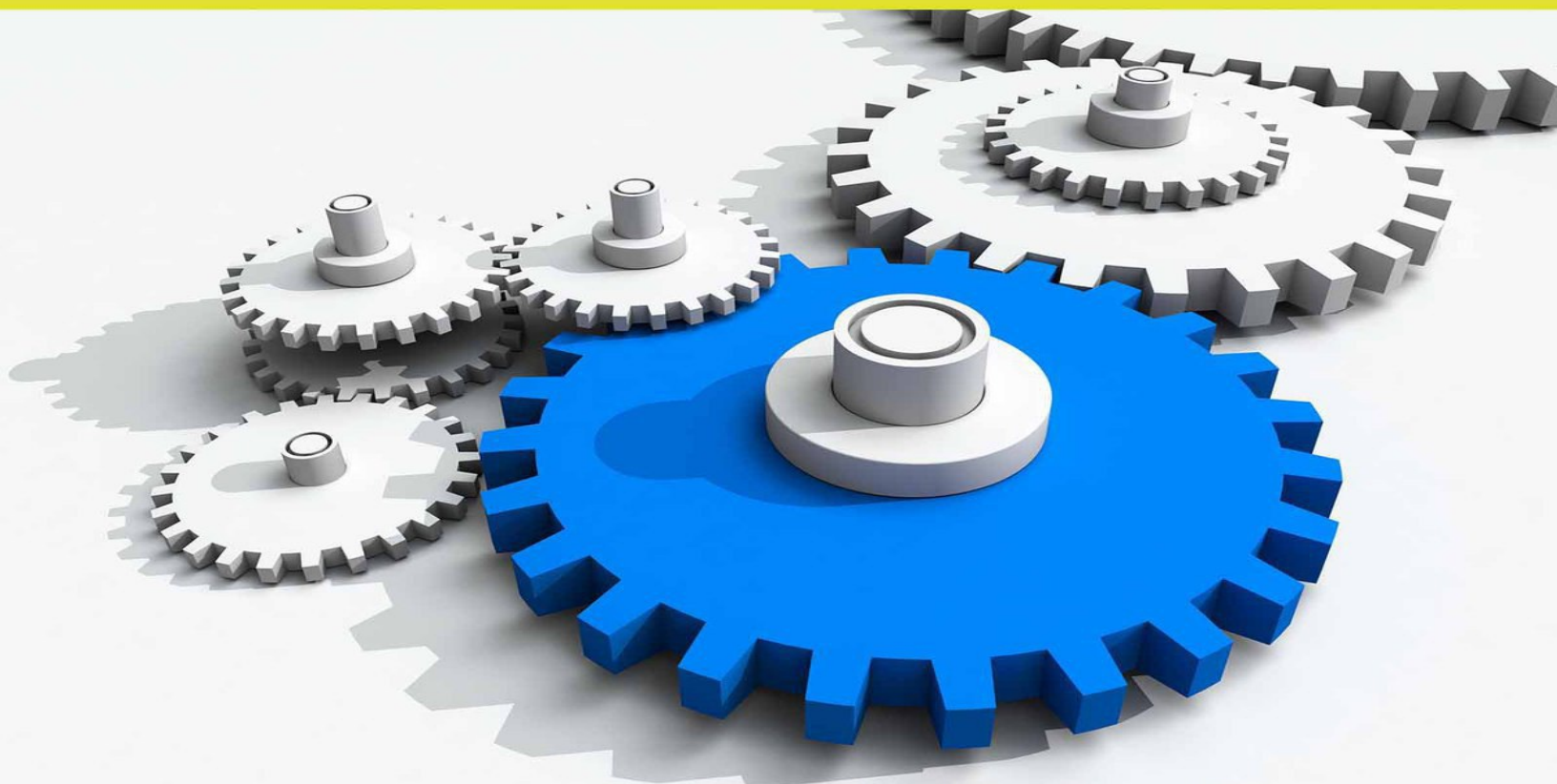


# MECHANICAL ENGINEERING HANDBOOK

**FOR FORMULAS**



(GATE, ESE, SSC JE AND OTHER COMPETITIVE EXAMS)



N A V Y   F E R O Z

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# **MECHANICAL ENGINEERING HANDBOOK FOR FORMULAS**

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**NAVY FEROZ**

## **MECHANICAL ENGINEERING FORMULA BOOK**

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## **ABOUT**

“ Mechanical Engineering Formulas - all subjects formulas with concepts and course outlines are given here. Select your desired course and you can revise all the Formulas within an hour only. When you are a mechanical engineer, you need to know the important formulas during the competitive exams like GATE, ESE and other exams to solve the answers easily using the formula. So, you must know the all-important formulas in the mechanical engineering Subjects. This book is specially prepared for the mechanical engineers”.

## **PREFACE**

In order to ignite your preparations for your Exams. This book providing the list of Important formulas for all subject of mechanical engineering, which was quite in demand and useful for all learners. Providing all the formulas in the single book will help the candidates for their preparation.

This combined formula book will help you to learn the all mechanical engineering formulas for GATE, ESE, SSC JE and other mechanical engineering exams.

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## SI MULTIPLES

### Metric System

VALUE	EXPONENT	SYMBOL	PREFIX
1 000 000 000 000	$10^{12}$	T	tera
1 000 000 000	$10^9$	G	giga
1 000 000	$10^6$	M	mega
1 000	$10^3$	k	kilo
100	$10^2$	h	hecto
10	$10^1$	da	deca
0.1	$10^{-1}$	d	deci
0.01	$10^{-2}$	c	centi
0.001	$10^{-3}$	m	milli
0.000 001	$10^{-6}$	$\mu$	micro
0.000 000 001	$10^{-9}$	n	nano
0.000 000 000 001	$10^{-12}$	p	pico

### Conversion Chart for Metric Units

To Convert		To Milli-	To Centi-	To Deci-	To Metre, Gram, Litre	To Deca-	To Hecto-	To Kilo-
	Kilo-	$\times 10^6$	$\times 10^5$	$\times 10^4$	$\times 10^3$	$\times 10^2$	$\times 10^1$	
	Hecto-	$\times 10^5$	$\times 10^4$	$\times 10^3$	$\times 10^2$	$\times 10^1$		$\times 10^{-1}$
	Deca-	$\times 10^4$	$\times 10^3$	$\times 10^2$	$\times 10^1$		$\times 10^{-1}$	$\times 10^{-2}$
	Metre, Gram, Litre	$\times 10^3$	$\times 10^2$	$\times 10^1$		$\times 10^{-1}$	$\times 10^{-2}$	$\times 10^{-3}$

	Deci-	$\times 10^2$	$\times 10^1$		$\times 10^{-1}$	$\times 10^{-2}$	$\times 10^{-3}$	$\times 10^{-4}$
	Centi-	$\times 10^1$		$\times 10^{-1}$	$\times 10^{-2}$	$\times 10^{-3}$	$\times 10^{-4}$	$\times 10^{-5}$
	Milli-		$\times 10^{-1}$	$\times 10^{-2}$	$\times 10^{-3}$	$\times 10^{-4}$	$\times 10^{-5}$	$\times 10^{-6}$

## BASIC UNITS (DISTANCE, AREA, VOLUME, MASS, DENSITY)

### BASIC UNITS

SI	IMPERIAL
<b>DISTANCE</b> 1 metre (1 m) = 10 decimetres (10 dm) =100 centimetres (100 cm) =1000 millimetres (1000 mm)  1 decametre (1 dam) = 10 m 1 hectometre (1 hm) = 100 m 1 kilometre (1 km) = 1000 m	12in.=1ft 3 ft = 1yd 5280 ft = 1 mile 1760 yd = 1 mile

### Conversions:

$$\begin{aligned}1 \text{ in.} &= 25.4 \text{ mm} \\1 \text{ ft} &= 30.48 \text{ cm} \\1 \text{ mile} &= 1.61 \text{ km} \\1 \text{ yd} &= 0.914 \text{ m} \\1 \text{ m} &= 3.28 \text{ ft}\end{aligned}$$

### AREA

1 sq metre (1 m <sup>2</sup> ) = 10 000 cm <sup>2</sup> = 1 000 000 mm <sup>2</sup>  1 sq hectometre (1 hm <sup>2</sup> ) = 10 000 m <sup>2</sup> = 1 hectare (1 ha) 1 sq km (1 km <sup>2</sup> ) = 1 000 000 m <sup>2</sup>	1 ft <sup>2</sup> = 144 in. <sup>2</sup> 1 yd <sup>2</sup> = 9 ft <sup>2</sup> 1 sq mile = 640 acre = 1 section
---	---

### Conversions:

$$\begin{aligned}1 \text{ in.}^2 &= 6.45 \text{ cm}^2 = 645 \text{ mm}^2 \\1 \text{ m}^2 &= 10.8 \text{ ft}^2 \\1 \text{ acre} &= 0.405 \text{ ha} \\1 \text{ sq mile} &= 2.59 \text{ km}^2\end{aligned}$$



SI	IMPERIAL
<b>VOLUME</b> $1 \text{ m}^3 = 1000000 \text{ cm}^3$ $= 1 \times 10^9 \text{ mm}^3$ $1 \text{ dm}^3 = 1 \text{ litre}$ $1 \text{ litre} = 1000 \text{ cm}^3$ $1 \text{ mL} = 1 \text{ cm}^3$ $1 \text{ m}^3 = 1000 \text{ litres}$	$1 \text{ ft}^3 = 1728 \text{ in.}^3$ $1 \text{ yd}^3 = 27 \text{ ft}^3$ $1(\text{liquid}) \text{ U.S. gallon} = 231 \text{ in.}^3$ $= 4 (\text{liquid}) \text{ quarts}$ $1 \text{ U.S. barrel (bbl)} = 42 \text{ U.S. gal.}$ $1 \text{ imperial gallon} = 1.2 \text{ U.S. gal.}$

### Conversions:

$1 \text{ in.}^3 = 16.4 \text{ cm}^3$   
 $1 \text{ m}^3 = 35.3 \text{ ft}^3$   
 $1 \text{ litre} = 61 \text{ in.}^3$   
 $1 \text{ U.S. gal.} = 3.78 \text{ litres}$   
 $1 \text{ U.S. bbl.} = 159 \text{ litres}$   
 $1 \text{ litre/s} = 15.9 \text{ U.S. gal/min}$

### Mass and Weight

$1 \text{ kilogram (1 kg)} = 1000 \text{ grams}$ $1000 \text{ kg} = 1 \text{ tonne}$	$2000 \text{ lb} = 1 \text{ ton (short)}$ $1 \text{ long ton} = 2240 \text{ lb}$
---	---

### Conversions:

1 kg (on Earth) results in a weight of 2.2 lb

### Density

$\text{Mass density} = \frac{\text{mass}}{\text{volume}}$ $\rho = \frac{m}{v} \left( \frac{\text{kg}}{\text{m}^3} \right)$	$\text{Weight density} = \frac{\text{weight}}{\text{volume}}$ $\rho = \frac{w}{v} \left( \frac{\text{lb}}{\text{ft}^3} \right)$
--	---

### Conversions:

(on Earth) a mass density of  $1 \text{ Kg/m}^3$   
 results in a weight density of  $0.0623 \text{ lb/ft}^3$

<b>S.I</b>	<b>IMPERIAL</b>
<b>RELATIVE DENSITY</b> In SI R.D. is a comparison of mass density to a standard. For solids and liquids the standard is fresh water.	In Imperial the corresponding quantity is <b>specific gravity</b> ; for solids and liquids a comparison of weight density to that of water.

### Conversion

In both systems the same numbers hold for R.D. as for S.G. since these are equivalent ratios.

<b>RELATIVE DENSITY ( SPECIFIC GRAVITY) OF VARIOUS SUBSTANCE</b>	
Water (Fresh).....1.00	Mica.....2.9
Water (Sea Average)....1.03	Nickel.....8.6
Aluminum.....2.56	Oil (linseed).....0.94
Antimony.....6.70	Oil (olive).....0.92
Bismuth.....9.80	Oil (petroleum).....0.76-0.86
Brass.....8.40	Oil (turpentine).....0.87
Brick.....2.1	Paraffin .....0.86
Calcium.....1.58	Platinum.....21.5
Carbon (Diamond).....3.4	Sand (dry).....1.42
Carbon (Graphite).....2.3	Silicon.....2.6
Carbon (Charcoal).....1.8	Silver.....10.57
Chromium.....6.5	Slate.....2.1-2.8
Clay.....1.9	Sodium.....0.97
Coal.....1.36-1.4	Steel (mild).....7.87
Cobalt.....8.6	Sulphur.....2.07
Copper.....8.77	Tin.....7.3
Cork.....0.24	Tungsten.....19.1
Glass (Crown).....2.5	Wood (ash).....0.75
Glass (Flint).....3.5	Wood (beech).....0.7-0.8
Gold.....19.3	Wood (ebony).....1.1-1.2
Iron (Cast).....7.21	Wood (elm).....0.66
Iron (Wrought).....7.78	Wood (lignum-vitae).....1.3
Lead.....11.4	Wood (oak).....0.7-1.0

Magnesium.....1.74  
Manganese.....8.0  
Mercury.....13.6

Wood (pine).....0.56  
Wood (teak).....0.8  
Zinc.....7.0

## Greek Alphabet

Alpha	$\alpha$	Iota	$\iota$	Rho	$\rho$
Beta	$\beta$	Kappa	$\kappa$	Sigma	$\Sigma, \sigma$
Gamm	$\gamma$	Lambd	$\lambda$	Tau	$\tau$
a Delta	$\Delta$	a Mu	$\mu$	Upsilon	$\upsilon$
Epsilon	$\epsilon$	Nu	$\nu$	Phi	$\Phi, \phi$
Zeta	$\zeta$	Xi	$\xi$	Kai	$\chi$
Eta	$\eta$	Omicron	$\omicron$	Psi	$\psi$
Theta	$\theta$	Pi	$\pi$	Omega	$\Omega, \omega$

## THERMODYNAMICS

Symbol/Formula	Parameter
M	Molar mass ( $M/\mu$ )
m	Mass (M)
$n = \frac{m}{M}$	Number of moles ( $\mu$ )
E	Energy or general extensive property
$e = \frac{E}{m}$	Specific molar energy (energy per unit mass) or general extensive property per unit mass
$\bar{e} = \frac{E}{n} = eM$	Specific energy (energy per unit mole) or general extensive property per unit mole
P	Pressure ( $ML^{-1}T^{-2}$ )
V	Volume ( $L^3$ ); Specific volume or volume per unit mass, $v$ ( $L^3M^{-1}$ ) and the volume per unit mole $v$ ( $L^3\mu^{-1}$ )
T	Temperature ( $\Theta$ )
$\rho$	Density ( $ML^{-3}$ ); $\rho = 1/v$ .
X	Quality
U	Thermodynamic internal energy ( $ML^2T^{-2}$ ); Internal energy per unit mass, $u$ ( $L^2T^{-2}$ ), and the internal energy per unit mole, $u$ ( $ML^2T^{-2}\mu^{-1}$ )
$H=U+PV$	Thermodynamic enthalpy ( $ML^2T^{-2}$ ); Enthalpy per unit mass, $h = u + Pv$ (dimensions: $L^2T^{-2}$ ) and the internal energy per unit mole $h$ ( $ML^2T^{-2}\mu^{-1}$ )
S	Entropy ( $ML^2T^{-2}\Theta^{-1}$ ); Entropy per unit mass, $s$ ( $L^2T^{-2}\Theta^{-1}$ ) and the internal energy per unit mole $s$ ( $ML^2T^{-2}\Theta^{-1}\mu^{-1}$ )
W	Work ( $ML^2T^{-2}$ )
Q	Heat transfer ( $ML^2T^{-2}$ )
$W_u$	The useful work rate or mechanical power ( $ML^2T^{-3}$ )
M	The mass flow rate ( $MT^{-1}$ )
$\frac{\vec{V}^2}{2}$	The kinetic energy per unit mass ( $L^2T^{-2}$ )
Gz	The potential energy per unit mass ( $L^2T^{-2}$ )
$E_{tot}$	The total energy = $m \left( u + \frac{\vec{V}^2}{2} + gz \right)$ ( $ML^2T^{-2}$ )
$\dot{Q}$	The heat transfer rate ( $ML^2T^{-3}$ )
$\frac{dE_{cv}}{dt}$	The rate of change of energy for the control volume. ( $ml^2t^{-3}$ )

## UNIT CONVERSION FACTOR

## For metric units

### Basic:

- $1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$  ;
- $1 \text{ J} = 1 \text{ N} \cdot \text{m}$ ;
- $1 \text{ W} = 1 \text{ J/s}$ ;
- $1 \text{ Pa} = 1 \text{ N/m}^2$  .

### Others:

- $1 \text{ kPa} \cdot \text{m}^3 = 1 \text{ kJ}$ ;
- $T(\text{K}) = T(^{\circ}\text{C}) + 273.15$ ;
- $1 \text{ L (liter)} = 0.001 \text{ m}^3$  ;
- $1 \text{ m}^2/\text{s}^2 = 1 \text{ J/kg}$ .

## Prefixes (and abbreviations):

- nano(n) –  $10^{-9}$ ;
- micro ( $\mu$ ) –  $10^{-6}$ ;
- milli(m) –  $10^{-3}$ ;
- kilo(k) –  $10^3$ ;
- mega(M) –  $10^6$ ;
- giga(G) –  $10^9$ .
- A metric ton (European word: tonne) is 1000 kg

## For engineering units

### Energy:

- $1 \text{ Btu} = 5.40395 \text{ psia} \cdot \text{ft}^3 = 778.169 \text{ ft} \cdot \text{lb}_f = (1 \text{ kWh})/3412.14 = (1 \text{ hp} \cdot \text{h})/2544.5 = 25,037 \text{ lb}_m \cdot \text{ft}^2/\text{s}^2$  .

### Pressure:

- $1 \text{ psia} = 1 \text{ lb}_f/\text{in}^2 = 144 \text{ psfa} = 144 \text{ lb}_f/\text{ft}^2$  .

### Others:

- $T(\text{R}) = T(^{\circ}\text{F}) + 459.67$ ;
- $1 \text{ lb}_f = 32.174 \text{ lb}_m \cdot \text{ft/s}^2$  ;
- $1 \text{ ton of refrigeration} = 200 \text{ Btu/min}$ .

## Concepts & Definitions

	Formula	Units
Pressure	$P = \frac{F}{A}$	Pa
Units	$1 \text{ Pa} = 1 \text{ N} / \text{m}^2$ $1 \text{ bar} = 10^5 \text{ Pa} = 0.1 \text{ Mpa}$ $1 \text{ atm} = 101325 \text{ Pa}$	
Specific Volume	$v = V/m$	$\text{m}^3 / \text{kg}$
Density	$\rho = \frac{m}{V} \rightarrow \rho = \frac{1}{v}$	$\text{kg} / \text{m}^3$
Static Pressure Variation	$\Delta P = \rho gh \quad \uparrow = -, \downarrow = +$	Pa
Absolute Temperature	$T (\text{K}) = T (^\circ\text{C}) + 273.15$	

## Properties of a Pure Substance

	Formula	Units
Quality	$x = \frac{m_{\text{vapor}}}{m_{\text{tot}}} \text{ (Vapour mass fraction)}$ $1 - x = \frac{m_{\text{liquid}}}{m_{\text{tot}}} \text{ (Liquid mass fraction)}$	
Specific Volume	$v = v_f + xv_{fg}$	$\text{m}^3 / \text{kg}$
Average Specific Volume	$v = (1 - x)v_f + xv_g \text{ (only two phase mixture)}$	$\text{m}^3 / \text{kg}$
Ideal -gas law	$P \ll P_c, T \ll T_c \quad Z = 1$	
• Equations	$Pv = RT \quad PV = mRT = n\bar{R}T$	
• Universal Gas Constant	$\bar{R} = 8.3145$	$\text{kJ} / \text{kmol K}$
• Gas Constant	$R = \frac{\bar{R}}{M} \quad M = \text{molecular mass}$	$\text{kJ} / \text{kg K}$
Compressibility Factor Z	$Pv = ZRT$	
Reduced Properties	$P_r = P/P_c, \quad T_r = T/T_c$	

## Work & Heat



	Formula	Units
Displacement Work	$W = \int_1^2 F dx = \int_1^2 P dV$	J
Integration	$W = \int_1^2 P dV = P(V_2 - V_1)$	J
Specific Work	$w = \frac{W}{m}$ (work per unit mass)	J/kg
Power (rate of work)	$W = F\bar{V} = P\dot{V} = T\omega$	W
• Velocity	$\bar{V} = r\omega$	Rad/s
• Torque	$T = Fr$	Nm
Polytropic Process ( $n \neq 1$ )	$PV^n = \text{const} = P_1 V_1^n = P_2 V_2^n$	
• Polytropic Exponent	$n = \frac{\ln\left(\frac{P_2}{P_1}\right)}{\ln\left(\frac{V_1}{V_2}\right)}$	
• $n=1$	$PV = \text{Const} = P_1 V_1 = P_2 V_2$	
Polytropic Process Work	$W_2 = \frac{1}{1-n} (P_2 V_2 - P_1 V_1) \quad n \neq 1$	J
• $n=1$	$W_2 = P_2 V_2 \ln\left(\frac{V_2}{V_1}\right)$	J
Adiabatic Process	$Q = 0$	
Conduction Heat Transfer	$\dot{Q} = -KA \frac{dT}{dx}, K = \text{conductivity}$	W
Convection Heat Transfer	$\dot{Q} = hA\Delta T, h$ = convection coefficient	W
Radiation Heat Transfer	$\dot{Q} = \epsilon\sigma A(T_S^4 - T_{\text{amb}}^4)$	W

## Terminology

$Q_1$  = heat

$Q_2$  = heat transferred during the process between state 1 and state 2

$Q$  = rate of heat transfer

$W$  = work

	Formula	Units
Total Energy	$E = U + KE + PE \rightarrow dE = dU + d(KE) + d(PE)$	J
Energy	$dE = \delta Q - \delta W \rightarrow E_2 - E_1 = {}_1Q_2 - {}_1W_2$	J
Kinetic Energy	$KE = 0.5m\bar{V}^2$	J
Potential Energy	$PE = mgZ \rightarrow PE_2 - PE_1 = mg(Z_2 - Z_1)$	J
Internal Energy	$U = U_{liq} + U_{vap} \rightarrow mu = m_{liq}u_f + m_{vap}u_g$	
Specific Internal Energy of Saturated Steam (two-phase mass average)	$u = (1-x)u_f + xu_g$ $u = u_f + xu_{fg}$	kJ / kg
Total Energy	$U_2 - U_1 + \frac{m(\bar{V}_2^2 - \bar{V}_1^2)}{2} + mg(Z_2 - Z_1) = Q_2 - W_1$	J
Specific Energy	$e = u + 0.5\bar{V}^2 + gZ$	
Enthalpy	$H = U + PV$	
Specific Enthalpy	$h = u + Pv$	kJ / kg
For Ideal Gasses	$Pv = RT$ and $u = f(T)$	
• Enthalpy	$h = u + Pv = u + RT$	
• R Constant	$u = f(T) \rightarrow h = f(T)$	
Specific Enthalpy for Saturation State (two-phase mass average)	$h = (1-x)h_f + xh_g$ $h = h_f + xh_{fg}$	kJ / kg
Specific Heat at Constant Pressure	$C_v = \frac{1}{m} \left( \frac{\delta Q}{\delta T} \right)_v = \frac{1}{m} \left( \frac{\delta U}{\delta T} \right)_v = \left( \frac{\delta u}{\delta T} \right)_v$ $(u_e - u_i) = C_v(T_e - T_i)$	
Specific Heat at Constant Pressure	$C_p = \frac{1}{m} \left( \frac{\delta Q}{\delta T} \right)_p = \frac{1}{m} \left( \frac{\delta H}{\delta T} \right)_p = \left( \frac{\delta h}{\delta T} \right)_p$ $(h_e - h_i) = C_p(T_e - T_i)$	
Solids & Liquids	Incompressible, so $v = \text{constant}$ $C = C_c = C_p$ (Tables A.3 & A.4) $u_2 - u_1 = C(T_2 - T_1)$ $h_2 - h_1 = u_2 - u_1 + v(P_2 - P_1)$	
Ideal Gas	$h = u + Pv = u + RT$ $u_2 - u_1 \cong C_v(T_2 - T_1)$ $h_2 - h_1 \cong C_p(T_2 - T_1)$	
Energy Rate	$E = Q - W(\text{rate} = +\text{in} - \text{out})$ $E_2 - E_1 = {}_1Q_2 - {}_1W_2(\text{change} = +\text{in} - \text{out})$	

W2 = work done during the change from state 1 to state 2

W = rate of work = Power. 1 W=1 J/s

## The First Law of Thermodynamics

### First-Law Analysis for Control Volume

	Formula	Units
<b>Volume Flow Rate</b>	$\dot{V} = \int \bar{V} dA = A\bar{V}$ (using average velocity)	
<b>Mass Flow Rate</b>	$\dot{m} = \int \rho \bar{V} dA = \rho A\bar{V} = A \frac{\bar{V}}{v}$ (using average values)	kg / s
<b>Power</b>	$\dot{W} = \dot{m} C_p \Delta T$ $\dot{W} = \dot{m} C_v \Delta T$ $\dot{m} = \frac{\dot{V}}{v}$	W
<b>Flow Work Rate</b>	$\dot{W}_{flow} = P\dot{V} = \dot{m}Pv$	
<b>Flow Direction</b>	From higher P to lower P unless significant KE or PE	
• <b>Total Enthalpy</b>	$h_{tot} = h + \frac{1}{2}\bar{V}^2 + gZ$	
<b>Instantaneous Process</b>		
• <b>Continuity Equation</b>	$\dot{m}_{C.V.} = \sum \dot{m}_i - \sum \dot{m}_e$	
• <b>Energy Equation</b>	$\dot{E}_{C.V.} = \dot{Q}_{C.V.} - \dot{W}_{C.V.} + \sum \dot{m}_i h_{toti} - \sum \dot{m}_e h_{toe} \rightarrow \text{First Law}$ $\rightarrow \dot{Q} + \sum \dot{m}_i (h_i + \frac{1}{2}\bar{V}_i^2 + gZ_i) = \frac{dE}{dt} + \sum \dot{m}_e (h_e + \frac{1}{2}\bar{V}_e^2 + gZ_e) - \dot{W}$	
<b>Steady State Process</b>		
• <b>No Storage</b>	$\dot{m}_{C.V.} = 0, \quad \dot{E}_{C.V.} = 0$	
• <b>Continuity Equation</b>	$\sum \dot{m}_i = \sum \dot{m}_e$ (in = out)	
• <b>Energy Equation</b>	$\dot{Q}_{C.V.} + \sum \dot{m}_i h_{toti} = \dot{W}_{C.V.} + \sum \dot{m}_e h_{toe}$ (in = out) $\rightarrow \text{First Law}$ $\rightarrow \dot{Q} + \sum \dot{m}_i (h_i + \frac{1}{2}\bar{V}_i^2 + gZ_i) = \dot{W} + \sum \dot{m}_e (h_e + \frac{1}{2}\bar{V}_e^2 + gZ_e)$	
• <b>Specific Heat Transfer</b>	$q = \frac{\dot{Q}_{C.V.}}{\dot{m}}$	kJ / kg
• <b>Specific Work</b>	$w = \frac{\dot{W}_{C.V.}}{\dot{m}}$	kJ / kg
• <b>SS Single Flow Eq.</b>	$q + h_{toti} = w + h_{toe}$ (in = out)	
<b>Transient Process</b>	Change in mass (storage) such as filling or emptying of a container.	

• <b>Continuity Equation</b>	$m_2 - m_1 = \sum m_i - \sum m_e$
• <b>Energy Equation</b>	$E_2 - E_1 = Q_{C.V.} - W_{C.V.} + \sum m_i h_{tot,i} - \sum m_e h_{tot,e}$ $E_2 - E_1 = m_2 \left( u_2 + \frac{1}{2} \bar{V}_2^2 + gZ_2 \right) - m_1 \left( u_1 + \frac{1}{2} \bar{V}_1^2 + gZ_1 \right)$
→	$Q_{C.V.} + \sum m_i h_{tot,i} = \sum m_e h_{tot,e} + \left[ m_2 \left( u_2 + \frac{1}{2} \bar{V}_2^2 + gZ_2 \right) - m_1 \left( u_1 + \frac{1}{2} \bar{V}_1^2 + gZ_1 \right) \right]_{C.V.} - W_{C.V.}$

## The Second Law of Thermodynamics

	Formula	Units
All $W$ , $Q$ can also be rates $\dot{W}$ , $\dot{Q}$		
<b>Heat Engine</b>	$W_{HE} = Q_H - Q_L$	
• <b>Thermal efficiency</b>	$\eta_{HE} = \frac{W_{HE}}{Q_H} = 1 - \frac{Q_L}{Q_H}$	
• <b>Carnot Cycle</b>	$\eta_{Thermal} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$	
• <b>Real Heat Engine</b>	$\eta_{HE} = \frac{W_{HE}}{Q_H} \leq \eta_{Carnot, HE} = 1 - \frac{T_L}{T_H}$	
<b>Heat Pump</b>	$W_{HP} = Q_H - Q_L$	
• <b>Coefficient of Performance</b>	$\beta'_{HP} = \frac{Q_H}{W_{HP}} = \frac{Q_H}{Q_H - Q_L}$	
• <b>Carnot Cycle</b>	$\beta'_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{T_H}{T_H - T_L}$	
• <b>Real Heat Pump</b>	$\beta_{HP} = \frac{Q_H}{W_{HP}} \leq \beta_{Carnot, HP} = \frac{T_H}{T_H - T_L}$	
<b>Refrigerator</b>	$W_{REF} = Q_H - Q_L$	
• <b>Coefficient of Performance</b>	$\beta_{REF} = \frac{Q_L}{W_{REF}} = \frac{Q_L}{Q_H - Q_L}$	
• <b>Carnot Cycle</b>	$\beta = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L}$	
• <b>Real Refrigerator</b>	$\beta_{REF} = \frac{Q_L}{W_{REF}} \leq \beta_{Carnot, REF} = \frac{T_L}{T_H - T_L}$	
<b>Absolute Temperature</b>	$\frac{T_L}{T_H} = \frac{Q_L}{Q_H}$	

## Entropy

	Formula	Units
<b>Inequality of Clausius</b>	$\oint \frac{\delta Q}{T} \leq 0$	
<b>Entropy</b>	$dS = \left( \frac{\delta Q}{T} \right)_{rev}$	$kJ / kgK$
<b>Change of Entropy</b>	$S_2 - S_1 = \int_1^2 \left( \frac{\delta Q}{T} \right)_{rev}$	$kJ / kgK$
<b>Specific Entropy</b>	$s = (1-x)s_f + xs_g$ $s = s_f + xs_{fg}$	$kJ / kgK$
<b>Entropy Change</b>		
<ul style="list-style-type: none"> <li><b>Carnot Cycle</b></li> </ul>	Isothermal Heat Transfer: $S_2 - S_1 = \frac{1}{T_H} \int_1^2 \delta Q = \frac{Q_2}{T_H}$  Reversible Adiabatic (Isentropic Process): $dS = \left( \frac{\delta Q}{T} \right)_{rev}$  Reversible Isothermal Process: $S_4 - S_3 = \int_3^4 \left( \frac{\delta Q}{T} \right)_{rev} = \frac{Q_4}{T_L}$  Reversible Adiabatic (Isentropic Process): Entropy decrease in process 3-4 = the entropy increase in process 1-2.	
<ul style="list-style-type: none"> <li><b>Reversible Heat-Transfer Process</b></li> </ul>	$s_2 - s_1 = s_{fg} = \frac{1}{m} \int_1^2 \left( \frac{\delta Q}{T} \right)_{rev} = \frac{1}{mT} \int_1^2 \delta Q = \frac{q_2}{T} = \frac{h_{fg}}{T}$	
<b>Gibbs Equations</b>	$Tds = du + Pdv$ $Tds = dh - vdP$	
<b>Entropy Generation</b>	$dS = \frac{\delta Q}{T} + \delta S_{gen}$ $\delta W_{irr} = PdV - T\delta S_{gen}$ $S_2 - S_1 = \int_1^2 dS = \int_1^2 \frac{\delta Q}{T} + S_{2gen}$	
<b>Entropy Balance Equation</b>	$\Delta Entropy = +in - out + gen$	
<b>Principle of the Increase of Entropy</b>	$dS_{net} = dS_{c.m.} + dS_{surr} = \sum \delta S_{gen} \geq 0$	
<b>Entropy Change</b>		
<ul style="list-style-type: none"> <li><b>Solids &amp; Liquids</b></li> </ul>	$s_2 - s_1 = c \ln \frac{T_2}{T_1}$  Reversible Process: $dS_{gen} = 0$ Adiabatic Process: $dq = 0$	

<ul style="list-style-type: none"> <li><b>Ideal Gas</b></li> </ul>	<p>Constant Volume: <math>s_2 - s_1 = \int_1^2 C_{v0} \frac{dT}{T} + R \ln \frac{v_2}{v_1}</math></p> <p>Constant Pressure: <math>s_2 - s_1 = \int_1^2 C_{p0} \frac{dT}{T} - R \ln \frac{P_2}{P_1}</math></p> <p>Constant Specific Heat: <math>s_2 - s_1 = C_{v0} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}</math>  <math>s_2 - s_1 = C_{p0} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}</math></p>
<b>Standard Entropy</b>	$s_T^0 = \int_{T_0}^T \frac{C_{p0}}{T} dT$ <span style="float: right;"><math>kJ / kgK</math></span>
<b>Change in Standard Entropy</b>	$s_2 - s_1 = (s_{T2}^0 - s_{T1}^0) - R \ln \frac{P_2}{P_1}$ <span style="float: right;"><math>kJ / kgK</math></span>
<b>Ideal Gas Undergoing an Isentropic Process</b>	$s_2 - s_1 = 0 = C_{p0} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$ $\rightarrow \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{R/C_{p0}}$ <p>but <math>\frac{R}{C_{p0}} = \frac{C_{p0} - C_{v0}}{C_{p0}} = \frac{k-1}{k}</math>,</p> <p><math>k = \frac{C_{p0}}{C_{v0}}</math> = ratio of specific heats</p> $\Rightarrow \frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{k-1}, \quad \frac{P_2}{P_1} = \left( \frac{v_1}{v_2} \right)^k$ <p>Special case of polytropic process where <math>k = n</math>: <math>Pv^k = const</math></p>
<b>Reversible Polytropic Process for Ideal Gas</b>	$PV^n = const = P_1 V_1^n = P_2 V_2^n$ $\rightarrow \frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^n, \quad \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = \left( \frac{V_1}{V_2} \right)^{n-1}$
<ul style="list-style-type: none"> <li><b>Work</b></li> </ul>	${}_1W_2 = \int_1^2 P dV = const \int_1^2 \frac{dV}{V^n} = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n}$
<ul style="list-style-type: none"> <li><b>Values for n</b></li> </ul>	<p>Isobaric process: <math>n = 0</math>, <math>P = const</math></p> <p>Isothermal Process: <math>n = 1</math>, <math>T = const</math></p> <p>Isentropic Process: <math>n = k</math>, <math>s = const</math></p> <p>Isochronic Process: <math>n = \infty</math>, <math>v = const</math></p>



## Second-Law Analysis for Control Volume

	Formula	Units
<b>2<sup>nd</sup> Law Expressed as a Change of Entropy</b>	$\frac{dS_{c.v.}}{dt} = \sum \frac{\dot{Q}}{T} + \dot{S}_{gen}$	
<b>Entropy Balance Equation</b>	<p>rate of change = +in - out + generation</p> $\rightarrow \frac{dS_{c.v.}}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{c.v.}}{T} + \dot{S}_{gen}$ <p>where <math>S_{c.v.} = \int \rho s dV = m_{c.v.} s = m_A s_A + m_B s_B + \dots</math></p> <p>and <math>\dot{S}_{gen} = \int \rho \dot{s}_{gen} dV = \dot{S}_{gen,A} + \dot{S}_{gen,B} + \dots</math></p>	
<b>Steady State Process</b>	$\frac{dS_{c.v.}}{dt} = 0$ $\rightarrow \sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum \frac{\dot{Q}_{c.v.}}{T} + \dot{S}_{gen}$	
• <b>Continuity equation</b>	$\dot{m}_i = \dot{m}_e = \dot{m} \quad \Rightarrow \quad \dot{m}(s_e - s_i) = \sum \frac{\dot{Q}_{c.v.}}{T} + \dot{S}_{gen}$	
• <b>Adiabatic process</b>	$s_e = s_i + s_{gen} \geq s_i$	
<b>Transient Process</b>	$\frac{d}{dt}(ms)_{c.v.} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{c.v.}}{T} + \dot{S}_{gen}$ $\rightarrow (m_2 s_2 - m_1 s_1)_{c.v.} = \sum m_i s_i - \sum m_e s_e + \int_0^t \frac{\dot{Q}_{c.v.}}{T} dt + {}_1\dot{S}_2_{gen}$	
<b>Reversible Steady State Process</b>		
<b>If Process Reversible &amp; Adiabatic</b>	$s_e = s_i$ $h_e - h_i = \int_i^e v dP$ $w = (h_i - h_e) + \frac{\bar{V}_i^2 - \bar{V}_e^2}{2} + g(Z_i - Z_e)$ $= -\int_i^e v dP + \frac{\bar{V}_i^2 - \bar{V}_e^2}{2} + g(Z_i - Z_e)$	
<b>If Process is Reversible and Isothermal</b>	$\dot{m}(s_e - s_i) = \frac{1}{T} \sum \dot{Q}_{c.v.} = \frac{\dot{Q}_{c.v.}}{T}$ <p>or <math>T(s_e - s_i) = \frac{\dot{Q}_{c.v.}}{\dot{m}} = q</math></p> $\rightarrow T(s_e - s_i) = (h_e - h_i) - \int_i^e v dP$	

<b>Incompressible Fluid</b>	$v(P_e - P_i) + \frac{\bar{V}_e^2 - \bar{V}_i^2}{2} + g(Z_e - Z_i) = 0 \rightarrow \text{Bernoulli Eq.}$
<b>Reversible Polytrophic Process for Ideal Gas</b>	$w = -\int_i^e v dP \quad \text{and} \quad Pv^n = \text{const} = C^n$ $w = -\int_i^e v dP = -C \int_i^e \frac{dP}{P^{1/n}}$ $= -\frac{n}{n-1} (P_e v_e - P_i v_i) = -\frac{nR}{n-1} (T_e - T_i)$
<b>Isothermal Process (n=1)</b>	$w = -\int_i^e v dP = -C \int_i^e \frac{dP}{P} = -P_i v_i \ln \frac{P_e}{P_i}$
<b>Principle of the Increase of Entropy</b>	$\frac{dS_{net}}{dt} = \frac{dS_{C.V.}}{dt} + \frac{dS_{surr}}{dt} = \sum \dot{S}_{gen} \geq 0$
<b>Efficiency</b>	
<b>Turbine</b>	$\eta = \frac{w_a}{w_s} = \frac{h_i - h_e}{h_i - h_{es}} \quad \text{Turbine work is out}$
<b>Compressor (Pump)</b>	$\eta = \frac{w_s}{w_a} = \frac{h_i - h_{es}}{h_i - h_e} \quad \text{Compressor work is in}$
<b>Cooled Compressor</b>	$\eta = \frac{w_T}{w}$
<b>Nozzle</b>	$\eta = \frac{\frac{1}{2} \bar{V}_e^2}{\frac{1}{2} \bar{V}_{es}^2} \quad \text{Kinetic energy is out}$

**Note:**

$$^{\circ}\text{F} = (^{\circ}\text{C} \times 9/5) + 32$$

$$^{\circ}\text{C} = (^{\circ}\text{F} - 32) \times (5/9)$$

$$^{\circ}\text{K} = ^{\circ}\text{C} + 273$$

$$Q = mC\Delta T \text{ thermal energy} = \text{mass} \times \text{specific heat} \times \text{change in } T$$

$$Q = mH_f \text{ thermal energy} = \text{mass} \times \text{heat of fusion}$$

$$Q = mH_v \text{ thermal energy} = \text{mass} \times \text{heat of vaporization}$$

$$\Delta L = \alpha L_i \Delta T \text{ change in length} = \text{coefficient of expansion} \times \text{initial length} \times \text{change in } T$$

$$\Delta V = \beta V_i \Delta T \text{ change in volume} = \text{coefficient of expansion} \times \text{initial volume} \times \text{change in } T$$

$$\Delta U = Q - W \text{ internal energy} = \text{heat energy} - \text{work}$$

Plausible Physical Situations		Name	State Variables	P	V	T	$\Delta E_{th}$	Q	$W_s$	W
<b>Insulated sleeve or rapid process</b>	Add weight to or push down on piston	Adiabatic compression	$PV^\gamma = \text{Const};$ $TV^{\gamma-1} = \text{Const}$	Up	Down	Up	$nC_v\Delta T > 0$	0	$-nC_v\Delta T < 0$	$nC_v\Delta T > 0$
<b>Insulated sleeve or rapid process</b>	Remove weight from or pull up on piston	Adiabatic expansion	$PV^\gamma = \text{Const};$ $TV^{\gamma-1} = \text{Const}$	Down	Up	Down	$nC_v\Delta T < 0$	0	$-nC_v\Delta T > 0$	$nC_v\Delta T < 0$
<b>Heat gas</b>	Locked piston or rigid container	Isochoric	V fixed; $P \propto T$	Up	Fixed	Up	$nC_v\Delta T > 0$	$nC_v\Delta T > 0$	0	0
<b>Cool gas</b>	Locked piston or rigid container	Isochoric	V fixed; $P \propto T$	Down	Fixed	Down	$nC_v\Delta T < 0$	$nC_v\Delta T < 0$	0	0
<b>Heat gas</b>	Piston free to move, load unchanged	Isobaric expansion	P fixed; $V \propto T$	Fixed	Up	Up	$nC_v\Delta T > 0$	$nC_p\Delta T > 0$	$P\Delta V > 0$	$-P\Delta V < 0$
<b>Cool gas</b>	Piston free to move, load unchanged	Isobaric compression	P fixed; $V \propto T$	Fixed	Down	Down	$nC_v\Delta T < 0$	$nC_p\Delta T < 0$	$P\Delta V < 0$	$-P\Delta V > 0$
<b>Immerse gas in large bath</b>	Add weight to piston	Isothermal compression	T fixed at temperature of bath, $PV = \text{Const}$	Up	Down	Fixed	$nC_v\Delta T = 0$	$nRT \ln(V_i/V_f) < 0$	$nRT \ln(V_i/V_f) < 0$	$-nRT \ln(V_i/V_f) > 0$
<b>Immerse gas in large bath</b>	Remove weight from piston	Isothermal expansion	T fixed at temperature of bath, $PV = \text{Const}$	Down	Up	Fixed	$nC_v\Delta T = 0$	$nRT \ln(V_i/V_f) > 0$	$nRT \ln(V_i/V_f) > 0$	$-nRT \ln(V_i/V_f) < 0$
<b>Unknown</b>	Unknown	No Name	$PV/T = \text{Const}$	?	?	?	$nC_v\Delta T$	$\Delta E_{th} + W_s$	$\int PdV = \pm$ area under curve in PV diagram	$-\int PdV = \pm$ area under curve in PV diagram

## THERMAL ENGINEERING

### I.C Engine

- **Mean Effective Pressure MEP**

$$W_{\text{net}} = P_{\text{MEP}} \Delta V$$

$$= \text{MEP} \times \text{Piston area} \times \text{Stroke}$$

$$= \text{MEP} \times \text{Displacement}$$

$$\text{MEP} = W_{\text{net}} / \text{Displacement} = W_{\text{net}} / (V_{\text{max}} - V_{\text{min}})$$

- **Otto Cycle Efficiency**

$$\eta_{\text{th, otto}} = 1 - (1/r^{k-1})$$

where  $r$  = compression ratio  $k = c_p/c_v$  for air at room T

- **Diesel Efficiency**

$$\eta_{\text{th, Diesel}} = 1 - (1/r^k - 1) [(r_c)^k - 1/k(r_c - 1)]$$

$r$  = compression ratio,  $r_c$  = cutoff ratio ( $r_c = V_3/V_2 = v_3/v_2$ )  $k = c_p/c_v$  for air at room T.

- **Indicated Thermal Efficiency**

$$\eta_{\text{ith}} = \frac{\text{indicated power}}{\text{Energy in fuel per second}} = \frac{I_p}{m_f \times c_v}$$

Here,  $I_p$  = Indicated power,  $C_v$  = Calorific value of fuel, and  $M_f$  = Mass of fuel/second

- **Brake Thermal Efficiency**

$$\eta_{\text{Brake}} = \frac{\text{Break Power}}{\text{Energy in fuel per second}} = \frac{b_p}{m_f \times c_v}$$

### Psychrometry

- **Partial Pressure of Water Vapour:**

$$P_v = (P_{vs})_{wb} \frac{-[p - (p_v)_{wb}](t_{db} - t_{wb})}{1527.4 - 1.3t_{wb}}$$

or

$$P_v = (P_{vs})_{wb} \frac{1.8 \times (t_{db} - t_{wb})}{2700}$$

$p_v$  = partial pressure of water vapour

$p_s$  = partial pressure of dry air

where,  $(p_{vs})_{wb}$  = saturation pressure of water vapour corresponding to wet bulb temperature

$p$  = atmospheric pressure of moist air

$T_{wb}$  = wet bulb temperature

$T_{db}$  = dry bulb temperature

- **Absolute Humidity:-**

$$= \frac{\text{Weight of Water Vapour}}{\text{Volume of air (Mixture)}}$$

- **Specific Humidity(w):-**

$$= \frac{\text{Mass of water vapour in air}}{\text{Mass of Dry air}}$$

$$\omega = 0.622 \frac{P_v}{P - P_v} \text{ Kg water vapour / kg of dry air}$$

- **Relative Humidity**

$$\phi = \frac{m_v}{m_s} = \frac{P_v}{P_{vs}}$$

- **Degree of Saturation (p)**

$$\mu = \frac{\text{Specific humidity of air}}{\text{Specific humidity of Saturated air}} = \frac{\omega}{\omega_s}$$

- **Dew Point Depression**

$$\text{DPD} = \text{DBT} - \text{DPT}$$

- **Enthalpy of Air (>)**

$$h = C_{pm} t_{ab} + W(h_g - C_{pV} t_{db})$$

- **Sensible Heat Factor**

$$= \frac{S_H}{S_H + S_L}$$

- **By Pass Factor**

$$m = \frac{t_2 - t_{\text{coil}}}{t_1 - t_{\text{coil}}}$$

Air temperature at entry and exit is  $t_1$  and  $t_2$  respectively

- **Volumetric Efficiency**

$$\eta_v = 1 + c - c(r_p)^{1/r}$$

$C$  = Compression ratio,  $r_p = p_2/p_1$

- **Vapour Absorption System C.O.P**

$$C.O.P = \left( \frac{T_e}{T_c - T_e} \right) \times \left( \frac{T_g - T_c}{T_g} \right)$$

$T_e$  = Evaporator Temperature

$T_g$  = Generator Temperature

$T_c$  = Condenser Temperature

- **Designation of Refrigerant**

For a hydrocarbon Chemical formula is  $C_r H_s F_t Cl_y$

If  $S + t + y = 2r + 2$ , designation of refrigerant is  $R(r-1)(S+1)t$

If  $S + t + y = 2r$ , designation of refrigerant is  $Rl(r-1)(S+1)t$

For an Inorganic Refrigerant

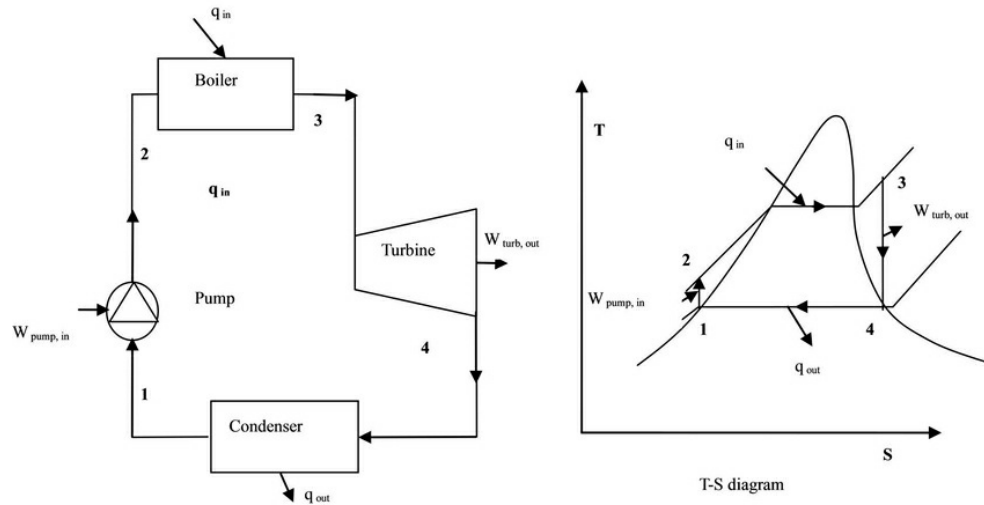
Designation is  $R(700 + \text{molecular weight})$

## POWER PLANT ENGINEERING

- **Ranking Cycle efficiency**

$$\eta = \frac{q_1 - q_2}{q_1} = \frac{(h_3 - h_2) - (h_4 - h_1)}{h_3 - h_2} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$$





- **Overall efficiency**

=Eff. Boiler x Eff. of Cycle x Eff. of turbine x Eff. of generator x Eff. of auxiliary

- **Efficiency of Brayton Cycle**

$$\eta_{th, \text{Brayton}} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}}$$

Where  $r_p$  is pressure ratio

## HEAT TRANSFER

### Fourier's Law of Heat Conduction

$$Q = -KA \frac{dT}{dx}$$

$Q$  = Heat transfer in given direction.

$A$  = Cross-sectional area perpendicular to heat flow direction.

$dT$  = Temperature difference between two ends of a block of thickness  $dx$

$dx$  = Thickness of solid body

$\frac{dT}{dx}$  = Temperature gradient in direction of heat flow.

### General Heat Conduction Equation

#### • Cartesian Coordinates (side parallel to $x$ , $y$ and $z$ -directions)

$$\frac{\partial}{\partial x} \left( k_x \frac{\partial t}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial t}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial t}{\partial z} \right) + q_g = \rho c \frac{\partial t}{\partial \tau}$$

$q_g$  = Internal heat generation per unit volume per unit time

$t$  = Temperature at left face of differential control volume

$k_x, k_y, k_z$  = Thermal conductivities of the material in  $x, y$  and  $z$  -directions respectively

$c$  = Specific heat of the material

$\rho$  = Density of the material

$\alpha$  = Thermal diffusivity

$d\tau$  = Instantaneous time

#### For homogeneous and isotropic material

$$k_x = k_y = k_z = k. \alpha = \frac{k}{\rho c}$$

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \frac{\partial t}{\partial \tau}$$

#### For steady state condition (Poisson's equation)

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = 0$$

#### For steady state and absence of internal heat generation (Laplace

equation)

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = 0$$

**For unsteady heat flow with no internal heat generation**

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = \frac{1}{\alpha} \frac{\partial t}{\partial \tau}$$

- **Cylindrical Coordinates**

**For homogeneous and isotropic material ,**

$$\left[ \frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} + \frac{1}{r^2} \frac{\partial^2 t}{\partial \phi^2} \frac{\partial^2 t}{\partial \tau^2} \right] + \frac{q_g}{k} = \frac{1}{\alpha} \frac{\partial t}{\partial \tau}$$

**For steady state unidirectional heat flow in radial direction with no internal heat generation ,**

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} = 0$$

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dt}{dr} \right) = 0$$

- **Spherical Coordinates**

**For homogeneous and isotropic materia l**

$$\frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial t}{\partial r} \right) + \frac{q_g}{k} = \frac{1}{\alpha} \frac{\partial t}{\partial \tau}$$

**For steady state uni-direction heat flow in radial direction with no internal heat generation ,**

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dt}{dr} \right) = 0$$

**Thermal resistance of hollow cylinders**

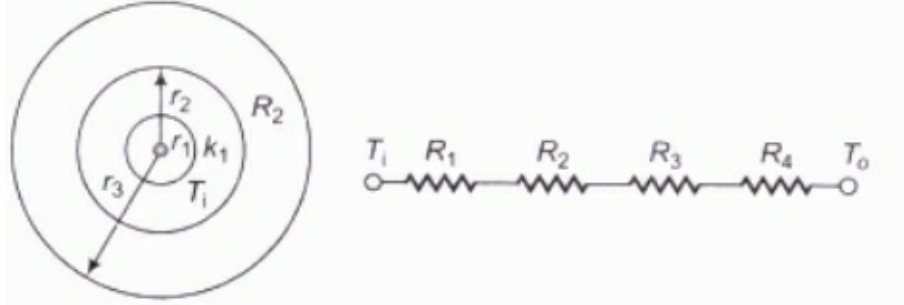
$$R = \frac{\log_e \frac{r_2}{r_1}}{2\pi KL}$$

$$Q = \frac{T_i - T_o}{R} = \frac{\Delta T}{R}$$

## Thermal Resistance of a Hollow Sphere

$$R = \frac{r_2 - r_1}{4\pi k r_1 r_2}$$

## Heat Transfer through a Composite Cylinder



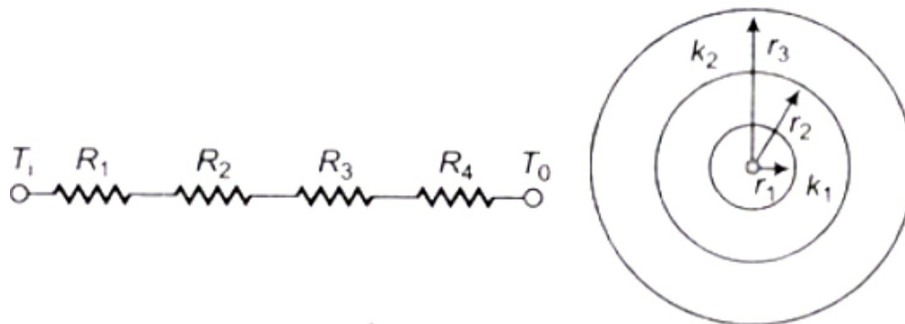
$$Q = \frac{T_i - T_o}{R_{eq}}$$

$$R_{eq} = R_1 + R_2 + R_3 + R_4$$

$$R_1 = \frac{1}{h_i 2\pi r_1 l}, R_2 = \frac{\ln r_2 / r_1}{2\pi k_1 l}$$

$$R_3 = \frac{\ln r_3 / r_2}{2\pi k_2 l}, R_4 = \frac{1}{h_o 2\pi r_2 l}$$

## Heat Transfer through a Composite Sphere



$$R_1 = \frac{1}{h_j A_i} = \frac{1}{h_j 4\pi r_1^2}$$

$$R_2 = \frac{r_2 - r_1}{4\pi k_1 r_1 r_2}, R_3 = \frac{r_3 - r_2}{4\pi k_2 r_2 r_3}$$

$$R_4 = \frac{1}{h_o 4\pi r_3^2}, Q = \frac{T_i - T_o}{R_1 + R_2 + R_3 + R_4}$$

## Critical Thickness of Insulation:

- In case of cylinder,

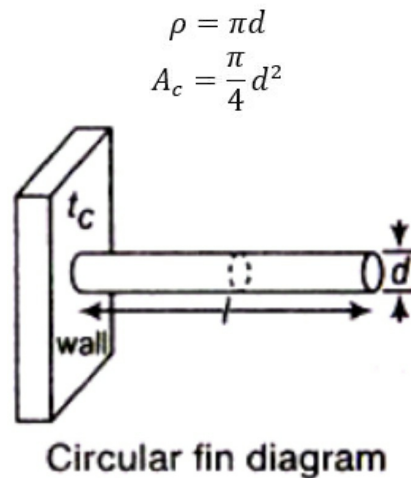
$$r_c = \frac{K_0}{h}$$

where,  $k_0$  = Thermal conductivity, and  $h$  = Heat transfer coefficient

- The drop in temperature across the wall and the air film will be proportional to their

resistances, =  $hL/k$ .

### Steady Flow of Heat along a Rod Circular fin



### Generalized Equation for Fin Rectangular fin

$$\frac{d^2\theta}{dx^2} + \frac{1}{A_c} \frac{dA_s}{dx} \frac{d\theta}{dx} - \frac{h}{kA_c} = 0$$

### Heat balance equation if $A_c$ constant and $A_s \propto P(x)$ linear

$$\frac{d^2t}{dx^2} - \frac{hp}{kA_c} (t - t_\alpha) = 0$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$m = \sqrt{\frac{hp}{kA_c}}$$

### General equation of 2<sub>nd</sub> order

$$\vartheta = c_1 e^{mx} + c_2 e^{-mx}$$

## Heat Dissipation from an Infinitely Long Fin ( $l \rightarrow \infty$ )

$$\frac{t - t_{\alpha_2}}{t_0 - t_{\alpha}} = e^{-mx}$$

## Heat transfer by conduction at base

$$Q_{fin} = \sqrt{kA_cPh}(t_0 - t_{\alpha})$$

## Heat Dissipation from a Fin Insulated at the End Tip

$$\frac{\theta}{\theta_0} = \frac{t - t_0}{t_0 - t_{\alpha}} = \left( \frac{\cosh m(l - x)}{\cosh ml} \right)$$

$$Q_{fin} = \sqrt{kA_cPh}(t_0 - t_{\alpha}) \tanh ml$$

## Heat Dissipation from a Fin losing Heat at the End Tip

$$-KA \left( \frac{dt}{dx} \right)_{x=l} = h A_s (t - t_{\alpha})$$

$$\theta = \theta_0 \left( \frac{\cosh m(l - x) + \frac{h}{mk} \sinh m(l - x)}{\cosh ml + \frac{h}{mk} \sinh ml} \right)$$

$$Q_{fin} = \sqrt{hPkA_c} \theta_0 \frac{\tanh ml + \frac{h}{mk}}{1 + \frac{h}{mk} \tanh ml}$$

## Fin Efficiency

- Fin efficiency is given by

$$\eta = \frac{\text{Actual heat rate from fin } Q}{\text{Maximum heat transfer rate } Q_{max}}$$

- If  $l \rightarrow \infty$  (infinite length of fin),

$$\eta = \frac{\sqrt{hPkA_c} \theta_0}{h(pl + b\delta) \theta_0} = \frac{1}{l} \sqrt{\frac{kA_c}{hP}}$$

$$\eta = \frac{\theta_0 \sqrt{hPkA_c} \tanh ml}{hPl \theta_0}$$

- If finite length of fin,

$$\eta = \frac{\theta_0 \sqrt{hPkA_c} \left[ \frac{\tanh ml + \frac{h}{mk}}{1 + \frac{h}{mk} \tanh ml} \right]}{h(Pl + b\delta) \theta_0}$$

### Fin Effectiveness

$$\varepsilon = \frac{\text{Actual heat transfer from the fin surface } Q}{\text{Rate of heat transfer without fin}}$$

$$\varepsilon = \frac{Q}{hA\theta_0} = \frac{\theta_0 \sqrt{hPkA_c} \tanh hl}{hA\theta_0} = \frac{\tanh hl}{\sqrt{\frac{hA_c}{Pk}}} \quad (\text{if } l \rightarrow \infty)$$

### Lumped Parameter System

$$Q = -\rho VC_p \frac{dT}{dt} = hA(T - T_a)$$

$$\int \frac{dT}{T - T_a} = \frac{hA}{\rho VC_p} \int dt$$

$$\ln(T - T_a) = -\frac{hA}{\rho VC_p} t + C_1$$

$$\ln(T - T_a) = -\frac{hA}{\rho VC_p} t + \ln(T_i - T_a)$$

$$\frac{T - T_a}{T_i - T_a} = \exp \left[ -\frac{hA}{\rho VC_p} t \right]$$

### Nusselt Number (Nu)

- It is a dimensionless quantity defined as  $hL/k$ ,

$h$  = convective heat transfer coefficient,

$L$  is the characteristic length

$k$  is the thermal conductivity of the fluid.

- The Nusselt number could be interpreted physically as the ratio of the temperature gradient

in the fluid immediately in contact with the surface to a reference temperature gradient

$(T_s - T_\infty) / L$ .

- Newton's Law of Cooling says that the rate of heat transfer per unit area by convection is given by

$$\dot{Q} / A = h(T_w - T_\infty)$$

$$\frac{\dot{Q}}{A} = h(T_w - T_\infty) = -k \frac{T_w - T_\infty}{\delta_t}$$

$$h = \frac{k}{\delta_t} \quad Nu = \frac{hL}{k} = \frac{L}{\delta_t}$$

$$Nu = \frac{\text{Rate of heat transfer by convection}}{\text{Rate of heat transfer by conduction}} = \frac{h.A.\Delta T}{KA \frac{\Delta T}{1}}$$

### Temperature distribution in a boundary layer: Nusselt modulus

• The heat transfer by convection involves conduction and mixing motion of fluid particles. At the solid fluid interface ( $y = 0$ ), the heat flows by conduction only, and is given by

$$\frac{\dot{Q}}{A} = -k \left( \frac{dT}{dy} \right)_{y=0}$$

$$h = \frac{\left( -K \frac{dT}{dy} \right)_{y=0}}{T_w - T_\infty}$$

$$\frac{hL}{k} = \frac{\left( \frac{dT}{dy} \right)_{y=0}}{(T_w - T_\infty)/L}, \text{ and}$$

In dimensionless form,

$$= \left( \frac{\frac{d(T_w - T)}{(T_w - T_\infty)}}{d\left(\frac{y}{L}\right)} \right)_{y=0}$$

**Reynold Number (Re):**

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}}$$

$$Re = \frac{\rho v l}{\mu}$$

$$Re = \frac{v l}{\nu}$$

**Critical Reynold Number:** It represents the number where the boundary layer changes from laminar to turbine flow.

**For flat plate,**



- $Re < 5 \times 10^5$  (laminar)
- $Re > 5 \times 10^5$  (turbulent)

**For circular pipes,**

- $Re < 2300$  (laminar flow)
- $2300 < Re < 4000$  (transition to turbulent flow)
- $Re > 4000$  (turbulent flow)

**Stanton Number (St)**

$$St = \frac{\text{Heat transfer coefficient}}{\text{Heat flow per unit temperature rise}}$$

$$St = \frac{Nu}{Re \times Pr}$$

**Grashof Number (Gr)**

If a body with a constant wall temperature  $T_w$  is exposed to a quiescent ambient fluid at  $T_\infty$ , the force per unit volume can be written as:

$$\rho g \beta (t_w - t_\infty)$$

where  $\rho$  = mass density of the fluid,  $\beta$  = volume coefficient of expansion and  $g$  is the acceleration due to gravity.

$$Gr = \frac{\text{Inertia force} \times \text{buoyancy force}}{\text{viscous force}}$$

$\beta$  = Coefficient of volumetric expansion =  $1/T$

$$\begin{aligned} Gr &= \frac{(\rho V^2 L^2) \times \rho g \beta (T_w - T_\infty) L^3}{(\mu V L)^2} \\ &= \frac{\rho^2 g \beta (T_w - T_\infty) L^3}{\mu^2} = g \beta L^3 (T_w - T_\infty) / \nu^2 \end{aligned}$$

The magnitude of Grashof number indicates whether the flow is **laminar or turbulent**.

- If the Grashof number is greater than  $10^9$ , the flow is turbulent and
- For Grashof number less than  $10^8$ , the flow is laminar.
- For  $10^8 < Gr < 10^9$ , It is the transition range.

**Prandtl Number (Pr):**

$$Pr = \frac{\text{Momentum diffusivity through the fluid}}{\text{Thermal diffusivity through the fluid}}$$

$$Pr = \frac{\mu C_p}{k} = \frac{v}{\alpha}$$

$$Pr = \frac{v}{\alpha} = \frac{\frac{\mu}{\rho}}{\frac{k}{\rho C_p}}$$

$$Pr = \frac{\mu C_p}{k}$$

- For liquid metal,  $Pr < 0.01$
- For air and gases,  $Pr \approx 1$
- For water,  $Pr \approx 10$
- For heavy oil and grease,  $Pr > 10^5$
- For  $Pr \ll 1$  (in case of liquid metals), the thickness of the thermal boundary layer will be much more than the thickness of the momentum boundary layer and vice versa.
- The product of Grashof and Prandtl number is called Rayleigh number. Or,

$$Ra = Gr \times Pr$$

### Rayleigh Number (Ra)

$$Ra = Gr \cdot Pr, Ra = \frac{g \beta l^3 \Delta t}{\nu \cdot \alpha}$$

Free or natural convection

- $10^4 < Ra < 10^9$  (laminar flow)
- $Ra > 10^9$  (turbulent flow)

### Turbulent flow over flat plate

$$Nu_x = 0.0292 (Re)^{0.8} (Pr)^{0.33}$$

$$\bar{Nu} = 0.036 (Re)^{0.8} (Pr)^{0.33}$$

$$\delta = \frac{0.37x}{(Re_x)^{\frac{1}{5}}}$$

### **Turbulent flow in tubes**

$$Nu = 0.023 (Re)^{0.8} (Pr)^n$$

where,  $n = 0.4$  if fluid is being heated,

$= 0.3$  if fluid is being cooled.

$$\bar{Nu} = 0.0036 (Re)^{0.8} (Pr)^n$$

### **Empirical Correlation for Free Convection**

Heated surface up or cooled surface down

#### **Laminar flow**

$$2 \times 10^5 < Gr.Pr < 2 \times 10^7$$

$$Nu = 0.54 (Gr Pr)^{0.25}$$

#### **Turbulent flow**

$$2 \times 10^7 < Gr.Pr < 3 \times 10^{10}$$

$$Nu = 0.14 (Gr Pr)^{0.33}$$

Heated surface down or cooled surface up

#### **Laminar flow**

$$3 \times 10^5 < Gr.Pr < 7 \times 10^8$$

$$Nu = 0.27 (Gr Pr)^{0.25}$$

#### **Turbulent flow**

$$7 \times 10^8 < Gr.Pr < 11 \times 10^{10}$$

$$Nu = 0.107 (Gr Pr)^{0.33}$$

Vertical plates and Large cylinder

#### **Laminar flow**

$$10^4 < GrPr < 10^9$$

$$Nu = 0.59 (GrPr)^{0.25}$$

#### **Turbulent flow**

$$10^9 < GrPr < 10^{12}$$

$$Nu = 0.13 (GrPr)$$

### **Empirical Correlation for Forced Convection**

## Laminar Flow over Flat Plate

$$Nu_x = \frac{hx}{k} = 0.332(Re_x)^{1/2}Pr^{1/3}$$

$$Nu_x = \frac{\bar{h}l}{k} = 0.64(Re_x)^{1/2}Pr^{1/3}$$

## Hydrodynamic boundary layer thickness

$$\rho = \frac{5x}{\sqrt{Re_x}}$$

## Laminar Flow over Inside Tube

$$Nu = \frac{hd}{k}, T_\infty = \frac{T_0 + T_i}{2}$$

Constant heat flux,  $Nu = 4.36$

## Fouling Factor ( $R_f$ )

$$R_f = \frac{1}{v_{dirty}} - \frac{1}{v_{clear}}$$

## Fin Efficiency and Fin Effectiveness

$\eta_{fin} = (\text{actual heat transferred}) / (\text{heat which would be transferred if the entire fin area were at the root temperature})$

**For a very long fin, effectiveness :**

$$E = \frac{\bar{Q}_{with fin}}{\bar{Q}_{without fin}}$$
$$\frac{(hpkA)^{1/2}\theta_0}{hA\theta_0} = \left(\frac{kp}{hA}\right)^{1/2}$$

And,

$$\eta_{fin} = (hpkA)^{1/2}\theta_0(hpL\theta_0) = \frac{(hpkA)^{1/2}}{hpL}$$

$$\frac{E}{\eta_{fin}} = \frac{\left(\frac{kp}{hA}\right)^{1/2}}{(hpkA)^{1/2}} \times hpL = \frac{pL}{A} = \frac{\text{Surface area of fin}}{\text{Cross-section area of the fin}}$$

i.e., effectiveness increases by increasing the length of the fin but it will decrease the fin efficiency.

• **Expressions for Fin Efficiency for Fins of Uniform Cross-section:**

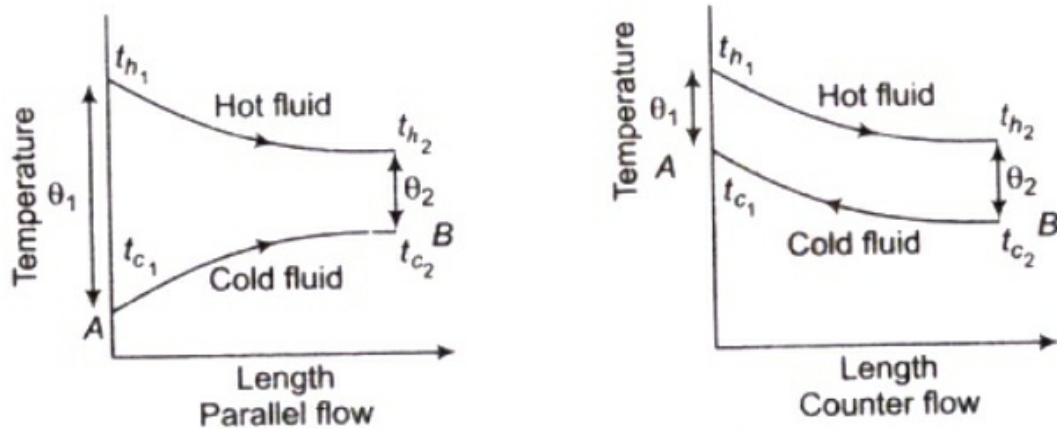
Very long fins:

$$\frac{(hpkA)^{\frac{1}{2}}(T_0 - T_\infty)}{[hpL(T_0 - T_\infty)]} = 1/mL$$

For fins having insulated tips

$$\frac{(hpkA)^{\frac{1}{2}}(T_0 - T_\infty) \tanh (mL)}{hpL(T_0 - T_\infty)} = \frac{\tanh (mL)}{mL}$$

## Logarithmic Mean Temperature Difference (LMTD)



Temperature distribution for parallel and counter flow heat exchanger

$$Q = UA \frac{\theta_1 - \theta_2}{\log_e \left( \frac{\theta_1}{\theta_2} \right)} = UA \theta_m$$

$$\theta_m = \frac{\theta_1 - \theta_2}{\log_e \left( \frac{\theta_1}{\theta_2} \right)}$$

## Capacity Ratio

Capacity ratio  $c = mc$ , where  $c$  = Specific heat

$$\text{If } m_h c_h > m_c c_c, R = \frac{m_c c_c}{m_h c_h}$$

$$\text{If } m_h c_h < m_c c_c, R = \frac{m_h c_h}{m_c c_c}$$

## Effectiveness of Heat Exchanger:

$$\varepsilon = \frac{(Q_{\text{act}})\text{actual heat transfer}}{(Q_{\text{max}})\text{maximum possible heat transfer}}$$

$$Q_{\text{act}} = m_h c_h (t_{h_1} - t_{h_2})$$

$$m_c c_c (t_{c_2} - t_{c_1})$$

$$Q_{\text{max}} = c_{\min} (t_{h_1} - t_{c_1})$$

$$\text{If } m_c c_c < m_h c_h \Rightarrow c_{\min} m_c c_c$$

$$Q_{\text{max}} = m_c c_c (t_{h_1} - t_{c_1})$$

$$\varepsilon = \frac{m_c c_c (t_{c_2} - t_{c_1})}{m_c c_c (t_{h_1} - t_{c_1})} = \frac{t_{c_2} - t_{c_1}}{t_{h_1} - t_{c_1}}$$

$$\text{If } m_c c_c < m_h c_h \Rightarrow c_{\min} m_h c_h$$

$$Q_{\text{max}} = m_h c_h (t_{h_1} - t_{c_1})$$

$$\varepsilon = \frac{m_h c_h (t_{h_1} - t_{h_2})}{m_h c_h (t_{h_1} - t_{c_1})} = \frac{t_{h_1} - t_{h_2}}{t_{h_1} - t_{c_1}}$$

## Number of Transfer Units (NTU):

$$NTU = \frac{UA}{c_{\min}}$$

$U$  = Overall heat transfer coefficient

$A$  = Surface area

$C_{\min}$  = Minimum capacity rate

$$\text{If } m_h c_h < m_c c_c \Rightarrow c_{\min} = m_c c_c$$

$$NTU = \frac{UA}{m_h c_c}$$

$$\text{If } m_h c_h < m_c c_c \Rightarrow c_{\min} = m_h c_h$$

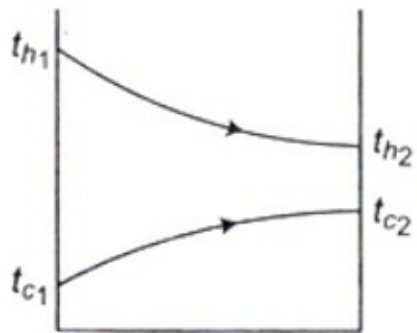
$$NTU = \frac{UA}{m_h c_h}$$

## Effectiveness for Parallel Flow Heat Exchanger

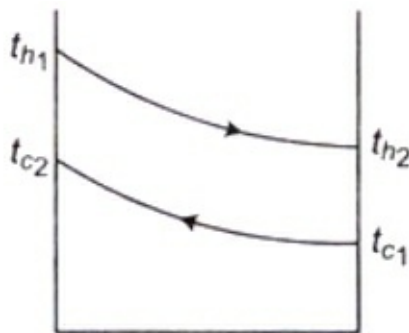
$$\square = \frac{1 - \exp[-NTU(1 + R)]}{1 + R}$$

$$R = \frac{c_{\min}}{c_{\max}}$$

$$NTU = \frac{UA}{C_{\min}}$$



Parallel flow



Counter flow

Heat exchanger for parallel and counter flow

### Effectiveness for the Counter Flow Heat Capacity:

$$\epsilon = \frac{1 - \exp[-NTU(1 - R)]}{1 + R \exp[-NTU(1 + R)]}$$

### Heat Exchanger Effectiveness Relation:

#### Concentric tube:

- Parallel flow:

$$\epsilon = \frac{1 - \exp[-N(1 + R)]}{(1 + R)}, R = \frac{C_{\min}}{C_{\max}}$$

- Counter flow:

$$\epsilon = \frac{1 - \exp[-N(1 - R)]}{1 - R \exp[-N(1 - R)]}, R < 1$$

$$\epsilon = \frac{N}{1 + R} \text{ for } R = 1$$

### Cross flow (single pass)

- Both fluids unmixed:

$$\epsilon = 1 - \exp\left[\left(\frac{1}{R}\right)(N)^{0.22}\{\exp(-R(N)^{0.78}) - 1\}\right]$$

- $C_{\max}$  mixed,  $C_{\min}$  unmixed:

$$\epsilon = \left(\frac{1}{R}\right)[1 - \exp\{-R(1 - \exp(-N))\}]$$

- $C_{\min}$  mixed,  $C_{\max}$  unmixed:

$$\epsilon = 1 - \exp[-R^{-1}\{1 - \exp(-RN)\}]$$

### Total Emissive Power (E)

- It is defined as the total amount of radiation emitted by a body per unit time and area.

$$E = \frac{\sigma T^4 W}{m^2}$$

$\sigma$  = Stefan Boltzmann constant

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$$

### **Monochromatic (Spectral) Emissive Power ( $E_\lambda$ )**

- It is defined as the rate of energy radiated per unit area of the surface per unit wavelength.

$$E = \frac{\sigma T^4 W}{m^2}$$

### **Emission from Real Surface**

- The emissive power from a real surface is given by

$$E = \sigma \varepsilon A T^4 W$$

$\varepsilon$  = Emissivity of the surface,

$T$  = Surface temperature

### **Emissivity ( $\varepsilon$ )**

- It is defined as the ratio of the emissive power of any body to the emissive power of a black body of same temperature.

$$\varepsilon = E/E_b$$

1. For black body,  $\varepsilon = 1$
2. For white body,  $\varepsilon = 0$
3. For gray body,  $0 < \varepsilon < 1$

### **Reflectivity ( $\rho$ )**

- It is defined as the fraction of total incident radiation that are reflected by material.

$$\text{Reflectivity} = \frac{\text{Energy reflected } (Q_r)}{\text{Total incident radiation } (Q)}$$

### **Absorptivity**

- It is defined as the fraction of total incident radiation that are absorbed by material.



$$\text{Absorbptivity} = \frac{\text{Energy absorbed } (Q_a)}{\text{Total incident radiation } (Q)}$$

### Transmissivity

- It is defined as the fraction of total incident radiation that are transmitted through the material.

$$\text{Transmissivity } (\tau) = \frac{\text{Energy absorbed } (Q_t)}{\text{Total incident radiation } (Q_w)}$$

$$Q_o = Q_a + Q_r + Q_t$$

$$\frac{Q_o}{Q_o} = \frac{Q_a}{Q_o} + \frac{Q_r}{Q_o} + \frac{Q_t}{Q_o}$$

$$\alpha + \rho + \tau = 1$$

**For black body**  $\alpha = 1, \rho = 0, \tau = 0$

• **For opaque body**  $\tau = 0, \alpha + \rho = 1$

• **For white body**  $\rho = 1, \alpha = 0$  and  $\tau = 0$

### Kirchoff's Law

- The emissivity  $\varepsilon$  and absorptivity  $\alpha$  of a real surface are equal for radiation with identical temperature and wavelength.

$$\alpha = \varepsilon = E/E_b$$

Emissive power of a black body is directly proportional to the fourth power of its absolute temperature.

$$E_b = \sigma T^4$$

$E_b$  = Emissive power of a black body,

$\sigma$  = Stefan Boltzmann constant ( $5.67 \cdot 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ ),

$T$  = Absolute temperature of the emitting surface, K.

### Wien's Displacement Law

- Wien's displacement law state that the product of  $\lambda_{\max}$  and  $T$  is constant

$$\lambda_{\max} T = \text{Constant}$$

$\lambda_{\max}$  = Wavelength at which the maximum value of monochromatic emissive power occurs.

## Gray Surfaces

The gray surface is a medium whose monochromatic emissivity ( $\epsilon_\lambda$ ) does not vary with wavelength.

$$\epsilon_\lambda = \frac{E_\lambda}{E_{\lambda,b}}$$

But, we know the following.

$$E = \int_0^\infty \epsilon_\lambda E_{\lambda,b} d\lambda$$

$$E_b = \int_0^\infty E_{\lambda,b} d\lambda = \sigma T^4$$

Therefore,

$$\epsilon = \frac{\int_0^\infty \epsilon_\lambda E_{\lambda,b} d\lambda}{\sigma T^4}$$

## View Factors:

- Define the view factor,  $F_{1 \rightarrow 2}$ , as the fraction of energy emitted from surface 1, which directly strikes surface 2.

$$F_{1 \rightarrow 2} = \frac{q_{1 \rightarrow 2}}{q_{\text{emitted}}} = \frac{\int_{A_2} \int_{A_1} \frac{I \cdot \cos\theta_1}{R^2} \cdot dA_1 \cdot \cos\theta_2 dA_2}{\pi \cdot I \cdot A_1}$$

$$F_{1 \rightarrow 2} - \frac{1}{A_1} = \int_{A_2} \int_{A_1} \frac{\cos\theta_1 \cdot \cos\theta_2 \cdot dA_1 \cdot dA_2}{\pi \cdot R^2}$$

## Reciprocity:

$$A_i \cdot F_{i \rightarrow j} = \int_{A_j} \int_{A_i} \frac{\cos\theta_i \cdot \cos\theta_j \cdot dA_i \cdot dA_j}{\pi \cdot R^2}$$

$$A_j \cdot F_{j \rightarrow i} = \int_{A_i} \int_{A_j} \frac{\cos\theta_j \cdot \cos\theta_i \cdot dA_j \cdot dA_i}{\pi \cdot R^2}$$

$$A_i \cdot F_{i \rightarrow j} = A_j \cdot F_{j \rightarrow i}$$

## Planck's Law:

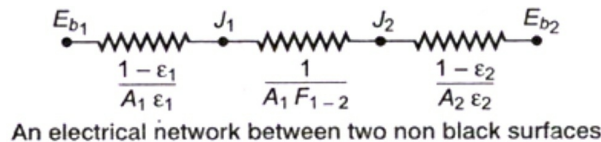
- Planck suggested following formula, monochromatic emissive power of a black body.

$$(E_{\lambda})_b = \frac{2\pi c^2 h \lambda^{-5}}{\exp\left(\frac{ch}{\lambda kT}\right) - 1}$$

- Total emissive power

$$E_b = \int_0^{\infty} (E_{\lambda})_b d\lambda$$

## Electrical Network Approach for Radiation Heat Exchange



$$(Q_{1-2})_{net} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{1-2}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$\text{or } (Q_{1-2})_{net} = (F_g)_{1-2} A_1 \sigma_b (T_1^4 - T_2^4)$$

## New Gray Body Factor

$$(G_g)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$E_1$  = Emissivity for body 1

$E_2$  = Emissivity for body 2

- In case of black surfaces,  $\varepsilon_1 = \varepsilon_2 = 1$ ,  $(F_g)_{12} = F_{1-2}$

$$Q_{net} = F_{1-2} A_1 \sigma (T_1^4 - T_2^4)$$

In case of parallel planes,  $A_1 = A_2$  and  $F_{1-2} = 1$

$$(F_g)_{1-2} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

In case of concentric cylinder or sphere,  $F_{1-2} = 1$

$$(F_g)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{1-\varepsilon_2}{\varepsilon_2} \frac{A_1}{A_2}}$$

Where,  $\frac{A_1}{A_2} = \frac{r_1}{r_2}$  for concentric cylinder

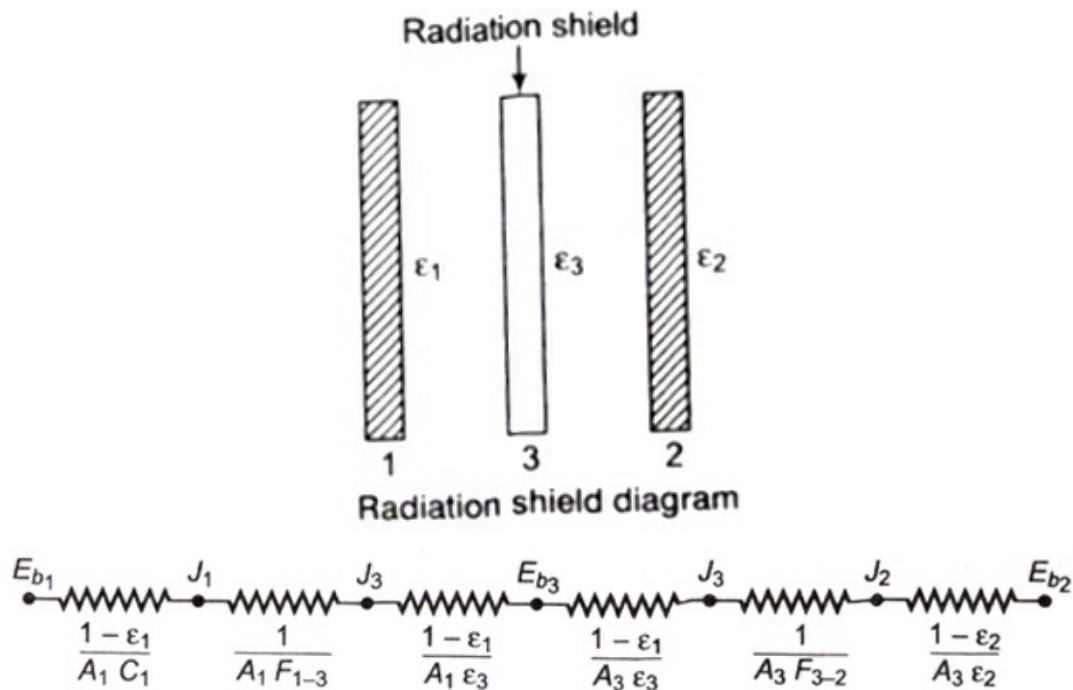
$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} \quad \text{for concentric sphere}$$

**When a small body lies inside a large enclosure**

$$F_{1-2}=1, A_1 \ll A_2 \Rightarrow \frac{A_1}{A_2} = 0$$

$$(F_\varepsilon)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1}$$

## Radiation Shield



**Radiation network for 2 parallel infinite plates separated by one shield**

$$(Q_{1-3})_{net} = (Q_{3-2})_{net} \quad (A_1 = A_3 = A_2)$$

$$\frac{A\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{A\sigma(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$(Q_{1-2})_{net} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right) + \left(\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1\right)}$$

$$\frac{[(Q_{1-2})_{net}] \text{ with shield}}{[(Q_{1-2})_{net}] \text{ without shield}}$$

$$= \frac{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right) + \left(\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1\right)}$$

If  $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$

Then,

$$(Q_{1-3})_{net} = (Q_{3-2})_{net} = \frac{1}{2}(Q_{1-2})_{net}$$

And

$$T_3^4 = \frac{1}{2}(T_1^4 - T_2^4)$$

## FLUID MECHANICS

### Pressure ( $P$ ):

- If  $F$  be the normal force acting on a surface of area  $A$  in contact with liquid, then pressure exerted by liquid on this surface is:  $P = F / A$
- **Units :**  $N / m^2$  or Pascal (S.I.) and Dyne/cm<sup>2</sup> (C.G.S.)
- **Dimension :**  $[P] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$
- **Atmospheric pressure:** Its value on the surface of the earth at sea level is nearly  $1.013 \times 10^5 N / m^2$  or Pascal in S.I. other practical units of pressure are atmosphere, bar and torr (*mm of Hg*)
- $1 \text{ atm} = 1.01 \times 10^5 \text{ Pa} = 1.01 \text{ bar} = 760 \text{ torr}$
- **Fluid Pressure at a Point:**  $\rho = \frac{dF}{dA}$

### Density ( $\rho$ ):

- In a fluid, at a point, density  $\rho$  is defined as:  $\rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$
- In case of homogenous isotropic substance, it has no directional properties, so is a scalar.
- It has dimensions  $[ML^{-3}]$  and S.I. unit  $kg/m^3$  while C.G.S. unit  $g/cc$  with  $1 g/cc = 10^3 kg/m^3$
- Density of body = Density of substance
- Relative density or specific gravity which is defined as :  $RD = \frac{\text{Density of body}}{\text{Density of water}}$
- If  $m_1$  mass of liquid of density  $\rho_1$  and  $m_2$  mass of density  $\rho_2$  are mixed, then as

$$m = m_1 + m_2 \text{ and } V = (m_1 / \rho_1) + (m_2 / \rho_2) \quad [\text{As } V = m / \rho]$$

$$\rho = \frac{m}{V} = \frac{m_1 + m_2}{(m_1 / \rho_1) + (m_2 / \rho_2)} = \frac{\sum m_i}{\sum (m_i / \rho_i)}$$

$$\text{If } m_1 = m_2, \rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = \text{Harmonic mean}$$

- If  $V_1$  volume of liquid of density  $\rho_1$  and  $V_2$  volume of liquid of density  $\rho_2$  are mixed, then as:  $m = \rho_1 V_1 + \rho_2 V_2$  and  $V = V_1 + V_2$  [As  $\rho = m / V$ ]

$$\text{If } V_1 = V_2 = V \quad \rho = (\rho_1 + \rho_2) / 2 = \text{Arithmetic Mean}$$

- With rise in temperature due to thermal expansion of a given body, volume will increase while mass will remain unchanged, so density will decrease, i.e.,

$$\frac{\rho}{\rho_0} = \frac{(m/V)}{(m/V_0)} = \frac{V_0}{V} = \frac{V_0}{V_0(1+\gamma\Delta\theta)} \quad [\text{As } V = V_0(1+\gamma\Delta\theta)]$$

or

$$\rho = \frac{\rho_0}{(1+\gamma\Delta\theta)} \approx \rho_0(1-\gamma\Delta\theta)$$

- With increase in pressure due to decrease in volume, density will increase, i.e.,

$$\frac{\rho}{\rho_0} = \frac{(m/V)}{(m/V_0)} = \frac{V_0}{V} \quad [\text{As } \rho = \frac{m}{V}]$$

- By definition of **bulk-modulus**:  $B = -V_0 \frac{\Delta p}{\Delta V}$  i.e.,  $V = V_0 \left[ 1 - \frac{\Delta p}{B} \right]$

$$\rho = \rho_0 \left( 1 - \frac{\Delta p}{B} \right)^{-1} \approx \rho_0 \left( 1 + \frac{\Delta p}{B} \right)$$

### Specific Weight ( $w$ ):

- It is defined as the weight per unit volume.
- Specific weight =  $\frac{\text{Weight}}{\text{Volume}} = \frac{m \cdot g}{\text{Volume}} = \rho \cdot g$

### Specific Gravity or Relative Density (s):

- It is the ratio of specific weight of fluid to the specific weight of a standard fluid. Standard fluid is water in case of liquid and H<sub>2</sub> or air in case of gas.

$$s = \frac{\gamma}{\gamma_w} = \frac{\rho \cdot g}{\rho_w \cdot g} = \frac{\rho}{\rho_w}$$

Where,  $\gamma_w$  = Specific weight of water, and  $\rho_w$  = Density of water specific.

### Specific Volume ( $v$ ):

- Specific volume of liquid is defined as volume per unit mass. It is also defined as the reciprocal of specific density.
- Specific volume =  $\frac{V}{m} = \frac{1}{\rho}$

$$\text{Inertial force per unit area} = \frac{dp/dt}{A} = \frac{v(dm/dt)}{A} = \frac{v \cdot A v \cdot \rho}{A} = v^2 \rho$$

**Viscous force per unit area:**  $F/A = \frac{\eta v}{r}$

**Reynold's number:**  $N_R = \frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}} = \frac{v^2 \rho}{\eta v / r} = \frac{v \rho r}{\eta}$

**Pascal's Law:**  $p_x = p_y = p_z$ ; where,  $p_x$ ,  $p_y$  and  $p_z$  are the pressure at point x,y,z respectively.

**Hydrostatic Law:**

- $\frac{\partial p}{\partial z} = pg$  or  $dp = pg \, dz$
- $\int_0^p dp = pg \int_0^h dz$
- $p = pgh$  and  $h = \frac{p}{pg}$ ; where, h is known as **pressure head**.

Pressure Energy	Potential energy	Kinetic energy
It is the energy possessed by a liquid by virtue of its pressure. It is the measure of work done in pushing the liquid against pressure without imparting any velocity to it.	It is the energy possessed by liquid by virtue of its height or position above the surface of earth or any reference level taken as zero level.	It is the energy possessed by a liquid by virtue of its motion or velocity.
Pressure energy of the liquid $PV$	Potential energy of the liquid $mgh$	Kinetic energy of the liquid $mv^2/2$
Pressure energy per unit mass of the liquid $P/\rho$	Potential energy per unit mass of the liquid $gh$	Kinetic energy per unit mass of the liquid $v^2/2$
Pressure energy per unit volume of the liquid $P$	Potential energy per unit volume of the liquid $\rho gh$	Kinetic energy per unit volume of the liquid $\rho v^2/2$

Quantities that Satisfy a Balance Equation						
Quantit y	mass	x momentum	y momentum	z momentum	Energy	Species
$\Phi$	m	mu	mv	mw	$E + mV^2/2$	$m^{(K)}$
$\phi$	1	u	v	w	$e + V^2/2$	$W^{(K)}$
In this table, u, v, and w are the x, y and z velocity components, E is the total thermodynamic internal energy, e is the thermodynamic internal energy per unit mass, and $m^{(K)}$ is the mass of a chemical species, K, $W^{(K)}$ is the mass fraction of species K.						



The other energy term,  $m\mathbf{V}^2/2$ , is the kinetic energy.

- $Storage = \frac{\partial \Phi}{\partial t} = \frac{\partial (m\phi)}{\partial t} = \frac{\partial (\rho \Delta x \Delta y \Delta z \phi)}{\partial t} = \frac{\partial (\rho \phi)}{\partial t} \Delta x \Delta y \Delta z$
- $Inflow = \rho u \phi|_x \Delta y \Delta z + \rho v \phi|_y \Delta x \Delta z + \rho w \phi|_z \Delta y \Delta x$
- $Outflow = \rho u \phi|_{x+\Delta x} \Delta y \Delta z + \rho v \phi|_{y+\Delta y} \Delta x \Delta z + \rho w \phi|_{z+\Delta z} \Delta y \Delta x$
- $Source = S_\phi \Delta x \Delta y \Delta z$
- $\frac{\partial \rho \phi}{\partial t} + \frac{\rho u \phi|_{x+\Delta x} - \rho u \phi|_x}{\Delta x} + \frac{\rho v \phi|_{y+\Delta y} - \rho v \phi|_y}{\Delta y} + \frac{\rho w \phi|_{z+\Delta z} - \rho w \phi|_z}{\Delta z} = S_\phi$
- $\frac{\partial \rho \phi}{\partial t} + \frac{\partial \rho u \phi}{\partial x} + \frac{\partial \rho v \phi}{\partial y} + \frac{\partial \rho w \phi}{\partial z} = S_\phi^*$
- $S_\phi^* = \lim_{\Delta x \Delta y \Delta z \rightarrow 0} S_\phi$

### The Mass Balance Equations:

- $\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$
- $\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$
- $\frac{\partial \rho}{\partial t} + u_i \frac{\partial \rho}{\partial x_i} + \rho \frac{\partial u_i}{\partial x_i} = 0$
- $\frac{\partial \rho}{\partial t} + \rho \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right] + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} = 0$
- $\frac{D\Psi}{Dt} = \frac{\partial \Psi}{\partial t} + u \frac{\partial \Psi}{\partial x} + v \frac{\partial \Psi}{\partial y} + w \frac{\partial \Psi}{\partial z} \quad \text{or} \quad \frac{D\Psi}{Dt} = \frac{\partial \Psi}{\partial t} + u_i \frac{\partial \Psi}{\partial x_i}$
- $\frac{D\rho}{Dt} + \rho \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right] = 0 \quad \frac{D\rho}{Dt} + \rho \frac{\partial u_i}{\partial x_i} = 0 \quad \frac{D\rho}{Dt} + \rho \Delta = 0$
- $\Delta \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{or} \quad \Delta \equiv \frac{\partial u_i}{\partial x_i} = 0$

- $\rho \frac{\partial \phi}{\partial t} + \phi \left[ \frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} \right] + \rho u_i \frac{\partial \phi}{\partial x_i} = S_\phi$
- $\rho \frac{\partial \phi}{\partial t} + \rho u_i \frac{\partial \phi}{\partial x_i} = S_\phi$

### Momentum Balance Equation:

- *Net j-direction source term*  $= \frac{\partial \sigma_{1j}}{\partial x_1} + \frac{\partial \sigma_{2j}}{\partial x_2} + \frac{\partial \sigma_{3j}}{\partial x_3} + \rho B_j = \frac{\partial \sigma_{ij}}{\partial x_i} + \rho B_j$
- $\frac{\partial \rho u_j}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_i} = \frac{\partial \sigma_{ij}}{\partial x_i} + \rho B_j \quad j = 1, \dots, 3$
- For a Newtonian fluid, the stress,  $\sigma_{ij}$ , is given by the following equation:

$$\sigma_{ij} = -P\delta_{ij} + \mu \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] + \left( \kappa - \frac{2}{3} \mu \right) \Delta \delta_{ij}$$

- $\frac{\partial \rho u_j}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_i} = \frac{\partial}{\partial x_i} \left[ -P\delta_{ij} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \left( \kappa - \frac{2}{3} \mu \right) \Delta \delta_{ij} \right] + \rho B_j \quad j = 1, \dots, 3$
- $\frac{\partial \rho u_j}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_i} = -\frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_i} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + \frac{\partial}{\partial x_j} \left[ \left( \kappa - \frac{2}{3} \mu \right) \Delta \right] + \rho B_j \quad j = 1, \dots, 3$
- $\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u u}{\partial x} + \frac{\partial \rho v u}{\partial y} + \frac{\partial \rho w u}{\partial z} = \rho B_x$
- $-\frac{\partial P}{\partial x} + 2 \frac{\partial}{\partial x} \mu \left( \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left[ \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \right] + \frac{\partial}{\partial z} \left[ \mu \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \right] + \frac{\partial}{\partial x} \left[ \left( \kappa - \frac{2}{3} \mu \right) \Delta \right]$

### Energy Balance Equation:

- This directional heat flux is given the symbol  $q_i$ :  $q_i = -k \frac{\partial T}{\partial x_i}$
- $\frac{\text{Net } x\text{Direction heat}}{\text{Unit Volume}} = -\frac{q_x|_{x+\Delta x} - q_x|_x}{\Delta x \Delta y \Delta z} \Delta y \Delta z = -\frac{q_x|_{x+\Delta x} - q_x|_x}{\Delta x}$
- $\lim_{\Delta x \rightarrow 0} \frac{\text{Net } x\text{Direction heat source}}{\text{Unit Volume}} = -\frac{\partial q_x}{\partial x}$

- $Heat\ Rate = -\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} - \frac{\partial q_z}{\partial z} = -\frac{\partial q_i}{\partial x_i}$
- $Body\text{-}force\ work\ rate = \rho(uB_x + vB_y + wB_z) = \rho u_i B_i$

- The work term on each face is given by the following equation:

$$y\text{-face surface force work} = (u\sigma_{yx} + v\sigma_{yy} + w\sigma_{yz})\Delta x \Delta z = u_i\sigma_{iy}\Delta x \Delta z$$

- $Net\ yFace\ Surface\ Force\ Work = \frac{\partial(u\sigma_{yx} + v\sigma_{yy} + w\sigma_{yz})}{\partial y} = \frac{\partial u_i\sigma_{yi}}{\partial y}$
- $Net\ Surface\ Force\ Work = \frac{\partial u_i\sigma_{xi}}{\partial x} + \frac{\partial u_i\sigma_{yi}}{\partial y} + \frac{\partial u_i\sigma_{zi}}{\partial z} = \frac{\partial u_i\sigma_{ji}}{\partial x_j}$
- Energy balance equation:

$$\frac{\partial \rho(e + \mathbf{V}^2/2)}{\partial t} + \frac{\partial \rho u_i(e + \mathbf{V}^2/2)}{\partial x_i} = -\frac{\partial q_i}{\partial x_i} + \frac{\partial u_i\sigma_{ji}}{\partial x_j} + \rho u_i B_i$$

### Substitutions for Stresses and Heat Flux:

Using only the Fourier Law heat transfer, the source term involving the heat flux in the energy balance equation:

- $-\frac{\partial q_i}{\partial x_i} = -\frac{\partial}{\partial x_i} \left( -k \frac{\partial T}{\partial x_i} \right) = \frac{\partial}{\partial x_i} k \frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x} k \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} k \frac{\partial T}{\partial y} + \frac{\partial}{\partial z} k \frac{\partial T}{\partial z}$
- $\frac{\partial \rho e}{\partial t} + \frac{\partial \rho u_i e}{\partial x_i} = \frac{\partial}{\partial x_i} k \frac{\partial T}{\partial x_i} + \left[ -P\delta_{ij} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \left( \kappa - \frac{2}{3}\mu \right) \Delta \delta_{ij} \right] \frac{\partial u_i}{\partial x_j}$
- $\delta_{ij} \frac{\partial u_i}{\partial x_j} = \frac{\partial u_j}{\partial x_j} = \Delta$
- $\frac{\partial \rho e}{\partial t} + \frac{\partial \rho u_i e}{\partial x_i} = \frac{\partial}{\partial x_i} k \frac{\partial T}{\partial x_i} - P\Delta + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} + \left( \kappa - \frac{2}{3}\mu \right) \Delta^2$

Dissipation to avoid confusion with the general quantity in a balance equation:

- $\Phi_D = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} + \left( \kappa - \frac{2}{3}\mu \right) \Delta^2$

- $$\frac{\partial \rho e}{\partial t} + \frac{\partial \rho u_i e}{\partial x_i} = \frac{\partial}{\partial x_i} k \frac{\partial T}{\partial x_i} - P \Delta + \Phi_D$$

The temperature gradient in the Fourier law conduction term may also be written as a gradient of enthalpy or internal energy:

- $$\frac{\partial T}{\partial x_i} = \frac{1}{c_v} \frac{\partial e}{\partial x_i} + \frac{1}{c_v} \left[ \frac{T \beta_p}{\kappa_T} - P \right] \frac{1}{\rho^2} \frac{\partial \rho}{\partial x_i}$$

- $$\frac{\partial T}{\partial x_i} = \frac{1}{c_p} \frac{\partial h}{\partial x_i} - \frac{1 - T \beta_p}{\rho c_p} \frac{\partial P}{\partial x_i}$$

- $$\frac{\partial \rho e}{\partial t} + \frac{\partial \rho u_i e}{\partial x_i} = \frac{\partial}{\partial x_i} \frac{k}{c_v} \frac{\partial e}{\partial x_i} - P \Delta + \Phi_D + \frac{\partial}{\partial x_i} \frac{1}{c_v} \left[ \frac{T \beta_p}{\kappa_T} - P \right] \frac{1}{\rho^2} \frac{\partial \rho}{\partial x_i}$$

- $$\frac{\partial \rho h}{\partial t} + \frac{\partial \rho u_i h}{\partial x_i} = \frac{\partial}{\partial x_i} \frac{k}{c_p} \frac{\partial h}{\partial x_i} + \Phi_D + \frac{\partial}{\partial x_i} \left[ \frac{1 - T \beta_p}{\rho c_p} \right] \frac{\partial P}{\partial x_i} + \frac{DP}{Dt}$$

- $$c_p \left[ \frac{\partial \rho T}{\partial t} + \frac{\partial \rho u_i T}{\partial x_i} \right] = \frac{\partial}{\partial x_i} k \frac{\partial T}{\partial x_i} + \Phi_D + \beta_p T \frac{DP}{Dt}$$

General Balance Equations			
$\Phi$	$c$	$\Gamma^{(\Phi)}$	$S^{(\Phi)}$
1	1	0	0
$u = u_x = u_1$	1	$\mu$	$-\frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \mu \frac{\partial u}{\partial x} + \frac{\partial}{\partial y} \mu \frac{\partial v}{\partial x} + \frac{\partial}{\partial z} \mu \frac{\partial w}{\partial x} + \frac{\partial}{\partial x} \left[ \left( \kappa - \frac{2}{3} \mu \right) \Delta \right] + \rho B_x$
$v = u_y = u_2$	1	$\mu$	$-\frac{\partial P}{\partial y} + \frac{\partial}{\partial x} \mu \frac{\partial u}{\partial y} + \frac{\partial}{\partial y} \mu \frac{\partial v}{\partial y} + \frac{\partial}{\partial z} \mu \frac{\partial w}{\partial y} + \frac{\partial}{\partial y} \left[ \left( \kappa - \frac{2}{3} \mu \right) \Delta \right] + \rho B_y$
$w = u_z = u_3$	1	$\mu$	$-\frac{\partial P}{\partial z} + \frac{\partial}{\partial x} \mu \frac{\partial u}{\partial z} + \frac{\partial}{\partial y} \mu \frac{\partial v}{\partial z} + \frac{\partial}{\partial z} \mu \frac{\partial w}{\partial z} + \frac{\partial}{\partial z} \left[ \left( \kappa - \frac{2}{3} \mu \right) \Delta \right] + \rho B_z$
e	1	$k/c_v$	$-P\Delta + \Phi_D + \frac{\partial}{\partial x_i} \frac{1}{c_v} \left[ \frac{T\beta_p}{\kappa_T} - P \right] \frac{1}{\rho^2} \frac{\partial \rho}{\partial x_i}$
h	1	$k/c_p$	$\Phi_D + \frac{\partial}{\partial x_i} \left[ \frac{1 - T\beta_p}{\rho c_p} \right] \frac{\partial P}{\partial x_i} + \frac{DP}{Dt}$
T	$c_p$	k	$\Phi_D + \beta_p T \frac{DP}{Dt}$
T	$c_v$	k	$\Phi_D + \frac{T\beta_p}{\kappa_T} \Delta$
$W^{(K)}$	1	$\rho D_{K,Mix}$	$\Gamma^{(K)}$

### Momentum equation:

$$\bullet \quad \frac{\partial \rho u_j}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_i} = \frac{\partial}{\partial x_i} \mu \frac{\partial u_j}{\partial x_i} + S^{(j)} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_i} \mu \frac{\partial u_j}{\partial x_i} + S^{*(j)}$$

General Momentum Equations			
$\Phi$	$c$	$\Gamma^{(\Phi)}$	$S^{*(\Phi)}$
1	1	0	0
$u = u_x = u_1$	1	$\mu$	$\frac{\partial}{\partial x} \mu \frac{\partial u}{\partial x} + \frac{\partial}{\partial y} \mu \frac{\partial v}{\partial x} + \frac{\partial}{\partial z} \mu \frac{\partial w}{\partial x} + \frac{\partial}{\partial x} \left[ \left( \kappa - \frac{2}{3} \mu \right) \Delta \right] + \rho B_x$
$v = u_y = u_2$	1	$\mu$	$+\frac{\partial}{\partial x} \mu \frac{\partial u}{\partial y} + \frac{\partial}{\partial y} \mu \frac{\partial v}{\partial y} + \frac{\partial}{\partial z} \mu \frac{\partial w}{\partial y} + \frac{\partial}{\partial y} \left[ \left( \kappa - \frac{2}{3} \mu \right) \Delta \right] + \rho B_y$
$w = u_z = u_3$	1	$\mu$	$\frac{\partial}{\partial x} \mu \frac{\partial u}{\partial z} + \frac{\partial}{\partial y} \mu \frac{\partial v}{\partial z} + \frac{\partial}{\partial z} \mu \frac{\partial w}{\partial z} + \frac{\partial}{\partial z} \left[ \left( \kappa - \frac{2}{3} \mu \right) \Delta \right] + \rho B_z$

### Bernoulli's Equation:

This equation has four variables: velocity ( $v$ ), elevation ( $z$ ), pressure ( $p$ ), and density ( $\rho$ ). It also has a constant ( $g$ ), which is the acceleration due to gravity. Here is Bernoulli's equation:

- $\frac{v^2}{2} + gz + \frac{p}{\rho} = \text{constant}$
- $P + \rho gh + \frac{1}{2} \rho v^2 = \text{constant}$
- $\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$ ;  $\frac{P}{\rho g}$  is called pressure head,  $h$  is called gravitational head and  $\frac{v^2}{2g}$  is called velocity head.

### Factors that influence head loss due to friction are:

- Length of the pipe ( $l$ )
- Effective diameter of the pipe ( $D_h$ )
- Velocity of the water in the pipe ( $v$ )
- Acceleration of gravity ( $g$ )
- Friction from the surface roughness of the pipe ( $\lambda$ )
- *The head loss due to the pipe is estimated by the following equation:*

$$h_{f,major} = \lambda \frac{lv^2}{2D_h g}$$

- To estimate the total head loss in a piping system, one adds the head loss from the fittings and the pipe:

$$h_{f,total} = \sum h_{f,minor} + \sum h_{f,major}$$

- Note that the summation symbol ( $\sum$ ) means to add up the losses from all the different sources. A less compact-way to write this equation is:

$$h_{f,total} = h_{f,minor1} + h_{f,minor2} + h_{f,minor3} + \dots$$
$$h_{f,major1} + h_{f,major2} + h_{f,major3} + \dots$$

### Combining Bernoulli's Equation With Head Loss:

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_{f,total}$$



### Relation between coefficient of viscosity and temperature:

$$\text{Andrade formula } \eta = \frac{A e^{C\rho/T}}{\rho^{-1/3}}$$

**Stoke's Law:**  $F = 6\pi\eta rv$

### Terminal Velocity:

- Weight of the body ( $W$ ) =  $mg = (\text{volume} \times \text{density}) \times g = \frac{4}{3}\pi r^3 \rho g$
- Upward thrust ( $T$ ) = weight of the fluid displaced  
$$= (\text{volume} \times \text{density}) \text{ of the fluid} \times g = \frac{4}{3}\pi r^3 \sigma g$$
- Viscous force ( $F$ ) =  $6\pi\eta rv$
- When the body attains terminal velocity the net force acting on the body is zero.
- $W - T - F = 0$  or  $F = W - T$
- $6\pi\eta rv = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g = \frac{4}{3}\pi r^3 (\rho - \sigma) g$
- Terminal velocity  $v = \frac{2 r^2 (\rho - \sigma) g}{9 \eta}$
- Terminal velocity depend on the radius of the sphere so if radius is made  $n$  - fold, terminal velocity will become  $n^2$  times.
- Greater the density of solid greater will be the terminal velocity
- Greater the density and viscosity of the fluid lesser will be the terminal velocity.
- If  $\rho > \sigma$  then terminal velocity will be positive and hence the spherical body will attain constant velocity in downward direction.
- If  $\rho < \sigma$  then terminal velocity will be negative and hence the spherical body will attain constant velocity in upward direction.

### Poiseuille's Formula:

- $V \propto \frac{P r^4}{\eta l}$  or  $V = \frac{K P r^4}{\eta l}$
- $V = \frac{\pi P r^4}{8 \eta l}$ ; where  $K = \frac{\pi}{8}$  is the constant of proportionality.

### Buoyant Force:

- Buoyant force = Weight of fluid displaced by body

- Buoyant force on cylinder = Weight of fluid displaced by cylinder
- $V_{\text{in}}$  = Value of immersed part of solid
- $F_B = \rho_{\text{water}} \times g \times \text{Volume of fluid displaced}$
- $F_B = \rho_{\text{water}} \times g \times \text{Volume of cylinder immersed inside the water}$
- $F_B = mg$
- $F_B = \rho_w g \frac{\pi}{4} d^2$  ( $\because w = mg = \rho V g$ )
- $V_{\text{in}} \rho_l g = V_i \rho_s g$
- $\rho_w g \frac{\pi}{4} d^2 x = \rho_{\text{cylinder}} g \frac{\pi}{4} d^2 h$
- $\rho_w x = \rho_{\text{cylinder}} h$

### Relation between B, G and M:

- $GM = \frac{I}{V} - BG$  ; where  $I$  = Least moment of inertia of plane of body at water surface,  $G$  = Centre of gravity,  $B$  = Centre of buoyancy, and  $M$  = Metacentre.
- $I = \min(I_{xx}, I_{yy})$ ,  $I_{xx} = \frac{bd^3}{12}$ ,  $I_{yy} = \frac{bd^3}{12}$
- $V = bcd$

### Energy Equations:

- $F_{\text{net}} = F_g + F_p + F_v + F_c + F_t$  ; where Gravity force  $F_g$ , Pressure force  $F_p$ , Viscous force  $F_v$ , Compressibility force  $F_c$ , and Turbulent force  $F_t$
- If fluid is incompressible, then  $F_c = 0$   
 $\therefore F_{\text{net}} = F_g + F_p + F_v + F_t$  ; This is known as **Reynolds equation** of motion.
- If fluid is incompressible and turbulence is negligible, then  
 $F_c = 0, F_t = 0 \therefore F_{\text{net}} = F_g + F_p + F_v$  ; This equation is called as **Navier-Stokes equation**.
- If fluid flow is considered ideal then, viscous effect will also be negligible. Then  
 $F_{\text{net}} = F_g + F_p$  ; This equation is known as **Euler's equation**.
- Euler's equation can be written as:  $\frac{dp}{\rho} + g dz + v dv = 0$



**Dimensional analysis:**

<b>Quantity</b>	<b>Symbol</b>	<b>Dimensions</b>
<b>Mass</b>	m	M
<b>Length</b>	l	L
<b>Time</b>	t	T
<b>Temperature</b>	T	$\theta$
<b>Velocity</b>	u	$LT^{-1}$
<b>Acceleration</b>	a	$LT^{-2}$
<b>Momentum/Impulse</b>	mv	$MLT^{-1}$
<b>Force</b>	F	$MLT^{-2}$
<b>Energy - Work</b>	W	$ML^2T^{-2}$
<b>Power</b>	P	$ML^2T^{-3}$
<b>Moment of Force</b>	M	$ML^2T^{-2}$
<b>Angular momentum</b>	-	$ML^2T^{-1}$
<b>Angle</b>	$\eta$	$M^0L^0T^0$
<b>Angular Velocity</b>	$\omega$	$T^{-1}$
<b>Angular acceleration</b>	$\alpha$	$T^{-2}$
<b>Area</b>	A	$L^2$
<b>Volume</b>	V	$L^3$
<b>First Moment of Area</b>	Ar	$L^3$
<b>Second Moment of Area</b>	I	$L^4$
<b>Density</b>	$\rho$	$ML^{-3}$
<b>Specific heat- Constant Pressure</b>	$C_p$	$L^2T^{-2}\theta^{-1}$
<b>Elastic Modulus</b>	E	$ML^{-1}T^{-2}$
<b>Flexural Rigidity</b>	EI	$ML^3T^{-2}$
<b>Shear Modulus</b>	G	$ML^{-1}T^{-2}$
<b>Torsional rigidity</b>	GJ	$ML^3T^{-2}$
<b>Stiffness</b>	k	$MT^{-2}$

Angular stiffness	$T/\eta$	$ML^{-2}T^{-2}$
Flexibility	$1/k$	$M^{-1}T^2$
Vorticity	-	$T^{-1}$
Circulation	-	$L^2T^{-1}$
Viscosity	$\mu$	$ML^{-1}T^{-1}$
Kinematic Viscosity	$\tau$	$L^2T^{-1}$
Diffusivity	-	$L^2T^{-1}$
Friction coefficient	$f/\mu$	$M^0L^0T^0$
Restitution coefficient		$M^0L^0T^0$
Specific heat-Constant volume	$C_v$	$L^2T^{-2}\theta^{-1}$

### Boundary layer:

- **Reynolds number**  $= \frac{\rho v \cdot x}{\mu} (Re)_x = \frac{v \cdot x}{\nu}$
- **Displacement Thickness ( $\delta^*$ ):**  $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$
- **Momentum Thickness ( $\theta$ ):**  $\theta = \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy$
- **Energy Thickness ( $\delta^{**}$ ):**  $\delta^{**} = \int_0^\delta \frac{u}{U} \left(1 - \frac{u^2}{U^2}\right) dy$
- **Boundary Conditions for the Velocity Profile:** Boundary conditions are as
  - (a) At  $y = 0, u = 0, \frac{du}{dy} \neq 0$ ; (b) At  $y = \delta, u = U, \frac{du}{dy} = 0$

### Turbulent flow:

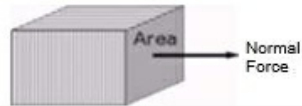
- **Shear stress in turbulent flow:**  $\tau = \tau_v + \tau_t = \mu \frac{d\bar{u}}{dy} + \eta \frac{d\bar{u}}{dy}$
- **Turbulent shear stress by Reynold:**  $\tau = \rho u'v'$
- **Shear stress in turbulent flow due to Prndtle :**  $\tau = \rho l^2 \left(\frac{du}{dy}\right)^2$

# STRENGTH OF MATERIALS

## Stress and Strain

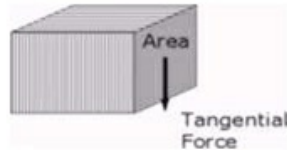
Stress = Force / Area

Normal stress



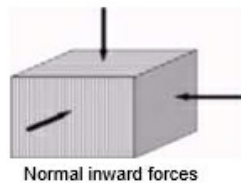
$$\text{Normal stress} = \frac{\text{Normal force}}{\text{area}} \sigma_n = \frac{F_n}{A}$$

Shear stress



$$\text{Shear stress} = \frac{\text{tangential force}}{\text{area}} \sigma_t = \frac{F_t}{A}$$

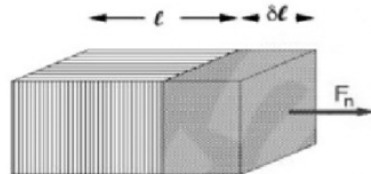
Bulk stress



$$\text{Bulk stress} = \frac{\text{normal inward force}}{\text{area}} \sigma_b = P$$

$$\text{Tensile strain } (e) = \frac{\Delta L}{L} = \frac{\text{Change in length}}{\text{original length}}$$

Normal strain

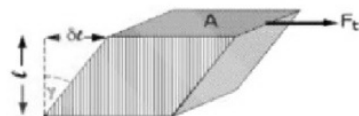


$$\text{Normal Strain} = \frac{\text{change in normal length}}{\text{original normal length}}$$

$$e_n = \frac{\delta l}{l}$$

Since strain is m/m it is dimensionless.

Shear strain



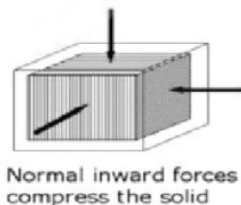
$$\text{Shear Strain} = \frac{\text{tangential displacement}}{\text{original normal length}}$$

$$e_t = \frac{\delta t}{t} = \gamma (\text{rad})$$

Note 1: the volume of the solid is not changed by shear strain.

Note 2: the angle is in radians, not degrees.

Bulk strain



$$\text{Bulk Strain} = \frac{- (\text{change in volume})}{\text{original volume}}$$

$$e_b = \frac{-\delta V}{V}$$

## Brinell Hardness Number (BHN)

$$HB = \frac{\text{Load (kgf)}}{\text{Surface area of indentation (mm}^2\text{)}} = \frac{P}{\frac{\pi d}{2} (D - \sqrt{D^2 - d^2})}$$

Where D: Diameter of the ball indenter.

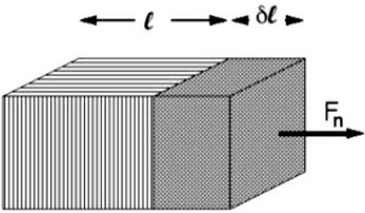
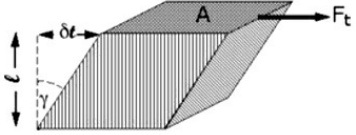
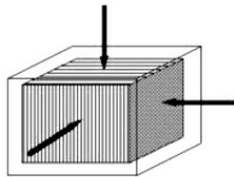
d : Diameter of the rim of the permanent impression,

P : Load

## Elastic constant:

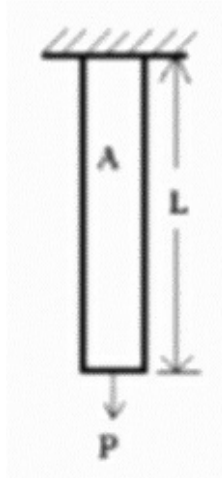
$$\frac{\text{Stress}}{\text{Strain}} = \text{Elastic modulus} = \frac{\sigma}{\epsilon}$$

where, P = Standard load, D = Diameter of steel ball, and d = Diameter of the indent.

<p>Young's modulus</p> 	$\frac{\text{normal stress}}{\text{normal strain}} = \text{Young's Modulus}$ $E = \frac{\sigma_n}{\epsilon_n} = \frac{F_n/A}{\delta l/l} = \frac{F_n \cdot l}{\delta l \cdot A}$
<p>Rigidity modulus</p> 	$\frac{\text{shear stress}}{\text{shear strain}} = \text{Shear Modulus}$ $G = \frac{\sigma_s}{\epsilon_s} = \frac{F_t/A}{\delta t/l} = \frac{F_t}{A \cdot \gamma}$
<p>Bulk modulus</p>  <p>Normal inward forces compress the solid</p>	$\frac{\text{bulk stress}}{\text{bulk strain}} = \text{Bulk Modulus}$ $K = \frac{\sigma_b}{\epsilon_b} = \frac{P}{-\delta V/V} = -\frac{PV}{\delta V}$

## Axial Elongation of Bar Prismatic Bar Due to External Load

$$\Delta = \frac{PL}{AE}$$



### Elongation of Prismatic Bar Due to Self-Weight

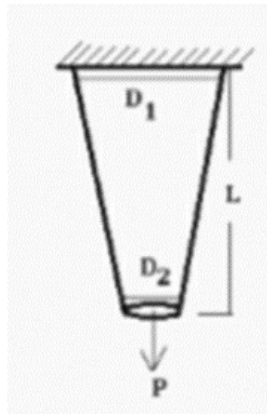
$$\Delta = \frac{PL}{2AE} = \frac{\gamma L^2}{2E}$$

Where  $\gamma$  is specific weight

### Elongation of Tapered Bar

#### • Circular Tapered

$$\Delta = \frac{4PL}{\pi D_1 D_2 E}$$



#### Rectangular Tapered

$$\Delta = \frac{PL \log_e \left( \frac{B_2}{B_1} \right)}{E \cdot t (B_2 - B_1)}$$

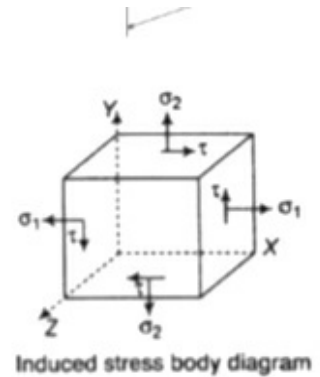
### Stress Induced by Axial Stress and Simple Shear

- Normal stress

$$\sigma_n = \sigma_1 \cos^2 \theta + \sigma_2 \sin^2 \theta + \tau \sin 2\theta$$

- Tangential stress

$$\sigma_t = -\left( \frac{\sigma_1 + \sigma_2}{2} \right) \sin 2\theta + \tau \cos 2\theta$$



### Principal Stresses and Principal Planes

- Major principal stress

$$\sigma_1' = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\frac{(\sigma_1 - \sigma_2)^2}{4} + \tau^2}$$

- Major principal stress

$$\sigma_2' = \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\frac{(\sigma_1 - \sigma_2)^2}{4} + \tau^2}$$

\

$$\tan 2\theta_p = \frac{2\tau}{\sigma_1 - \sigma_2}$$

$$\sigma_1 + \sigma_2 = \sigma_1' + \sigma_2'$$

where  $2\theta_p = 0$

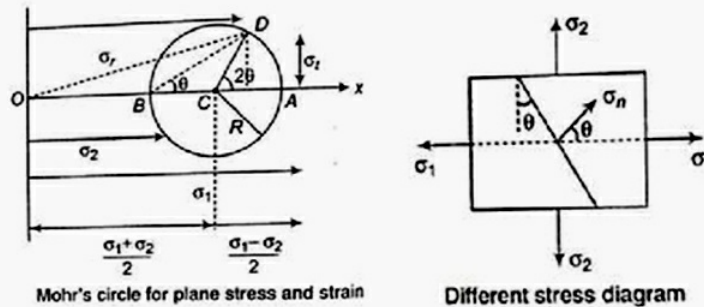
$$\sigma_1 = \sigma_1' \text{ and } \sigma_2 = \sigma_2'$$

### Principal Strain

$$\varepsilon_I = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{1}{2} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

$$\varepsilon_{II} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{1}{2} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

#### Mohr's Circle-



## STRAIN ENERGY

### Energy Methods:

#### (i) Formula to calculate the strain energy due to axial loads (tension):

$$U = \int P^2 / (2AE) dx \quad \text{limit } 0 \text{ to } L$$

Where, P = Applied tensile load, L = Length of the member, A = Area of the member, and E = Young's modulus.

#### (ii) Formula to calculate the strain energy due to bending:

$$U = \int M^2 / (2EI) dx \quad \text{limit } 0 \text{ to } L$$

Where, M = Bending moment due to applied loads, E = Young's modulus, and I = Moment of inertia.

#### (iii) Formula to calculate the strain energy due to torsion:

$$U = \int T^2 / (2GJ) dx \quad \text{limit } 0 \text{ to } L$$

Where, T = Applied Torsion, G = Shear modulus or Modulus of rigidity, and J = Polar moment of inertia

#### (iv) Formula to calculate the strain energy due to pure shear:

$$U = K \int V^2 / (2GA) dx \quad \text{limit } 0 \text{ to } L$$

Where, V = Shear load

G = Shear modulus or Modulus of rigidity

A = Area of cross section.

K = Constant depends upon shape of cross section.

**(v) Formula to calculate the strain energy due to pure shear, if shear stress is given:**

$$U = \tau^2 V / (2G)$$

Where,  $\tau$  = Shear Stress

G = Shear modulus or Modulus of rigidity

V = Volume of the material.

**(vi) Formula to calculate the strain energy , if the moment value is given:**

$$U = M^2 L / (2EI)$$

Where, M = Bending moment

L = Length of the beam

E = Young's modulus

I = Moment of inertia

**(vii) Formula to calculate the strain energy , if the torsion moment value is given:**

$$U = T^2 L / (2GJ)$$

Where, T = Applied Torsion

L = Length of the beam

G = Shear modulus or Modulus of rigidity

J = Polar moment of inertia

**(viii) Formula to calculate the strain energy, if the applied tension load is given:**

$$U = P^2 L / (2AE)$$

Where,

P = Applied tensile load.

L = Length of the member

A = Area of the member

E = Young's modulus.

**(ix) Castigliano's first theorem:**



$$\delta = \partial U / \partial P$$

Where,  $\delta$  = Deflection,  $U$  = Strain Energy stored, and  $P$  = Load

**(x) Formula for deflection of a fixed beam with point load at centre:**

$$\delta = - wl^3 / 192EI$$

This deflection is  $\frac{1}{4}$  times the deflection of a simply supported beam.

**(xi) Formula for deflection of a fixed beam with uniformly distributed load:**

$$\delta = - wl^4 / 384EI$$

This deflection is 5 times the deflection of a simply supported beam.

**(xii) Formula for deflection of a fixed beam with eccentric point load:**

$$\delta = - wa^3 b^3 / 3 EIl^3$$

Stress due to

**Gradual Loading :**

$$\sigma = \frac{F}{A}$$

**Sudden Loading :**

$$\sigma = \frac{2F}{A}$$

**Impact Loading:**

$$\sigma = \frac{P}{A} \left( 1 + \sqrt{1 + \frac{2AEH}{PL}} \right)$$

Deflection

$$\text{If } \Delta_{st} = \frac{PL}{AE}$$

$$\Delta = \Delta_{st} + \sqrt{(\Delta_{st})^2 + 2h\Delta_{st}}$$

$$\text{If } h \text{ is very small than } \Delta \approx \sqrt{2h\Delta_{st}}$$

Thermal Stress,

$$\Delta L = \alpha L \Delta T$$

$$\sigma = \alpha E \Delta T$$

When bar is not totally free to expand and can be expand free by “a”

$$\sigma = E \alpha \Delta T - \frac{aE}{L}$$

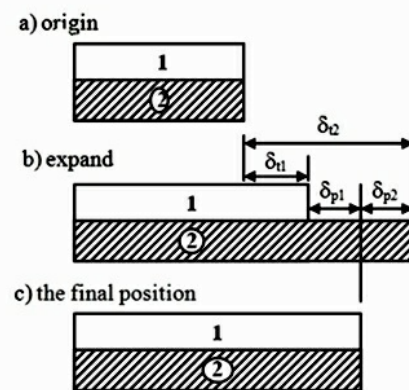
Temperature Stresses in Taper Bars:-

$$\text{Stress} = \alpha L \Delta T = \frac{4PL}{\pi d_1 d_2 E}$$

Temperature Stresses in Composite Bars

$$\begin{aligned} \delta t_2 &= \delta_{t1} + \delta_{p1} + \delta_{p2} \\ \delta t_2 - \delta_{t1} &= \delta_{p1} + \delta_{p2} \\ \Delta t (\alpha_1 L_1 - \alpha_2 L_2) &= P \left( \frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} \right) \end{aligned}$$

$$P = \frac{\Delta t (\alpha_2 - \alpha_1)}{\left( \frac{1}{A_1 E_1} + \frac{1}{A_2 E_2} \right)}$$



**Hooke's Law (Linear elasticity):**

Hooke's Law stated that within elastic limit, the linear relationship between simple stress and strain for a bar is expressed by equations.

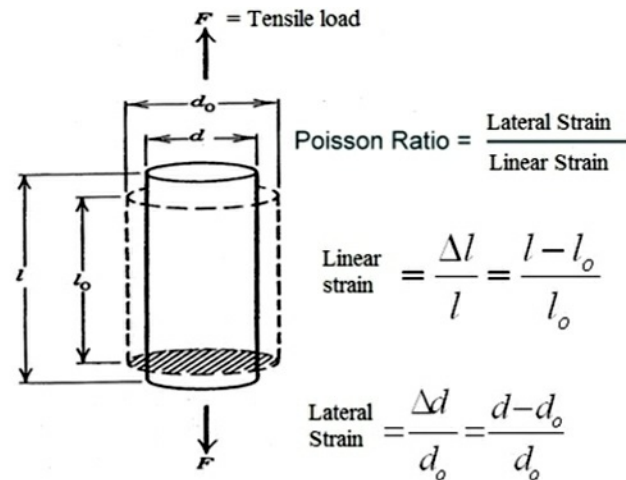
$$\sigma \propto \epsilon$$

$$\sigma = E \epsilon$$

$$\frac{P}{A} = E \frac{\Delta L}{L}$$

Where, E = Young's modulus of elasticity P = Applied load across a cross-sectional area  $\Delta l$  = Change in length l = Original length

**Poisson's Ratio**



## Volumetric Strain:

$$\text{Volumetric Strain} = \frac{\text{Change in volume}(\delta V)}{\text{Original Volume}(V)}$$

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_3}{E}$$

$$\epsilon_3 = \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$\text{Further Volumetric strain} = \epsilon_1 + \epsilon_2 + \epsilon_3$$

$$= \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{E} - \frac{2\mu(\sigma_1 + \sigma_2 + \sigma_3)}{E}$$

$$= \frac{(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu)}{E}$$

hence the

$\text{Volumetric strain} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu)}{E}$
---

## Relationship between E, G, K and $\mu$ :

### Modulus of rigidity:

$$\text{Modulus of rigidity, } G = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\gamma}$$

### Bulk modulus:

$$K = \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$$

$$K = \frac{dp}{dv} = -V \frac{dp}{dv}$$

Negative sign shows decrease in volume.

## Shear stress in Rectangular Beam:

### Compound Stresses

- Equation of Pure Bending

$$\frac{\sigma_a}{y} = \frac{M}{I} = \frac{E}{R}$$

- Section modulus

$$z = \frac{I}{y_{\max}} \rightarrow \frac{M}{I} = \frac{\sigma}{y}$$

$$M = \sigma_{\max} \frac{I}{y_{\max}} \rightarrow M = \sigma_{\max} z$$

- Shearing stress

$$T = \frac{VA\bar{y}}{Ib}$$

Where,

V= Shearing force

$A\bar{y}$  = First moment of area

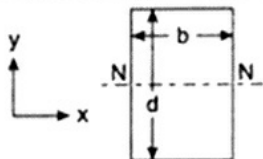
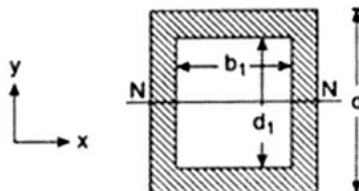
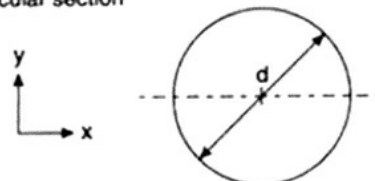
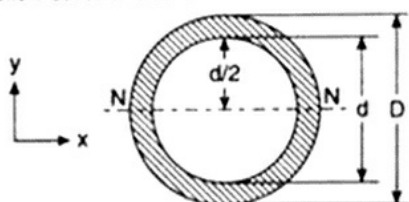
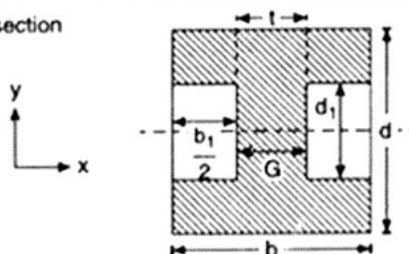

$$T_{\max} = \frac{3V}{2A}$$

$$T_{\max} = 1.5T_{\text{avg}}$$

### Shear Stress Circular Beam

$$T_{\max} = \frac{4V}{3A} = \frac{4}{3}T_{\text{avg}}$$

## Moment of Inertia and Section Modulus

Table 11.2.1			
Type of section	Moment of Inertia	$y_{\max}$	Section modulus (Z)
Rectangle or parallelogram 	$I_{xx} = \frac{bd^3}{12}$ $I_{yy} = \frac{db^3}{12}$	$\frac{d}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{bd^2}{6}$ $Z_{yy} = \frac{db^2}{6}$
Hollow rectangular section 	$I_{xx} = \frac{bd^3}{12} - \frac{b_1d_1^3}{12}$ $I_{yy} = \frac{db^3}{12} - \frac{d_1b_1^3}{12}$	$\frac{d}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{1}{6d}(bd^3 - b_1d_1^3)$ $Z_{yy} = \frac{1}{6b}(db^3 - d_1b_1^3)$
Circular section 	$I_{xx} = \frac{\pi}{64} d^4$ $I_{yy} = \frac{\pi}{64} d^4$	$\frac{d}{2}$ $\frac{d}{2}$	$Z_{xx} = \frac{\pi}{32} d^3$ $Z_{yy} = \frac{\pi}{32} d^3$
Hollow circular section 	$I_{xx} = I_{yy} = I$ $I_{yy} = \frac{\pi}{64} (D^4 - d^4)$	$\frac{D}{2}$	$Z_{xx} = Z_{yy} = Z$ $Z = \frac{\pi}{32D} (D^4 - d^4)$
I-section 	$I_{xx} = \frac{bd^3}{12} - \frac{b_1d_1^3}{12}$ $I_{yy} = \frac{db^3}{12} - \frac{d_1b_1^3}{12}$ or $I_{xx} = \frac{1}{12} (bd^3 - (b-t)d_1^3)$	$\frac{d}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{1}{6d}(bd^3 - b_1d_1^3)$ $Z_{yy} = \frac{1}{6b}(db^3 - d_1b_1^3)$
Triangle 	$I_G = \frac{bh^3}{36}$	$\frac{2}{3}h$	$Z_G = \frac{bh^2}{24}$

- Direct Stress

$$\sigma = \frac{P}{A}$$

where P = axial thrust, A = area of cross-section

- **Bending stress**

$$\sigma_b = \frac{My}{I}$$

where M = bending moment, y- distance from neutral axis, I = moment of inertia.

- **Torsional Shear Stress**

$$\tau = \frac{Tr}{J}$$

where T = torque, r = radius of shaft, J = polar moment of inertia.

- **Equivalent Torsional Moment**

$$\sqrt{M^2 + T^2}$$

- **Equivalent Bending Moment**

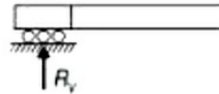
$$M + \sqrt{M^2 + T^2}$$

**Support:** Supports are used to provide suitable reaction (Resisting force) to beam or anybody. Following types of support are used.

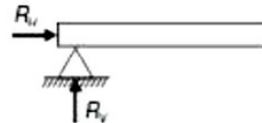
1. Simple support



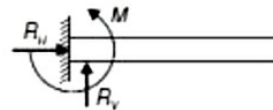
2. Roller support



3. Hinged (Pin) support



4. Fixed support

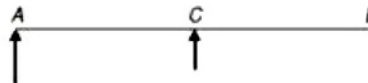


## Types of Beams:

1. Simply supported beams



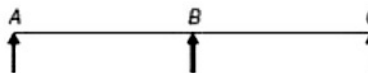
2. Over hanging beam



3. Cantilever beams

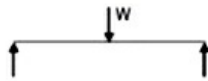


4. Continuous beams

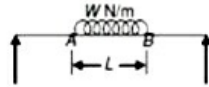


## Types of Loads:

1. Point load



2. Uniformly distributed load (UDL)



Value of UDL =  $w \times L$  KN point of application  $\rightarrow$  mid point of AB

3. Uniformly varying load (UVL)



Value of UVL =  $\frac{1}{2} \times W \times L$  KN point of application = CG of triangle formed

$$\Rightarrow \frac{2}{3} L \text{ from A, } \frac{L}{3} \text{ from B}$$

Shear force and Bending Moment Relation  $\frac{dv}{dx} = -M$

Load	0 	0 	Constant 
Shear	Constant 	Constant 	Linear 
Moment	Linear 	Linear 	Parabolic 
Load	0 	Constant 	Linear 
Shear	Constant 	Linear 	Parabolic 
Moment	Linear 	Parabolic 	Cubic 

## Euler's Buckling Load

$$P_{\text{Critical}} = \frac{\pi^2 EI}{l_{\text{equi}}^2}$$



For both end hinged  $l_{equi} = l$

For one end fixed and other free  $l_{equi} = 2l$

For both end fixed  $l_{equi} = l/2$

For one end fixed and other hinged  $l_{equi} = l/\sqrt{2}$

## Slenderness Ratio ( $\lambda$ )

$$\lambda = \frac{L_0}{r_{\min}}$$

$L_0$  = Effective length

$$r_{\min} = \sqrt{\frac{I_{\min}}{A}}$$

$r_{\min}$  = Least radius of gyration

## Rankine's Formula for Columns

$$\frac{1}{P_R} = \frac{1}{P_{CS}} + \frac{1}{P_E}$$

- $P_R$  = Crippling load by Rankine's formula
- $P_{CS} = \sigma_{CS} A$  = Ultimate crushing load for column
- $P_E = \frac{\pi^2 EI}{l_{\text{eff}}^2}$  Crippling load obtained by Euler's formula

## Deflection in different Beams

### BEAM BENDING

$L$ = overall length $W$ = point load, $M$ = moment $w$ = load per unit length	End Slope	Max Deflection	Max bending moment
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$	$M$
	$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$	$WL$
	$\frac{wL^3}{6EI}$	$\frac{wL^4}{8EI}$	$\frac{wL^2}{2}$
	$\frac{ML}{2EI}$	$\frac{ML^2}{8EI}$	$M$
	$\frac{WL^2}{16EI}$	$\frac{WL^3}{48EI}$	$\frac{WL}{4}$
	$\frac{wL^3}{24EI}$	$\frac{5wL^4}{384EI}$	$\frac{wL^2}{8}$
	$\theta_B = \frac{Wac^2}{2LEI}$ $\theta_A = \frac{L+b}{L+a} \theta_B$	$\frac{Wac^3}{3LEI}$ (at position c)	$\frac{Wab}{L}$ (under load)

## Torsion:

$$\frac{\tau_1}{r} = \frac{T}{J} = \frac{G\theta}{L}$$

Where,

T = Torque,

J = Polar moment of inertia

G = Modulus of rigidity,

$\theta$  = Angle of twist

L = Length of shaft,

### Total angle of twist

$$\phi = \frac{Tl}{GJ}$$

GJ = Torsional rigidity

GJ/l = Torsional stiffness

l/GJ = Torsional flexibility

EA/l = Axial stiffness

l/EA = Axial flexibility

### Moment of Inertia About polar Axis

#### 1. Moment of Inertia About polar Axis

$$J = \frac{\pi d^4}{32}, \quad T_{\max} = \frac{16T}{\pi d^3}$$

#### 2. For hollow circular shaft

$$J = \frac{\pi}{32}(d_o^4 - d_i^4)$$

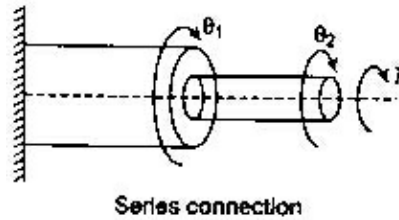
### Compound Shaft

- Series connection

$$\theta = \theta_1 + \theta_2$$

$$T = T_1 = T_2$$

$$\theta = \frac{TL_1}{G_1J_1} + \frac{TL_2}{G_2J_2}$$

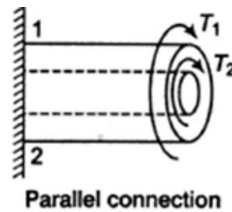


Where,

$\theta_1$  = Angular deformation of 1<sup>st</sup> shaft

$\theta_2$  = Angular deformation of 2<sup>nd</sup> shaft

- **Parallel Connection**



$$\theta_1 = \theta_2$$

$$T = T_1 + T_2$$

$$\frac{T_1 L}{G_1 J_1} = \frac{T_2 L}{G_2 J_2}$$

## Strain Energy in Torsion

$$U = \frac{1}{2} T \theta = \frac{1 T^2 L}{4 G J}$$

For solid shaft,

$$U = \frac{T^2}{4G} \times \text{Volume of shaft}$$

For hollow shaft,

$$U = \frac{T^2}{4G} \left( D^2 + \frac{d^2}{D^2} \right) \times \text{Volume of shaft}$$

### Thin Cylinder

- **Circumferential Stress /Hoop Stress**

$$\sigma_h = \frac{pd}{2t} \rightarrow \sigma_h = \frac{pd}{2t\eta}$$

$\eta$  = Efficiency of joint

- **Longitudinal Stress**

$$\sigma_t = \frac{pd}{4t} \rightarrow \sigma_t = \frac{pd}{4t\eta}$$

- **Hoop Strain**

$$\varepsilon_h = \frac{pd}{4tE} (2 - \mu)$$

- **Longitudinal Strain**

$$\varepsilon_L = \frac{pd}{4tE} (1 - 2\mu)$$

- **Ratio of Hoop Strain to Longitudinal Strain**

$$\varepsilon_v = \frac{pd}{4tE} (5 - 4\mu)$$

### Stresses in Thin Spherical Shell

- **Hoop stress/longitudinal stress**

$$\sigma_L = \sigma_h = \frac{pd}{4t}$$

- **Hoop stress/longitudinal strain**

$$\varepsilon_L = \varepsilon_h = \frac{pd}{4tE} (1 - \mu)$$

- **Volumetric strain of sphere**

$$\varepsilon_v = \frac{3pd}{4tE} (1 - \mu)$$

### Thickness ratio of Cylindrical Shell with Hemisphere Ends

$$\frac{t_2}{t_1} = \frac{1 - \nu}{2 - \nu}$$

Where  $\nu$  = Poisson Ratio

## THEORY OF MACHINES

### Instantaneous Centre of Velocity (I-centre)

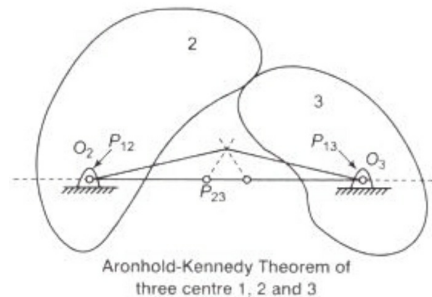
- The instantaneous centre of velocity can be defined as a point which has no velocity with respect to the fixed link.

### Centro

- Instantaneous centre is also called Centro
- Primary Centro One which can be easily located by a mere observation of the mechanism.
- Secondary Centro Centros that cannot be easily located.

### Aronhold-Kennedy Theorem of Three Centre

- It states that if three bodies are in relative motion with respect to one another, the three relative instantaneous centres of velocity are collinear.



### Number of Centros in a Mechanism

- For a mechanism of  $n$  links, the number of Centros (Instantaneous centre)  $N$  is

$$N = \frac{1}{2}n(n - 1)$$

### Linkages are the basic building blocks of all mechanisms

- Links: rigid member having nodes.
- Node: attachment points.
- Binary link: 2 nodes
- Ternary link: 3 nodes
- Quaternary link: 4 nodes

- Joint: connection between two or more links (at their nodes) which allows motion;
- (Joints also called kinematic pairs)

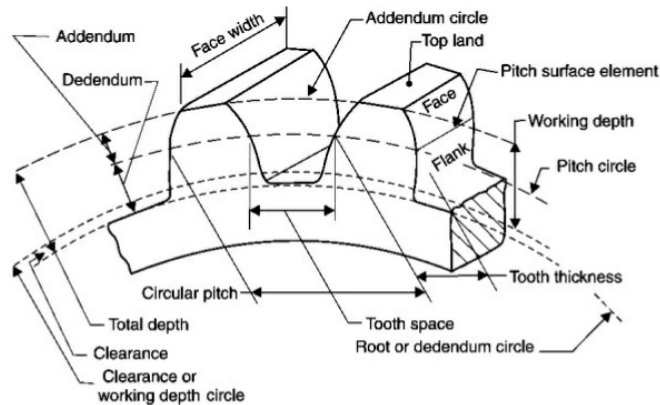
## D'Alembert's Principle and Inertia Forces

D'Alembert's principle states that the reverse-effective forces and torques and the external forces and torques on a body together give statical equilibrium.

$$F + (-ma_G) = 0$$

$$T_{eG} + (-I_G a) = 0$$

## Gear Terminology



### Circular Pitch (p):

It is a distance measured along the circumference of the pitch circle from a point on one tooth to the corresponding point on the adjacent tooth.

$$p = \frac{\pi d}{T}$$

### Diametrical Pitch (P)

It is the number of teeth per unit length of the pitch circle diameter in inches.

$$P = \frac{T}{d}$$

### Module (m)

It is the ratio of pitch diameter in mm to the number of teeth. The term is used SI

units in place of diametrical pitch.

$$m = \frac{d}{T}$$

$$p = \pi m$$

### **Gear Ratio (G)**

It is the ratio of the number of teeth on the gear to that on the pinion.

$$G = \frac{T}{t}$$

where, T = number of teeth on the gear

t = number of teeth on the pinion

### **Velocity Ratio**

The velocity ratio is defined as the ratio of the angular velocity of the follower to the angular velocity of the driver gear.

$$VR = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$$

### **Gear Train**

A gear train is a combination of gears used to transmit motion from one shaft to

another. Gear trains are used to speed up or stepped down the speed of driven shaft. The following are main types of gear trains.

#### **Simple Gear Train**

Series of gears, capable of receiving and transmitting motion from one gear to

another is called a simple gear train.

#### **Train value**

$$\frac{N_3}{N_2} = \frac{T_1}{T_3}$$

Number of teeth on driving gears

Number of teeth on driver gears

#### **Speed ratio**

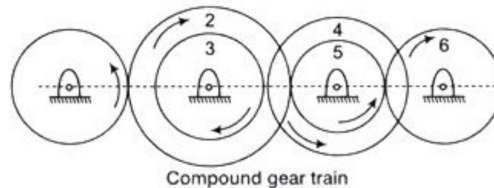
$$\frac{1}{\text{Train value}}$$

## Gears-and-gear-trains

The intermediate gears have no effect on the speed ratio and therefore they are known as idlers.

### Compound Gear Train

When a series of gears are connected in such a way that two or more gears rotate about an axis with the same angular velocity.



### Train value

$$= \frac{\text{Product of number of teeth on driving gears}}{\text{Product of number of teeth on driven gears}}$$

### Planetary or Epicyclic Gear Train

- A gear train having a relative motion of axes is called a planetary or an epicyclic gear train. In an epicyclic train, the axis of at least one of the gears also moves relative to the frame.

- If the arm a is fixed the wheels S and P constitute a simple train. However if the wheel S is fixed so that arm a can rotate about the axis of S. The P would be moved around S therefore it is an epicyclic train

### Flywheel

A flywheel is used to control the variations in speed during each cycle of an operation. A flywheel acts as a reservoir of energy which stores energy during the period when the supply of energy is more than the requirement and releases the energy during the period when the supply energy is less than the requirement.

### Maximum fluctuation of energy (e) ,



$$e = (\Delta KE) = \frac{1}{2}(\omega_{\max}^2 - \omega_{\min}^2)$$

$$e = I\omega(\omega_1 - \omega_2) \quad \therefore \left(\omega = \frac{\omega_1 + \omega_2}{2}\right)$$

$$e = I\omega^2 k_s \quad \therefore \left[k_s = \frac{\omega_1 - \omega_2}{2}\right]$$

$$k_s = \frac{e}{\frac{1}{2}I\omega^2 \times 2}, k_s = \frac{e}{2E}$$

where,

$\omega_{\max}$  and  $\omega_{\min}$  are the maximum and minimum angular speed respectively.

$E$  = kinematic energy of the flywheel at mean speed.

### Flywheel in Punching Press

- Generally, flywheel is used to reduce fluctuation of speed where the load on the
- crank shaft constant while the applied torque varies. However, the flywheel can also be used to reduce fluctuation of speed when the torque is constant but load varies during the cycle *e.g.* , in punching press in riveting machine.
- Let  $E$  be energy required for one punch energy supplied to crank shaft from the motor during punching.

$$= E \left( \frac{\theta_2 - \theta_1}{2\pi} \right)$$

### Governors

The function of a governor is to maintain or regulate the speed of an engine within specified limits whenever there is variation of load.

### Types of Governors

The broadly classification of the governors are given below.

#### Centrifugal Governor

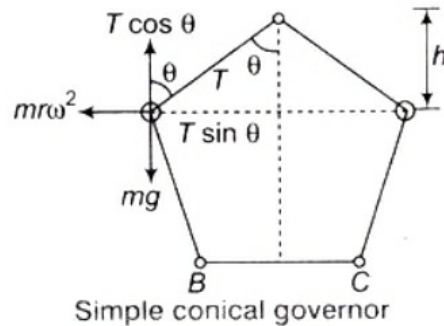
In this type of governor, the action of governor depends upon the centrifugal effects produced by the masses of two balls.

#### Inertia Governor

In this type of governor, positions of the balls are effected by the forces set up by an angular acceleration or deceleration of the given spindle in addition

to centrifugal forces on the balls.

## Pendulum Type Watt Governor



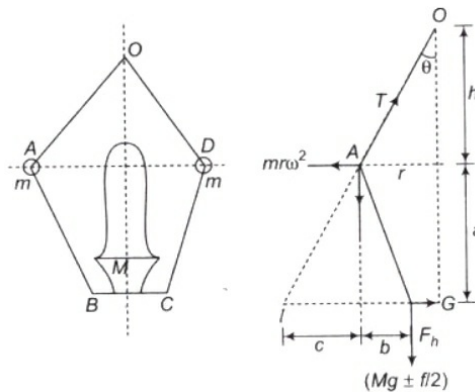
h=height of each ball

$$h = \frac{g}{w^2}$$

## Porter Type Governor

Porter governor can be shown as

$$h = \frac{895}{N^2} \left[ \frac{2mg + (mg \pm f)(1 + k)}{2mg} \right]$$



## Wilson Hardness Governor

- Main spring force

$$F'_{s_2} - F'_{s_1} = 4s(r_2 - r_1)$$

- Net auxiliary spring force

$$F'_{s_2} - F'_{s_1} = h_2 S_a = \left( h_1 \frac{y}{x} \right) \frac{s}{a}$$

$$(r_2 - r_1) \frac{by}{ax} S_a$$

## Pickering Governor

$$f = \frac{m(e + f)\omega^2}{192 EI}$$

Where, E = modulus of elasticity of the spring material

I = moment of inertia of the cross-section of the spring about neutral axis.

## Sensitiveness of a Governor

- The governor is said to be sensitive when it readily responds to a small change of speed.
- Sensitiveness of a governor is defined as the ratio of difference between the maximum and minimum speeds to the mean equilibrium speed.

$$\text{Sensitiveness} = \frac{\text{range of speed}}{\text{mean speed}}$$

$$2 \frac{N_2 - N_1}{N_1 + N_2}$$

where, N = mean speed

N1 = minimum speed corresponding to full load conditions

N2 = maximum speed corresponding to no load conditions.

## Hunting

- Sensitiveness of a governor is a desirable quality. However, if a governor is too sensitive, it may fluctuate continuously. This phenomenon of fluctuation is known as hunting.

## Isochronism

- If a governor is at equilibrium only for a particular speed, it is called isochronous governor, for which we can say that an isochronous governor is infinitely sensitive.

$$\frac{dF}{dr} = m\omega^2$$

## Stability

- A governor is said to be stable if it brings the speed of the engine to the required value and there is not much hunting. Obviously, the stability and sensitivity are two opposite characteristics.

## Cam

- A cam is a mechanical member used to impart desired motion (displacement) to a follower by direct contact (either point or line contact).
- Cam mechanisms belong to higher pair mechanism.
- A driver member known as cam.
- A driven member called the follower.
- A frame is one which supports the cam and guides the follower.

### Key Points for Cams

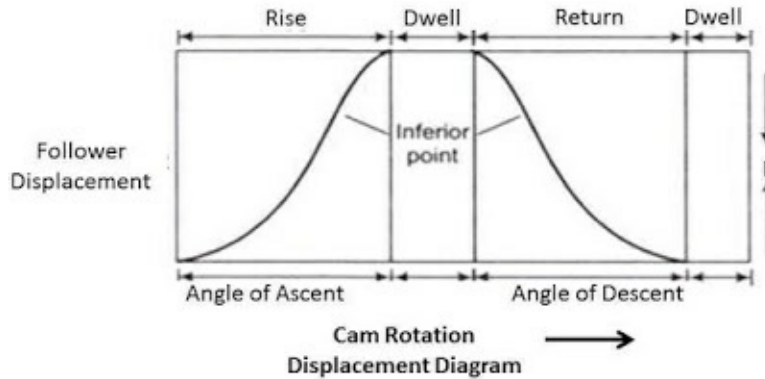
For a roller follower, the trace point is at the roller centre.

- For a flat-face follower, it is at the point of contact between the follower and cam surface when the contact is along the base circle of the cam.
- During a complete rotation, the pressure angle varies from its maximum to its minimum value.
- The greater the pressure angle, the higher will be side thrust and consequently the chances of the translating follower jamming in its guide will increase.
- It is not desirable to increase the pressure angle.

### Follower Displacement Diagram

The following terms are used with reference to the angular motion of the cam

- **Angle of Ascent ( $\phi_a$ ):** It is the angle through which the cam turns during the time the follower rises.
- **Angle of Dwell ( $\phi$ )** Angle of dwell is the angle through which the cam turns while the follower remains stationary at the highest or the lowest position.



## Balancing

- Balancing is defined as the process of designing a machine in which unbalance force is minimum.
- If the centre of mass of rotating machines does not lie on the axis of rotation, the inertia force is given by  $F_1 = m_1 \omega_2 e_1$

## Static Balancing

If a shaft carries a number of unbalanced masses such that the centre of mass of the system is said to be statically balanced.

## Dynamic Balancing

A system of rotating masses in dynamic balance when there does not exist any resultant centrifugal force as well as resultant couple.

## Vibrations

### Time Period for Simple Pendulum

**Vibrations:** Vibration refers to mechanical oscillations about an equilibrium point. In its simplest form, vibration can be considered to be the oscillation or repetitive motion of an object around an equilibrium position.

Vibrations or mechanical oscillations are of many types as given below:

- **Free Vibration** (Natural vibration) Vibration over an interval of time during which the system is free from excitation is known as free vibration.
- **Damped and Undamped Vibration:** Energy of a vibrating system is gradually dissipated by friction and other resistance.
- **Forced Vibration** When a repeated force continuously acts on a system, the vibrations are said to be forced.
- **Linear Vibration :** If all the basic components of a vibratory system- the spring, the mass, and the damper, behave linearly, the resulting

vibration is known as linear vibration. The differential equations that govern the behaviour of vibratory linear systems are linear. Therefore, the principle of superposition holds.

- **Nonlinear vibration:** If however, however, any of the basic components behave nonlinearly, the
- **Harmonic Vibration:** Vibration in which the motion is a sinusoidal function of time.
- **Fundamental Vibration:** Harmonic component of a vibration with the lowest frequency.
- **Steady State Vibration:** When the particles of the body move in steady state condition or continuing period vibration is called steady state vibration.
- **Transient Vibration:** Vibratory motion of a system other than steady state.
- **Longitudinal Vibration:** Vibration parallel to the longitudinal axis of a member.
- **Transverse Vibration:** Vibration in a direction perpendicular to the longitudinal axis or central plane of a member.
- **Torsional Vibration:** Vibration that involves torsion of a member.

**Mode of Vibration:** Configuration of points of a SHM is called the mode of vibration.

**Natural Frequency:** Frequency of free simple harmonic vibration of an undamped linear system.

**Time Period:** Time taken for one oscillation is called time period.

**Simple Pendulum:** If time period of the pendulum is 1s, then pendulum is called simple pendulum.

$$\left| \frac{\theta}{\alpha} \right| = \frac{1}{g} \left( \text{where, } \alpha = \frac{d^2\theta}{dt^2} \right)$$

Time period is given by

$$T = 2\pi \sqrt{\left| \frac{\theta}{\alpha} \right|}$$

$$T = 2\pi \sqrt{\frac{1}{g}}$$

(for small amplitude  $\sin \theta \approx \theta$ )

$$T = 2\pi \sqrt{\frac{1}{g} \left( 1 + \frac{\theta_0^2}{16} \right)}$$

[large amplitude ( $\theta_0$ )]

## Free Vibration of Damped One Degree-of-Freedom Systems

Damping factor:

$$C_0 = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{km}}$$

**The system response when under-damped:  $\xi < 1$**

$$x(t) = e^{-\xi\omega_n t} \left( x_0 \cos\omega_d t + \frac{\dot{x}_0 + \xi_0\omega_n x_0}{\omega_d} \sin\omega_d t \right)$$

**The system response when critically damped:  $\xi = 1$**

$$x(t) = e^{-\omega_n t} (x_0 + \dot{x}_0 + \omega_n x_0)t$$

**The system response when over-damped:  $\xi > 1$**

$$x(t) = C_1 e^{(-\xi + \sqrt{\xi^2 - 1})\omega_n t} + C_2 e^{(-\xi - \sqrt{\xi^2 - 1})\omega_n t}$$

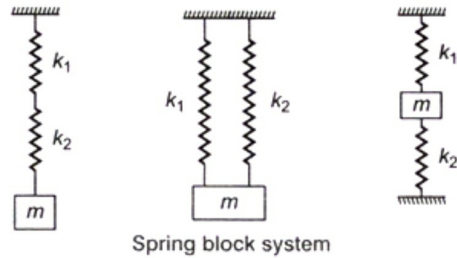
C1 and C2 are the constants that are lengthy in closed-form. They can be found numerically by the initial conditions.

### • Spring Block System

$$ma = -kx$$

$$\left| \frac{x}{a} \right| = \frac{m}{k} \Rightarrow T = 2\pi \sqrt{\frac{m}{k}}$$

- **Equivalent force constant ( $k$ )** is given by

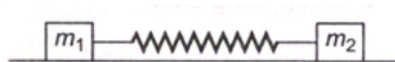


$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

$$k = k_1 + k_2$$

If spring has a mass  $m_s$  and a mass  $m$  is suspended from it,



$$T = 2\pi \sqrt{\frac{m + \frac{m_s}{3}}{k}}$$

### Mass spring system

$$m\ddot{x} + kx = 0$$

$$\omega_n = \sqrt{\frac{k}{m}} : \text{Natural Frequency}$$

$$T = \frac{2\pi}{\omega_n} : \text{Period of motion}$$

### Damping

Any influence which tends to dissipate the energy of a system.

**Damping Factor or Damping Ratio:** It is the ratio of actual to critical damping coefficient.

$$\varepsilon = \sqrt{\frac{\left(\frac{c}{2m}\right)^2}{\frac{k}{m}}} = \frac{c}{2\sqrt{mk}}$$



- $\zeta = 1$  , the damping is known as critical under critical damping condition Critical damping coefficient

$$C_c = 2\varepsilon\sqrt{mk}$$

$$C_c = 2m\omega_n$$

$\xi < 1$  i.e ., system is underdamped.

$$\alpha_{1,2} = (-\varepsilon \pm i\sqrt{1-\varepsilon^2})\omega_n$$

$$x = Ae^{(-\varepsilon \pm i\sqrt{1-\varepsilon^2})\omega_n t} + Be^{(-\varepsilon - i\sqrt{1-\varepsilon^2})\omega_n t}$$

- Damped frequency

$$\omega_d = \sqrt{1-\varepsilon^2}\omega_n$$

### Logarithmic Decrement

- In an underdamped system, arithmetic ratio of two successive oscillations is called logarithmic decrement (constant).  
Since,

$$\frac{X_n}{X_{n+1}} = e^{\varepsilon\omega_n T_d} = \frac{X_0}{X_1} = \frac{X_1}{X_2} = \frac{X_2}{X_3}$$

### Logarithmic decrement ,

$$\delta = \ln\left(\frac{X_n}{X_{n+1}}\right) = \ln e^{\varepsilon\omega_n T_d} = \varepsilon\omega_n T_d$$

$$\delta = \frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}}$$

$$\delta = \frac{1}{n} \ln\left(\frac{X_0}{X_n}\right)$$

### Forced Vibration

- Amplitude of the steady state response is given by case of steady state response first term zero ( $e^{-\infty} = 0$ ).

$$A = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$A = \frac{\frac{F_0}{k}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\varepsilon \frac{\omega}{\omega_n}\right)^2}} \tan\phi = \left(\frac{c\omega}{k - m\omega^2}\right)$$

### **Magnification Factor**

- Ratio of the amplitude of the steady state response to the static deflection under the action force  $F_0$  is known as magnification factor.

$$MF = \frac{\frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}}{\frac{F_0}{k}} = \frac{K}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$= \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\varepsilon \frac{\omega}{\omega_n}\right)^2}}$$

### **Resonance**

- When the frequency of external force(driving frequency) is equal to the natural frequency of a vibrating body, the amplitude of vibration becomes excessively large. This phenomenon is known as resonance.

### **Critical Speed**

- Critical or whirling or whipping speed is the speed at which the shaft tends to vibrate violently in transverse direction.

### **Critical speed essentially depends on**

- The eccentricity of the C.G of the rotating masses from the axis of rotation of the shaft.
- Diameter of the disc
- Span (length) of the shaft, and
- Type of supports connections at its ends

## **MACHINE DESIGN**

### **Static Load**

- A static load is a mechanical force applied slowly to an assembly or object. Load does not change in magnitude and direction and normally increases gradually to a steady value.
- This force is often applied to engineering structures on which people's safety depends on because engineers need to know the maximum force a structure can support before it will collapse.

### **Dynamic load**

1. A dynamic load, results when loading conditions change with time. Load may change in magnitude for example, traffic of varying weight passing a bridge.
2. Load may change in direction, for example, load on piston rod of a double acting cylinder.
3. Vibration and shock are types of dynamic loading.

### **Factor of safety (F.O.S):**

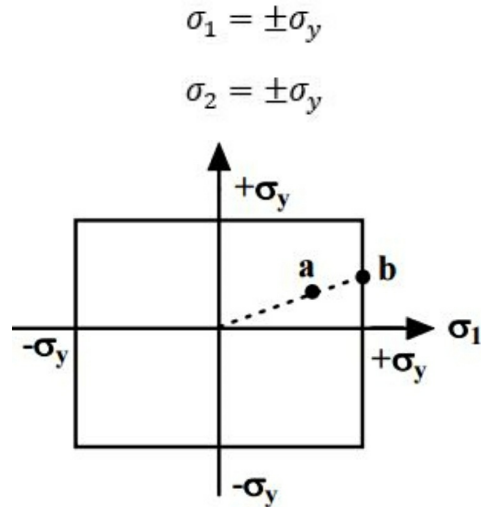
1. The ratio of ultimate to allowable load or stress is known as factor of safety i.e. The factor of safety can be defined as the ratio of the material strength or failure stress to the allowable or working stress.
2. The factor of safety must be always greater than unity. It is easier to refer to the ratio of stresses since this applies to material properties.

**F.O.S = failure stress / working or allowable stress**

### **Static Failure Theories**

#### **Maximum Principal Stress Theory (Rankine Theory):**

1. The principal stresses  $\sigma_1$  (*maximum principal stress*) ,  $\sigma_2$  (*minimum principal stress*) or  $\sigma_3$  exceeds the yield stress, yielding would occur.
2. For two dimensional loading situation for a ductile material where tensile and compressive yield stress are nearly of same magnitude:



Yielding occurs when the state of stress is at the boundary of the rectangle.

**Maximum Principal Strain Theory (St. Venant's theory):**

- If  $\varepsilon_1$  and  $\varepsilon_2$  are maximum and minimum principal strains corresponding to  $\sigma_1$  and  $\sigma_2$ , in the limiting case:

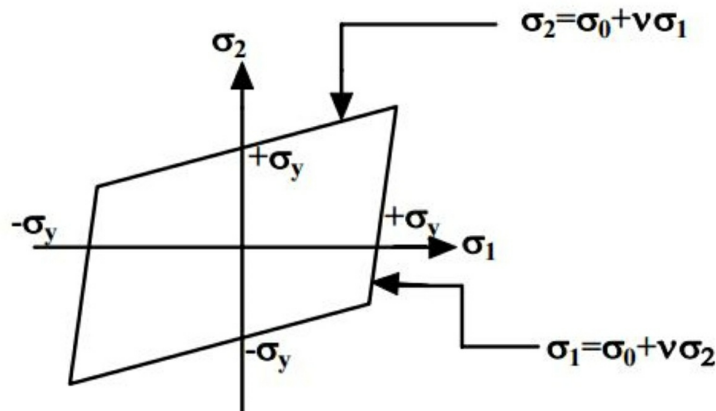
$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \quad |\sigma_1| \geq |\sigma_2|$$

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \quad |\sigma_2| \geq |\sigma_1|$$

$$E\varepsilon_1 = \sigma_1 - \nu\sigma_2 = \pm\sigma_0$$

$$E\varepsilon_2 = \sigma_2 - \nu\sigma_1 = \pm\sigma_0$$

Boundary of a yield surface in Maximum Strain Energy Theory is given below



**Maximum Shear Stress Theory (Tresca Theory):**

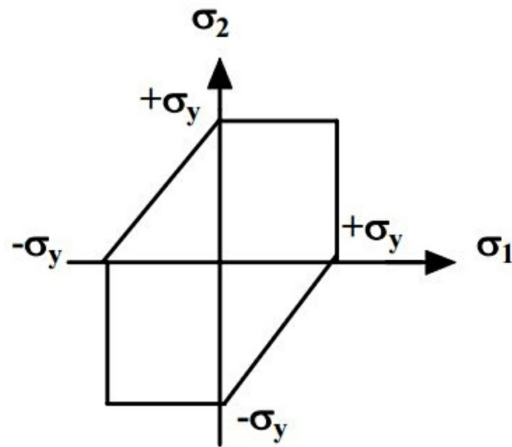
- At the tensile yield point  $\sigma_2 = \sigma_3 = 0$  and thus maximum shear stress is  $\sigma_y/2$ .

$$\sigma_1 - \sigma_2 = \pm \sigma_y$$

$$\sigma_2 - \sigma_3 = \pm \sigma_y$$

$$\sigma_3 - \sigma_1 = \pm \sigma_y$$

- Yield surface corresponding to maximum shear stress theory in biaxial stress situation is given below :



### Maximum strain energy theory ( Beltrami's theory):

- Failure would occur when the total strain energy absorbed at a point per unit volume exceeds the strain energy absorbed per unit volume at the tensile yield point.

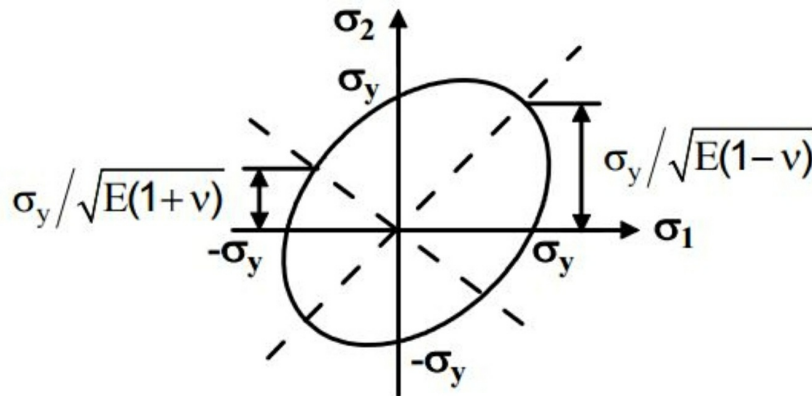
$$\frac{1}{2}(\sigma_1 \epsilon_1 + \sigma_2 \epsilon_2 + \sigma_3 \epsilon_3) = \frac{1}{2} \sigma_y \epsilon_y$$

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1) = \sigma_y^2$$

$$\left(\frac{\sigma_1}{\sigma_y}\right)^2 + \left(\frac{\sigma_2}{\sigma_y}\right)^2 - 2\nu\left(\frac{\sigma_1 \sigma_2}{\sigma_y^2}\right) = 1$$

$$\left(\frac{\sigma_1}{\sigma_y}\right)^2 + \left(\frac{\sigma_2}{\sigma_y}\right)^2 - 2\nu\left(\frac{\sigma_1 \sigma_2}{\sigma_y^2}\right) = 1$$

Above equation results in Elliptical yield surface which can be viewed as:



### **Distortion energy theory (Von Mises yield criterion):**

- Yielding would occur when total distortion energy absorbed per unit volume due to applied loads exceeds the distortion energy absorbed per unit volume at the tensile yield point.

Total strain energy ET and strain energy for volume change EV can be given as:

$$E_T = \frac{1}{2}(\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3) \text{ and } E_V = \frac{3}{2} \sigma_{av} \varepsilon_{av}$$

$$E_d = E_T - E_V = \frac{2(1+V)}{6E}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1 \sigma_2 - \sigma_2 \sigma_3 - \sigma_3 \sigma_1)$$

At the tensile yield point,  $\sigma_1 = \sigma_y$ ,  $\sigma_2 = \sigma_3 = 0$  which gives,

$$E_{dy} = \frac{2(1+V)}{6E} \sigma_y^2$$

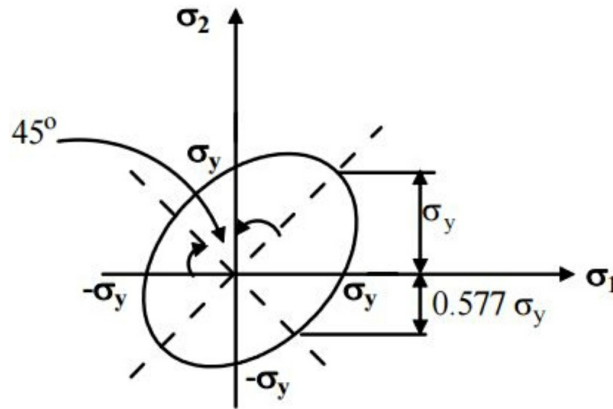
The failure criterion is thus obtained by equating  $E_d$  and  $E_{dy}$ , which gives

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2$$

In a 2-D situation if  $\sigma_3 = 0$ , so the equation reduces to,

$$\left(\frac{\sigma_1}{\sigma_y}\right)^2 + \left(\frac{\sigma_2}{\sigma_y}\right)^2 - \left(\frac{\sigma_1}{\sigma_y}\right)\left(\frac{\sigma_2}{\sigma_y}\right) = 1$$

1. This is an equation of ellipse and yield equation is an ellipse.
2. This theory is widely accepted for ductile materials.



### Cotter and Knuckle Joints

A cotter joint is a temporary fastening and is used to connect rigidly two co-axial rod or bars which are subjected to axial tensile or compressive forces.

### Socket and Spigot Cotter Joints

In a socket and spigot cotter joint, one end of the rods is provided with a socket type of end as shown in figure and the other end of the rod is inserted into a socket. The end

of the rod which goes into a socket is also called spigot.

## Failures in Socket and Spigot Cotter Joints

Failure Cases	Tensile Force
Failure of the rod in tension	$P = \frac{\pi}{4} \times d^2 \times \sigma_t$
Failure of spigot in tension across the weakest section	$P = \left[ \frac{\pi}{4} (d_2)^2 - d_1 \times t \right] \sigma_t$
Failure of the rod or cotter in crushing	$P = d_1 \times t \times \sigma_t$
Failure of the socket in tension across the slot	$P = \left\{ \frac{\pi}{4} [(d_1)^2 - (d_2)^2] - (d_1 - d_2) t \right\} \sigma_t$
Failure of cotter in shear	$P = 2b \times t \times \tau$
Failure of the socket collar in crushing	$P = (d_4 - d_2) t \times \sigma_c$
Failure of rod end in shear	$P = 2(d_4 - d_2) c \times \tau$
Failure of spigot collar in crushing	$P = 2a \times d_2 \times \tau$
Failure of the spigot collar in shearing	$P = \frac{\pi}{4} [(d_3)^2 - (d_2)^2] \sigma_c$
	$P = \pi d_2 \times t_1 \times \tau$

## Failures in Sleeve and Cotter Joints

Failure Cases	Tensile Force
Failure of the rod in tension	$P = \frac{\pi}{4} \times d^2 \times \sigma_t$
Failure of rod in tension across the weakest section	$P = \left[ \frac{\pi}{4} (d_2)^2 - d_2 \times t \right] \sigma_t$
Failure of the rod or cotter in crushing	$P = d_2 \times t \times \sigma_t$
Failure of sleeve in tension across the slot	$P = \left\{ \frac{\pi}{4} [(d_1)^2 - (d_2)^2] - (d_1 - d_2) t \right\} \sigma_t$
Failure of cotter in shear	$P = 2b \times t \times \tau$
Failure of rod end in shear	$P = 2a \times d_2 \times \tau$
Failure of sleeve end in shearing	$P = 2(d_1 - d_2) c \times \tau$

## Knuckle Joint

1. It is used to connect two rods whose axis either coincide or intersect and lie in one plane.
2. This joint generally found in the link of a cycle chain tie rod joint for roof truss, valve rod joint with eccentric rod tension link in bridge structure, lever and rod connection of various types.
3. It is sometimes also called forked pin joint.



## Failures in Knuckle Joint

Failure Cases	Tensile Force
Failure of the solid rod in tension	$P = \frac{\pi}{4} \times d^2 \times \sigma_t$
Failure of the knuckle pin in shear	$P = 2 \frac{\pi}{4} (d_2)^2 \tau$
Failure of the single eye or rod end in tension	$P = (d_2 - d_1) t \times \sigma_t$
Failure of the single eye or rod end in shearing	$P = (d_2 - d_1) t \times \tau$
Failure of forked end in tension	$P = (d_2 - d_1) 2t_1 \times \sigma_t$
Failure of the forked end in crushing	$P = d_1 \times 2t_1 \times \sigma_c$

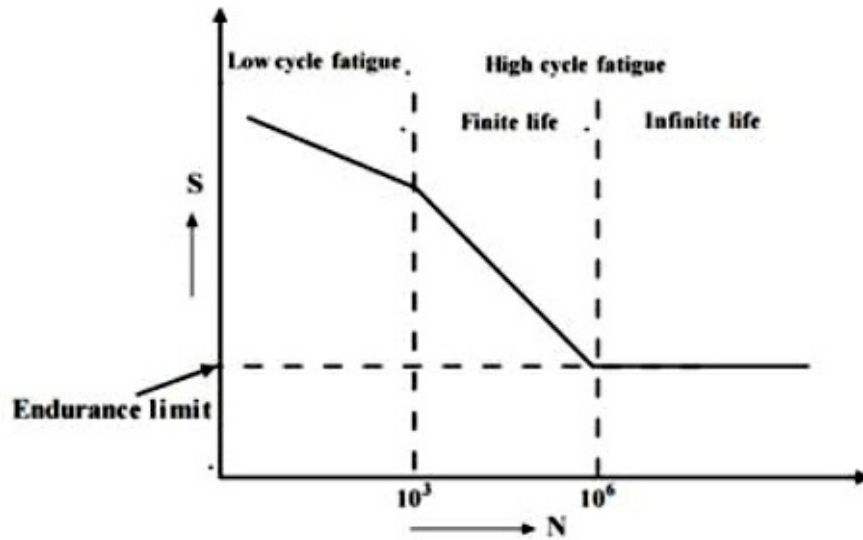
1. To connect the transmission shaft to rotating machine elements like pulley, gear, sprocket or flywheel.
2. Cotter and knuckle joints are not used for connect.

## Fatigue

1. Fatigue loading is primarily the type of loading which causes cyclic variations in the applied stress or strain on a component.
2. Variable loading due to: -Change in the magnitude of applied load Example: punching or shearing operations- -Change in direction of load application Example: a connecting rod - Change in point of load application Example: a rotating shaft.

## Fatigue Failure:

1. Machine elements subjected to fluctuating stresses usually fail at stress levels much below their ultimate strength and in many cases below the yield point of the material too.
2. These failures occur due to very large number of stress cycle and are known as fatigue failure.
3. Fatigue failures are influenced by
  - Nature and magnitude of the stress cycle
  - Endurance limit
  - Stress concentration
  - Surface characteristics



### Riveted joints:

1. There are two basic components of riveted joints:

Rivets

Two or more plates.

2. The popular materials for the rivets are: *Steel, Brass, Aluminium & Copper* as per the

requirement of the application for fluid tight joints the steel rivets are used.

### Welded Joints

3. It is a permanent joint.
4. When the two parts are joined by heating to a suitable temperature with or without

application of pressure.

### Welding Processes

1. Fusion Welding
2. Thermit Welding
3. Gas Welding
4. Electric Arc Welding
5. Forge Welding

### Types of Welded Joints:

## Lap Joint or Fillet Joint

1. In lap joint, overlapping the plate and welding the edge of the plates takes place in welding process.
2. The strength of different types of fillet joint can be given according to their welding process.

### Shear strength in parallel fillet weld ,

$$T = P/0.707hl \text{ or } P = 0.707h/\tau$$

where,  $P$  = Tensile force on the plates

$h$  = Leg of the weld

$l$  = Length of the weld

$\tau$  = Permissible shear stress

For double parallel fillet weld,

$$P = 1.414 h/\tau$$

### Strength of Transverse Fillet Weld

$$p = \text{Throat area Allowable tensile stress} \\ = 0.707 s \times l \times \sigma_t$$

### For double transverse fillet joint

$$P = 1.414 h l \sigma_t$$

## Special Cases of Fillet Welded Joint

### Circular Fillet Weld Subjected to Torsion

$$\tau = \frac{2T}{\pi t d^2}$$

Shear stress

$$\tau_{\max} = \frac{2.83T}{\pi h d^2}$$

where,  $T$  = Torque acting on rod

$h$  = size of weld

$t$  = Throat thickness

### Circular Fillet Weld Subjected to Bending Moment

- Bending stress:

$$\sigma_b = \frac{4M}{\pi t d^2}$$

$$\sigma_{b(max)} = \frac{5.66M}{\pi h d^2}$$

### Long Fillet Weld Subjected to Torsion

$$\tau = \frac{3T}{t l^2}$$

- **Shear stress:**

$$\tau_{max} = \frac{4.242T}{h l^2}$$

### Butt Joint

#### Strength of Butt Joint

For single V – butt joint,

$$P = t \times l \times \sigma_t$$

For double V-butt joint,

$$P = (t_1 + t_2) l \times \sigma_t$$

### Eccentric Loaded Welded Joints

When the shear and bending stresses are simultaneously present in a joint.

Maximum normal stress

$$\sigma_{t_{max}} = \frac{\sigma_b}{2} + \frac{1}{2} \sqrt{(\sigma_b)^2 + 4\tau^2}$$

Maximum shear stress

$$\tau_{max} = \frac{1}{2} \sqrt{(\sigma_b)^2 + 4\tau^2}$$

Direct or primary shear stress

$$\tau_1 = \frac{\text{Load}}{\text{Throat Area}} = \frac{P}{2tl}$$

$$\tau_1 = \frac{P}{1.414 hl}$$

### Strength of Bolted Joint

Maximum tensile stress in the bolt

$$\sigma_t = \frac{P}{\left(\frac{\pi}{4} d_c^2\right)}$$

where,  $d_c$  = Core diameter

$$\frac{\sigma_{yt}}{f_s} = \frac{P}{\frac{\pi}{4} d_c^2}$$

### **Torque Requirement for Bolt Tightening**

$$M_t = \frac{P_i d_m}{2} \left( \frac{\mu \sec \theta + \tan \alpha}{1 - \mu \sec \theta \tan \alpha} \right)$$

where,

$P_i$  = Pretension in bolt,  $d_m = 0.9 d$

$d$  = Nominal diameter

For ISO metric screw thread  $\theta = 30^\circ$

For ISO metric  $\alpha = 25^\circ$

### **Eccentric Load on Bracket with Circular Base**

If there are  $n$  number of bolts, then load in a bolt

$$w_{b_2} = \frac{2wL (R - r \cos \alpha)}{n(2R^2 + r^2)}$$

In above case when  $n = 4$

$$w_b = \frac{w.L (R - r \cos \alpha)}{2(2R^2 + r^2)}$$

### **Maximum load in bolt**

$$(w_{b_2})_{max} = \frac{2wL}{n} \left( \frac{R + r}{2R^2 + r^2} \right)$$

where,  $\cos \alpha = -1$

### **Factor of Safety (FOS) in Bolted Joints**

It is defined as the ratio of failure stress to allowable stress.

$$FOS = \frac{\text{Failure stress}}{\text{Allowable stress}}$$

For ductile material,

$$FOS = \frac{S_{yt}}{\sigma}$$

For brittle material,

$$FOS = \frac{S_{ut}}{\sigma}$$

where,  $S_{yt}$  = Yield strength of component material

$S_{ut}$  = Ultimate tensile stress of components material

$\sigma$  = Allowable stress.

### Stress concentration Factor

It is defined as the ratio of highest value of actual stress near discontinuity to nominal stress obtained by elementary equations for minimum cross-section. It is denoted by  $k_t$ .

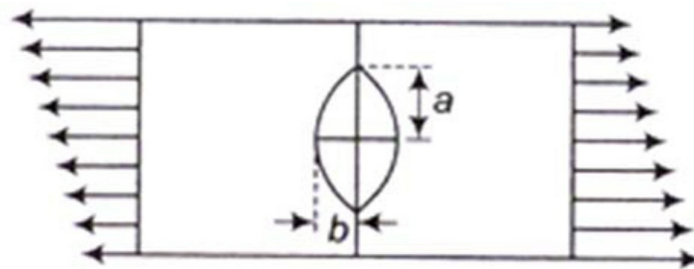
$$K_t = \frac{\text{Highest value of actual stress near discontinuity}}{\text{Nominal stress obtained by elementary equations}}$$

$$\text{or } K_t = \frac{\sigma_{max}}{\sigma_0} = \frac{\tau_{max}}{\tau_0}$$

where,  $\sigma_0, \tau_0$  = Nominal stresses

The magnitude of stress concentration factor depends upon the geometry of the component.

In this case,  $k_t = 1 + 2(a/b)$



Design against fluctuating load

where,

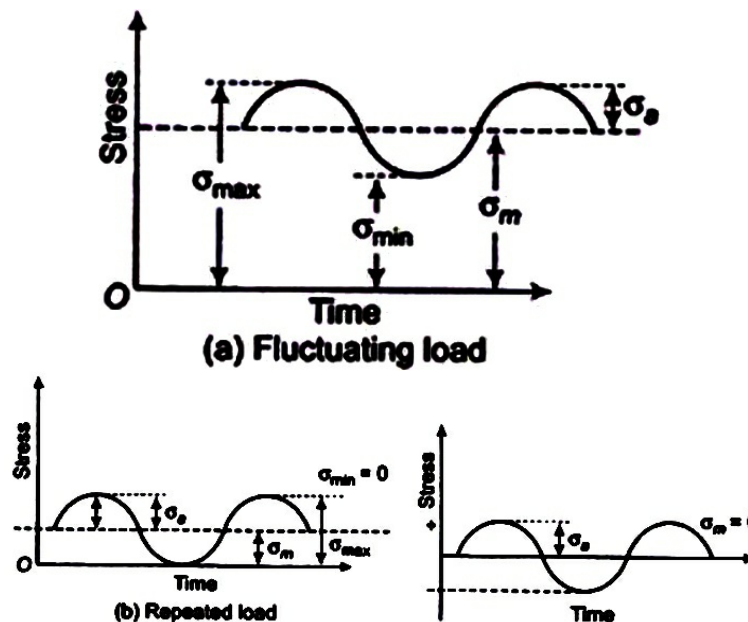
$a$  = Semi-axis of ellipse perpendicular to the direction of load

$b$  = Semi-axis of ellipse parallel to the direction of load

- If  $b = 0$  then, hole is like as very sharp crack then,  $k_t = \infty$
- If  $a = b$  then, hole becomes a circular hole then,  $k_t = 1 + 2 = 3$

### Fluctuating Load

It is defined as the load, of which magnitude and direction both changes with respect to time.



$\sigma_m$  = Mass stress,  $\sigma_a$  = Stress amplitude

- Mass stress and stress amplitude

$$\sigma_m = \frac{1}{2}(\sigma_{\max} + \sigma_{\min})$$

$$\sigma_a = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

For repeated stress,

$$\sigma_m = 0, \sigma_m = \frac{\sigma_{\max}}{2} \quad \text{and} \quad \sigma_a = \frac{\sigma_{\max}}{2}$$

$$\sigma_m = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 0$$

For reversed stress,

$$\sigma_a = \frac{1}{2}(\sigma_{\max} + \sigma_{\max}) = \sigma_{\max}$$

## Gears

Gear can be defined as the mechanical element used for transmitted power and rotary motion from one shaft to another by means of progressive engagement of projections called teeth.

## Classification of Gears

- Spur Gear
- Helical Gear

- Bevel Gear
- Worm Gear

### Spur Gear

In spur gears, teeth are cut parallel to axis of the gear.

Circular pitch

$$P = \frac{\pi d}{Z}$$

Diametrical pitch

$$P = \frac{Z}{d}$$

Module

$$m = \frac{d}{Z}$$

Torque transmitted by gear

$$M_t = \frac{60 \times 10^6 (KW)}{2\pi n}$$

Dynamic load or incremental dynamic load

$$P_d = \frac{21v(ceb + P_t)}{21v + \sqrt{ceb + P_t}}$$

Where,  $v$  = Pitch line velocity

$c$  = Deformation factor

$b$  = Face width of tooth

$P_t$  = Tangential force due to rated torque.

$e$  = Sum of errors between two meshing teeth

### Estimation of module based on beam strength

$$m = \left[ \frac{60 \times 10^6}{\pi} \left\{ \frac{(kw) c_s (f_s)}{Z n c_v \left( \frac{b}{m} \right) \left( \frac{S_w}{3} \right) Y} \right\} \right]^{-1/3}$$

Where,  $c_s$  = Service factor,

$c_v$  = Velocity factor

$f_s$  = Factor of safety,

$n$  = Speed (rpm)

### Estimation of module based on wear strength



$$m = \left[ \frac{60 \times 10^6}{\pi} \left\{ \frac{(kw) c_s (f_s)}{z_p^2 n_p c_v \left( \frac{b}{m} \right) p K} \right\} \right]^{-1/3}$$

## Helical Gear

The teeth of helical gear cut in the form of helix or an angle on the pitch cylinder.

$$P_n = \frac{p}{\cos \psi}$$

Where,  $P_n$  = Normal diametrical pitch

$P$  = Transverse diametrical pitch

$\Psi$  = Helix angle

$mn = m \cos \psi$

$mn$  = Normal module

$m$  = transverse module

Axial pitch

$$P_a = \frac{p}{\tan \varphi}$$

Pitch circular diameter

$$d = \frac{zm_n}{\cos \varphi}$$

## Tooth proportions

Addendum  $ha = mn$

Dedendum  $hf = 1.25 mn$

Clearance  $c = 0.25 mn$

- Addendum circle diameter

$$d_a = d + 2ha \quad \text{or}$$

$$d_a = \frac{zm_n}{\cos \varphi} + 2.5m_n$$

- Dedendum circle diameter

$$d_f = \frac{zm_n}{\cos \varphi} + 2.5m_n$$

- Component of tooth forces

$$P_t = \frac{2M_t}{d_p}$$

$$P_v = P_t \left[ \frac{\tan \alpha_n}{\cos \phi} \right]$$

$$P_a = P_t \tan \phi$$

- Beam strength of helical gear

$$S_b = m n b \sigma_b Y$$

Where,  $m$  = Module,

$\sigma_b$  = Permissible bending stress

$y$  = Lewis form factor

- Dynamic load or incremental dynamic load,  $P_d$

$$P_d = \frac{21v(ceb \cos^2 \phi + P_t) \cos \phi}{21v + \sqrt{(ceb \cos^2 \phi + P_t)}}$$

Where,  $e$  = Sum of errors,

$C$  = Deformation factor

- Wear strength of helical gear

$$S_w = \frac{bQd_pK}{\cos^2 \phi}$$

## Herringbone Gear

In order to avoid an axial thrust on the shaft and the bearings, the double helical gears or

Herringbone gears are used.

## Bevel Gears

- Use to transmit power between two intersecting shafts.
- High speed high power transmission.

## Classification of Bevel Gear

- Mitre Gear: When two bevel gears are mounted on shafts that are intersecting at right angle.
- Crown Gear: In pair of bevel gear, when one of the gear has a pitch angle of  $90^\circ$ .
- Internal Bevel Gear: When the teeth of bevel gear are cut on the inside of the pitch.
- Skew Bevel Gear: Mounted on non-parallel and non-intersecting shafts. It consists of straight teeth.
- Hypoid Gear: Similar to skew bevel gear, non-parallel and non-intersecting shafts. It consists of curved teeth.

- Zerol Gear: Spiral bevel gear with zero spiral angle.
- Force Gear: Consists of a spur or helical pinion meshing with a conjugate gear or disk form.

- **Beam strength of bevel gear**

$$S_b = mb\sigma_b Y \left[ 1 - \frac{b}{A_0} \right]$$

$$\text{Where, } \left[ 1 - \frac{b}{A_0} \right] = \text{bevel factor}$$

- **Wear strength of bevel gears**

$$S_w = \frac{0.75bQD_p K}{\cos \gamma}$$

$$K = 0.16 \left( \frac{BHN}{100} \right)^2$$

Where, K = Material constant,

### **Bearing**

A bearing is a mechanical element that permits relative motion between two components or parts, such as the shaft and housing, with minimum friction.

### **Plain Bearings (Sliding Contact Bearings)**

A plain bearing is any bearing that works by sliding action, with or without lubricant. This group encompasses essentially all types other than rolling-element bearings.

### **Journal or Sleeve Bearings**

These are cylindrical or ring-shaped bearings designed to carry radial loads. The simplest and most widely used types of sleeve bearings are cast-bronze and porous-bronze (powdered-metal) cylindrical bearings.

### **Thrust Bearings**

This type of bearing differs from a sleeve bearing in that loads are supported axially rather than radially which is shown in the following figure. Thin, disk like thrust bearings are called *thrust washers*.

### **Bearing Materials**

- Babbitts
- Bronzes and Copper Alloys
- Aluminium
- Porous Metals

- Plastics

### **Anti-Friction Bearings**

- *Ball, roller, and needle bearings* are classified as antifriction bearings since friction has been reduced to a minimum.

### **Bearing Loads**

- **Radial Load**

Loads acting perpendicular to the axis of the bearing are called *radial loads*. Although radial bearings are designed primarily for straight radial service, they will withstand considerable thrust loads when deep ball tracks in the raceway are used.

- **Thrust Load**

Loads applied parallel to the axis of the bearing are called *thrust loads*.  
Thrust

bearings are not designed to carry radial loads.

- **Ball Bearings**

Angular-contact bearings are used for combined radial and thrust loads and where precise shaft location is needed. Uses of the other two types are described by their names: radial bearings for radial loads and thrust bearings for thrust loads (See the following figure).

- **Radial Bearings**

*Deep-groove* bearings are the most widely used ball bearings. In addition to radial loads, they can carry substantial thrust loads at high speeds, in either direction.

*Self-aligning* bearings come in two types: internal and external. In internal bearings, the outer-ring ball groove is ground as a spherical surface. Externally self-aligning bearings have a spherical surface on the outside of the outer ring, which matches a concave spherical housing.

*Double-row, deep-groove* bearings embody the same principle of design as single-row bearings. Double-row bearings can be used where high radial and thrust rigidity is needed and space is limited.

*Angular-contact thrust* bearings can support a heavy thrust load in one direction combined with a moderate radial load.

- **Thrust Bearings**

*Flat-race* bearings consist of a pair of flat washers separated by the ball complement and a shaft-piloted retainer, so load capacity is limited. Contact stresses are high, and torque resistance is low.

*One-directional, grooved-race* bearings have grooved races very similar to those found in radial bearings.

*Two-directional, groove-race* bearings consist of two stationary races, one rotating race, and two ball complements.

- **Roller Bearing (Rolling Contact Bearings)**

The principal types of roller bearings are *cylindrical*, *needle*, *tapered*, and *spherical*.

They have higher load capacities than ball bearings of the same size and are widely used in heavy-duty, moderate-speed applications.

- **Cylindrical Bearings**

Cylindrical roller bearings have high radial capacity and provide accurate guidance to the rollers. Their low friction permits operation at high speed, and thrust loads of some magnitude can be carried through the flange-roller end contacts.

### **Needle Bearings**

Needle bearings are roller bearings with rollers that have high length-to-diameter ratios.

Compared with other roller bearings, needle bearings have much smaller rollers for a given bore size.

*Loose-needle* bearings are simply a full complement of needles in the annular space between two hardened machine components, which form the bearing raceways. They provide an effective and inexpensive bearing assembly with moderate speed capability, but they are sensitive to misalignment.

*Caged assemblies* are simply a roller complement with a retainer, placed between two hardened machine elements that act as raceways. Their speed capability is about 3 times higher than that of loose-needle bearings, but the smaller complement of needles reduces load capacity for the caged assemblies.

*Thrust bearings* are caged bearings with rollers assembled like the spokes of a wheel in a wafer like retainer.

### **Tapered Bearings**

Tapered roller bearings are widely used in roll-neck applications in rolling mills, transmissions, gear reducers, geared shafting, steering mechanisms, and machine-tool spindles. Where speeds are low, grease lubrication suffices, but high speeds demand oil lubrication, and very high speeds demand special lubricating arrangements.

### **Spherical Bearings**

Spherical roller bearings offer an unequalled combination of high load capacity, high tolerance to shock loads, and self-aligning ability, but they are speed-limited.

*Single-row* bearings are the most widely used tapered roller bearings. They have a high radial capacity and a thrust capacity about 60 percent of radial capacity.

*Two-row* bearings can replace two single-row bearings mounted back-to-back or face-to face when the required capacity exceeds that of a single-row bearing.

### **Brake**

A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine.

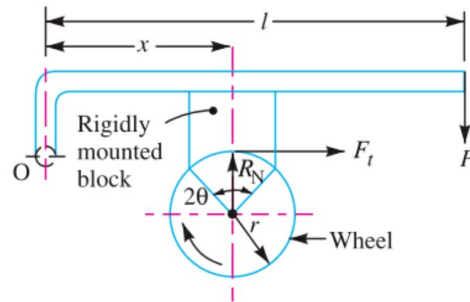
The most commonly brakes use friction to convert kinetic energy into heat, though other methods of energy conversion may be employed.

### **Single Block or Shoe Brake**

It consists of a block or shoe which is pressed against the rim of revolving brake wheel drum.

The block is made of a softer material than the rim of the wheel.

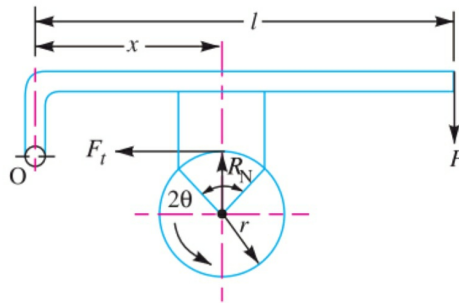
If the angle of contact is less than  $60^\circ$  then, it may be assumed that normal pressure or force between the block and the wheel is uniform.



(a) Clockwise rotation of brake wheel.

**Case I:** When the line of action of tangential braking force passes through the fulcrum O of the lever.

- If wheel is rotating in clockwise direction then, Free Body Diagram (FBD) of wheel and block is



(b) Anticlockwise rotation of brake wheel.

- If wheel is rotating in anticlockwise direction then, FBD of wheel and block is
- Braking force

$$\mu R_N = \frac{\mu Pl}{x}$$

- Braking torque

$$T_B = \frac{\mu Plr}{x}$$

When wheel is rotating in anticlockwise direction then, the braking torque is same as above

$$T_B = \frac{\mu Plr}{x}$$

**Case II:** When the line of acting of the tangential braking force ( $F_t$ ) passes through a distance a below the fulcrum O. Then, there are two cases:

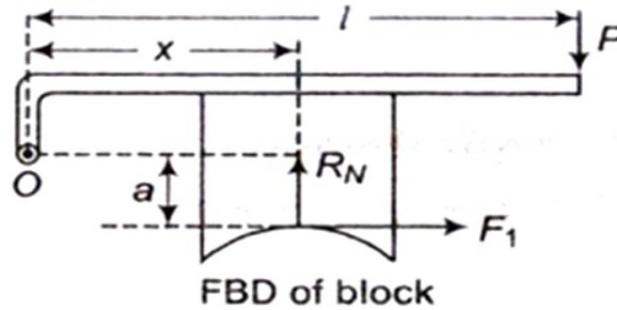
**For Clockwise :**

- Braking force

$$F_t = \mu R_N = \frac{\mu Pl}{x + \mu a}$$

- Braking torque

$$T_B = \frac{\mu Plr}{x + \mu a}$$



**For Anti Clockwise:**

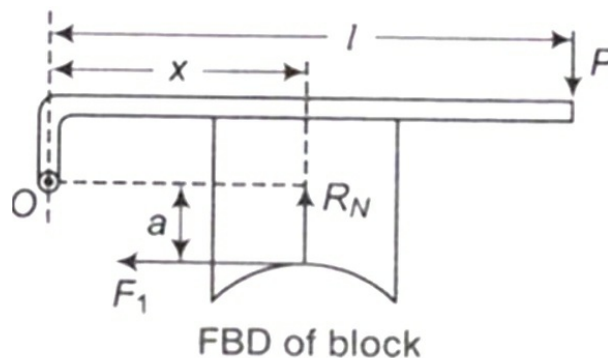
- Braking force

$$F_t = \frac{\mu Pl}{x - \mu a}$$

- Braking torque

$$T_B = \frac{\mu Plr}{x - \mu a}$$

$$\text{as } (T_B = F_t \times r)$$



**Case III:** When the line of action of tangential braking force ( $F_t$ ) passes through a distance 'a' above the fulcrum O.

**For clockwise,**

- Braking force

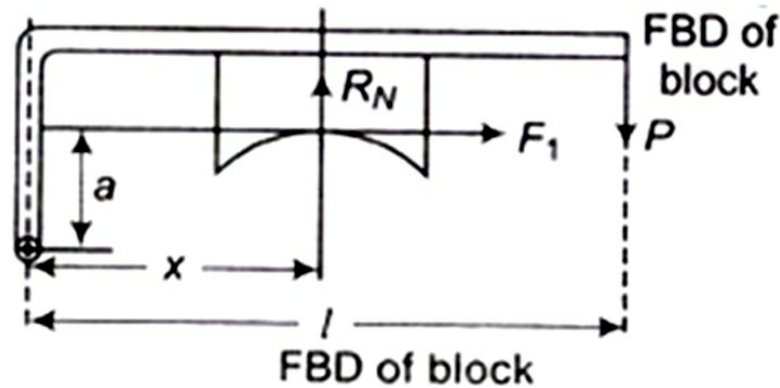
$$F_t = \frac{\mu Pl}{x - \mu a}$$



- Braking torque

$$T_B = \frac{\mu Plr}{x - \mu a}$$

$$(T_B = F_t \times r)$$



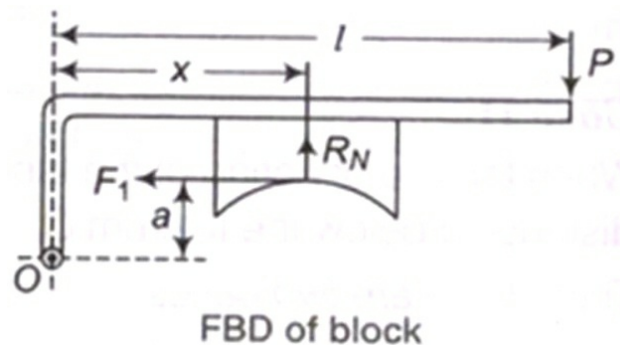
### For Anti- Clockwise

- Braking force

$$F_t = \frac{\mu Pl}{x + \mu a}$$

- Braking torque

$$T_B = \frac{\mu Plr}{x - \mu a}$$



- When the frictional force helps to apply the brakes then, such type of brakes are said to self-energizing brakes.
- When P is negative or equal to zero then, these are known as self-locking brakes.

### Simple Band Brake

- A band brake consists of a flexible band of leather, one or more ropes, or steel lined with friction material, which embraces a part of the circumference of the drum is called simple band brake.

We know,

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\theta = 360^\circ - \theta'$$

$$\text{or } 2.3 \log \left( \frac{T_1}{T_2} \right) = \mu(360^\circ - \theta') = \mu\theta$$

- Braking force on the drum =  $(T_1 - T_2)$
- Braking torque on the drum  $(T_B) = (T_1 - T_2) \times r$
- When wheel rotates in the clockwise direction and taking moment about fulcrum O

$$Pl = T_1 \times b$$

$$T_1 = \frac{Pl}{b}$$

- For anticlockwise rotation of the drum  $Pl = T_2 b$

$$T_2 = \frac{Pl}{b}$$

where,

$b$  = Perpendicular distance from O to the line of action  $T_1$  or  $T_2$

$l$  = Length of the lever from the fulcrum

$T_1 = \sigma t w$

$w$  = Width of the band

$t$  = Thickness of the band

$\sigma$  = Permissible stress in the band.

### Clutch

A clutch is a mechanical device that provides for the transmission of power (and therefore usually motion) from one component (the driving member) to another (the driven member) when engaged, but can be disengaged.

### Friction Clutch

The friction clutch is used to transmit power of shafts and machines which must be started and stopped frequently.

Friction surfaces of a clutch remain in contact to each other by applying an axial thrust or load  $w$ .

### Considering Uniform Pressure

- The uniform pressure  $p$  can be evaluated as,

$$p = \frac{w}{\pi(r_1^2 - r_2^2)}$$

- Total frictional torque given in this case,

$$T = \frac{2}{3} \mu w \left( \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right) = \mu w R_m$$

where,  $R_m$  = Mean radius of friction surfaces

$$R_m = \frac{2}{3} \left( \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right)$$

### Considering Uniform Wear

1. Total frictional torque acting on clutch

$$T = \frac{1}{2} \mu w (r_1 + r_2) = \mu w R_m$$

$$\frac{(r_1 - r_2)}{2}$$

where  $R_m$  = Mean radius of friction surfaces

1. In uniform wear theory, Maximum pressure acts at the inner radius and minimum pressure

acts at the outer radius.

$$P_{\max} \times r_2 = c \cdot P_{\min} \times r_1 = c$$

$$\frac{P_{\max}}{P_{\min}} = \frac{r_1}{r_2}$$

1. Average pressure on the friction surfaces

$$p_{\text{avg}} = \frac{w}{\pi(r_1^2 - r_2^2)}$$

### Multiple Disc Clutch

1. Number of pairs of contact surfaces

$$n = n_1 + n_2 - 1$$

where,  $n_1$  = Number of discs on the driving shaft

$n_2$  = Number of discs on the driven shaft

**Total frictional torque** acting on the frictional surface

$$T = \eta \mu w R_m$$

Where,

$$R_m = \frac{2}{3} \left( \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right) \text{ in case of uniform pressure}$$

$$\frac{1}{2} (r_1 + r_2) \text{ in case of uniform wear}$$

where,  $r_1$  and  $r_2$  are outer and inner radii of the friction plates.

### Cone Clutch

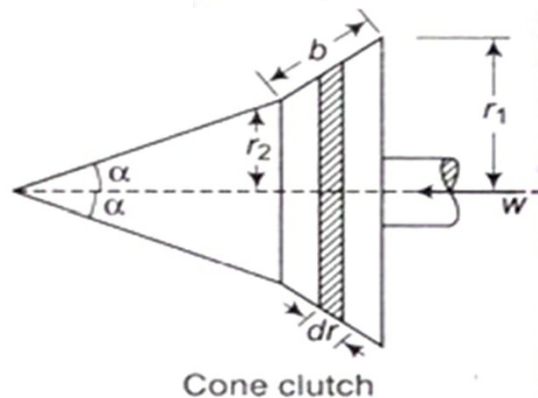
- In cone clutch, driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven.
- Total torque on the clutch,

$$T = \frac{2}{3} \mu w \operatorname{cosec} \left[ \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right] \text{ for uniform pressure}$$

$$\frac{1}{2} \mu w \operatorname{cosec} \alpha (r_1 + r_2) \text{ for uniform wear}$$

$\alpha$  = Semi angle of cone or face angle of the cone

$$w_n = \frac{w}{\sin \alpha}$$



where,  $w$  = Axial load or thrust

Axial force required for **engaging** the clutch,

$$= w_n (\sin \alpha + \mu \cos \alpha)$$

Axial force required to **disengage** the clutch

$$wd = wn (\mu \cos \alpha - \sin \alpha)$$

If face width  $b$  and mean radius of cone clutch is  $R_m$ .

Then,

$$R_m = \frac{r_1 + r_2}{2}$$

$$R_n = \frac{W}{2\pi R_m b \sin \alpha}, T = 2\pi \mu P_n R_m b$$

$$= \frac{2\pi NT}{60}$$

Power transmitted by clutch

- **Centrifugal Clutch**

Centrifugal force acting on each shoe at running speed

$$p_c = m \omega^2 r$$

Where,

$$\omega = \frac{2\pi N}{60}$$

Friction force acting on each shoe =  $\mu (P_c - P_s)$

The direction of force is perpendicular to the radius of the rim pulley.

Frictional torque on each shoe =  $\mu (P_c - P_s) \times R$

Total torque transmitted = Number of shoes  $\mu (P_c - P_s) \times R$

$$= n\mu (P_c - P_s) \times R$$

$$Arc = Angle (in radian) \times Radius l = \theta R$$

Where, area of contact =  $lb$

Force exerted on each shoe =  $plb$

$$P_c - P_s = lbp$$

Where,  $l$  = Contact length of the shoe

$b$  = Width of the shoe

$p$  = Pressure intensity on shoe

$q$  = Angle made by shoe at the centre of spider in radian

$R$  = Contact radius of shoe = inside radius of the rim of the pulley

## MANUFACTURING

### Crystal Structure of Materials

- When metals solidify from molten state, the atoms arrange themselves into various orderly configurations called crystal.
- There are seven basic crystal structures, they are

Crystal system	Relation between primitives	Interface Angles	Examples
Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90^\circ$	<i>Fe, Al, Cu</i>
Tetragonal	..	$\alpha = \beta = \gamma = 90^\circ$	<i>Sn, SO<sub>2</sub></i>
Orthogonal	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90^\circ$	<i>KNO<sub>3</sub>, BaSO<sub>4</sub></i>
Hexagonal	$a = b \neq c$	$\alpha = \beta = 90^\circ \gamma = 120^\circ$	<i>SiO<sub>2</sub>, AgCl, Zn</i>
Rhombohedral	$a = b = c$	$\alpha = \beta = \gamma \neq 90^\circ$	<i>CaSO<sub>4</sub>, CaCO<sub>3</sub></i>
Monoclinic	$a \neq b \neq c$	$\alpha = \beta = 90^\circ \neq \gamma$	<i>FeSO<sub>4</sub>, NaSO<sub>4</sub></i>
Friclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90^\circ$	<i>CuSO<sub>4</sub>, K<sub>2</sub>Cr<sub>2</sub>O<sub>7</sub></i>

### Simple Cubic Cell (SCC)

- The total number of atoms present in crystal structure,

$$n = \frac{1}{8} \times 8 = 1$$

- Atomic Packing Factor (APF)

Volume of atoms in the cell

Volume of unit cell

$$= \frac{4\pi a^3}{8 \times 3 \times a^3}$$

$$\frac{\pi}{6} = \frac{3.14}{6}$$

$$= 0.52$$

Percentage APF = **74%**

Percentage of voids =  $100 - 74 = 26\%$

### Gibbs phase rule :

- $F = C - P + 2$
- Number of external factors = 2 (pressure and temperature) .
- For metallurgical system, pressure has no appreciable effect on

phase equilibrium and  
hence,

$$F = C - P + 1$$

### **Engineering and True Stress-Strain Diagrams:**

- When we calculate the stress on the basis of the original area, it is called the engineering or nominal stress.
- If we calculate the stress based upon the instantaneous area at any instant of load it is then termed as true stress.
- If we use the original length to calculate the strain, then it is called the engineering strain.

$$\begin{aligned}\text{True stress}(\sigma_t) &= \frac{\text{Instantaneous load}}{\text{Instantaneous cross section area}} \\ &= \frac{P}{A_i}\end{aligned}$$

### **Brittleness:**

It may be defined as the property of a metal by virtue of which it will fracture without any appreciable deformation.

- This property is just opposite to the ductility of a metal.

### **Toughness:**

- It may be defined as the property of a metal by virtue of which it can absorb maximum energy before fracture takes place.
- Toughness is also calculated in terms of area under stress-strain curve.
- Toughness is the property of materials which enables a material to be twisted, bent or stretched under a high stress before rupture.

### **Resilience:**

- This may be defined as the property of a metal by virtue of which it stores energy and resists shocks or impacts.
- It is measured by the amount of energy absorbed per unit volume, in stressing a material up to elastic limit.

### **Endurance:**

- This is defined as the property of a metal by virtue of which it can withstand varying stresses (same or opposite nature).
- The maximum value of stress, which can be applied for indefinite times without causing its failure, is termed as its endurance limit.

### **Anelastic Behaviour:**

- Recoverable deformation that takes place as a function of time is termed an-elastic deformation.
- Due to some relaxation process within the material, the elastic deformation of the material continues even after the application of the load.

### **Isomorphous system**

#### **There are 5 invariant reactions occurring in binary phase system**

**Eutectic reaction :** When a liquid phase changes into two different solid phases during cooling or two solid phases change into a single liquid phase during heating, this point is known as eutectic point.

**Eutectoid reaction:** When a solid phase changes into two solid phases during cooling and vice-versa that point is known as eutectoid point.

**Peritectic reaction:** A binary system when solid and liquid phases changes solid phase on cooling and vice-versa on heating, then state of system is known as peritectic point.

**Peritectoid reaction :** If a binary phase diagram when two solid phases changes to one solid phase, then state of system is known as peritectoid point.

### **Normalising**

- For this process, the metal is placed in the furnace and heated to just above its 'Upper Critical Temperature'.
- When the new grain structure is formed it is then removed from the furnace and allowed to cool in air as it cools new grains will be formed.
- These grains, although similar to the original ones, will in fact be smaller and more evenly spaced.
- Normalising is used to relieve stresses and to restore the grain structure to normal.

### **Quenching**

- It is a heat treatment when metal at a high temperature is rapidly cooled by immersion in water or oil.
- Quenching makes steel harder and more brittle, with small grains



structure.

### **Annealing (Softening)**

- Annealing is a heat treatment procedure involving heating the alloy and holding it at a certain temperature (annealing temperature), followed by controlled cooling.
- Annealing results in relief of internal stresses, softening, chemical homogenising and transformation of the grain structure into more stable state.
- The annealing process is carried out in the same way as normalising, except that the component is cooled very slowly. This is usually done by leaving the component to cool down in the furnace for up to 48 hours.

### **Hardening**

- Hardening also requires the steel to be heated to its upper critical temperature (plus 50°C) and then quenched.
- The quenching is to hold the grains in their solid solution state called Austenite; cooling at such a rate (called the critical cooling rate) is to prevent the grains forming into ferrite and pearlite.
- Hardening is a process of increasing the metal hardness, strength, toughness, fatigue resistance.

### **Tempering**

As there are very few applications for very hard and brittle steel, the hardness and brittleness needs to be reduced. The process for reducing hardness and brittleness is called tempering.

- Tempering consists of reheating the previously hardened steel.
- During this heating, small flakes of carbon begin to appear in the needle like structure. (See below) This has the effect of reducing the hardness and brittleness.

### **Stress Relieving**

- When a metal is heated, expansion occurs which is more or less proportional to the temperature rise. Upon cooling a metal, the reverse reaction takes place. That is, a contraction is observed.
- When a steel bar or plate is heated at one point more than at another, as

in welding or during forging, internal stresses are set up.

- During heating, expansion of the heated area cannot take place unhindered, and it tends to deform.
- On cooling, contraction is prevented from taking place by the unyielding cold metal surrounding the heated area.

## Casting

- **Solidification Time**

$$T_s = k \left( \frac{V}{S.A} \right)^2$$

V=Volume of casting, S.A= Surface area of casting, k=Solidification time (sec/m<sup>2</sup>)

- **Caine's Formula**

$$X = \frac{a}{Y - b} + C$$

Freezing Ratio(F. R) X

$$= \frac{\left( \frac{V}{S.A} \right)_R}{\left( \frac{V}{S.A} \right)_C} \text{ and } Y = \frac{V_R}{V_C} \quad a, b, \text{ and } c \text{ are constants}$$

- **Shape Factor**

$$S.F = \frac{L + W}{T}$$

L- Length of casting, W- Width of the casting and T- Avg. thickness of section

- **Modulus Method**

$$M_R = 1.2M_C$$

$$M = \left( \frac{V}{S.A} \right)$$

## Gating Design-

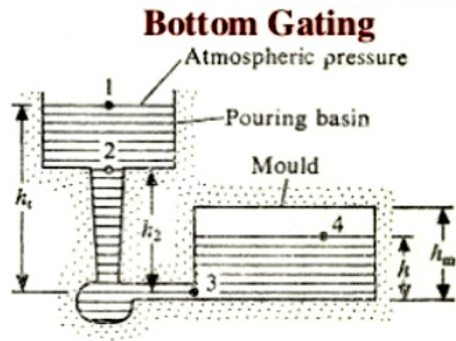
### 1. Vertical Gating

$$t = \frac{V}{A_g V_g}$$

where V= Volume of mould, Ag=Cross-section area

## 2. Bottom Gating

$$t = \frac{A_m}{A_g \sqrt{2g}} 2(\sqrt{h_t} - \sqrt{h_t - h_m})$$



## Aspiration Effect

$$R = \frac{A_2}{A_3} = \sqrt{\frac{h_c}{h_t}}$$

## Forging Flow Stress

$$\sigma = k \epsilon^n$$

## Rolling Bite Angle

$$\cos \theta = 1 - \frac{\Delta h}{2R}$$

## Reduction (Draft)

$$\Delta h \leq \mu^2 R$$

$$\text{Max Draft } \Delta h = \mu^2 R$$

## Roll Separating Force

## Extrusion and Wire Drawing

$$\sigma_d = K \ln \frac{A_i}{A_f}$$

Where K=Constant, Ai & Af = Initial and final Area

## Bending

$$\text{Bend Allowance} = \theta(R + Kt)$$

t = Sheet thickness

$\theta$  = angle in radians, K = Stretch factor (0.33 when  $R < 2t$  and 0.5 when  $R > 2t$ )

R = bend radius

## Punching and Blanking

$$\text{Punch load} = \pi D t \times k' \frac{\text{Penetration}}{\text{Shear on punch}}$$

K' = Yield strength in shear

## Deep Drawing

$$\frac{D}{d} = \text{Draw ratio}$$

$$\frac{\pi}{4} D^2 = \frac{\pi}{4} d^2 + \pi d h$$

D = Blank Diameter, d = Cup diameter

## Metal Cutting

### Cutting Speed in Turning

$$V = \pi D N$$

Where D is the diameter of the work piece, m; N is the rotational speed of the work

piece, rev/s

## Economics of Machining

### 1) Total Minimum Cost

$$V_{\text{opt}} = \frac{C}{\left[ \left( \frac{C_e}{C_m} + T \right) \left( \frac{1}{n} - 1 \right) \right]^n}$$

$$t_{\text{opt}} = \left[ \left( \frac{C_e}{C_n} + T_c \right) \left( \frac{1}{n} - 1 \right) \right]$$

where,  $C_m$  = Machine cost in /time,  $T_c$  = Tool changing time

C = Total cost,  $C_e$  = Cost of tool/grind.

### 2) Maximum Production Rate

$$V_{\text{opt}} = \frac{C}{\left[ \left( \frac{1}{n} - 1 \right) T_c \right]^n}$$

$$T_{\text{opt}} = \left[ \left( \frac{1}{n} - 1 \right) T_c \right]$$

### 3)Maximum Profit Rate

$$\text{Profit rate} = \frac{R - C_0}{T_n + T_n + \frac{T_m}{T} \times T_c}$$

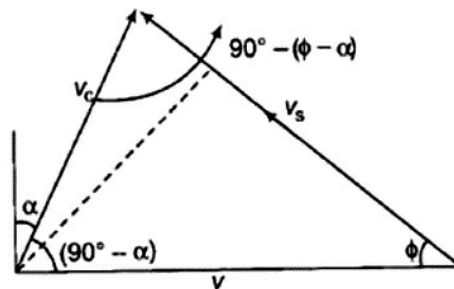
$$T_m = \frac{1}{fN}$$

### Shear angle ( $\beta_0$ )

$$\tan \beta_0 = r \cos \gamma_0 / 1 - r \sin \gamma_0$$

Rake angle,  $\gamma_0$  (in orthogonal plane), r-Chip thickness before and after cut.

### Velocity Ratio



Velocity ratio diagram

$$\frac{V_s}{\cos \alpha} = \frac{V}{\cos (\phi - \alpha)} = \frac{V_c}{\sin \phi}$$

$$\phi = \frac{\pi}{2} + \frac{\alpha}{2} - \frac{\beta}{2}$$

$\Phi$ = Shear angle,  $\alpha$ =Rake angle,  $\beta$ = Friction angle

### Shear Strain

$$\gamma = \tan \phi + \cot (\phi - \alpha)$$

## Tool Life

$$VT^n = C$$

V = Cutting speed, T = Tool life in minutes, n = A exponent which depends on cutting condition, C = Constant.

$VT^n d^x f^y = \text{Constant}$  Where d = Depth of cut, f = Feed rate (in mm/rev) in turning.

## Yield Shear Strength ( $\tau_s$ )

$$\tau_s = \frac{P}{A_s}$$

$$A_s = \frac{tS_0}{\sin \beta_0}$$

t-Thickness,  $S_0$  -feed  $\beta_0$ =Shear angle

## Specific Power Consumption -

$$U_s = \frac{P_z V_c}{MRR} = \frac{P_z}{tS_0}$$

Cutting force,  $V_c$  = Cutting Velocity

## Cutting Force Expression

$F = F_c \sin \alpha + F_t \cos \alpha$   
 $N = F_c \cos \alpha - F_t \sin \alpha$

The coefficient of friction will be then given as :

$$\mu = \frac{F}{N} = \frac{F_c \tan \alpha + F_t}{F_c - F_t \tan \alpha}$$

$$\lambda = \tan^{-1} \mu$$

On Shear plane,

$$F_s = F_c \cos \phi - F_t \sin \phi$$

$$F_n = F_c \sin \phi + F_t \cos \phi$$

Now,

$$F_t = F_n \cos \phi - F_s \sin \phi$$

$$F_c = F_n \sin \phi + F_s \cos \phi$$

## Peak to Valley Height (H Max)

$$H_{\max} = \frac{f^2}{8R}$$

f- feed, R-Nose radius

$$H_{\max} = \frac{f}{\tan \delta + \cot \delta_1}$$

$\delta$ =Side Cutting Edge Angle,  $\delta_1$ =End Cutting Edge Angle

## Non-Conventional Machining

### 5. Electrochemical Machining-

$$\text{Mass Removal Rate in} = \frac{AI}{ZF}$$

$$\text{Volumetric Removal Rate } Q = \frac{AI}{\rho ZF}$$

A=Gram atomic weight of the metallic ions

I=Current (Amps) P= density of the anode (g/cm<sup>3</sup>)

Z= Valency of the cation,

F=mFaraday (96,500 coulombs)

### 6. Electro Discharge Machining-

$$V = V_0(1 - e^{-t/RC})$$

V<sub>0</sub>=Max. Voltage, t=time, R=Resistance, C= Capacitance

## Welding

- Dilution**

$$\frac{A_p}{A_p + A_R}$$

A<sub>p</sub>=Area of Penetration

A<sub>R</sub>= Area of reinforcement

- Relation in Voltage and Arc Length**

$$V = A + Bl$$

l=Arc length, V=Voltage, A&B= Constant

## Duty Cycle

$$I^2 D = \text{Constant}$$

D=Duty cycle time, I=current

- **Open Circuit Voltage (OCV) and Short-Circuit Current(SCC) relation-**

$$V_{\text{arc}} = \text{OCV} - \left[ \frac{\text{OCV}}{\text{SCC}} \right] I$$

## Resistance Welding

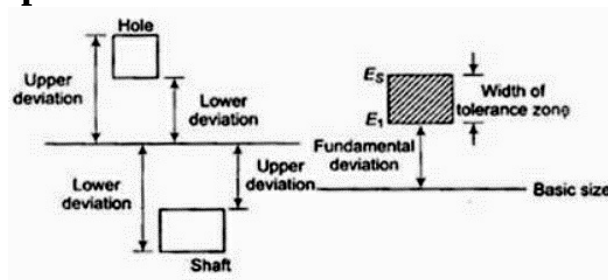
$$H = I^2 R t$$

For spot welding, diameter of nugget  $d_n = 6\sqrt{t}$

Height of the nugget  $H_n = 2(t - \text{indentation})/\text{sheet}$

## Metrology

- **Assembly Representation**



## Tolerances

$$i = 0.45D^{\frac{1}{3}} + 0.001D$$

$$D = \sqrt{D_{\text{max}} \times D_{\text{min}}}$$

## Tolerance table



IT01 $0.3 + 0.008 D$	IT0 $0.5 + 0.012 D$	IT1 $\frac{0.8 + 0.02D}{a}$	IT2 $ar$
IT3 $ar^2$	IT4 $ar^3$	IT5 $= 7i = ar^4$	IT6 $10i$
IT7 $10(10)^{1/5_i}$ $= 16i$	IT8 $10(10)^{2/5_i}$ $= 25i$	IT9 $10(10)^{3/5_i}$ $= 40i$	IT10 $10(10)^{4/5_i}$ $= 64i$
IT11 $100i$	IT12 $160i$	IT13 $250i$	IT14 $400i$
IT15 $640i$	IT16 $1000i$		

## INDUSTRIAL ENGINEERING

### Forecasting

#### Simple Moving Average

$$F_n = \frac{1}{N}(D_n + D_{n-1} + D_{n-2} + D_{n-3} + \dots + D_{n-N+1})$$

#### Moving Weight Average

$$\text{Weight Moving Average} = \frac{\sum_{i=1}^n W_i D_i}{\sum_{i=1}^n W_i}$$

#### Single (Simple) Exponential Smoothing

$$F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1})$$
$$\text{or } F_t = (1 - \alpha)F_{t-1} + \alpha D_{t-1}$$

if previous forecasting is not given

$$F_t = \alpha D_t + \alpha(1 - \alpha)D_{t-1} + \alpha(1 - \alpha)^2 D_{t-2} \dots$$

Where  $F_t$  = Smoothed average forecast for period  $t$

$F_{t-1}$  = Previous period forecast

$\alpha$  = Smoothing constant

#### Linear Regression

$$Y = a + bX$$

$$\sum y = na + b \sum x$$

$$\sum xy = n \sum x + b \sum x^2$$

#### Forecasting Error

$$e_t = (D_t - F_t)$$

#### Bias

$$\text{Bias} = \frac{1}{N} \sum_{t=1}^n (D_t - F_t)$$

#### Mean Absolute Deviation

$$\text{MAD} = \frac{1}{N} \sum_{t=1}^n |D_t - F_t|$$

#### Mean Square Error

$$MSE = \frac{1}{N} \sum_{t=1}^n (D_t - F_t)^2$$

### Mean Absolute Percentage Error

$$MAPE = \frac{1}{N} \sum_{t=1}^n \frac{|D_t - F_t|}{D_t} \times 100$$

### Inventory

If  $D$ = demand/year,  $C_o$ =Order cost,  $C_c$ = Carrying cost,  $P$ = Purchase price/unit  
 $Q^*$ =Economic Order Quantity,  $K$ = Production Rate and  $C_s$ =Shortage Cost/unit/period.

### Case-1 Purchase Model With Instantaneous Replenishment and Without Shortage

$$EOQ \quad Q^* = \sqrt{\frac{2DC_o}{C_c}} \quad \text{at EOQ Inventory cost} = \text{Order cost}$$

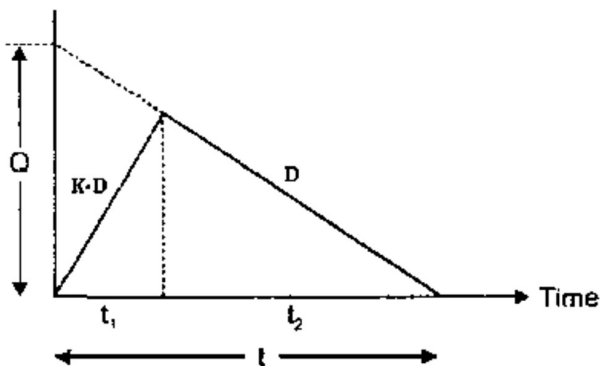
$$\text{No of Order} = \frac{D}{Q^*}$$

$$\text{Time Taken Per Order} = \frac{Q^*}{D}$$

$$\text{Total cost} = \text{Unit cost} + \text{Inventory cost} + \text{Order cost}$$

$$= (D \times P) + \left( \frac{Q}{2} \times C_c \right) + \left( \frac{D}{Q} \times C_o \right) = (D \times P) + \sqrt{2DC_cC_o}$$

### Case-2 Manufacturing Model Without Shortage



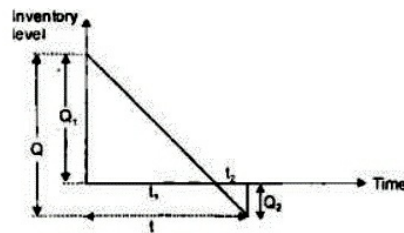
$$EOQ = Q^* = \sqrt{\frac{2C_0D}{C_c \left(1 - \frac{D}{K}\right)}}$$

$$t_1 = \frac{Q^*}{k}$$

$$t_2 = \frac{Q^* \left(1 - \frac{D}{K}\right)}{D}$$

$$\text{Total optimum cost} = \sqrt{2DC_cC_0 \left(1 - \frac{D}{K}\right)}$$

### Case 3 Purchase Model With Shortage



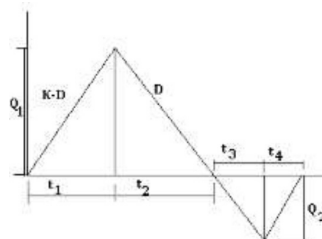
$$Q = EOQ = \sqrt{\frac{2DC_0}{C_c} \left( \frac{C_s + C_c}{C_s} \right)}$$

$$Q_1 = \sqrt{\frac{2DC_0}{C_c} \left( \frac{C_s}{C_s + C_c} \right)}, \quad Q_2 = Q - Q_1$$

$$t = \frac{Q}{D}, \quad t_1 = \frac{Q_1}{D} \text{ and } t_2 = \frac{Q_2}{D}$$

$$\text{Total optimum cost} = \sqrt{2DC_cC_0 \left( \frac{C_s}{C_s + C_c} \right)}$$

### Case 4 Manufacturing Model With Shortfall



$$Q = EOQ = \sqrt{\frac{2DC_0}{C_c \times \left(1 - \frac{D}{K}\right)} \left(\frac{C_s + C_c}{C_s}\right)}$$

$$Q_1 = \sqrt{\frac{2DC_0}{C_c} \left(\frac{C_s}{C_s + C_c}\right) \left(1 - \frac{D}{K}\right)}$$

$$Q_1 = \left(1 - \frac{D}{K}\right) Q - Q_2$$

$$t = \frac{Q}{D}, t_1 = \frac{Q_1}{K - D}, t_2 = \frac{Q_1}{D}, t_3 = \frac{Q_3}{D} \text{ and } t_4 = \frac{Q_2}{K - D}$$

Lead Time Demand + Safety Stock = Reorder Point

### PERT and CPM

EFT = EST + activity time

LFT = LST + Duration of activity

### Total Float

$$TF_{ij} = L_j - (E_i + t_{ij}) = LF_{ij} - EF_{ij}$$

$$L_j - t_{ij} = E_i$$

$$LS_{ij} - ES_{ij}$$

### Free Float

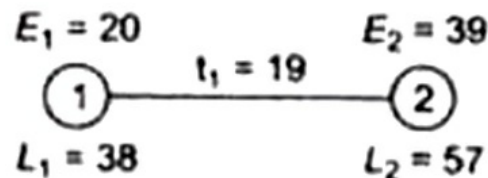
$$FF_{ij} = (E_j - E_i) - T_{ij}$$

### Independent Float

$$IF_{ij} = (E_j - L_j) - t_{ij}$$

$$FF_j - (\text{Slake of event } i)$$

### Example



$$1. \text{ Total float} = L_2 - (E_1 + t_{12}) = 57 - (20 + 19) = 18$$

$$2. \text{ Free float} = E_2 - E_1 - t_{12} = 0$$

$$3. \text{ Independent float} = E_2 - (L_1 + t_{12}) = -18$$

### PERT Expected time

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

1.  $t_o$  = Optimistic time i.e., shortest possible time to complete the activity if all goes well.
2.  $t_p$  = Pessimistic time i.e., longest time that an activity could take if everything goes wrong.
3.  $t_m$  = Most likely time i.e., normal time of an activity would take.

### Standard deviation

$$\sigma = \frac{t_p - t_o}{6}$$

### Variance

$$\sigma^2 = \left( \frac{t_p - t_o}{6} \right)^2$$

### Crashing

$$\text{Cost Slope (CS)} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

### Standard Normal Variation

$$Z = \frac{T_s - T_e}{\sigma}$$

### Linear Programming

#### *Simplex Method Case 1. Maximization Problem*

$$\text{Max } Z = 3x_1 + 5x_2$$

$$\text{s / t } 3x_1 + 2x_2 \leq 18 \quad \text{---(I)}$$

$$x_1 \leq 4 \quad \text{---(II)}$$

$$x_2 \leq 6 \quad \text{---(III)}$$

$$x_1, x_2 \geq 0$$

#### **Standard Form:**

$$\text{Max } Z = 3x_1 + 5x_2 + 0w_1 + 0w_2 + 0w_3$$

$$3x_1 + 2x_2 + w_1 + 0w_2 + 0w_3 = 18$$

$$x_1 + 0x_2 + 0w_1 + w_2 + 0w_3 = 4$$

$$0x_1 + x_2 + 0w_1 + 0w_2 + w_3 = 6$$

**To prepare initial Table:**

**Table – I**

$c_j$		3	5	0	0	0	
$C_i$	$X_i$	$x_1$	$x_2$	$w_1$	$w_2$	$w_3$	$b_i$
0	$w_1$	3	2	1	0	0	18
0	$w_2$	1	0	0	1	0	4
0	$w_3$	0	1	0	0	1	6
	$I_j$	-3	-5	0	0	0	$Z=0$

$$I_j = (Z_j - c_j) = \left( \sum a_{ij} \cdot c_i \right) - c_j$$

**Interpretation of Simplex Table**

**Table – I**

$c_j$		3	5	0	0	0	
$C_i$	$X_i$	$x_1$	$x_2$	$w_1$	$w_2$	$w_3$	$b_i$
0	$w_1$	3	2	1	0	0	18 (18/2=9)
0	$w_2$	1	0	0	1	0	4 (4/0=infinity)
0	$w_3$	0	1	0	0	1	6 (6/1=6)
	$I_j$	-3	-5	0	0	0	$Z=0$

Key Column  $\rightarrow$  Min  $I_j$  [ Most Negative ]

• Key Row  $\rightarrow$  Min positive ratio.

**How to get next table ?**

- Leaving variable :  $w_3$
- Entering variable :  $x_2$
- Key no. = 1
- For old key row : New No.= Old No./key No.
- For other rows:

$$\begin{aligned} \text{New no} \\ &= \text{Old no} - \frac{(\text{Corresponding key Row No}) \times (\text{Corresponding key Row No})}{\text{Key No}} \end{aligned}$$

- $18 \rightarrow 18 - (6 \cdot 2)/1 = 6$
- $I(w_3) = 0 \rightarrow 0 - [1 \cdot (-5)]/1 = 5$

**Table – II**

$c_j$		3	5	0	0	0	
$C_i$	$X_i$	$x_1$	$x_2$	$w_1$	$w_2$	$w_3$	$b_i$ Ratio
0	$w_1$	3	0	1	0	-2	6 (6/3=2)
0	$w_2$	1	0	0	1	0	4 (4/1=4)
5	$x_2$	0	1	0	0	1	6 (6/0=infinity)
	$I_j$	-3	0	0	0	5	$Z=30$

Key Column  $\rightarrow$  Min  $I_j$

Key Row  $\rightarrow$  Min positive ratio

**Table – III**

$c_j$		<b>3</b>	<b>5</b>	<b>0</b>	<b>0</b>	<b>0</b>	
$C_i$	$x_i$	$x_1$	$x_2$	$w_1$	$w_2$	$w_3$	$b_i$ Ratio
3	$x_1$	1	0	$1/3$	0	$-2/3$	2
0	$w_2$	0	0	$-1/3$	1	$2/3$	2
5	$x_2$	0	1	0	0	1	6
	$I_j$	0	0	0	0	3	$Z=36$

**This is the final Table**

**The Optimal Solution is  $x_1 = 2$ ,  $x_2 = 6$   
giving  $Z = 36$**

**Type of Solutions** : Basic, Feasible/Infeasible, Optimal/ Non-Optimal, Unique/Alternative Optimal, Bounded/Unbounded, Degenerate/Non-Degenerate

### **Analysis of Solution**

1. This is a Basic solution, as values of basic variables are Positive.
2. This is a feasible solution, as values of basic variables, not containing Artificial Variable, are Positive and all constraints are satisfied.
3. This Feasible solution is an Optimal, as all values in Index Row are positive.
4. If there is an Artificial Variable, as Basic variable in final table, it is called as Infeasible solution
5. This solution is unique Optimal, as the number of zeroes are equal to number of basic variables in Index Row in final Table.
6. If the number of zeroes are more than number of basic variables in Index Row in final Table, it is a case of more than one optimal solutions.
7. This is a Bounded Solution, as the values of all Basic variables in final table, are finite positive.
8. This is a Non-degenerate Solution, as value of none of the basic variables is Zero, in final table.
9. If value of at least one of the basic variables is Zero in Index Row in final Table, it is a Degenerate Solution.

### **Duality With Example**



### 1. Case-1

$$\text{Max. } Z = x_1 - x_2 + 3x_3$$

$$\text{s/t } x_1 + x_2 + x_3 \leq 10$$

$$2x_1 - x_2 - x_3 \leq 2$$

$$2x_1 - 2x_2 - 3x_3 \leq 6$$

$$x_1, x_2, x_3 \geq 0$$

Dual of this would be

$$\text{Min } Z = 10y_1 + 2y_2 + 6y_3$$

$$\text{s/t } y_1 + 2y_2 + 2y_3 \geq 1$$

$$y_1 - y_2 - 2y_3 \geq -1$$

$$y_1 - y_2 - 3y_3 \geq 3$$

$$y_1, y_2, y_3 \geq 0,$$

### 2. Case -2

$$\text{Min } Z = 20x_1 + 23x_2$$

$$\text{s/t } -4x_1 - x_2 \leq -8$$

$$5x_1 - 3x_2 \leq -4$$

$$x_1, x_2 \geq 0$$

Dual of this would be

$$\text{Max } Z = 8y_1 + 4y_2$$

$$\text{s/t } 4y_1 - 5y_2 \leq 20$$

$$y_1 + 3y_2 \leq 23$$

$$y_1, y_2 \geq 0,$$

### 3. Case-3

$$\text{Max. } Z_p = x_1 + 2x_2 - 3x_3$$

$$\text{s/t } 2x_1 + x_2 + x_3 \leq 10$$

$3x_1 - x_2 + 2x_3 \geq 110$  (this needs to be converted in less than form)

$$x_1 + 2x_2 - x_3 = 4$$

$$x_1, x_3 \geq 0, x_2 \text{ unrestricted}$$

Dual of this would be

$$\text{Min. } Z_d = 10y_1 - 110y_2 + 4y_3$$

$$\text{s/t } 2y_1 - 3y_2 + y_3 \geq 1$$

$$y_1 + y_2 + 2y_3 = 2$$

$$y_1 - 2y_2 - y_3 \geq -3$$

$$y_1, y_2 \geq 0, y_3 \text{ unrestricted}$$

## Assignment

### Steps of Solving Assignment Problem:

1. Identify the minimum element in each row and subtract it from every element of that row.
2. Identify the minimum element in each column and subtract it from every element of that column.
3. Make the assignments for the reduced matrix obtained from steps 1 and 2 in the following way:

- I. For each row or column with a single zero value cell that has not been assigned or eliminated, box that zero value as an assigned cell.
- II. For every zero that becomes assigned, cross out (X) all other zeros in the same row and the same column.
- III. If for a row and a column, there are two or more zeros and one cannot be chosen by inspection, choose the cell arbitrarily for assignment.
- IV. The above process may be continued until every zero cell is either assigned or crossed (X)

4. An optimal assignment is found, if the number of assigned cells equals the number of rows (and columns). In case you have chosen a zero cell arbitrarily, there may be alternate optimal solutions. If no optimal solution is found, go to step 5.

5. Draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced matrix obtained from step 3 by adopting the following procedure:

- I. Mark all the rows that do not have assignments.
- II. Mark all the columns (not already marked) which have zeros in the marked rows.
- III. Mark all the rows (not already marked) that have assignments in marked columns.
- IV. Repeat steps 5 (ii) and (iii) until no more rows or columns can be marked.
- V. Draw straight lines through all unmarked rows and marked columns.

1. Select the smallest element (i.e., 1) from all the uncovered elements. Subtract this smallest element from all the uncovered elements and add it to the elements, which lie at the intersection of two lines. Thus, we obtain another reduced matrix for fresh assignment.

**Note:** For maximization problem the matrix is converted into minimization problem by subtracting each element by the element having maximum value in the matrix.

## Queuing Theory

Arrival rate is  $\lambda$  and mean service rate is denoted by  $\mu$ .

### • Kendall's notation

(a/b/c) : (d/e)

a = Probability law for the arrival time

b = Probability law according to which the customers are being served.

c = number of channels

d = capacity of the system

e = queue discipline.

**Traffic Intensity(  $\rho$  )** = Mean arrival rate/ Mean service rate =  $\lambda / \mu$

### Formulas of Queuing Theory

1. Expected number of customers in the system ( $L_s$ ) =  $\lambda / (\mu - \lambda)$
2. Expected number of customers in the queue ( $L_q$ ) =  $\lambda^2 / \mu (\mu - \lambda)$
3. Expected waiting time for a customer in the queue ( $W_q$ ) =  $\lambda / \mu (\mu - \lambda)$
4. Expected waiting time for a customer in the system ( $W_s$ ) =  $1 / (\mu - \lambda)$
5. Probability that the queue is non-empty  $P(n > 1) = (\lambda / \mu)^2$
6. Probability that the number of customers ,n in the system exceeds a given number,  $P(n \geq k) = (\lambda / \mu)^k$
7. Expected length of non-empty queue =  $\mu / (\mu - \lambda)$ .

## Sequencing and Scheduling

### Sequencing

#### 1. Johnson's Problem

##### 1.2 machines and n jobs

**Step 1** : Find the minimum among various  $t_{i1}$  and  $t_{i2}$ .

**Step 2a** : If the minimum processing time requires machine 1, place the associated job in the first available position in sequence. Go to Step 3.

**Step 2b** : If the minimum processing time requires machine 2, place the associated job in the last available position in sequence. Go to Step 3.

**Step 3** : Remove the assigned job from consideration and return to Step 1 until all positions in sequence are filled.

##### 2.3 machines and n jobs

Two conditions of this approach

- The smallest processing time on machine A is greater than or equal

to the greatest processing time on machine B, i.e.,

$$\text{Min. } (A_i) \geq \text{Max. } (B_i)$$

- The smallest processing time on machine C is greater than or equal to the greatest processing time on machine B, i.e.,

$$\text{Max. } (B_i) \leq \text{Min. } (C_i)$$

If **either or both** of the above conditions are satisfied, then we replace the three machines by two fictitious machines G & H with corresponding processing times given by

$$G_i = A_i + B_i$$

$$H_i = B_i + C_i$$

Where  $G_i$  &  $H_i$  are the processing times for  $i$ th job on machine G and H respectively.

### **Time Study**

Normal time = Observed time x Rating factor

Standard time = Normal time + allowances